# The Collateral Rule: Evidence from the Credit Default Swap Market*

Agostino Capponi† W. Allen Cheng‡ Stefano Giglio§ Richard Haynes¶

Appendices A and B by Chuan Du

**Abstract**

We explore a novel dataset of daily cleared credit default swap (CDS) positions along with the posted margins to study how collateral varies with portfolio risks and market conditions. Contrary to many theoretical models, which assume that collateral constraints follow Value-at-Risk rules, we find strong evidence that collateral requirements are set *an order of magnitude larger* than what standard Value-at-Risk rules imply. The panel variation in collateralization rates of CDS portfolios (over time and across participants) is well explained by measures of extreme tail risks, related to the maximal potential loss of the portfolio. We develop a model of endogenous collateral in CDS markets to interpret these empirical findings. The model predicts that the conservativeness of collateral levels can be explained through disagreement of market participants about the extreme states of the world, in which CDSs pay off and counterparties default.

*We appreciate insightful comments from Georgy Chabakauri (discussant), Ian Dew-Becker, Darrell Duffie, Zorka Simon (discussant), Dimitri Vayanos (discussant), Pietro Veronesi, and seminar participants at the American Finance Association Annual Meeting, Adam Smith workshop, the Financial Intermediation Research Society Conference, and at the Federal Reserve Board. We thank Aref Bolandnazar, CFTC Intern, for excellent research assistance. The research of Allen Cheng was supported by the National Science Foundation Graduate Research Fellowship under Grant No. DGE-11-44155. The research of Agostino Capponi was partly supported by the NSF CAREER 1752326. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation. Author Capponi is a CFTC consultant. Authors Cheng and Giglio were CFTC consultants between 02/2015-07/2017 and 02/2016-04/2018, respectively, and during those times they accessed data and conducted research for this paper. Both are now, however, affiliated with registrants of the CFTC. Author Haynes is a current CFTC employee. The research was produced in each author's official capacity. The analyses and conclusions expressed in this paper are those of the authors and do not reflect the views of other members of the Office of Chief Economist, other Commission staff, or the Commission itself.

†Department of Industrial Engineering and Operations Research, Columbia University, New York, NY 10027, USA, `ac3827@columbia.edu`. Research supported by a grant from the Global Risk Institute.

‡Department of Industrial Engineering and Operations Research, Columbia University, New York, NY 10027, USA, `wc2232@columbia.edu`.

§Yale School of Management, NBER, and CEPR. `stefano.giglio@yale.edu`.

¶Division of Clearing and Risk, U.S. Commodity Futures Trading Commission, Washington, DC 20581, USA. `rhaynes@cftc.gov`.

# 1 Introduction

Collateral plays a central role for sustaining risk sharing in the economy. However, as highlighted during the financial crisis of 2008-2009, it can also amplify fundamental shocks and create self-reinforcing death spirals that can affect the entire economy.

The role of the "collateral channel" in amplifying fundamental shocks has been studied by a vast theoretical literature. At the core of models in this literature is the *collateral rule*, which determines how margins are set and how they respond to changes in economic conditions. Different models have made different assumptions about the collateral rule. Some studies have specified the rule exogenously (e.g., Brunnermeier and Pedersen (2009) assume a Value-at-Risk (VaR) criterion; Gromb and Vayanos (2002) assume a maximum-loss constraint). Other works have determined collateral levels endogenously via general equilibrium models (e.g., Geanakoplos (1997) and Fostel and Geanakoplos (2015)).

Understanding how collateral is set, and how it responds to changes in market or portfolio conditions, is central for understanding the mechanisms through which shocks propagate and are amplified. Yet, empirical evidence on the collateral rule is scarce.

In this paper, we aim to fill this gap by studying the collateral rule in depth, first empirically and then theoretically, in a large market that was at the very center of the financial crisis: the credit default swap (CDS) market. Using a novel dataset of positions and corresponding margins in the cleared CDS market, we document several novel facts about the way collateral is determined, and the way it is adjusted in response to changes in individual and macroeconomic risks. Appendix A constructs a theoretical model of the endogenous collateral rule in derivatives markets, building on the work of Geanakoplos (1997), Fostel and Geanakoplos (2015), and Simsek (2013), that allows us to interpret our empirical findings and connect them to the existing literature (Appendices A and B reproduce results from Du et al. (2020)).

The CDS market is a trillion-dollar market for credit risk transfer, which has experienced remarkable growth in the years before the global financial crisis. Over the past decade, the CDS market has transitioned towards mandatory clearing: after two parties enter a bilateral CDS contract, all counterparty obligations are transferred to a clearinghouse. That is, the clearinghouse becomes the counterparty to each the original trading parties, referred to as *clearing members*. Operating as a *central counterparty* (CCP), the clearinghouse insulates members from default risk, but requires them to post daily-settled collateral (margin). Our dataset, collected and maintained by the U.S. Commodity Futures Trading Commission (CFTC), is built from the universe of CDS trades cleared by Ice Clear Credit (ICC), the largest clearinghouse for these contracts. In our data, we can observe each CDS position of every member in this market, covering a total of more than 26,000 contracts. More importantly, we also observe the amount of collateral posted each day by each member to the clearinghouse. Our sample covers the period 2014-2020 at the daily frequency.

Using this data set, we document several new empirical findings on collateral setting in the cleared CDS market. First, we show that there is a large amount of heterogeneity (both across clearing members and over time) in the collateralization rate – the value of collateral posted as a fraction of the total size of the members' portfolios. This indicates that clearing members trade portfolios with very different risk characteristics, which command different levels of collateralization.

Next, we study what determines the amount of collateral posted by clearing members. The natural benchmark to evaluate the level of collateralization of a portfolio is the widely-used Value-at-Risk (VaR) rule, under which collateral levels should be set to cover a certain fraction of losses occurring over a limited period of time. The VaR rule is especially important in this setting, not only because it is the rule assumed by many theoretical models,[1] but also because most clearinghouses (including ICC itself) explicitly use it when they describe their collateral requirements (Ivanov and Underwood (2011)). The benchmark rule in this context is a 5-day 99%-level VaR, according to which collateral should be sufficient to cover 99% of the 5-day loss distribution of each member's portfolio.[2] The VaR rule tries to strike a balance between the ability to recover payments in case of counterparty default and the cost of keeping cash immobilized as collateral. Under this rule, we should therefore observe losses from individual member portfolios that exceed the posted collateral in about 1% of all 5-day periods.

The second finding of our empirical analysis is that, in the cleared CDS market, collateral levels *far exceed* those implied by the benchmark VaR. For our entire sample period, 2014-2020, exceedances (i.e., losses above the posted collateral) occurred only in 0.008% of cases (specifically, only on two consecutive days, for a single member), and were observed during the COVID-19 pandemic period.

More strikingly, this conservativeness in collateral levels holds true even when we incorporate in our analysis the history of CDS price movements since 2004, therefore including the large shocks occurred to the CDS market during the financial crisis. To perform this analysis, we collect historical CDS prices since 2004, and for each portfolio observed in our sample we build the time series of returns that the portfolio would have realized over that much longer time period. We can therefore ask, for each portfolio, how that portfolio would have performed on any 5-day interval since 2004. While in this case a small number of exceedances would have been observed, they still represent a fraction that is two orders of magnitudes smaller than the one implied by the standard 99% VaR. To sum up, our second finding is that collateral levels are set orders of magnitude more conservatively than standard benchmarks imply.

Motivated by this result, and guided by the theoretical literature, we then explore in our data what factors determine margins in our panel. We begin by estimating a panel model relating margins to VaR as well as other portfolio risk measures that have been proposed in the theoretical literature: maximum shortfall (maximum potential loss obtained from historical simulations), and aggregate short notional (total notional of short positions only).

We show that two variables explain a significant portion of the panel variation with an overall $R^2$ of 72% (and 85% if member fixed effects are added), and they significantly improve over standard VaR, which falls short to explain not only the average level of margins, but also the panel variation. Interestingly, Duffie et al. (2015) *assume* a theoretical collateral rule based precisely on a combination

[1] For example, see Figlewski (1984), Brunnermeier and Pedersen (2009), Hull (2012) and Glasserman et al. (2016).

[2] As discussed later, ICC Europe instead uses a 99.5% level instead. Throughout the paper we use 99% as baseline, but the results apply similarly to a 99.5% level.

of these two variables; our empirical analysis provides direct evidence in favor of that assumption. These two variables have an interesting economic interpretation: they are much more related to extreme tail risks than VaR. In comparison, VaR focuses on less extreme losses, and has a correlation of 0.98 with simple volatility (see Table A.2).

Given that the maximum shortfall of a portfolio is based on its historical performance, it captures the *experienced* maximal loss in the data (which, of course, is more severe than the 99th loss quantile used by VaR). Aggregate short notional, on the other hand, captures the fact that in the CDS market the biggest tail losses occur on the short side: if a default occurs suddenly ("jump-to-default") the liability of the short side can jump to the notional value of the CDS (less the recovery). Hence, the aggregate short notional represents the maximum *potential* loss of a portfolio if *all* short positions default simultaneously (and the recovery rate of the underlying bond is zero). In this sense, aggregate short notional captures the most extreme form of tail risk, and provides a measure that is also less sensitive to the exact specification of the portfolio loss distribution compared to other tail risk measures.

Finally, we explore how *market variables* enter the collateral rule: because collateral rules adapt to market conditions, we expect collateral levels to respond to variables that capture the state of the economy. We incorporate in our panel analysis measures of aggregate risk and risk premia, such as VIX and the average CDS spread of all dealers, and measures of individual member funding opportunity costs. We find that margins increase for all members when risk in the economy increases, suggesting that the collateral rule adapts to the state of the economy.

To sum up, we find that maximal shortfall and aggregate short notional – two extreme tail risk measures – outperform VaR in describing the collateral rule in this market; in addition, the collateral rule also depends on aggregate market conditions.

A few caveats are important in interpreting our results. First, the fact that the collateral rule depends on extreme loss variables does not imply that it will cover *every* possible loss in practice; instead, it shows that it is the very extreme tail risks that the clearinghouse is concerned about, much farther out the tails than standard VaR (which instead is closely related to simple volatility). Second, it goes without saying that our results only apply to the specific market that our data covers (the cleared CDS market, whose main clearinghouse is ICC); different collateral rules may be applied in other markets. We believe that the setting we study is particularly interesting because the potential for large and sudden changes in values of CDS contracts makes collateral especially important to prevent default.

Finally, it is worth emphasizing that the goal of this paper is *not* to reverse-engineer the procedure that ICC uses to set collateral. As we describe in greater detail in the next sections, ICC follows a complex set of procedures to determine collateral levels, that include calibration of different scenarios and simulations; the methodology also includes a discretionary component, like the choice of several parameters, that can change over time. Our aim is to identify and quantify the *main economic determinants* of the variation in collateral over time and across members that are induced by this methodology. That is, our goal is to produce a simple empirical characterization of the collateral rule that allows us to distill its main economic drivers, and that can be mapped into the simple rules that are used in theoretical models. Note that the collateral rule we estimate, being a reduced-form estimate of the relationship between collateral posted and portfolio and market conditions, reflects not only the (exogenous) set of calculations ICC makes to determine the collateral level, but also the endogenous choice of parameters that ICC uses when applying the procedure, which can change over time as market conditions vary.

Our empirical results have direct implications for models of financial markets and intermediation, in which the collateral constraint plays an important role. For models in which the collateral constraint is specified *exogenously*, our findings provide support for defining it as a function of extreme tail risks, such as a maximum-loss constraint (Gromb and Vayanos (2002)), instead of a standard, less conservative VaR (as in Brunnermeier and Pedersen (2009)). The difference is significant: empirically, VaRs are very closely related to volatility, and do not fully capture the extreme tail risks; in models that assume VaR rules, the dynamics of collateral requirements will mostly reflect movements in portfolio volatility. Instead, as we have shown, collateral levels are driven by extreme tail events, that capture nonlinear effects beyond volatility (for example, jumps): collateral demand may not be very sensitive to small changes in portfolio or market risks, but may change significantly if large shocks occur.

Our results also have implications for models of the *endogenous* collateral rule, like Fostel and Geanakoplos (2015). A key prediction of Fostel and Geanakoplos (2015) is that in a binomial economy (i.e., when there are two states of nature only), any collateral equilibrium is equivalent to one in which there is no default – that is, where collateral covers the most extreme losses. Intuitively, therefore, our results support the key intuition of this model, which argues for very conservative collateral rules. However, it is difficult to link tightly our empirical results on the CDS markets with this model, because (1) the conclusions only hold if there are two states of nature, and (2) defaults on CDS obligations do sometimes arise in practice.

Appendix A overcomes these two limitations by developing a new equilibrium model, which explains the conservativeness of collateral levels through disagreement of market participants about the extreme states of the world, in which CDSs pay off and counterparties default. The model features a continuum of states, as opposed to just two, and non-zero counterparty default probability in equilibrium. Trade in this model occurs because of differences in beliefs. The model is a natural adaptation of Simsek (2013), where belief disagreements are central to asset prices and endogenous margin requirements. Unlike Simsek (2013) who considers standard debt contracts, allowing short selling and a completely unrestricted contract space (whereby every Arrow-Debreu equilibrium is equivalent to a general equilibrium), the model in Appendix A analyzes the role of belief differences in an economy where the only contracts available for trading are state contingent promises (CDSs) backed by risk-free collateral. In the model, optimists naturally sell insurance (CDS protection) to pessimists. Pessimists require that the sellers post collateral in the form of cash. The collateralization rate on CDS contracts arises endogenously in the model. Appendix A shows that this model can generate high margin requirements (akin to what we see in the data) and low default probabilities, and that this occurs when the pessimists attach a large weight to the negative tail events where the seller of the CDS contract is expected to pay out the most.

A related literature on the role of collateral in risk management and enforcement of payments (see for instance Rampini and Viswanathan (2010) and Gorton and Ordonez (2014)) takes a different approach in modeling the collateral constraints that arise in equilibrium. In these models, agents' decisions endogenously affect the total amount of collateral available in the economy; but the amount one can borrow against each unit of collateral is set exogenously. In contrast, the model in Appendix A has a fixed supply of collateral, but allows the amount of collateral required to back a given promise to be determined endogenously in equilibrium. Since in this paper we are interested in the determinants of margin requirements for a given portfolio of CDS contracts, the approach adopted in Appendix A is better suited for our purposes.

In summary, the model in Appendix A and data together suggest that the clearinghouse, i.e., the pessimist in the model, focuses on the very extreme events when setting collateral rules; in equilibrium, these collateral requirements may be viewed as onerous from the members, i.e., the optimists in the model, but are not so onerous as to fully prevent default when viewed from the clearinghouse's perspective. To sum up, our empirical results provide new evidence in favor of models where collaterals is determined endogenously.[3]

Our paper relates to a large theoretical literature on the relation between margin requirements and asset prices, and the collateral equilibrium. Noticeable contributions include Brunnermeier and Pedersen (2009), Gromb and Vayanos (2002), Coen-Pirani (2005), and Chabakauri (2013), in which the collateral rule is exogenous; and Geanakoplos (1997), Holmström and Tirole (1997), Brunnermeier and Sannikov (2014), and Simsek (2013) in which collateral requirements arise endogenously, and depend both on market conditions and specific characteristics of market participants. A recent theoretical literature has focused specifically on cleared derivative markets, like Koeppl et al. (2012) and Biais et al. (2016). The impact of central clearing reforms on the collateral demand for derivatives transactions is investigated in Heller and Vause (2012), Sidanius and Zikes (2012), and Loon and Zhong (2014), assuming exogenously specified margin requirements based on VaR, expected shortfall, or a mix of the two.[4]

Empirical work on the *determinants* of collateral is scarce, mostly for the difficulty of obtaining data on positions and collateral.[5] The papers in this area mostly focus on margining in the futures market (Figlewski (1984), Gay et al. (1986), Fenn and Kupiec (1993), Hedegaard (2014)). Our study of collateral in the cleared CDS space enriches the existing empirical evidence on collateral rules

[3] Of course, there are differences between the theoretical models of collateral and our empirical setting. Among them, it is worth noting that we analyze empirically a clearinghouse that determines collateral rules in an oligopolistic setting (given that ICC is the largest clearinghouse with a certain degree of market power), whereas the theoretical models of endogenous collateral assume a competitive market. That said, we still believe existing theoretical models give us important insights on the determinants of collateral.

[4] Other work (Cruz Lopez et al. (2017), Menkveld (2017)) has studied the determination of collateral requirements accounting for systemic interdependencies.

[5] Collateral data for non-cleared OTC markets is often scattered among a variety of participants, with no centralized data sets available. Clearinghouse data contain proprietary information of large market participants and are often disclosed only under strict confidentiality and anonymity arrangements. Due to such data limitations, there is little empirical work focusing on portfolio-level margins (as opposed to individual-security collateral requirements), and on how well conventional risk measures relate to the required collateral levels.

along several dimensions. First, while prior studies have looked at headline margin requirements for individual securities, their approaches are less applicable in the modern setting of portfolio margining, where margins are set at the portfolio level rather than for individual contracts (as in the case of CDS clearinghouses). Our disaggregated, granular CDS data provide a valuable source of information for analyzing portfolio-level collateral requirements and the associated systemic risk implications. Second, we consider a market where payoffs are highly skewed (default probabilities can jump upward suddenly, and defaults can occur instantaneously), which implies that collateral plays a crucial role in allowing this market to function properly. Third, while existing studies focus mostly on the cross-sectional dimension of margins, we focus on both the cross-sectional and time-series variation. Fourth, we consider not only portfolio-specific risk measures, but also aggregate risk and funding measures as potential determinants of collateral – all factors that can play an important role in the amplification of aggregate shocks via the collateral-feedback channel. Fifth, we document that margins are best captured by using a combination of tail risk measures (maximum shortfall and short notional). As the short notional does not depend on historical probabilities and correlations nor on the state of the market, this shows that clearinghouses use a rule that is robust to the exact specification of the model for tail events.

## 2 Institutional Details and Data

In this section, we introduce the main institutional details of our setting. We also describe our data, and show our first finding: collateralization rates vary substantially across clearinghouse members and over time.

### 2.1 Clearinghouse Margining in Practice

Clearinghouses have significant discretion over modeling assumptions and parameters used to generate and justify margin requirements. They set them taking into account market conditions, the demand for trading, and collateral quality. In practice, margining rules involve a wide range of scenarios and simulations to arrive at a portfolio loss distribution, requiring the clearinghouses to make various modeling and statistical assumptions.

Most clearinghouses, *including ICC* (e.g., Ivanov and Underwood (2011)), state that their margins are broadly "set to cover five days of adverse price/credit spread movements for the portfolio positions with a confidence level of 99%", which we refer to as a 5-day 99% Value-at-Risk (VaR) margining rule.[6] However, this is only a simplified description of their actual margining rules. Scenario-specific add-ons are often applied to produce the final margin requirement (CME Group (2010), ICE Clear US (2015)).[7] In particular, the margin requirement set by ICC is the sum of seven components. In

[6] The CDS initial margin methodology adopted by ICE Clear Europe provides coverage for index, single Name and Western European Sovereign CDS products using at least, a 5-day for house positions, and 7-day for client positions, 99.5% Value-at-Risk measure. See https://www.theice.com/publicdocs/ICE_CDS_Clearing_Margin_Calculator_Overview.pdf for details.

[7] These rule are not described in detail in publicly available documents, but they are available to CFTC officials.

addition to considering (i) losses due to credit quality (changing credit spreads), the methodology also considers losses due to (ii) changing recovery rates and (iii) interest rates. There are additional charges capturing (iv) bid-offer spreads, (v) large, concentrated positions, (vi) basis risk arising from different trading behavior of indices and their constituents. Finally, there is (vii) an additional jump-to-default requirement due to the potential large payouts associated with selling credit protection on single name contracts. Similar to the Basel capital requirements, the ICC margin framework follows a bucket approach. It first calculates each of the seven components ("buckets" ), and the final collateral requirement is simply the sum of these components. Importantly, even if clearinghouses were restricted to using VaR based margining rules, the confidence level, margin period of risk, and the distributional assumption of losses are inputs that give the clearinghouse significant freedom in setting the actual margin levels.

Overall, clearinghouses employ complex rules to determine the amount of required margins. These rules make it difficult to understand what the main economic determinants of collateral requirements are, partly because they depend on the interactions of several variables and calibration choices, and partly because they do not explicitly take into account variables that may still matter indirectly: for example, aggregate volatility or default risks do not directly enter into the calculations, but may still affect the collateral rule because they affect the scenarios used by the clearinghouse to simulate portfolio losses, or affect the choice of discretionary parameters. The goal of this paper is not to reverse-engineer this complicated procedure, but rather to identify and quantify the main economic determinants of collateral both in the level and in its panel variation.

## 2.2 Data and Summary Statistics

In this section, we provide an overview of our data and present descriptive statistics for the key variables. We construct a database of the entire universe of CDS positions cleared by ICE Clear Credit (ICC), for the period between May, 1 2014 and May, 08, 2020. ICC managed a significant fraction of the U.S. cleared CDS market, totaling 56% in 2015 and 2016, 52% in 2017, and 70% in 2018 and 2019. The absolute value of the cleared amount increased from nearly 9 billion in 2015, 2016, and 2017 to about 12 billion in 2018 and 2019.[8] Hence, it has always remained the largest CDS clearinghouse throughout our sample.

**Clearinghouse collateral data: the Part 39 data set.** The Dodd–Frank Wall Street Reform and Consumer Protection Act grants the U.S. Commodity Futures Trading Commission (CFTC) authority over Derivative Clearing Organizations (DCOs). As a result, major clearinghouses recognized as DCOs are required to report confidential swap trade data to CFTC on a daily basis. The data are collectively referred to as "Part 39 data," as the relevant rules and regulations are codified in Title 17, Chapter I,

For ICC's public disclosure, see https://www.theice.com/publicdocs/clear_credit/ICE_CDS_Margin_Calculator_Presentation.pdf.

[8] These percentages are computed from the quantitative disclosure statements of the three clearinghouses in the CDS market, i.e., ICE Europe ICC, and LCH CDSClear. These documents disclose the notional of cleared positions for the house accounts of their members.

Part 39 of the Code of Federal Regulations. Part 39 data provides a complete overview of the centrally cleared swaps in the U.S..

We obtain clearing member data from the CFTC Part 39 database. Our data set consists of both positions data and account summary data for CDS trades cleared by ICE Clear Credit (ICC) (combined with ICEU, the European arm of ICE's CDS clearing, they account for over 90% of the cleared CDS market registered in the database). Our sample period covers nearly five years, from 05/01/2014 to 05/08/2020, for a total of 1,554 business days.

**CDS positions data.** Credit default swaps (CDS) are credit derivatives used to trade the credit risk of a reference entity (a bond). The protection buyer (the long side of the contract) is obligated to pay a quarterly premium to the protection seller (the short side of the contract) up until contract maturity or the arrival of a credit event for the reference entity, whichever occurs earlier. Upon arrival of the credit event, the seller pays to the buyer the difference between the face value and the market value of the reference obligation. If the reference entity is a sovereign or corporate entity, the CDS is referred to as a single name CDS, and is uniquely identified by its coupon rate (the quarterly premium), maturity, reference bond seniority, and doc clause (which defines what constitutes a credit event), typically rolled out quarterly. If the reference entity is a weighted basket of bonds from various sovereign or corporate entities, it is referred to as an index CDS, typically rolled out semiannually. When components of the reference basket default, the protection seller pays a pro rata cash flow depending on the weights of the components. The index contract is then reversioned (i.e., the basket is updated), and coupon payments and the contract notional are reduced accordingly. A CDS index contract is identified by its notional, coupon rate, maturity, reference basket, version, and doc clause. Our data set includes both single name and index contracts.

The CDS position component of the Part 39 data set contains daily reports of each account's end-of-day (EOD) position in each cleared CDS contract, for each account used by a clearinghouse member (see below). For each day/account/contract combination, we observe long/short gross notional, EOD prices for the contract, the currency denomination and exchange rates, and the mark-to-market (MtM) value of the position.[9]

In the sample period considered, the most liquid CDS index contracts were mandatorily required to be cleared through a clearinghouse. Many other CDS contracts were cleared as well, but only voluntarily; as a result, our data presents only a partial view of the entire CDS market (which is inconsequential for the goals of this paper). Our data set includes CDSs on 614 distinct reference entities, 603 single names and 11 indices. A total of 26,603 distinct contracts referencing these reference entities were cleared during our sample period (multiple CDS contracts are written on each underlying reference entity, since CDS contracts differ by seniority, doc clause, etc., as described above).

We adjust for changes in reference entities due to spin-offs, split-offs, or combined firms from mergers and acquisitions. After accounting for this, we are left with a total of 614 distinct reference entities.

[9] We report in Appendix C a brief overview of standardized CDS price quote conventions.

**Account and margin data.** The cleared CDS market is dominated by a handful of *clearing members* who act as dealers to the outside market. Smaller clearing participants access the cleared market by becoming customers to clearing members. Each clearing member may have several accounts with ICC. The account is designated as a "customer account" if the account positions are taken on behalf of a customer, and designated as a "house" account if the positions are proprietary. Customer accounts are commingled; that is, they consist of multiple sub-accounts for many customers, and segregated customer specific data are not reported. We observe 45 accounts in total, each identified by a distinct clearing firm identification number. Of these accounts, 14 are designated as customer accounts and 31 are house accounts.

Many house accounts are set up to help with the processing of client trades, but have little open interest, as clearing members usually use one house account to hold the majority of their proprietary positions. We thus define a house account to be "auxiliary" if there are little to no positions associated with them. We refer to the remaining house accounts as "active" house accounts.[10]

For each clearing member account, the account summary portion of the Part 39 data set contains daily reports of EOD information. For each day/account combination, we observe the so-called *initial margin* requirement, the initial margin posted, the currency denomination and exchange rates, and the MtM value of the portfolio. The initial margin requirement is the level of collateral the clearinghouse demands from the account holders, whereas the margin posted is the actual amount that account holders supply; the two are almost always the same, or extremely close.[11]

It is important to emphasize that what is referred to as the initial margin in this market – the collateral requirement we study in this paper – is the collateral kept by the clearinghouse with the purpose of buffering against potential *future* losses in case clearing members default on their obligations. Despite the name "initial" margin, this margin is *not* just posted at initiation of a CDS position: instead, it is updated every day and it covers the entire portfolio of an account. It therefore corresponds directly to what is typically referred to as collateral requirement in standard theoretical models. We will use the terms initial margin, margin requirements, and collateral requirements interchangeably.[12]

We provide descriptive statistics for each of the three account categories (active house, customer, and auxiliary house) in Table 1. Table 1 reports, for each account, the pooled averages of the key variables over our sample period. Pooled averages are computed by averaging point observations within the account categories and across the sample time period. The table shows that we identify 15 active house accounts, which on average own more than 3,500 distinct contracts, for a total average notional

[10] To be precise, a house account is auxiliary if (i) the average gross notional is less than \$15 billion USD, (ii) the average number of distinct CDS contracts traded is less than 500, or (iii) the number of distinct CDS reference entities traded is less than 100. Our empirical conclusions are robust to changes in these thresholds.

[11] Margin requirements are reported separately in USD and Euro; we combine them using the appropriate exchange rate and express the total initial margin requirement of the entire portfolio in USD. The actual collateral posted is often reported entirely in USD, and covers both the USD and Euro margin requirements.

[12] All cleared contracts are marked to market daily, so that the change in *current* value of the portfolio is transferred to the clearinghouse by the next day. This transfer is referred to as *variation margin*, and is distinct from the initial margin as it does not represent a stock of collateral meant to cover for future changes in the value of the portfolio, but rather a cash flow reflecting the mark-to-market process. So the variation margin will play no role in our analysis, as it is different from what we typically refer to as collateral.

position of $136bn, and post an average amount of collateral to ICC of $654m. Auxiliary accounts have activity levels that are one order of magnitude smaller.[13] Customer accounts have similarly small activity levels, but post higher amounts of collateral because of the lower diversification. In the remainder of the paper, we will focus exclusively on on the active house accounts. This is because customer accounts are commingled and margins information aggregated. Therefore, the observed margins are not associated with a specific institution's portfolio in our data set, so that we cannot study the relationship between collateral posted and portfolio characteristics. We also exclude auxiliary house accounts because there are little to no positions associated with them.

To gain further insight into active house account margins, we compute the level of collateralization for cleared portfolios. We measure this with the *margin to net notional* ratio, which accounts for varying sizes of cleared portfolios; it is computed by taking the ratio of the initial margins requirement and aggregate net notional. The results are reported as a histogram in Figure 1. The figure shows that there is substantial heterogeneity in collateralization rate across time and accounts, from a low close to zero to a high above 20% (the observations that cluster around 15% all belong to one specific clearing member). In the remainder of this paper, we analyze what characteristics of the members' portfolios drive this significant heterogeneity in collateralization rates.

**Market events during our sample period.** Mandatory clearing of standardized CDS contracts was imposed only after the financial crisis. Thus, our data set does not include the years of the crisis in which the financial system (and the CDS market) underwent significant stress, which are particularly interesting times for understanding collateral requirements and their interaction with the broader economy. However, several important events happened during this period, among which commodity market events (e.g., the drop in oil prices in November 2014), currency market events (the plunge in the Euro in 2015), political events (the Brexit referendum and Trump's election in 2016), credit events (Venezuela's default in 2017), and the COVID 19 pandemic. Appendix D reviews in detail the most important events that occurred during our sample period.

We also remark that, while we do not have *positions* data going back to the financial crisis, we do observe CDS spreads going back to 2004. This allows us to do counterfactual simulations of portfolio returns, and to assess how well current collateral buffers can absorb shocks of magnitudes as large as those observed in 2008–2009.

## 3 Collateral requirements and the *Value-at-Risk* rule

In this section, we test whether the standard VaR margining rule is a good description of the collateral rule in the cleared CDS market. Using two different approaches, we show that that is not the case: actual collateral levels are orders of magnitude more conservative than predicted by the standard VaR rule.

[13] In fact, five auxiliary house accounts had zero margins throughout, indicating no trading activity at all. We excluded these accounts when calculating descriptive statistics of auxiliary house accounts.

### 3.1 Notation

Consider a set of dates $\mathcal{T} := \{1, \ldots, T\}$, a set of contracts $\mathcal{I} := \{1, \ldots, I\}$, and a set of market participants (clearing members) $\mathcal{N} := \{1, \ldots, N\}$. The portfolio held by participant $n$ at time $t$ is a vector $\mathbf{X}_t^n \in \mathbb{R}^I$, whose $i$-th component $X_{i,t}^n$ is the portfolio's *notional position* in contract $i$. $X_{i,t}^n$ can be positive or negative, depending on whether $n$ has a long or a short position in the contract $i$.

We denote the end-of-day (EOD) quoted prices of cleared contracts at time $t$ by $\mathbf{P}_t$, whose $i$-th component, $P_{i,t}$, is the EOD quoted price of contract $i$. As explained in detail in Appendix C, the market value of a position with one dollar notional and quoted price $P_{i,t}$ is simply $(1 - P_{i,t})$; we follow this conventional notation here, and express all quantities in terms of quoted prices $P_{i,t}$.

The mark-to-market value of the portfolio $\mathbf{X}_t^n$ held by market participant $n$ at time $t$, denoted by $MtM_t(\mathbf{X}_t^n)$, can be computed as[14]

$$MtM_t(\mathbf{X}_t^n) = \sum_i X_{i,t}^n \,(1 - P_{i,t}) \tag{1}$$

The profit and loss ($P\&L$) between times $t$ and $t + M$ (for a given time-$t$ portfolio $\mathbf{X}_t^n$) is given by

$$\begin{aligned}\Psi_{M,t}(\mathbf{X}_t^n) &:= MtM_{t+M}(\mathbf{X}_t^n) - MtM_t(\mathbf{X}_t^n) \\ &= \mathbf{X}_t^n \cdot (\mathbf{P}_t - \mathbf{P}_{t+M})\,.\end{aligned}$$

We use $VaR_t^{M,\alpha}(\cdot)$ to denote the $\alpha-$th quantile of the profit-and-loss (P&L) distribution of the portfolio $\mathbf{X}_t^n$ held by market participant $n$ at time $t$ over an $M-$day period starting at $t$.[15] Hence, Value-at-Risk ($VaR$) is defined by

$$\mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -VaR_t^{M,\alpha}(\mathbf{X}_t^n)|\mathcal{F}_t) = \alpha,$$

where $\mathcal{F}_t$ represents the information set available at time $t$. $M$ is commonly referred to as the margin period of risk (or liquidation period), and $1 - \alpha$ is the confidence level.

### 3.2 Testing the *Value-at-Risk* rule

The standard VaR collateral rule assumed in the literature stipulates that collateral requirements (initial margins) at time $t$ are set equal to $VaR_t^{M,\alpha}(\cdot)$, for a certain confidence level $\alpha$ and margin period of risk $M$. That is, under the VaR rule, initial margins are set as

$$H_0 : IM_t(\mathbf{X}_t^n) = VaR_t^{M,\alpha}(\mathbf{X}_t^n),$$

where $IM_t(\mathbf{X}_t^n)$ is the margin required by the clearinghouse at time $t$ for holding portfolio $\mathbf{X}_t^n$.

[14] There is an additional adjustment factor for CDS indices that have been reversioned after the default of a component, which we omit here for ease of exposition but account for in our empirical analysis. The adjustment factor is smaller than one and accounts for a proportional decrease in effective notional due to the contract payout.

[15] By definition, this assumes that margins cannot be increased *during* the $M$ days between $t$ and $t + M$.

We test the VaR hypothesis $H_0$ using two different approaches. The first approach can be applied if $\alpha$ and $M$ are known. For instance, CDS clearinghouses typically claim that initial margins are set to cover 5-day losses with 99% confidence (Ivanov and Underwood (2011)), so $\alpha = 1\%$ and $M = 5$. If initial margins are set to be a certain conditional quantile of the returns distribution (say the 1% quantile), the fraction of times the portfolio loss exceeds the posted collateral is expected to be on average equal to that quantile (1% of the time). We refer to this approach as the *time-series* test of the VaR hypothesis; this is in fact equivalent to the "backtesting" procedure advocated by the Basel Accords (Hull (2012)).

A second approach can be considered in the cases that $\alpha$ and $M$ are unknown. Rather than testing the rule jointly across all counterparties, this test looks at whether the *same* VaR rule is applied to all counterparties, similar to the approach implemented by Gay et al. (1986). That is, no matter what $\alpha$ and $M$ are, under VaR margining we would expect the same margining rule to be applied to all counterparties. This test reveals whether the proposed rule ($VaR$) is able to capture all portfolio- and counterparty-specific factors that are relevant for determining margin requirements. We refer to this as the *cross-sectional* test of the VaR hypothesis.

### 3.3 Time-series test of the VaR hypothesis

We start with the time-series test of the $VaR$ hypothesis, using the null $H_0$ described by the clearinghouse: $M = 5$ days and $\alpha = 1\%$. Since $IM_t(\mathbf{X}_t^n)$ is observable in our data set, the main step of our analysis is to estimate the empirical distribution of the P&L of the portfolio over $M$ days $(\Psi_{M,t})$ . We consider two different approaches, one using the *realized* P&Ls during our sample period, 2014-2020; the other using *simulated* P&Ls for the period 2004-2020.

**Using realized returns.** The first tests asks the question: how often do we see a 5-day portfolio P&L $\Psi_{5,t}(\mathbf{X}_t^n)$ negative enough to exceed the collateral that had been posted against it (we refer to this as an *exceedance*)? If we define the *return on margin* as the ratio of the realized 5-day P&L to the initial margin, we can restate the question as: how often do we see a 5-day return on margin below -100%? Under the null of a VaR rule, we should see these exceedances approximately 1% of the time; we can the simply test the null by looking at the empirical proportion of exceedances in our data. Note that this test does not require us to observe or specify the information set $\mathcal{F}_t$ – since under the null, the initial margin fully takes it into account.

Figure 2 reports the empirical distribution of realized 5-day-ahead returns on margins. We compute returns on margins for each account/day in our sample and obtain $23,310$ observations. A few interesting patterns emerge from the figure. First, the dispersion of returns in our sample period is quite small relative to the amount of collateral posted. Second, the distribution of returns on margins does not exhibit distinctly heavy tails, despite the fact that several important events occurred during our sample period, including the recent COVID crash (see Appendix D). With probability 99.45%, the return on margins lies in the $[-26\%, 26\%]$ interval. Third, and most relevant for the analysis of margining and losses, only in 2 out of $23,310$ cases (that is, 0.008% of the time) an exceedance occurred, far from the 1% frequency corresponding to the 99% VaR rule. In fact, in our sample margins have been about 7 times as high as what the 99% VaR rule would imply (the 99th percentile of the empirical distribution of losses).

In order to formally test the hypothesis $H_0$, we perform a statistical test comparing the observed empirical frequency (0.008%) to the one predicted by the model ($\alpha$). The distribution of the test statistic (the difference between the empirical frequency and $\alpha$) is derived in Appendix E. Not surprisingly, our statistical test strongly rejects the VaR null (p-value<0.001). Overall, this first test provides a strong indication that margins are set more conservatively than the standard VaR rule.

**Counterfactual return estimates (historical simulation).** Next, we use historical simulation methods to estimate the distribution of returns on margins over a longer time period. Such a time period covers the financial crisis, the most significant period of market distress since the Great Depression. More specifically, the idea of our counterfactual simulation is as follows. First, we collect CDS price data on all the CDS in our Part 39 sample, going back to 2004, from Markit and Bloomberg. For each day in our sample period (2014-2020), we observe the portfolio held by each member, $\mathbf{X}_t^n$. By looking at the history of (joint) price movements for all the constituents of those portfolios, we can then ask what the historical distribution of returns of that specific portfolio *would have been,* starting from 2004 and up until $t+5$, that is, including the realized 5-day return that begins on date $t$. The resulting distribution of P&L therefore includes the large price changes that occurred during the financial crisis, and incorporates the dramatic increase in correlations observed in those years. In other words, we backtest the ability of current (2014-2020) initial margins to prevent exceedances due to CDS price movements similar to those observed since 2004.

Implementing this test involves a few additional steps. Since there are new contracts issued and old contracts expiring every quarter, historical prices for a currently traded contract are not always available. To deal with this practical obstacle, we follow exactly the methodology of Duffie et al. (2015), designed specifically for this purpose. We first aggregate net exposures by name (reference entity), and then use the historical 5-year CDS spread on those names (for which we have accurate spreads data) to compute counterfactual returns for all days for which CDS spreads are available, starting from 2004/01/01. We review the details of the methodology in Appendix F.[16]

Using this approach, for each observed portfolio held by account $n$ on each day $t$, we compute all returns on margins that could have occurred to that portfolio in each 5-day window since 2004 and up to $t+5$. This procedure yields a total of $81,130,350$ simulated 5-day returns on margins.

We report the distribution of counterfactual returns on margins in Figure 3. Due to the large number of observations clustering around zero, we only display the histogram in the range $[-50\%, +50\%]$ in Figure 3a, and zoom in on the left tail of the histogram in Figure 3b. We see that the distribution is sharply peaked, and that most returns on margins lie between $\pm 20\%$. The frequency of returns decreases rapidly as we move away from the mean.

[16] A day is included in our data analysis only if prices are observed for at least 250 out of the 593 reference entities; this filter excludes few days in the early part of the sample for which price information was not uniformly available across contracts.

When we consider counterfactual returns, we again observe a very small number of margin exceedances: the portfolios held during 2014-2020 would have sometimes experienced losses larger than the posted collateral if prices moved as they did during the financial crisis, but only in 0.007% of all 5-day periods. This fraction is two orders of magnitude smaller than the 1% predicted by the standard VaR. On average, posted margins were 7 times higher than the empirical 99st percentile of 5-day simulated losses starting in 2004.

We also extend the formal statistical test of the previous section to include all the historical counterfactual returns for each portfolio held in each day by each account. As before, the test compares the empirical frequency of margin exceedances (0.007%) with that predicted by the VaR rule (1%). To fully account for potential time-series and cross-sectional correlation of the errors, we double-cluster the standard errors of the test statistic at both the day and the account levels (as described in Petersen (2009)). Appendix G reports the details of the test statistic and its distribution. Not surprisingly, the test again strongly rejects the null of VaR (p-value<0.001).

To sum up, both versions of the time-series test strongly reject the null that collateral is set by a 5-day, 99% VaR rule. We observe only two exceedances in the period 2014-2020 both of which occur during the COVID-19 period, and historical simulations that include the financial crisis imply exceedances only in 0.007% of cases. Instead, we find that posted collateral levels are one order of magnitude (700%) higher than the 99% percentile of realized and simulated losses. The collateral rule in this market appears *very* conservative.

### 3.4 Cross-sectional test of the VaR hypothesis

We conclude by performing the cross-sectional test of the VaR hypothesis, that requires no assumptions about the confidence level ($\alpha$).[17] Recall that the margining rule requires that

$$\mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)) = \alpha \quad \text{for all}\, n,$$

that is, the exceedance ratios should be the same across clearing members. We analyze the validity of this rule by testing that the empirical frequencies of exceedances across accounts are the same, using a G-test described in Appendix H.

The test strongly rejects the null (p-value <0.001) that exceedance ratios are the same across clearing members. There is therefore direct evidence against equality of exceedance probabilities, and thus against the null hypothesis that there exists a VaR rule which can explain observed initial margins for all counterparties.

[17] We perform the test using the counterfactual returns over the period 2004-2019. The results obtained using only the period 2014-2019 are trivial: there were no exceedances for any account, so a VaR with 100% confidence level fits the cross-section perfectly.

# 4 The Determinants of the Collateral Rule: Portfolio Risk and Market Risk

The previous section has shown that a simple VaR rule fails to capture observed margins along different dimensions. This suggests that other variables (both at the level of the individual member's portfolio and marketwide) might enter the collateral rule.

In this section, we consider two groups of potential explanatory variables. *Portfolio variables* are those that are specific to the portfolio that an account holds with the clearinghouse, and are conventionally used to measure the risk of positions. These include, in addition to VaR, expected shortfall (ES), maximum shortfall (MS), aggregate net notional (AN), aggregate short notional (AS), and the volatility of the portfolio (SD). We describe these in detail below. *Market variables* are those that are determined by market forces, and include the clearing members' CDS spreads, the LIBOR-OIS spread, the average clearing member CDS spread, and aggregate volatility as measured by VIX. Table 2 summarizes the full list of variables for convenience.

## 4.1 Description of portfolio and market variables

**Portfolio variables.** Let $\Omega_k$ denote the set of CDS contracts with reference entity $k$ (that differ by maturity, doc clauses, etc...). For each reference entity $k$, net notionals are defined by

$$Y^n_{k,t} := \sum_{i\in\Omega_k} X^n_{t,i}.$$

The aggregate net notional $AN^n_t$ is then defined as

$$AN^n_t := \sum_{k\in K} |Y^n_{k,t}|,$$

i.e., as the sum of absolute net notional values across reference entities. The aggregate *short* notional, $AS^n_t$, is instead defined as

$$AS^n_t := \sum_{Y^n_{k,t}<0} |Y^n_{k,t}|.$$

The aggregate short notional plays an important role because of the highly asymmetric nature of CDS payoffs. While the premium leg makes fixed payments, the protection leg (i.e. the short side of the CDS position) is exposed to jump-to-default risk. Such an asymmetry induces strong left skewness in the payoff function of a short position, which is why it is typically the seller of protection that posts the most collateral.

Duffie et al. (2015) propose to approximate the initial margin rule in the CDS market with the following combination of portfolio variables:

$$DSV^n_t = MS_5(\mathbf{X}^n_t) + 0.02 \times AS(\mathbf{X}^n_t), \tag{2}$$

where $MS_M(\cdot)$ represents the maximum shortfall of the portfolio for a $M-$day margin period of risk, computed using historical simulations. This formula – clearly different from Value at Risk – is assumed by Duffie et al. (2015) to approximately model the much more complex procedure that ICC uses; we refer to this rule as the "DSV model". Note that whereas in Duffie et al. (2015) this rule was directly assumed to hold, we evaluate here how well it captures the panel variation in initial margins in practice, comparing it to alternative models like VaR.[18]

Finally, we also consider a modified version of the DSV model, MDSV:

$$MDSV_t^n = w_1 \times MS_5(\mathbf{X}_t^n) + w_2 \times AS(\mathbf{X}_t^n), \tag{3}$$

that estimates the weights $w_1$ and $w_2$ from the data instead of using the ones calibrated by Duffie et al. (2015). The corresponding estimates are reported in Table 5, and given by $w_1 = 1.121$ and $w_2 = 0.025$.

We estimate the empirical distribution of the simulated series $\psi := \left\{\hat{\Psi}_{5,\mathrm{t}}(\mathbf{X}_t^n)\right\}_{t=1}^T$ of 5-day ahead P&L via the historical simulation approach discussed in Section 3.3. Using the empirical distribution, we form estimates of volatility (SD, standard deviation), 99% Value-at-Risk (VaR), expected shortfall (ES, the expected loss conditional on exceeding the VaR), and maximum shortfall of the portfolio (MS, the maximum loss experienced in simulations). All portfolio variables are in millions of USD to conform with the level of initial margins.

**Market variables.** We collect from Bloomberg time series data of the 3-month Overnight Index Swap (OIS) spread, the 3-month USD LIBOR rates, clearing member 5-year CDS spreads, and aggregate volatility as measured by VIX. CDS spreads on the members themselves can be interpreted as a measure of individual member counterparty risk, or potentially as a measure of funding cost for a clearing member, because higher spreads make it more costly for a member to borrow funds. We include in our analysis both individual CDS spreads and average CDS spreads of the members. As an alternative to the average credit spread $ACDS_t$, we also consider the LIBOR-OIS spread to control for market distress. The LIBOR-OIS spread

$$LOIS_t := LIBOR_t - OIS_t.$$

is typically viewed as a measure of financial sector stress, capturing mainly the interest rate differential between uncollateralized and collateralized loans. All market variables are recorded in basis points (bps) to conform with market convention.

**Summary statistics.** Table 3 displays summary statistics of our key portfolio variables and initial margins, in millions of USD. Note that each portfolio variable (like SD, VaR, etc) is computed sepa-

[18] While Duffie et al. (2015) compute maximum shortfall for a fixed look-back period of 1000 days, we use a longer price series starting from the year 2004. As both ours and their time series data cover the years of the crisis, when the largest losses occurred, the difference between the initial margins computed by the two approaches is negligible. We also explore robustness with respect to this choice in Section 4.4.

rately for each time $t$ and each member $n$; in the table, in addition to the pooled mean and standard deviations (which we also refer to as dispersions) across all $n$ and $t$, we also describe other measures of portfolio dispersion in the time series and in the cross-section.

We observe that all measures of dispersion increase in the order of extreme tail risk captured. That is, as more weight is put into the tail of the distribution, there is more variability in the computed measures both across time and across accounts. The measure with the smallest value is the standard deviation ($SD$), followed in order by Value-at-Risk ($VaR$), expected shortfall ($ES$), and maximum shortfall ($MS$).

Consistent with the results of the previous section, Table 3 shows that $VaR$ is about one order of magnitude smaller than initial margins on average. Interestingly, even $ES$ and $MS$, that do capture more extreme tails, are much smaller than the posted margins – suggesting that these variables alone should not be able to explain the observed level of margins either. On the other hand, the table also shows that the $DSV$ model matches well not only the level of margins, but also all the dispersion measures; the modified $DSV$ model does even better.

Table A.1 in the Appendix reports summary statistics of our key market variables and initial margins, in basis points and millions of USD.

### 4.2 Margins and Portfolio-specific Risks

In this section we perform a panel analysis relating observed margins to portfolio variables. In particular, we estimate the following panel regression model with time and account fixed effects:

$$IM_t^n = \alpha^n + \eta_t + \sum_{v \in PV} \beta_v v_t^n + u_t^n, \tag{4}$$

where $PV$ is the set of portfolio variables included in the panel regression. Note that in the model specification of Eq. (4), the regression coefficients do not depend on the specific clearing member $n$, a necessary condition if margining rules are implemented uniformly across accounts.

We start by examining the set of portfolio variables to include in the regression. First, we note that aggregate net notional ($AN$) serves primarily as a measure of portfolio size. As portfolio size is already accounted for by risk measures such as $VaR$, $MS$ and $AS$, all expressed in dollar units, we drop this variable from our regression.[19] Second, we perform a check for multicollinearity, reported in Appendix Table A.2. The table shows that $VaR$ explains more than 96% of the variation of both expected shortfall and standard deviation. This strongly points to multicollinearity issues, and thus we leave out standard deviation and expected shortfall in our panel model specification.

Our final set of portfolio variables includes Value-at-Risk, maximum shortfall, aggregate short notional, and the DSV model given in Eq. (2); we also construct the MDSV variable by adjusting the DSV weights (0.5 on MS, 0.02 on AS) to maximize the in-sample fit (see (3)). We report all the results in Table 5. We use double-clustered standard errors (by time and account) as in Petersen (2009), thus

[19] We have conducted a regression analysis including $AN$ as an explanatory variable, and found that, qualitatively, our results are largely unaffected.

accounting for potential correlation in the errors, both within each account over time, and across accounts within each day. The signs of all the coefficients are in line with intuition: because larger values for each of the explanatory variables point to a riskier portfolio, all coefficients are expected to be positive.

Columns (1) and (2) of Table 4 show that Value-at-Risk alone can explain 56% of the variation in initial margins, and 80% of the variation if fixed effects are added to the regression. The estimated slope coefficient, however, is much higher than unity in either case. In particular, a multiplier of at least 360% is needed for the regression fit, again showing that collateral requirements are set much more conservatively than what would be implied by the conventional 5-day 99% VaR rule. Columns (3) and (4) introduce maximum shortfall ($MS$) and aggregate short notional ($AS$) as explanatory variables in conjunction with Value-at-Risk. The addition of these variables brings the $R^2$ to 73%, even without fixed effects. Moreover, the magnitude of the VaR slope coefficients are much closer to unity once these variables are included. Our results therefore show that initial margins depend on risk characteristics which cannot be captured only by VaR, and in particular they depend on more extreme tail risks.

It is worth remarking that while both MS and AS relate to the extreme tails of the distribution, they differ significantly in their nature. MS represents the maximum *experienced* loss in historical simulations. As such, it depends on the experienced realization of shocks and historical correlations across portfolio components. AS, instead, captures the maximal *potential* loss if all short CDS positions jump to default simultaneously, and recovery rates are zero; in this sense, it represents a theoretical worst-case scenario for the potential loss, and represents a measure of tail risk that is less sensitive to the specification of the loss distribution (as well as of the correlations between portfolio components).

Motivated by the DSV model that only features MS and AS, we drop Value-at-Risk as an explanatory variable in columns (5) and (6), and find that there is little loss in explanatory power compared to columns (3) and (4). Maximum shortfall is positively correlated with Value-at-Risk, and dropping Value-at-Risk increases the statistical significance of the maximum shortfall loading.[20] Interestingly, the aggregate short notional coefficient estimate remains very stable (in the 2–3% range) and highly significant for all the estimated models.

Columns (7)–(12) investigate the usefulness of the DSV initial margin model in explaining empirically observed margins. Columns (7) and (8) show that the DSV model captures a significant portion of the variation in initial margins; it strongly outperforms Value-at-Risk in terms of explanatory power (columns (1) and (2)). The significance of the DSV slope coefficient persists when we introduce Value-at-Risk, and the explanatory power remains roughly the same, showing again that Value-at-Risk has little explanatory power beyond that already captured by DSV. The overall explanatory power improves marginally when we consider MDSV (columns (11) and (12)), whose coefficients are based on the estimates reported in columns (5) and (6) (so we should expect a similar explanatory power). Again, Value-at-Risk still has little explanatory power beyond that already captured by our modified

[20] Given that maximum shortfall corresponds to the realized maximal loss, it is not surprising that its estimate can be noisy, which can sometimes affect its statistical significance.

DSV model.

To sum up, our empirical results provide strong support for the DSV model (in which the parameters were calibrated, not estimated), against alternatives such as VaR. More generally, the results show that tail risk variables like maximum shortfall and aggregate net notional work significantly better than VaR in explaining the observed collateral rule.

While our proposed measures capture significant variation in initial margins, the explanatory power is obviously not 100%. On the one hand, that's to be expected, since we are estimating a simple approximation of the true collateral rule, which is far more complex. On the other hand, it leaves open the possibility that other factors might help explain the margin setting, which are not captured by standard portfolio risk measures.

### 4.3 Funding Cost, Collateral Rates and other Market Variables

In this section we incorporate market variables into our panel analysis and assess their ability to explain margin requirements. The included market variables are chosen to capture variation in margins that is due to changes in default risk and funding costs. We consider the following panel regression model:

$$IM_t^n = \alpha^n + \eta_t \sum_{v \in PV} \beta_v v_t^n + \sum_{v \in MV} \beta_v v_t^n + u_t^n, \tag{5}$$

where $PV = \{VaR, MS, AS, DSV, MDSV\}$ and $MV = \{LOIS, CDS, ACDS, VIX\}$ are, respectively, the portfolio and market variables included in the panel regression. Because market variables are often not account-specific (e.g. the LIBOR-OIS spread), time fixed effects cannot be included in the regression. Thus, in this section we only consider time fixed effects when non-account-specific variables are excluded.

We estimate the model in Eq. (5) using least squares regressions, choosing initial margins as the dependent variable and portfolio and market variables as explanatory variables. The results with double-clustered (by account and time) standard errors are reported in Table 5.

Columns (1) and (2) report the results when member CDS spreads, average CDS spreads, and the VIX are included, and the portfolio variables VaR and MDSV are controlled for. There is a small but significant increase in explanatory power compared to models that include only portfolio variables (Table 4). Both the VIX and the average CDS spreads do not appear significant. There is strong evidence that funding costs, proxied by the individual members' CDS spread, matter when we account for fixed effects. Overall, while market variables do seem to influence margin levels, their effects seem to be much smaller than that of portfolio variables.

Columns (3) and (4) report our results when we replace the average CDS spread with the LIBOR-OIS spread. There is no change in explanatory power and the loadings on the portfolio variables remains very similar; the LIBOR-OIS spread appears insignificant. Columns (5) and (6) report the results when maximum shortfall and aggregate short notional are used together instead of MDSV. The increase in explanatory power is small when compared to the results in columns (1) and (2). Interestingly, the estimated coefficient for aggregate short notional is significant and in the range of

2-3%, again showing the robustness of our previous results in Table 5 (MS has a magnitude comparable with that of the previous table).

Across columns, we find that among all market measures, the VIX is the only one that robustly seems to affect the collateral rule. The magnitude of the VIX's effect, however, is substantial. In our estimates, a one-point increase in the VIX increases required margins by an amount between $1.5 and $3 millions. During crisis episodes movements of the VIX of even 50 points are possible, these estimates imply large potential effects on prices and systemic stability through the collateral channel, consistent the model of Brunnermeier and Pedersen (2009).[21]

### 4.4 Robustness

Two of the portfolio variables (Value-at-Risk and maximum shortfall) used in our analysis were based on P&L generated from our entire sample of credit spreads. Because our data set covered the financial crisis, the risk measures captured extreme movements and may thus be viewed as overly conservative for estimating portfolio losses. In Appendix I, we consider using only the last 1000 days (approximately 4 years) of credit spreads data to generate P&L, as in Duffie et al. (2015). The results of our analysis presented in the appendix show that the results remain qualitatively similar to those reported in Tables 4 and 5.

## 5 Conclusions

We study the empirical determinants of collateral requirements in a large market in which counterparty risk plays an important role – the cleared CDS market. Our analysis exploits the availability of a unique data set on clearing members' portfolio exposures and associated margin levels. Margins in this market are set at the portfolio level rather than at the individual security level; this allows us to study how risk measures like VaR and other portfolio characteristics affect margins. We also study how market variables – in addition to portfolio variables – affect collateral requirements, highlighting the implications of our findings for models of the collateral feedback channel.

We document four novel empirical results on the collateral rule in this market. First, we show that there is large variability in the collateralization rate across accounts and over time, suggesting corresponding variability in the risk characteristics of the clearinghouse members' portfolios.

Second, we show that collateral in this market is set much more conservatively than what would be implied by a standard VaR rule – a rule that is at the core of many theoretical models with collateral constraints and that clearinghouses themselves state they use. In fact, the amount of collateral appears about 7 times larger than what would be needed to cover 99% of 5-day losses (which is the standard for VaR in this context).

Third, we find that other portfolio variables dominate VaR in explaining the time-series and cross-sectional variation in margins. In particular, two measures of extreme tail risk suggested by Duffie

[21] Of course, these estimates are obtained in a relatively calm period, so it is hard to extrapolate the estimates to times of crisis; however, they give a sense of the magnitude of these effects.

et al. (2015), maximum shortfall (the largest experienced portfolio loss) and aggregate short notional (the notional amount held by a member in short CDS positions, representing the theoretical maximal loss on all CDS positions simultaneously) dominate VaR in explaining the panel variation of required collateral. These two variables alone account for almost 72% of the entire variation in margins in our panel. Whereas empirically VaR is strongly related to simple portfolio volatility, these measures capture much more extreme tail risks, that are more strongly related to jump-to-default risk and less related to volatility. These measures are conceptually (and empirically) quite different from VaR.

Finally, we find that shocks to some market variables (particularly the VIX) increase the total amount of required collateral, even after controlling for portfolio-level risks, suggesting that average margin levels vary with aggregate market conditions beyond what individual portfolio measures capture.

Our findings have several implications for theoretical models of the collateral feedback channel. First, the fact that extreme tail risk measures explain margins better than VaR indicates that the clearinghouse is worried about more extreme losses than what the standard VaR captures. Given that standard VaR is highly correlated with volatility, it suggests that the margin spiral mechanisms examined in many theoretical models (in which the collateral rule is exogenously specified) could operate through the direct effects of shocks on the extreme tail of the distribution, rather than through changes in volatility. In other words, our results suggest that collateral levels may respond little to small changes in risks (like an increase in the variance of the portfolio), but may spike if the probability of an extreme event increases or the worst-case-scenario worsens. This nonlinearity can potentially play an important role in general equilibrium models, amplifying the largest shocks but dampening moderate-sized shocks.

Second, our empirical results are consistent with some of the key results of the theoretical literature on endogenous collateral, like Geanakoplos (1997) and Fostel and Geanakoplos (2015). These models cannot directly be mapped to our data (because they counterfactually assume a binomial world and feature no default in equilibrium). Appendix A therefore develops a new model of the endogenous collateral equilibrium, based on Simsek (2013), that is specifically suited for the CDS market. The model features a continuum of states and a non-zero default probability in equilibrium. In this model, trade occurs because of difference in beliefs, that also determine the equilibrium collateralization rate. The model can rationalize the observed empirical patterns of extremely high collateralization rates with disagreement about the extreme events in which members would default on their obligations to the clearinghouse.

Finally, collateral requirements are directly affected by market conditions: increases in aggregate risks directly induce an increase in collateral requirements, holding the portfolios fixed. Our empirical analysis therefore documents the existence of two channels for the amplification of fundamental shocks (studied, for example, in Brunnermeier and Pedersen (2009)): at the portfolio level, where an increase in perceived tail risk following a shock may affect the member's margin requirement; and at the macro level, where an increase in aggregate risk can increase the collateral requirements of all members.

Taken together, our empirical findings and theoretical results in Appendices A and B provide guidance for building empirically grounded models of the collateral feedback channel.

# References


Biais, B., Heider, F., and Hoerova, M. (2016). Risk-sharing or risk-taking? Counterparty risk, incentives, and margins. *The Journal of Finance*, 71(4):1669–1698. 1

Brunnermeier, M. K. and Pedersen, L. H. (2009). Market liquidity and funding liquidity. *Review of Financial studies*, 22(6):2201–2238. 1, 4.3, 5

Brunnermeier, M. K. and Sannikov, Y. (2014). A macroeconomic model with a financial sector. *The American Economic Review*, 104(2):379–421. 1

Chabakauri, G. (2013). Dynamic equilibrium with two stocks, heterogeneous investors, and portfolio constraints. *Review of Financial Studies*, 26(12):3104–3141. 1

CME Group (2010). CME SPAN: Standard portfolio analysis of risk. Available online at http://www.cmegroup.com/clearing/files/span-methodology.pdf. 2.1

Coen-Pirani, D. (2005). Margin requirements and equilibrium asset prices. *Journal of Monetary Economics*, 52(2):449–475. 1

Cruz Lopez, J., Hurlin, C., Harris, J. H., and Pérignon, C. (2017). Comargin. *Journal of Financial and Quantitative Analysis*, 52(50):2183–2215. 4

Du, C., Capponi, A., and Giglio, S. (2020). The collateral rule: Theory for the credit default swap market. *Working Paper, Yale University*. 1

Duffie, D., Scheicher, M., and Vuillemey, G. (2015). Central clearing and collateral demand. *Journal of Financial Economics*, 116(2):237–256. 1, 3.3, 4.1, 4.1, 4.1, 18, 4.4, 5, 2, F, G, I

Fenn, G. W. and Kupiec, P. (1993). Prudential margin policy in a futures-style settlement system. *Journal of Futures Markets*, 13(4):389–408. 1

Figlewski, S. (1984). Margins and market integrity: Margin setting for stock index futures and options. *Journal of Futures Markets*, 4(3):385–416. 1

Fostel, A. and Geanakoplos, J. (2015). Leverage and default in binomial economies: a complete characterization. *Econometrica*, 83(6):2191–2229. 1, 5, A.1

Gay, G. D., Hunter, W. C., and Kolb, R. W. (1986). A comparative analysis of futures contract margins. *Journal of Futures Markets*, 6(2):307–324. 1, 3.2

Geanakoplos, J. (1997). Promises promises. In Arthur, W. B., Durlauf, S. N., and A., L. D., editors, *The Economy as an Evolving Complex System II*, pages 285–320. Addison Wesley Longman. 1, 5

Glasserman, P., Moallemi, C. C., and Yuan, K. (2016). Hidden illiquidity with multiple central counterparties. *Operations Research*, 64(5):1143–1158. 1

Gorton, G. and Ordonez, G. (2014). Collateral crisis. *American Economic Review*, 104(2):343–378. 1

Gromb, D. and Vayanos, D. (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics*, 66(2-3):361–407. 1

Hedegaard, E. (2014). Causes and consequences of margin levels in futures markets. *Arizona State University Working Paper*. 1

Heller, D. and Vause, N. (2012). Collateral requirements for mandatory clearing of over-the-counter derivatives. *BIS Working Paper*. 1

Hoey, J. (2012). The two-way likelihood ratio (g) test and comparison to two-way chi squared test. *arXiv preprint arXiv:1206.4881*. H

Holmström, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics*, 112(3):663–691. 1

Hull, J. (2012). *Risk Management and Financial Institutions*, volume 733. John Wiley & Sons. 1, 3.2

ICE Clear US (2015). Span margin system. Available online at https://www.theice.com/publicdocs/clear_us/SPAN_Explanation.pdf. 2.1

Ivanov, S. and Underwood, L. (2011). *CDS Clearing at ICE: A Unique Methodology*, pages 31–33. Futures Industry Magazine. 1, 2.1, 3.2

Koeppl, T., Monnet, C., and Temzelides, T. (2012). Optimal clearing arrangements for financial trades. *Journal of Financial Economics*, 103(1):189–203. 1

Loon, Y. and Zhong, Z. (2014). The impact of central clearing on counterparty risk, liquidity, and trading: Evidence rom the credit default swap market. *Journal of Financial Economics*, 112:91–115. 1

Menkveld, A. J. (2017). Crowded trades: an overlooked systemic risk for central clearing counterparties. *Review of Asset Pricing Studies*, 7(2):209–242. 4

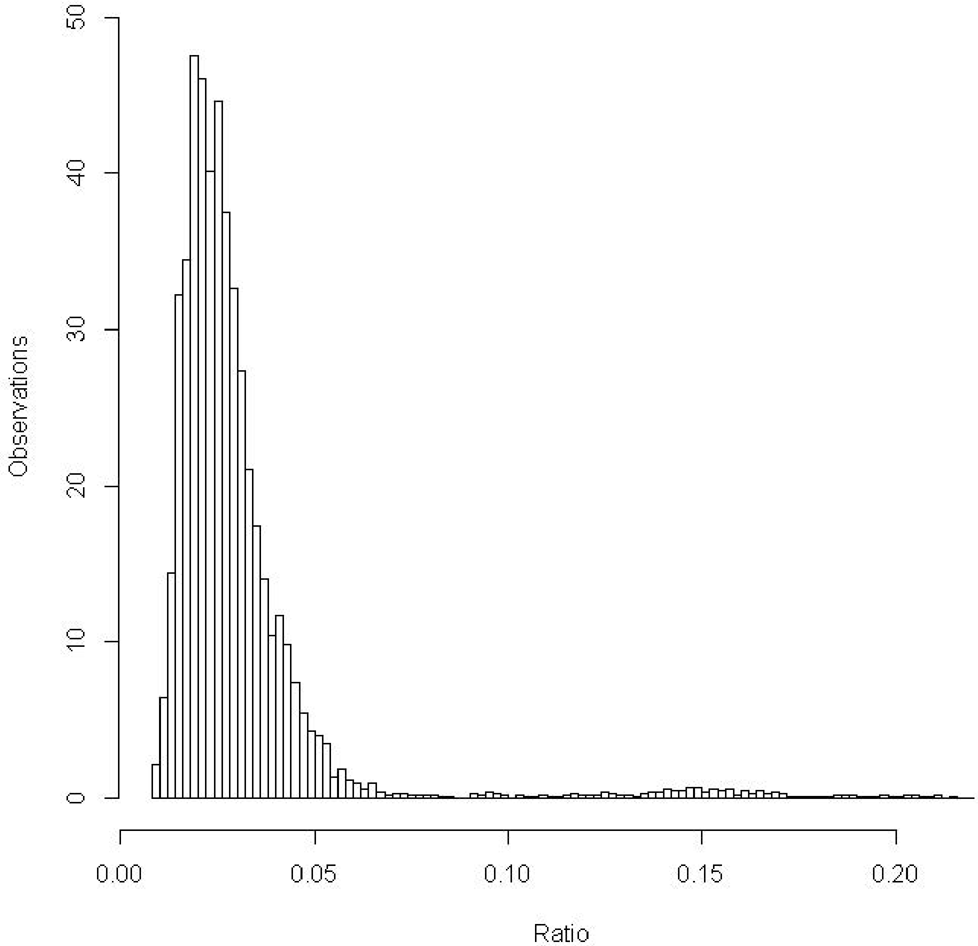
Petersen, M. A. (2009). Estimating standard errors in finance panel data sets: Comparing approaches. *Review of Financial Studies*, 22(1):435–480. 3.3, 4.2, G

Rampini, A. and Viswanathan, S. (2010). The impact of central clearing on counterparty risk, liquidity, and trading: Evidence rom the credit default swap market. *Journal of Finance*, 65:2293–2322. 1

Sidanius, C. and Zikes, F. (2012). OTC derivatives reform and collateral demand impact. *Financial Stability Paper*, (18). 1

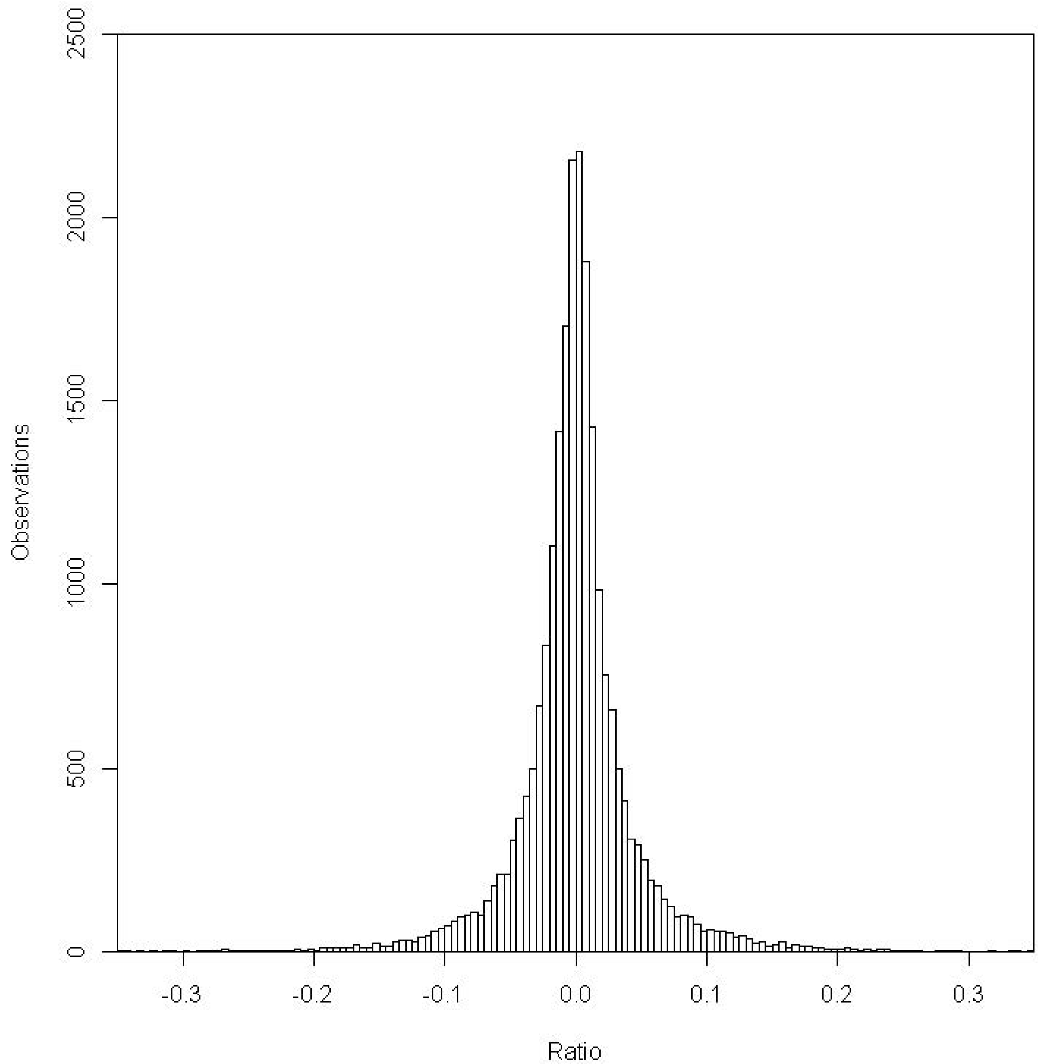
Simsek, A. (2013). Belief disagreements and collateral constraints. *Econometrica*, 81(2):1–53. 1, 5, 22, A.1, A.2, A.3

Figure 1: Histogram of margin/notional ratio observations.




**Note:** For each active house account/day combination, we compute the margin to notional ratio by dividing initial margins with aggregate net notional. We obtain the aggregate net notional by computing the net notional for each reference name and then summing the absolute net notional values across names. The figure reports the histogram of margin/notional ratio across all 23,310 account/day observations.

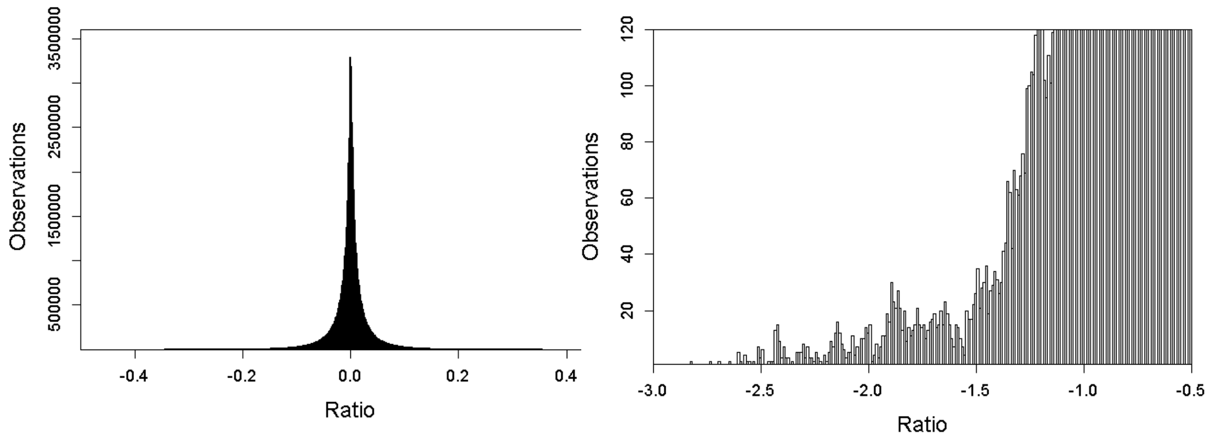
Figure 2: Histogram of realized return on margins for cleared portfolios.




**Note:** We compute the realized 5-day ahead returns on margins as the 5-day ahead $P\&L$ divided by posted margins. We compute this for each account/day and obtain 23,310 observations. The figure plots the histogram of the return on margins.

Figure 3: Histogram for historically simulated return on margins (left), with zoom on left tail (right)

(a) (b)




**Note:** The figure shows the histogram of simulated returns on margins. Due to the large number of observations clustering around zero, we only display observations between ±50% in Figure 3a, and report the left tail of the histogram in Figure 3b. We use the DV01 formula to approximate the 5-day ahead $P\&L$ with the product of net exposures to a reference name and the change in 5-year credit spreads for rolling 5-day windows from 2004/01/01 up to each position date $t$ + 5 days (for all business days from 2014/05/01 to 2020/05/08, adjusted for an average duration of $d = 3$. We compute this for each account/day/historical 5-day window and obtain 81,130,350 observations.

Table 1: Descriptive statistics for different account categories.

| | Active House | Customer | Auxiliary House |
|---|---|---|---|
| Number of accounts | 15 | 14 | 12(16) |
| Number of contracts | 3571.3 | 305.9 | 166.3 |
| Number of names | 302.3 | 101 | 63.1 |
| Gross notional (billions $) | 136.2 | 33 | 9.2 |
| Initial margins (millions $) | 654 | 1038.7 | 61.6 |

**Note:** The table reports the pooled averages of key variables within our data set depending on account type over our sample period. The number of contracts/names for each account counts those contracts/names for which the account has a non-zero position. Gross notional is computed by summing the absolute notional exposure for all contracts in the account. Margins are computed by summing the USD margin requirement and the Euro margin requirement, after adjusting for the historical exchange rates.

†Five auxiliary house had zero margins throughout, indicating no trading activity at all. We excluded these accounts when calculating auxiliary house account descriptive statistics.

Table 2: Portfolio and market variables.

| Notation | Units | Definition |
|---|---|---|
| $\psi$ | millions \$ | Empirical 5-day distribution of profit and losses for a portfolio |
| $IM$ | millions \$ | Observed initial margins posted for a portfolio |
| $Y$ | millions \$ | Net notional aggregated over reference names for a portfolio |
| **Portfolio Variables** | | |
| $VaR$ | millions \$ | 1 percent quantile of $\psi$ |
| $ES$ | millions \$ | Average of profit and losses less than equal to $VaR$ |
| $MS$ | millions \$ | Minimum of $\psi$ |
| $SD$ | millions \$ | Sample standard deviation of $\psi$ |
| $AN$ | millions \$ | Aggregate net notional (by reference entity) of portfolio |
| $AS$ | millions \$ | Aggregate short notional (by reference entity) of portfolio |
| $DSV$ | millions \$ | Initial margin estimate used by Duffie et al. (2015), equal to $MS + 0.02 \times AS$ |
| $MDSV$ | millions \$ | Adjusted initial margin from $DSV$, with estimated weights |
| **Market Variables** | | |
| $OIS$ | bps | End of day 3-month Overnight Index Swap spreads |
| $LOIS$ | bps | End of day 3 month USD LIBOR-OIS spreads |
| $CDS$ | bps | End of day market quote for clearing member specific 5-year CDS spread |
| $ACDS$ | bps | Average end of day clearing member 5-year CDS spread |
| $DCDS$ | bps | Deviation of end of day 5-year CDS spreads from the average, equal to $CDS - ACDS$ |
| $VIX$ | bps | End of day CBOE Volatility Index |

**Note:** This table displays the key variables and notation we use in our regression analyses. Portfolio variables are those that are specific to the portfolio that an account holds with the clearinghouse, and are conventionally used to measure the risk of positions. Market variables are those that are determined by market forces. Portfolio variables estimated from the empirical distribution via the historical simulation method outlined in Section 3.3 include Value-at-Risk, expected shortfall, maximum shortfall, and standard deviation. Portfolio variables estimated directly from positions include aggregate net notional and aggregate short notional. We record portfolio variables in millions USD. Market variables include the Overnight Index Swap (OIS) spread, the LIBOR-OIS spread, clearing member CDS spreads, the average clearing member CDS spread, and the aggregate volatility as measured by VIX. We record market variables in basis points.

Table 3: Initial margins and portfolio variables summary statistics

| Summary Statistic | *Portfolio variables, in millions \$* In. Margins ($IM_{n,t}$) | Portfolio SD ($SD_{n,t}$) | VaR ($VaR_{n,t}$) | Exp. Shortfall ($ES_{n,t}$) | Max Shortfall ($MS_{n,t}$) |
|---|---|---|---|---|---|
| Pooled mean (over all n and t): $\mu\left(x_{n,t}\right)$ | 654.0 | 25.3 | 78.2 | 116.3 | 240.3 |
| Std. deviation (over all n and t): $\sigma\left(x_{n,t}\right)$ | 427.4 | 17.7 | 57.0 | 83.3 | 205.9 |
| Time-series variation of cross-sectional averages: $\sigma\left(\bar{x}_t\right)$ | 108.4 | 3.2 | 8.4 | 18.9 | 83.0 |
| Mean cross-sectional dispersion: $\mu\left(\sigma_t\left(x_{n,t}\right)\right)$ | 421.0 | 17.8 | 57.7 | 81.8 | 171.5 |
| Cross-sectional dispersion of time-series averages: $\sigma\left(\bar{x}_n\right)$ | 360.7 | 14.5 | 46.7 | 67.3 | 137.2 |
| Mean time-series dispersion: $\mu\left(\sigma_n\left(x_{n,t}\right)\right)$ | 225.4 | 9.3 | 29.9 | 44.8 | 131.3 |

| Summary Statistic | *Portfolio variables, in millions \$* Aggr. Notional ($AN_{n,t}$) | Aggr. Short Notional ($AS_{n,t}$) | Duffie et al. ($DSV_{n,t}$) | Modified DSV ($MDSV_{n,t}$) |
|---|---|---|---|---|
| Pooled mean (over all n and t): $\mu\left(x_{n,t}\right)$ | 25,058.4 | 12,436.9 | 489.1 | 512.6 |
| Std. deviation (over all n and t): $\sigma\left(x_{n,t}\right)$ | 15,733.3 | 9,333.1 | 348.5 | 361.6 |
| Time-series variation of cross-sectional averages: $\sigma\left(\bar{x}_t\right)$ | 2,731.6 | 2,238.2 | 118.6 | 108.1 |
| Mean cross-sectional dispersion: $\mu\left(\sigma_t\left(x_{n,t}\right)\right)$ | 15,988.8 | 9,169.7 | 324.9 | 348.3 |
| Cross-sectional dispersion of time-series averages: $\sigma\left(\bar{x}_n\right)$ | 14,890.4 | 6,926.6 | 264.1 | 277.4 |
| Mean time-series dispersion: $\mu\left(\sigma_n\left(x_{n,t}\right)\right)$ | 5,823.1 | 5,334.9 | 204.8 | 206.8 |

**Note:** Table 3 displays summary statistics of our key portfolio variables and initial margins, in millions of USD. Definitions of portfolio variables are reported in Table 2. In addition to the pooled mean and standard deviations (dispersions), we report panel statistics that describe properties of panel variables both across accounts and time. In particular, for panel data $x_{n,t}$, we define

$$\bar{x}_t := \frac{1}{N}\sum_{n=1}^{N} x_{n,t}, \quad \bar{x}_n := \frac{1}{T}\sum_{t=1}^{T} x_{n,t}, \quad \sigma_t^2(x) := \frac{1}{N-1}\sum_{n=1}^{N}(x_{n,t}-\bar{x}_t)^2, \quad \sigma_n^2(x) := \frac{1}{T-1}\sum_{t=1}^{T}(x_{n,t}-\bar{x}_n)^2.$$

Above, we refer to $\sigma(\bar{x}_t)$ as the *time-series variation of cross-sectional averages*, $\bar{\sigma}_t(x_{n,t})$ as the *mean cross-sectional dispersion*, $\sigma(\bar{x}_n)$ as the *cross-sectional dispersion of time-series averages*, and $\bar{\sigma}_n(x_{n,t})$ as the *mean time-series dispersion*.

Table 4: Regression results for explaining initial margins with portfolio variables

| | *Dependent variable:* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Initial margins (IM) - Daily Frequency | | | | | | | | | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| Value-at-Risk (VaR) | 5.624*** | 3.618*** | 1.867** | 1.621*** | | | | | 1.446 | 1.125 | 1.526* | 1.419*** |
| | (1.018) | (1.049) | (0.866) | (0.501) | | | | | (0.974) | (0.797) | (0.882) | (0.484) |
| Maximum shortfall (MS) | | | 0.328** | 0.274** | 0.632*** | 0.519*** | | | | | | |
| | | | (0.163) | (0.127) | (0.199) | (0.123) | | | | | | |
| Aggregate short notional (AS) | | | 0.025*** | 0.018*** | 0.029*** | 0.020*** | | | | | | |
| | | | (0.002) | (0.003) | (0.004) | (0.004) | | | | | | |
| Duffie et al. model (DSV) | | | | | | | 1.016*** | 0.738*** | 0.818*** | 0.615*** | | |
| | | | | | | | (0.116) | (0.090) | (0.115) | (0.148) | | |
| Modified DSV model (MDSV) | | | | | | | | | | | 0.808*** | 0.588*** |
| | | | | | | | | | | | (0.052) | (0.103) |
| Number of Observations | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 |
| Adjusted $R^2$ | 0.563 | 0.798 | 0.734 | 0.862 | 0.716 | 0.853 | 0.686 | 0.845 | 0.698 | 0.850 | 0.731 | 0.861 |
| Account Fixed Effect | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* |
| Time Fixed Effect | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* |

*Note:* *p<0.1; **p<0.05; ***p<0.01
Two-Way Clustered Standard Errors (by Time and Account)

**Note:** We perform least squares regressions using initial margins as the dependent variable and portfolio variables as explanatory variables. Two-way clustered (by time and account) standard errors are reported in parentheses and used for the significance tests. We consider both the case with and without (time and account) fixed effects.

Table 5: Regression results for explaining initial margins with portfolio and market variables

| | *Dependent variable:* | | | | | |
|---|---|---|---|---|---|---|
| | Initial margins (IM) - Daily Frequency | | | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Value-at-Risk (VaR) 1% | 1.686** | 1.412*** | 1.754** | 1.404*** | 2.264*** | 1.648*** |
| | (0.851) | (0.517) | (0.823) | (0.445) | (0.765) | (0.508) |
| Modified DSV Model (MDSV) | 0.779*** | 0.626*** | 0.764*** | 0.622*** | | |
| | (0.051) | (0.076) | (0.040) | (0.054) | | |
| Maximum Shortfall (MS) | | | | | 0.208* | 0.286** |
| | | | | | (0.116) | (0.118) |
| Aggregate Short Notional (AS) | | | | | 0.025*** | 0.019*** |
| | | | | | (0.002) | (0.003) |
| CBOE Volatility Index (VIX) | 0.016 | 0.033* | 0.015 | 0.022* | 0.032** | 0.039** |
| | (0.013) | (0.018) | (0.014) | (0.011) | (0.013) | (0.016) |
| Member CDS Spread (DCDS) | 2.485** | 1.064 | 2.496** | 1.048 | 2.615** | 1.142 |
| | (1.163) | (0.771) | (1.126) | (0.774) | (1.142) | (0.734) |
| Average CDS Spread (ACDS) | 0.549 | 0.017 | | | 0.277 | −0.064 |
| | (1.092) | (1.110) | | | (1.124) | (1.106) |
| LIBOR-OIS Spread (LOIS) | | | 0.573 | 0.876 | | |
| | | | (0.869) | (1.023) | | |
| Number of Observations | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 |
| Adjusted $R^2$ | 0.758 | 0.859 | 0.758 | 0.859 | 0.765 | 0.859 |
| Account Fixed Effect | *N* | *Y* | *N* | *Y* | *N* | *Y* |
| Time Fixed Effect | *N* | *N* | *N* | *N* | *N* | *N* |
| *Note:* | | | | | | *p<0.1; **p<0.05; ***p<0.01 |
| | | | | | | Two-Way Clustered Standard Errors (by Time and Account) |

**Note:** We perform least squares regressions using initial margins as the dependent variable and portfolio and market variables as explanatory variables. Two-way clustered (by time and account) standard errors in parentheses are reported and used for the significance tests. We consider both the case of with and without fixed effects. Because market variables are often dependent only on time, we consider only account fixed effects when such variables are introduced.

# Appendices

## A Theoretical Model of Collateral Requirements

The empirical results presented in the main body of this paper demonstrate that extreme tail risk measures have a higher explanatory power for observed collateral requirements than standard VaR. To interpret these findings via models of endogenous collateral requirements, Du et al. (2020) constructs an equilibrium model where optimists post cash as collateral when they sell CDS insurance to pessimists (given the asymmetry of CDS payoffs, there is little need for collateral from a buyer - both in the theory and in the data). We present the model developed by Du et al. (2020) in this Appendix.

The model is an adaptation of the analytical framework of Simsek (2013) to the CDS market. In Du et al. (2020), the only contracts available for trading are state contingent promises (CDSs)[22], and the only way to enforce delivery is to post cash as collateral. Counterparties in the CDS market trade due to differences in beliefs about the uncertain states of the world. Du et al. (2020) shows that it is the nature - rather than the degree - of these belief differences that determines collateral requirements in equilibrium. In other words, what matters for collateral requirements is what the market participants disagree on, rather than the extent of such disagreements per se. In line with the empirical results presented in this paper, Du et al. (2020) suggests that the high collateral requirements we observe in practice may be explained by the clearinghouse's concerns over extreme tail events.

### A.1 Model Setup

Consider an economy with two periods $t = \{0, 1\}$ and two risk-neutral agents: one optimist and one pessimist. All agents trade in period $t = 0$ and consume in period $t = 1$. Uncertainty is captured by a continuum of states $s \in \mathcal{S} = [s^{min}, s^{max}]$ realized in period $t = 1$, with $s^{min}$ normalized to zero for simplicity. The pessimist, denoted by $i = 0$, holds prior beliefs over $\mathcal{S}$ given by the distribution $F_0$ and corresponding density $f_0$. The optimist, denoted by $i = 1$, has prior beliefs characterized by the distribution $F_1$ and density $f_1$. The optimist has a higher expectation than the pessimist on the state in period 1, i.e., $E_1[s] > E_0[s]$. These prior beliefs are common knowledge for all agents.

At the start of period $t = 0$, each agent $i = \{0, 1\}$ is endowed with $n_i$ units of the numeraire consumption good which can be safely stored without depreciation for consumption at period $t = 1$. We assume that the only other asset available is cash, which also yields one unit of the consumption good in period $t = 1$, but - unlike the the consumption good - cash can be used as collateral in CDS contracts. At $t = 0$, the entire endowment of cash (normalized to 1) is held by an un-modeled third party, who can sell cash in exchange for the numeraire consumption good at the equilibrium price $p$. The price of the consumption good is normalized to 1.

The optimist and pessimist have identical (linear) preferences over the consumption good, so trading between the two is driven purely by differences in beliefs. We assume that the only class

[22] In comparison, Simsek (2013) studies a different set of contract spaces that include: (1) simple debt contracts, (2) short selling; and (3) a completely unrestricted contract space whereby every Arrow-Debreu equilibrium is equivalent to a general equilibrium

of financial contracts available to trade is a simple CDS contract. Recall that the payoff of a CDS contract is zero if there is no default of the underlying (in our model, when $s$ high), and $1 - R$ in the case of default, where $R$ is the recovery rate of the underlying bond. Since the recovery $R$ worsens as the fundamentals of the underlying deteriorate, the payoff of the CDS becomes larger as the state $s$ becomes worse. We model the promised payoff of a CDS in a simple way, i.e., as $s^{max} - s$. We can think of the case $s = s^{max}$ as the event in which the underlying bond does not default, so that the CDS does not pay anything; $0 < s < s^{max}$ as the intermediate case in which the underlying bond defaults, but there is positive recovery, so that the the CDS pays off some amount; and $s = 0$ as the extreme case of zero recovery, where the payoff of the CDS is maximal (and equal to $s^{max}$).

To enforce payment of the promise of $s^{max} - s$, the seller of CDSs needs to post some amount of collateral. Following the endogenous collateral literature, we consider the *family* of CDS contracts $\mathcal{B}^{CDS} = \left\{[s^{max} - s]_{s\in\mathcal{S}}, \gamma\right\}$, each composed of a promise of $(s^{max} - s)$ units of the consumption good in state $s$ at $t = 1$, backed by $\gamma$ units of cash as collateral. We denote by $q(\gamma)$ the $t = 0$ price of such a CDS contract with collateral level $\gamma$. In general, multiple CDS contracts may coexist in equilibrium: they have the same promised payment $(s^{max} - s)$, but different amounts of collateral posted $\gamma$, and – as a consequence – trade for different prices $q(\gamma)$. Thus we can index the different CDS contracts in $\mathcal{B}^{CDS}$ by $\gamma$.

Since the promised payment on a CDS contract is enforceable only through the potential of seizing the collateral, the *actual delivery* on each contract in state $s$ is given by the minimum of the promised payment and the value of the collateral in that state: $\delta(s, \gamma) := \min\{s^{max} - s, \gamma\}$. In other words, in any state $\tilde{s}$ such that $s^{max} - \tilde{s} > \gamma$, the seller of the CDS contract would default on her promise, and the buyer would only receive the value of the collateral $\gamma$.

Denote by $\mu_i^+, \mu_i^-$, respectively, agent $i$'s long and short positions on CDS contracts (where a long position means that the agent has purchased the corresponding CDS contract). Let $a_i \in \mathbb{R}_+$ denote agent $i$'s holding of the numeraire consumption good; and $c_i \in \mathbb{R}^+$ be her cash holdings. Then agent $i$'s budget constraint can be written as:

$$a_i + pc_i + \int_{\gamma\in\mathcal{B}^{CDS}} q(\gamma)\, d\mu_i^+ \leq n_i + \int_{\gamma\in\mathcal{B}^{CDS}} q(\gamma)\, d\mu_i^-, \tag{A.1}$$

where the left hand side represents the total value of the agent's portfolio, comprised of her holding of the numeraire good $a_i$, the value of her cash holding $pc_i$ and her long-position in CDS contracts $\int_{\gamma\in\mathcal{B}^{CDS}} q(\gamma)\, d\mu_i^+$. The right hand side represents the total value of the funding available to the agent, comprising of her endowment of the consumption good $n_i$ and the amount she can raise by shorting CDS contracts, $\int_{\gamma\in\mathcal{B}^{CDS}} q(\gamma)\, d\mu_i^-$. If an agent $i$ has a short position on the CDS contracts, then she is also subject to the collateral constraint:

$$\int_{\gamma\in\mathcal{B}^{CDS}} \gamma d\mu_i^- \leq c_i, \tag{A.2}$$

which means that agent $i$ must have sufficient cash holdings to satisfy the collateral requirements for the CDS contracts sold. In contrast, the purchaser of the CDS contracts (the party who is long) is not subject to any collateral requirements.

The optimization problem for each agent $i$ is given by:

$$\max_{\left(a_i,c_i,\mu_i^+,\mu_i^-\right)\in\mathbb{R}_+^4} a_i + c_i + E_i\left[\int_\gamma \min\left\{s^{max}-s,\gamma\right\}d\mu_i^+\right] - E_i\left[\int_\gamma \min\left\{s^{max}-s,\gamma\right\}d\mu_i^-\right] \quad \text{(A.3)}$$

subject to the budget constraint (A.1) and the collateral constraint (A.2).

**Definition 1** A *collateral equilibrium* is a set of portfolio choices $\left(\hat{a}_i,\hat{c}_i,\hat{\mu}_i^+,\hat{\mu}_i^-\right)_{i\in\{0,1\}}$ and a set of prices $(p\in\mathbb{R}_+, q:\gamma\to\mathbb{R}_+)$ such that the portfolio choices solve the optimization problem (A.3) of each agent $i\in\{0,1\}$; and the prices are such that the market for cash clears $\sum_{i\in\{0,1\}}\hat{c}_i = 1$, and the CDS markets clear $\sum_{i\in\{0,1\}}\mu_i^+ = \sum_{i\in\{0,1\}}\mu_i^-$.

We will show that although the entire family of CDS contracts will be priced, only one contract will be traded in equilibrium. This result is line with the literature on collateral equilibrium (Fostel and Geanakoplos (2015), Simsek (2013)). Furthermore, the collateral level $\gamma$ of the actively traded CDS contract, and the price of cash $p$, will be determined endogenously. The equilibrium level of collateral requirement $\gamma$ will depend on the nature of belief differences between the optimist and the pessimist.

## A.2 Existence and Uniqueness of the Collateral Equilibrium

Following the approach in Simsek (2013), it is possible to show that (under suitable assumptions over initial endowments and beliefs) the collateral equilibrium exists, is unique, and is equivalent to a principal-agent equilibrium where the optimist chooses her cash holdings and the optimal CDS contract to sell, subject to the pessimist's participation constraint. To this end, we impose the following assumptions on initial endowments and prior beliefs, that parallel similar assumptions in Simsek (2013) (Assumption A1 of Simsek (2013), page 12):

**Assumption A1**: **[Restriction on Initial Endowments]**

$$n_1 < \frac{E_1[s]}{s^{max}}$$
$$\text{and } n_0 > \frac{E_0\left[s^{max}-s\right]}{E_1\left[s^{max}-s\right]} - n_1$$

The first inequality ensures that the optimist's initial endowment is not large enough to purchase the entire supply of cash in the economy with her own resources alone (that is, she will need to raise some more of the numeraire consumption good by selling CDS contracts).[23] The second inequality ensures that the initial endowment of the pessimist ($n_0$) is large enough that the pessimist will always have some residual consumption after paying for the CDS.[24]

Since the pessimist is risk neutral, this implies that her expected return on any CDS contracts purchased must also be equal to 1 in equilibrium. Thus the equilibrium price of a CDS contract with

[23] For a more detailed discussion of this, see Appendix B.1.

[24] More specifically, this inequality implies that the sum of the endowments needs to be greater than the maximum price cash can take in equilibrium (which we will show is bounded from above by $\left(\frac{E_0[s^{max}-s]}{E_1[s^{max}-s]}\right)$).

collateral $\gamma$ must be given by:

$$q(\gamma) = E_0 \left[ \min \left( s^{max} - s, \gamma \right) \right].$$

This pricing equation serves as a convenient characterization of the pessimist's participation constraint (as in Simsek (2013)).

We can now formulate the optimist's problem as choosing the level of cash holdings $c_1$, and the CDS contract $\gamma$ to sell, so as to maximize the expected payoffs, subject to the pessimist's participation constraint of achieving an expected return of 1 on the CDS contract sold:

$$\max_{(c_1,\gamma)\in\mathbb{R}^2_+} c_1 - \frac{c_1}{\gamma} E_1 \left[ \min \left\{ s^{max} - s, \gamma \right\} \right] \tag{A.4}$$
$$\text{s.t. } pc_1 = n_1 + \frac{c_1}{\gamma} E_0 \left[ \min \left\{ s^{max} - s, \gamma \right\} \right]$$

This leads us to define the *principal-agent equilibrium* as follows:

**Definition 2** A *principal-agent equilibrium* is a pair of optimist's portfolio choices $(c_1^*, \gamma^*)$ and price for cash $(p^*)$ such that the optimist's portfolio solves her optimization problem (A.4), and the market for cash clears: $c_1^* = 1$.

In order to show equivalence between the principal-agent equilibrium outlined here and the collateral equilibrium defined previously, we need to impose further restrictions on the nature of belief differences:

**Assumption A2: [Restrictions on Prior Beliefs]** The probability densities of the optimist's and the pessimist's beliefs satisfy the monotone likelihood ratio property:

$$\frac{f_1(s_1)}{f_0(s_1)} > \frac{f_1(s_0)}{f_0(s_0)} \quad \text{for every } s_1 > s_0$$

Note that this assumption implies: (1) first-order stochastic dominance: $F_1(s) < F_0(s), \forall s \in (s^{min}, s^{max})$; (2) monotone hazard rate: $\frac{f_1(s)}{1-F_1(s)} < \frac{f_0}{1-F_1(s)}, \forall s \in (s^{min}, s^{max})$; and (3) $\frac{f_0(s)}{F_0(s)} < \frac{f_1(s)}{F_1(s)}$ $\forall s \in (s^{min}, s^{max})$ (which in turn implies $\frac{d}{ds}\frac{F_0(s)}{F_1(s)} < 0$).

We can then prove the following Proposition:

**Proposition 1.** ***[Existence, Uniqueness, and Equivalence of Equilibria]*** *Under Assumptions A1 and A2:*

1. There exists a unique principal-agent equilibrium $(p^*, (c_1^*, \gamma^*))$ s.t. $p^* > 1$.
2. There exists a collateral general equilibrium, $\left(\hat{a}_i, \hat{c}_i, \hat{\mu}_i^+, \hat{\mu}_i^-\right)_{i\in\{0,1\}}$, whereby the optimist sells CDS to the pessimist (i.e. $\hat{\mu}_1^+ = \hat{\mu}_0^- = 0$), and only a single CDS contract is actively traded (i.e. $\hat{\mu}_0^+$ is a measure that puts weight only at one contract $\hat{\gamma} \in \mathcal{B}^{CDS}$). This collateral equilibrium is unique in the sense that the price of cash $\hat{p}$ and the price of the traded CDS contract $q(\hat{\gamma})$ are uniquely determined.

3. The collateral equilibrium and the principal-agent equilibrium are equivalent:

$$\hat{p} = p^*, \quad \hat{c}_1 = c_1^*, \quad \hat{\gamma} = \gamma^*$$
$$\text{and } q(\hat{\gamma}) = E_0\left[\min\left(s^{max} - s, \gamma^*\right)\right]$$

The detailed proof is reported in Appendix B. Intuitively, because under assumption A1 the pessimist will hold a surplus of the consumption good in equilibrium (he has a larger endowment than what the optimist would want to borrow), he must be indifferent between holding the consumption good (with a sure return of 1) and holding the CDS sold by the optimist. Hence, the optimist effectively holds all the bargaining power when deciding which CDS they should trade, and will only trade in the contract that maximizes the optimist's expected return. Assumption A2 provides the sufficient conditions for there to exist a unique contract $\gamma^*$ that solves the optimist's principal agent problem.

## A.3 Characterizing the equilibrium

This section shows that, in equilibrium, the optimist will wish to sell CDS contracts to the pessimist (so as to bet on the events she thinks are more likely). But, to do so, the optimist must first obtain more units of the numeraire good from the pessimist by selling CDS contracts, in order to purchase the cash required to collateralize the CDS contracts.[25] Because cash is the only asset that can be used as collateral, its equilibrium price will exceed its fundamental value ($p > 1$), so cash is held exclusively by the optimist in equilibrium.

To formally characterize the equilibrium, we first substitute $c_1 = \frac{n_1}{\left(p - \frac{1}{\gamma} E_0[\min\{s^{max} - s, \gamma\}]\right)}$ from the optimist's constraint into his objective function. This reduces the dimension of the problem by one, and allows us to restrict attention to choosing only the optimal contract $\gamma$. The resulting first order condition characterizes the optimal contract choice for given $p$:

**Proposition 2.** *Under assumptions A1 and A2, and fixing a price for cash p, the optimal CDS contract, $\bar{s}$, with respect to the optimist's problem (A.4) is given by the unique solution to:*

$$p = F_0\left(s^{max} - \bar{s}\right) + \left(1 - F_0\left(s^{max} - \bar{s}\right)\right) \frac{E_0\left[s^{max} - s | s \geq s^{max} - \bar{s}\right]}{E_1\left[s^{max} - s | s \geq s^{max} - \bar{s}\right]} =: p^{opt}\left(\bar{s}\right) \tag{A.5}$$

Inverting $p^{opt}(\bar{s})$ gives the optimal CDS contract $\bar{s}$ for the optimist, for given price of cash $p$. This first order condition effectively determines how much collateralization the optimist (seller) chooses for the CDS contract, since the states $s \leq s^{max} - \bar{s}$ are the ones where the optimist is defaulting on the promise of delivering the CDS payment and is instead relinquishing the collateral. Once the equilibrium price of cash $p^*$ is known (see Eq. (A.6) below), we can use Eq. (A.5) to derive the equilibrium level of collateral $\gamma^*$ by setting $p^* = p^{opt}(\gamma^*)$.

Equation (A.5) also implies that the price of cash is composed of two parts: (a) the pessimist's assessment of the probability of default on the CDS: $(F_0(s^{max} - \bar{s}))$, multiplied by the pessimist's

[25] This mechanism is analogous to a mortgage contract, where the borrower is raising funds from the lender in order to purchase the collateral (i.e. the house) required to back the mortgage.

valuation of cash in the default state (1); plus (b) the pessimist's assessment of the probability of the no-default state, multiplied by the value of cash to the optimist in the non-default state (each unit of cash allowed the optimist to borrow $\frac{1}{\bar{s}}E_0\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]$ from the pessimist, with an expected actual delivery of $\frac{1}{\bar{s}}E_1\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]$). Given the differences in beliefs, the optimist expects to deliver less than what the pessimist envisages on the CDS contract: $\frac{E_0[s^{max}-s|s\geq s^{max}-\bar{s}]}{E_1[s^{max}-s|s\geq s^{max}-\bar{s}]}\geq 1$. Hence, $p\geq 1$, i.e., cash generates collateral value. Consistent with the "Asymmetric Disciplining of Optimism" result in Simsek (2013), the pessimist's beliefs are used in assigning weights to the default and non-default states. The belief of the optimist enters only in determining the value of collateral in the non-default state.

Further, it is shown in the proof of Proposition 2 that $p^{opt}\left(\bar{s}\right)$ is an increasing function of $\bar{s}$. To see this intuitively, denote the optimist's perceived interest rate on (borrowing through selling) the CDS contract as

$$1+r_1^{per}\left(\bar{s}\right):=\frac{E_1\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]}{E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]}$$

Under assumption A2, it can be shown that the perceived interest is decreasing in $\bar{s}$ ($\frac{d\left(1+r_1^{per}(\bar{s})\right)}{d\bar{s}}<0$). In other words, given the nature of belief differences, offering a CDS contract with higher margin requirements is more attractive for the optimist. Thus the higher the margins $\bar{s}$ posted, the greater the discrepancy between expected deliveries becomes, and the more attractive selling the CDS is to the optimist. Thus, increasing $\bar{s}$ increases the collateral value of cash.

To close the model, one needs to impose the market clearing condition for cash. The budget constraint in equation (A.4) implies that the optimist's demand for cash is given by:

$$c_1=\frac{n_1}{p-\frac{1}{\bar{s}}E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]}$$

Since the supply of cash is normalized to 1, it holds:

$$p=n_1+\frac{1}{\bar{s}}E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]=:p^{mc}\left(\bar{s}\right) \tag{A.6}$$

It can be shown that $p^{mc}\left(\bar{s}\right)$ is a decreasing function of $\bar{s}$,[26] with boundary conditions $p^{mc}\left(s^{min}\right)>p^{opt}\left(s^{min}\right)$ and $p^{mc}\left(s^{max}\right)<p^{opt}\left(s^{max}\right)$. Hence, the unique intersection between $p^{opt}\left(\bar{s}\right)$ and $p^{mc}\left(\bar{s}\right)$ pins down the equilibrium price for cash $p^*$ and margin requirement $\gamma^*$.

**Proposition 3.** *Under assumptions A1 and A2, there is a unique principal-agent equilibrium* $(c_1^*,\gamma^*,p^*)$

[26] $p^{mc}\left(\bar{s}\right)$ is downward sloping because $\frac{1}{\bar{s}}E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]$ is decreasing in $\bar{s}$. Intuitively, even though a CDS contract with higher margin requirements demands a higher price (i.e. $E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]$ is increasing in $\bar{s}$), the need to post more margins reduces the number of such contracts the optimist can sell with a single unit of cash as collateral. The overall effect is that posting more margins reduces the total amount that the optimist can borrow, $\frac{1}{\bar{s}}E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]$, and this reduction in the purchasing power of optimists reduces the market clearing price of cash.

*characterized by:*

$$c_1^* = 1$$
$$p^* = p^{mc}(\gamma^*) = p^{opt}(\gamma^*)$$

*where $p^{opt}(\cdot)$ and $p^{mc}(\cdot)$ are respectively defined in equations (A.5) and (A.6).*

*Proof.* See Appendix B. □

### A.4 Comparative statics and illustrative example

This section presents a simple example to illustrate the collateral equilibrium, and performs comparative statics before presenting the more general results as propositions.

Suppose that the starting endowments of the numeraire consumption good are $n_0 = 3$ and $n_1 = 0.5$ for the pessimist and the optimist respectively. Let the set of states and the prior beliefs be given by: $\mathcal{S} = [0, 1]$ and

$$F_0(s) = s^{\frac{1}{2}} \quad \forall s \in \mathcal{S}$$
$$F_1(s) = s^3 \quad \forall s \in \mathcal{S}$$

Panel A of Figure A.1 plots the upward sloping cash price schedule $\{p^{opt}(\bar{s})\}_{\bar{s}\in[0,s^{max}]}$ (equation (A.5)), and the downward sloping cash price schedule $\{p^{mc}(\bar{s})\}_{\bar{s}\in[0,s^{max}]}$ schedule (equation (A.6)), the intersection of which characterizes the equilibrium price of cash $p^*$ and the equilibrium margin requirement $\gamma^*$.

Suppose instead that the optimist becomes less optimistic, such that:

$$\tilde{F}_1(s) = s^{\frac{3}{2}} \quad \forall s \in \mathcal{S}$$

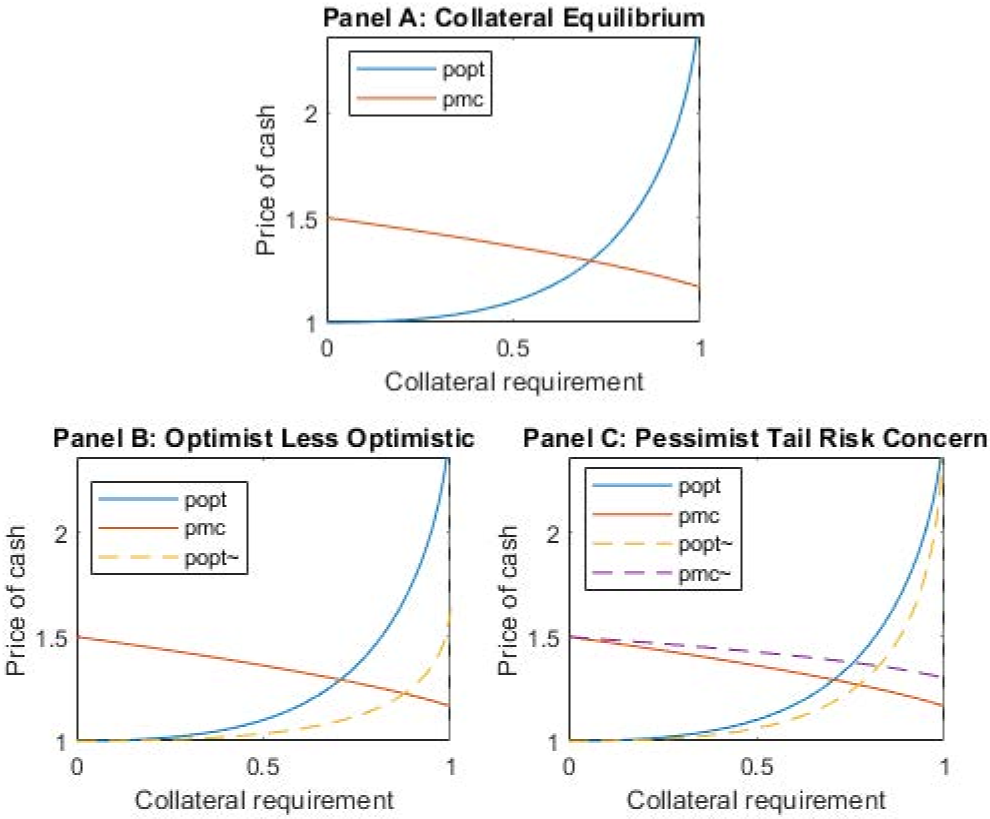
Then one can see from equation (A.5) that the decreased optimism of the optimist shifts the $p^{opt}(\bar{s})$ schedule downwards and leaves the $p^{mc}(\bar{s})$ schedule untouched. The resulting equilibrium, as illustrated in panel B of Figure A.1, is comprised of a lower price for cash $\tilde{p}^*$ and higher margin requirements $\tilde{\gamma}^*$.

This intuition is generalized in the following general proposition:

**Proposition 4.** *[**Change in optimist's beliefs**] Suppose that the optimist's beliefs becomes 'less optimistic' in the sense that it changes from $F_1$ to $\tilde{F}_1$, s.t. the monotone likelihood ratio condition is satisfied: $\frac{f_1(s_1)}{\tilde{f}_1(s_1)} > \frac{f_1(s_0)}{\tilde{f}_1(s_0)}$ for every $s_0, s_1 \in [s^{min}, s^{max}]$ and $s_1 > s_0$ . Then the equilibrium level of collateral $(\gamma^*)$ increases, and the equilibrium collateral value $(p^*)$ falls. Conversely, when the optimist's beliefs becomes 'more optimistic' s.t. $\frac{f_1(s_1)}{\tilde{f}_1(s_1)} < \frac{f_1(s_0)}{\tilde{f}_1(s_0)}$ for every $s_0, s_1 \in [s^{min}, s^{max}]$ and $s_1 > s_0$ , then the equilibrium level of collateral falls and the equilibrium collateral value rises.*

*Proof.* See Appendix B. □

Figure A.1: Collateral Equilibrium and Comparative Statics




**Note:** Panel A illustrates the upward-sloping $p^{opt}(\bar{s})$ schedule (eqn A.5, blue line) and the downward-sloping $p^{mc}(\bar{s})$ schedule (eqn A.6, red line), the intersection of which gives the collateral equilibrium $(\gamma^*, p^*)$. The $p^{opt}(\bar{s})$ schedule is increasing in the level of collateralization $(\bar{s})$ because the pessimist is willing to pay a higher price for a better collateralized CDS, which in turn increases the collateral value of cash $(p)$ for the optimist. The $p^{mc}(\bar{s})$ schedule is decreasing in the level of collateralization $(\bar{s})$ because even though each unit of a more collateralized CDS contract is worth more, the scarcity of collateral implies that a fewer number of such contracts can be written and the total amount of funding the optimist can raise from the pessimist is lower. Therefore, in order for the market of collateral (i.e., cash) to clear, its equilibrium price $(p)$ must fall as the level of collateralization increases.

Panel B illustrates the case where the optimist becomes "less optimistic", in the sense that her beliefs changes from $F_1(s)$ to $\tilde{F}_1(s)$ where $\frac{f_1(s_1)}{\tilde{f}_1(s_1)} > \frac{f_1(s_0)}{\tilde{f}_1(s_0)}$ for every $s_0, s_1 \in \left[s^{min}, s^{max}\right]$ and $s_1 > s_0$ . When the optimist becomes more pessimistic, the $p^{opt}(\bar{s})$ schedule shifts down to $\tilde{p}^{opt}(\bar{s})$ (yellow dashed line). Intuitively, this is because trading in CDSs becomes less attractive for both as the optimist and pessimist's beliefs converge, thus the optimist's demand for cash (the required collateral) falls (weakly) for every level of collateralization. The resulting equilibrium is one where the price of cash (its collateral value) is lower and the level of collateralization is higher.

Panel C illustrates the case where the pessimist becomes more concerned about the tail risks associated with the default states $\{s \in S \ : \ s < s^{max} - \gamma^*\}$. Specifically, the pessimist's beliefs changes from $F_0(s)$ to $\tilde{F}_0(s)$, where $f_0(s) > \tilde{f}_0(s) \quad \forall s \in [s^{max} - \gamma^*, s^{max}]$ . A greater concern for tail events increases the value of highly collateralized CDS contracts, and decreases the value of less collateralized contracts, so the collateral value of cash for the optimist is higher only when it is used to back highly collateralized CDS contracts (the $p^{opt}$ schedule pivots to $\tilde{p}^{opt}$, yellow upward-sloping dashed line). The total amount of the numeraire good the optimist can raise through selling such CDS contracts also rises, pushing up the market clearing price for cash (the $p^{mc}$ schedule shifts up to $\tilde{p}^{mc}$, purple downward-sloping dashed line). The resulting equilibrium is one where the equilibrium level of collateralization $(\gamma^*)$ is higher (see Proposition 5). Note that under the specific beliefs outlined in the illustrative example section of the main text, the equilibrium level of collateral in Panel C is 0.8251, and the optimist believes default will occur with probability 0.0053.

While the changes in the optimist's beliefs lead to unambiguous changes in the equilibrium, the same is not true for the pessimist's beliefs. Since the pessimist's beliefs feature in the cash price schedule of both the optimist (i.e, $p^{opt}$) and the pessimist (i.e., $p^{mc}$), a given change in the pessimist's beliefs can either increase or decrease equilibrium collateral levels depending on the initial equilibrium. Intuitively, when the pessimist becomes more pessimistic, she is willing to pay more for any given CDS contracts (i.e. $E_0 [\min \{s^{max} - s, \bar{s}\}]$ increases). This means that the optimist is now both: a) able to raise more funding from selling CDS contracts; and b) willing to pay more for each unit of collateral. The former effect shifts up the market clearing schedule $p^{mc}$, and the latter effect can push up the $p^{opt}$ schedule. Therefore, whether the equilibrium collateral level increases depends on the interaction of these two effects. Moreover, since the $p^{opt}$ schedule also depend on the pessimist's evaluation of the default probability (i.e. $F_0 (s^{max} - \bar{s})$)[27], the effect of increased pessimism is not always monotone along $\mathcal{S} = [0, 1]$. For low levels of collateral $\bar{s}$, the protection offered may be deemed insufficient, so the new $p^{opt}$ schedule may be lower than before; but for high levels of collateral, the new $p^{opt}$ schedule might be higher. In short, the $p^{opt}$ schedule may pivot as well as shift in response to a change in pessimist's beliefs, creating further ambiguity in the direction of the equilibrium collateral level $\gamma^*$.

Instead, it can be shown that a sufficient (but not necessary) condition for the equilibrium collateral level to increase is for the pessimist to become more concerned about the tail risks (i.e. to place a larger weight on the default states $\{s \in S \ : \ \gamma < s^{max} - s\}$ ) such that the probability density she attaches to all the non-default states are consistently lower than before $\tilde{f}_0 (s) < f_0 (s) \quad \forall s \in [s^{max} - \gamma^*, s^{max}]$.

For illustration, consider a change in pessimist's beliefs from $F_0$ (eqn. A.4) to $\tilde{F}_0$:

$$\tilde{F}_0 (s) = s^{\frac{1}{4}} \quad \forall s \in \mathcal{S}$$

Then the equilibrium collateral level and collateral value both increase (Panel C of Figure A.1). Note that under the specific beliefs outlined here, the equilibrium level of collateral in Panel C is 0.8251, and the optimist believes default will occur with probability 0.0053. This is consistent with our empirical finding that extreme tail risk measures are important in explaining the high collateral levels and low default frequencies observed in practice. This result is stated more formally in the following proposition.

**Proposition 5.** *[**Pessimist's concern for tail events**]: For any given initial equilibrium $(\gamma^*, p^*)$, if the pessimist becomes "more concerned about tail events" in the sense that her beliefs changes from $F_0$ to $\tilde{F}_0$, such that: $\tilde{f}_0 (s) < f_0 (s) \quad \forall s \in [s^{max} - \gamma^*, s^{max}]$, then the equilibrium collateral level $(\gamma^*)$ increases.*

*Proof.* See Appendix B. □

Finally, note that the key to high collateral requirements in the model is the nature – rather than the degree – of the belief difference between the two agents. The equilibrium level of collateral increases

[27] Recall from equation (A.5) the $p^{opt}$ is the weighted average of 1 and $\frac{E_0[s^{max}-s|s \geq s^{max}-\bar{s}]}{E_1[s^{max}-s|s \geq s^{max}-\bar{s}]} > 1$, with weights $F_0 (s^{max} - \bar{s})$ and $(1 - F_0 (s^{max} - \bar{s}))$ respectively.

both when the optimist becomes more pessimistic (which reduces the extent of the disagreement), and when the pessimist becomes more concerned about the tail events (which increases the extent of the disagreement). What the market participant disagree about matters more than how much they disagree per se.

### A.5 Summary of the theoretical results

In the previous sections, it has been shown that under suitable assumptions, a unique collateral equilibrium exists for the trading of CDS contracts between optimists and pessimists. In this equilibrium, the pessimist buys CDS contracts from the optimist. Since selling a CDS requires collateralizable cash, the optimist buys this cash on the market to use it as collateral. In equilibrium, the price of cash will be higher than the non-collateralizable value of the numeraire consumption good, reflecting its value as collateral.

In the equilibrium, only one CDS contract with a particular level of collateral $\gamma^*$ will be actively traded. Default on the CDS obligations will, in general, happen with positive probability. The level of collateral $\gamma^*$, the price of the CDS contract $q(\gamma^*)$ and the price of cash $p^*$ all depend on the belief disagreement between the optimist and the pessimist. In particular, the higher the disagreement about the probability of states in which *counterparty defaults* may occur (i.e. the left tail of the distribution), the higher the margin requirements $\gamma^*$ in equilibrium, and the higher the CDS price $q(\gamma^*)$ the buyer is willing to pay.

The model can therefore explain the particularly high collateral levels ($\gamma^*$) that are observed in the data described in this paper. Viewed through the lens of this endogenous collateral framework, the empirical results in this paper point to disagreement about the states of the world in which CDS sellers would default on their obligations to the clearinghouse as the fundamental reason why collateral levels are set so high.

## B Proofs, Comparative Statics, and Discussion of Assumptions of Theoretical Model

### B.1 Additional discussion of the assumptions

This section discusses in greater detail some of the assumptions in the model. The first inequality (A.2) states that the optimist's initial endowment is not large enough to purchase the entire supply of cash by issuing only fully collateralized CDS contracts.

To see this, note that the value of one unit of cash to the optimist when selling contract $\gamma$ is given by:

$$1 + \frac{1}{\gamma}\left[E_0\left[\min\left\{s^{max} - s, \gamma\right\}\right] - E_1\left[\min\left\{s^{max} - s, \gamma\right\}\right]\right]$$

The total amount of funding the optimist can raise by selling riskless CDS contracts $\gamma = s^{max}$ (the contract with the maximum collateral) using 1 unit of collateral (i.e. cash) is:

$$\frac{1}{s^{max}} E_0 \left[\min \left\{ s^{max} - s, s^{max} \right\}\right]$$

It is assumed that the optimist's endowment is not large enough to finance the purchase of cash using riskless CDS contracts:

$$\begin{aligned} n_1 + \frac{1}{s^{max}} E_0 \left[\min \left\{ s^{max} - s, s^{max} \right\}\right] < 1 + \frac{1}{s^{max}} \left[ E_0 \left[\min \left\{ s^{max} - s, s^{max} \right\}\right] - E_1 \left[\min \left\{ s^{max} - s, s^{max} \right\}\right] \right] \\ n_1 < 1 - \frac{1}{s^{max}} E_1 \left[\min \left\{ s^{max} - s, s^{max} \right\}\right] \\ = 1 - \frac{1}{s^{max}} \left[ s^{max} - E_1 [s] \right] = \frac{E_1 [s]}{s^{max}} \end{aligned}$$

Since $n_1 < \frac{E_1[s]}{s^{max}} \leq 1$, this in turn ensures that the optimist cannot simply purchase the entire stock of cash (normalized to 1) without borrowing from the pessimist.

## B.2 Proof of Proposition 2

Returning to the optimist's problem under the principal-agent formulation (A.4), one can substitute the pessimist's participation constraint into the objective function to reduce dimension of the choice variable to one:

$$\begin{aligned} &\max_{\gamma \in \mathbb{R}_+} \frac{n_1}{\left( p - \frac{1}{\gamma} E_0 \left[\min \left\{ s^{max} - s, \gamma \right\}\right] \right)} \left[ 1 - \frac{1}{\gamma} E_1 \left[\min \left\{ s^{max} - s, \gamma \right\}\right] \right] \\ \Leftrightarrow &\max_{\gamma \in \mathbb{R}_+} n_1 R_1^{CDS} (\gamma) \end{aligned}$$

where $R_1^{CDS}(\gamma)$ is defined as the expected return to the optimist who buys one unit of cash and uses it to back the sale of the CDS contract:

$$R_1^{CDS} (\gamma) := \frac{1 - \frac{1}{\gamma} E_1 \left[\min \left\{ s^{max} - s, \gamma \right\}\right]}{p - \frac{1}{\gamma} E_0 \left[\min \left\{ s^{max} - s, \gamma \right\}\right]} = \frac{\gamma - E_1 \left[\min \left\{ s^{max} - s, \gamma \right\}\right]}{p\gamma - E_0 \left[\min \left\{ s^{max} - s, \gamma \right\}\right]}$$

(the numerator is the expected $t = 1$ payoff from purchasing the cash whilst simultaneously selling the CDS; the denominator is the down payment required to purchase the cash).

Note that since $E_i \left[\min \left\{ s^{max} - s, \gamma \right\}\right] \equiv \gamma F_i \left( s^{max} - \gamma \right) + \int_{s^{max}-\gamma}^{s^{max}} \left( s^{max} - s \right) dF_i (s)$, it holds

$$\begin{aligned} \frac{dE_i \left[\min \left\{ s^{max} - s, \gamma \right\}\right]}{d\gamma} &= F_i \left( s^{max} - \gamma \right) - \gamma f_i \left( s^{max} - \gamma \right) - (-1) \left( s^{max} - \left( s^{max} - \gamma \right) \right) f_i \left( s^{max} - \gamma \right) \\ &= F_i \left( s^{max} - \gamma \right) \end{aligned}$$

Therefore the derivative of $R_1^{CDS}(\gamma)$ with respect to $\lambda$ can be expressed as:

$$\begin{aligned}
\frac{dR_1^{CDS}(\gamma)}{d\gamma} &= \frac{(1-F_1(s^{max}-\gamma))(p\gamma - E_0[\min\{s^{max}-s,\gamma\}])}{(p\gamma - E_0[\min\{s^{max}-s,\gamma\}])^2} \\
&\quad - \frac{(p-F_0(s^{max}-\gamma))(\gamma - E_1[\min\{s^{max}-s,\gamma\}])}{(p\gamma - E_0[\min\{s^{max}-s,\gamma\}])^2} \\
&= \frac{1}{(p\gamma - E_0[\min\{s^{max}-s,\gamma\}])}\left[(1-F_1(s^{max}-\gamma)) - (p-F_0(s^{max}-\gamma))R_1^{CDS}(\gamma)\right]
\end{aligned}$$

The first order condition for the optimist's problem simplifies to:

$$(1-F_1(s^{max}-\bar{s})) = (p-F_0(s^{max}-\bar{s}))\frac{\bar{s}-E_1[\min\{s^{max}-s,\bar{s}\}]}{p\bar{s}-E_0[\min\{s^{max}-s,\bar{s}\}]}$$

Re-arranging and simplifying notation (let $F_i \coloneqq F_i(s^{max}-\bar{s})$) gives:

$$\begin{aligned}
p &= \frac{(1-F_1)E_0[\min\{s^{max}-s,\bar{s}\}] + F_0E_1[\min\{s^{max}-s,\bar{s}\}] - \bar{s}F_0}{E_1[\min\{s^{max}-s,\bar{s}\}] - \bar{s}F_1} \\
&= \frac{(1-F_1)\left[\bar{s}F_0 + \int_{s^{max}-\bar{s}}^{s^{max}}(s^{max}-s)f_0(s)\,ds\right] + F_0\left[\bar{s}F_1 + \int_{s^{max}-\bar{s}}^{s^{max}}(s^{max}-s)f_1(s)\,ds\right] - \bar{s}F_0}{\left[\bar{s}F_1 + \int_{s^{max}-\bar{s}}^{s^{max}}(s^{max}-s)f_1(s)\,ds\right] - \bar{s}F_1} \\
&= F_0 + \frac{(1-F_1)\int_{s^{max}-\bar{s}}^{s^{max}}(s^{max}-s)f_0(s)\,ds}{\int_{s^{max}-\bar{s}}^{s^{max}}(s^{max}-s)f_1(s)\,ds} \\
&= F_0 + \frac{(1-F_1)(1-F_0)E_0[s^{max}-s|s\geq s^{max}-\bar{s}]}{(1-F_1)E_1[s^{max}-s|s\geq s^{max}-\bar{s}]} \\
&= F_0(s^{max}-\bar{s}) + (1-F_0(s^{max}-\bar{s}))\frac{E_0[s^{max}-s|s\geq s^{max}-\bar{s}]}{E_1[s^{max}-s|s\geq s^{max}-\bar{s}]} \\
&=: p^{opt}(\bar{s})
\end{aligned}$$

as required.

**Lemma 1.** *The price of cash (i.e. the collateral) is strictly increasing in the optimist's desired level of collateralization:* $\frac{dp^{opt}(\bar{s})}{d\bar{s}} > 0$ *for* $\bar{s} \in (s^{min}, s^{max})$.

*Proof.* Using the fact that

$$
\begin{aligned}
&\frac{d}{d\bar{s}} E_i \left[s^{max} - s | s \geq s^{max} - \bar{s}\right] \\
=&\frac{d}{d\bar{s}} \left( \frac{1}{1 - F_i \left(s^{max} - \bar{s}\right)} \int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) f_1 \left(s\right) ds \right) \\
=&\frac{\left(\bar{s} f_i \left(s^{max} - \bar{s}\right)\right) \left(1 - F_i \left(s^{max} - \bar{s}\right)\right) - f_i \left(s^{max} - \bar{s}\right) \int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) f_1 \left(s\right) ds}{\left(1 - F_i \left(s^{max} - \bar{s}\right)\right)^2} \\
=&\frac{\bar{s} f_i \left(s^{max} - \bar{s}\right) - f_i \left(s^{max} - \bar{s}\right) E_i \left[s^{max} - s | s \geq s^{max} - \bar{s}\right]}{1 - F_i \left(s^{max} - \bar{s}\right)} \\
=&\frac{f_i \left(s^{max} - \bar{s}\right)}{1 - F_i \left(s^{max} - \bar{s}\right)} \left(\bar{s} - E_i \left[s^{max} - s | s \geq s^{max} - \bar{s}\right]\right)
\end{aligned}
$$

Differentiating $p^{opt}\left(\bar{s}\right)$ (and using short-hands: $f_i := f_i\left(s^{max} - \bar{s}\right)$, $F_i := F_i\left(s^{max} - \bar{s}\right)$, and $E_i := E_i\left[s^{max} - s | s \geq s^{max} - \bar{s}\right]$) yields:

$$
\begin{aligned}
\frac{dp^{opt}\left(\bar{s}\right)}{d\bar{s}} &= -f_0 + \frac{\left[f_0 E_0 + \left(1 - F_0\right) \left(\frac{f_0}{1-F_0} \left(\bar{s} - E_0\right)\right)\right] \left[E_1\right] - \left[\frac{f_1}{1-F_1} \left(\bar{s} - E_1\right)\right] \left[\left(1 - F_0\right) E_0\right]}{\left(E_1\right)^2} \\
&= -f_0 + \frac{\bar{s} f_0}{E_1} - \bar{s} f_1 \frac{1 - F_0}{1 - F_1} \frac{E_0}{\left(E_1\right)^2} + f_1 \frac{1 - F_0}{1 - F_1} \frac{E_0}{E_1} \\
&= f_0 \left(\frac{\bar{s}}{E_1} - 1\right) + f_1 \frac{1 - F_0}{1 - F_1} \frac{E_0}{E_1} \left(1 - \frac{\bar{s}}{E_1}\right) \\
&= \underbrace{\left(\frac{\bar{s}}{E_1} - 1\right)}_{>0} \underbrace{\left(f_0 - f_1 \frac{\int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) f_0 \left(s\right) ds}{\int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) f_1 \left(s\right) ds}\right)}_{>0}
\end{aligned}
$$

where the first inequality follows from $\bar{s} > E_1\left[s^{max} - s | s \geq s^{max} - \bar{s}\right] \quad \forall \bar{s} \in \left(s^{min}, s^{max}\right)$, and the second inequality holds due to assumption A2:

$$
\begin{aligned}
&f_0 \left(s^{max} - \bar{s}\right) \int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) f_1 \left(s\right) ds - f_1 \int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) f_0 \left(s\right) ds \\
=&\int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) \left[f_0 \left(s^{max} - \bar{s}\right) f_1 \left(s\right) - f_1 \left(s^{max} - \bar{s}\right) f_0 \left(s\right)\right] ds \\
>&0 \quad \text{since by A2: } \frac{f_1\left(s\right)}{f_0\left(s\right)} > \frac{f_1\left(s^{max} - \bar{s}\right)}{f_0\left(s^{max} - \bar{s}\right)} \quad \forall s > \left(s^{max} - s\right)
\end{aligned}
$$

**Lemma 2.** *The optimist's perceived interest rate on the CDS is strictly decreasing in the level of collateral* $\bar{s}$*:* $\frac{d\left(1+r_1^{per}\left(\bar{s}\right)\right)}{d\bar{s}} < 0$ *for* $\bar{s} \in \left(s^{min}, s^{max}\right)$*.*

Recall from equation (A.3) that the perceived interest rate is defined as $1+r_1^{per}\left(\bar{s}\right) := \frac{E_1\left[\min\{s^{max}-s,\bar{s}\}\right]}{E_0\left[\min\{s^{max}-s,\bar{s}\}\right]}$.

Differentiating with respect to $\bar{s}$ (using the result $\frac{dE_i[\min\{s^{max}-s,\bar{s}\}]}{d\bar{s}} = F_i\left(s^{max}-\bar{s}\right)$) yields:

$$\frac{d\left(1+r_1^{per}\left(\bar{s}\right)\right)}{d\bar{s}} = \frac{F_1\left(s^{max}-\bar{s}\right)E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right] - F_0\left(s^{max}-\bar{s}\right)E_1\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]}{\left(E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]\right)^2}$$

Because the denominator in the above expression is always positive, to show that $\frac{d\left(1+r_1^{per}(\bar{s})\right)}{d\bar{s}} < 0$ it is enough to show that for $\bar{s} \in \left(s^{min}, s^{max}\right)$, it holds that $\frac{E_1[\min\{s^{max}-s,\bar{s}\}]}{E_0[\min\{s^{max}-s,\bar{s}\}]} > \frac{F_1(s^{max}-\bar{s})}{F_0(s^{max}-\bar{s})}$. Next, one proceed as follows:

$$\begin{aligned}
\frac{E_1\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]}{E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]} &= \frac{\bar{s}F_1\left(s^{max}-\bar{s}\right) + \int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_1\left(s\right)ds}{\bar{s}F_0\left(s^{max}-\bar{s}\right) + \int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_0\left(s\right)ds} \\
&> \frac{\frac{F_1(s^{max}-\bar{s})}{F_0(s^{max}-\bar{s})}\bar{s}F_0\left(s^{max}-\bar{s}\right) + \int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_0\left(s\right)\frac{F_1(s)}{F_0(s)}ds}{\bar{s}F_0\left(s^{max}-\bar{s}\right) + \int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)dF_0\left(s\right)} \\
&> \frac{\frac{F_1(s^{max}-\bar{s})}{F_0(s^{max}-\bar{s})}\bar{s}F_0\left(s^{max}-\bar{s}\right) + \frac{F_1(s^{max}-\bar{s})}{F_0(s^{max}-\bar{s})}\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_0\left(s\right)ds}{\bar{s}F_0\left(s^{max}-\bar{s}\right) + \int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)dF_0\left(s\right)} \\
&= \frac{F_1\left(s^{max}-\bar{s}\right)}{F_0\left(s^{max}-\bar{s}\right)}
\end{aligned}$$

where the first inequality follows from Assumption A2 ($f_0\left(s\right)\frac{F_1(s)}{F_0(s)} < f_1\left(s\right)$), and the second inequality follows from Assumption A2 ($\frac{d}{ds}\frac{F_0(s)}{F_1(s)} < 0$) . □

### B.3 Proof of Proposition 3

In Appendix A, it has been argued that the principal-agent equilibrium is given by the intersection of the optimality condition $p^{opt}\left(\bar{s}\right) = F_0\left(s^{max}-\bar{s}\right) + \left(1-F_0\left(s^{max}-\bar{s}\right)\right)\frac{E_0[s^{max}-s|s>s^{max}-\bar{s}]}{E_1[s^{max}-s|s>s^{max}-\bar{s}]}$ derived from the optimist's optimization problem, and the market clearing condition for cash: $p^{mc}\left(\bar{s}\right) = n_1 + \frac{1}{\bar{s}}E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]$. From the proof to Proposition 2, it has also been established that $p^{opt}\left(\bar{s}\right)$ is strictly increasing in $\bar{s}$ over $\bar{s} \in \left(s^{min}, s^{max}\right)$. It remains to show that (i) $p^{mc}\left(\bar{s}\right)$ is strictly decreasing in $\bar{s}$ over $\bar{s} \in \left(s^{min}, s^{max}\right)$; and (ii) the boundary conditions are such that an intersection exists: $p^{mc}\left(s^{min}\right) > p^{opt}\left(s^{max}\right)$ and $p^{mc}\left(s^{max}\right) < p^{opt}\left(s^{max}\right)$.

1. Show that $p^{mc}\left(\bar{s}\right)$ is strictly decreasing in $\bar{s}$ over $\bar{s} \in \left(s^{min}, s^{max}\right)$ .

   Given $p^{mc}\left(\bar{s}\right) = n_1 + \frac{1}{\bar{s}}E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]$, differentiating with respect to $\bar{s}$ yields:

$$\begin{aligned}
\frac{dp^{mc}\left(\bar{s}\right)}{d\bar{s}} &= \frac{F_0\left(s^{max}-\bar{s}\right)\bar{s} - E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]}{\bar{s}^2} \\
&= \frac{\bar{s}F_0\left(s^{max}-\bar{s}\right) - \left(\bar{s}F_0\left(s^{max}-\bar{s}\right) + \int_{s^{max}-\bar{s}}^{s^{max}} s dF_0\right)}{\bar{s}^2} \\
&= -\frac{\int_{s^{max}-\bar{s}}^{s^{max}} s dF_0}{\bar{s}^2} < 0 \quad \forall \bar{s} \in \left(s^{min}, s^{max}\right)
\end{aligned}$$

2. Consider the boundary conditions for $p^{mc}\left(\bar{s}\right)$ and $p^{opt}\left(\bar{s}\right)$:

(a) For $\bar{s} = s^{max}$, it holds:

$$\begin{aligned}
p^{mc}\left(s^{max}\right) &= n_1 + \left(\frac{s^{max} - E_0\left[s\right]}{s^{max}}\right) \\
&< \frac{E_1\left[s\right]}{s^{max}} + \left(\frac{s^{max} - E_0\left[s\right]}{s^{max}}\right) \quad \text{by assumption A1} \\
&= 1 + \frac{E_0\left[s^{max} - s\right] - E_1\left[s^{max} - s\right]}{s^{max}} \\
&< 1 + \frac{E_0\left[s^{max} - s\right] - E_1\left[s^{max} - s\right]}{E_1\left[s^{max} - s\right]} \\
&= \frac{E_0\left[s^{max} - s\right]}{E_1\left[s^{max} - s\right]} = p^{opt}\left(s^{max}\right).
\end{aligned}$$

(b) For $\bar{s} = s^{min} \equiv 0$, using L'Hospital's rule, one gets:

$$\begin{aligned}
\lim_{\bar{s}\downarrow 0} p^{mc}\left(\bar{s}\right) &= n_1 + \lim_{\bar{s}\downarrow 0} \frac{\frac{d}{d\bar{s}}\left(E_0\left[\min\left\{s^{max} - s, \bar{s}\right\}\right]\right)}{\frac{d}{d\bar{s}}\left(\bar{s}\right)} \\
&= n_1 + \lim_{\bar{s}\downarrow 0} \frac{F_0\left(s^{max} - \bar{s}\right)}{1} = n_1 + 1
\end{aligned}$$

and

$$\begin{aligned}
\lim_{\bar{s}\downarrow 0} p^{opt}\left(\bar{s}\right) &= \lim_{\bar{s}\downarrow 0} \left[F_0\left(s^{max} - \bar{s}\right) + \left(1 - F_0\left(s^{max} - \bar{s}\right)\right) \frac{E_0\left[s^{max} - s | s \geq s^{max} - \bar{s}\right]}{E_1\left[s^{max} - s | s \geq s^{max} - \bar{s}\right]}\right] \\
&= \lim_{\bar{s}\downarrow 0} F_0\left(s^{max} - \bar{s}\right) + \lim_{\bar{s}\downarrow 0} \frac{\int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) f_0\left(s\right) ds}{\frac{1}{1-F_1\left(s^{max}-\bar{s}\right)} \int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) f_1\left(s\right) ds} \\
&= 1 + \lim_{\bar{s}\downarrow 0}\left(1 - F_1\left(s^{max} - \bar{s}\right)\right) \lim_{\bar{s}\downarrow 0} \frac{\frac{d}{d\bar{s}} \int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) f_0\left(s\right) ds}{\frac{d}{d\bar{s}} \int_{s^{max}-\bar{s}}^{s^{max}} \left(s^{max} - s\right) f_1\left(s\right) ds} \\
&= 1 + 0 \times \lim_{\bar{s}\downarrow 0} \frac{\bar{s} f_0\left(s^{max} - \bar{s}\right)}{\bar{s} f_1\left(s^{max} - \bar{s}\right)} = 1 + 0 \times \frac{f_0\left(s^{max}\right)}{f_1\left(s^{max}\right)} \\
&= 1
\end{aligned}$$

so

$$p^{mc}\left(s^{min}\right) = n_1 + 1 > 1 = p^{opt}\left(s^{min}\right)$$

3. Because $p^{opt}\left(\gamma\right)$ and $p^{mc}\left(\gamma\right)$ are both continuous functions, by the intermediate value theorem they intersect at some interior point $\gamma^* \in \left(s^{min}, s^{max}\right)$ and $p^* \in [1, \frac{E_0\left[s^{max}-s\right]}{E_1\left[s^{max}-s\right]})$. Since $p^{mc}\left(\bar{s}\right)$ is strictly decreasing, and $p^{opt}\left(\bar{s}\right)$ is strictly increasing, the intersection is unique.

### B.4 Proof of Proposition 1

The existence of a unique Principal-Agent equilibrium has been shown in the proof of propositions 2 and 3. In this section, it is shown that a collateral general equilibrium as defined in the main body exists and is equivalent to the principal-agent equilibrium.

1. Step 1: Simplifying observations for solving the collateral general equilibrium:

   (a) Without loss of generality, one can show that the equilibrium price of cash satisfies $\hat{p} \in \left[1, 1 + \frac{[E_1[s] - E_0[s]]}{s^{max}}\right)$. Since cash guarantees a safe return of 1, its equilibrium price will never fall below 1. The optimist attach a higher value to cash, because by using cash as collateral in selling CDS contracts $\gamma$, an optimist also gains the difference in the expected delivery $\frac{1}{\gamma}\left[E_0\left[\min\left\{s^{max} - s, \gamma\right\}\right] - E_1\left[\min\left\{s^{max} - s, \gamma\right\}\right]\right]$[28]. This difference in beliefs is fully exploited when the CDS is fully collateralized at $\gamma = s^{max}$. So the maximum price an optimist is willing to pay for cash is equal to: $1 + \frac{1}{s^{max}}\left[E_0\left[\min\left\{s^{max} - s, s^{max}\right\}\right] - E_1\left[\min\left\{s^{max} - s, s^{max}\right\}\right]\right] = 1 + \frac{[E_1[s] - E_0[s]]}{s^{max}}$, at which point the optimist would also weakly prefer to hold the illiquid asset instead. $\frac{[E_1[s] - E_0[s]]}{s^{max}}$ can be interpreted as the upper bound on the equilibrium collateral value for cash. For the rest of the proof, the focus is on the more interesting cases where $\hat{p}$ is strictly less than $1 + \frac{[E_1[s] - E_0[s]]}{s^{max}}$.

   (b) Note that each agent's optimization problem (A.3) is linear in the objective variables, thus their value functions will take the form $v_i n_i$, where $v_i$ denotes the return on agent $i$'s endowment $n_i$.

   (c) Since the agents can always just hold their endowment of the illiquid asset or buy cash in order to sell the fully collateralized CDS contract $\gamma = s^{max}$, one must have:

$$v_i \geq \max\left\{1, \frac{1 - \frac{1}{s^{max}} E_i\left[\min\left\{s^{max} - s, s^{max}\right\}\right]}{p - \frac{1}{s^{max}} E_0\left[\min\left\{s^{max} - s, s^{max}\right\}\right]}\right\} \quad \forall i = \{0, 1\} \tag{B.7}$$

   In the above expression, 1 represents the rate of return on the illiquid asset; and

$$\left(\frac{1 - \frac{1}{s^{max}} E_i\left[\min\left\{s^{max} - s, s^{max}\right\}\right]}{p - \frac{1}{s^{max}} E_0\left[\min\left\{s^{max} - s, s^{max}\right\}\right]}\right)$$

   represents the expected rate of return on buying cash to use as collateral in selling the CDS contract $\gamma = s^{max}$. For the latter, the expected payoff in the second period is 1 (from the cash) minus $\frac{1}{s^{max}} E_i\left[\min\left\{s^{max} - s, s^{max}\right\}\right]$ (the expected delivery on the CDS contract $\gamma$). The down-payment on this transaction is $\left(p - \frac{1}{s^{max}} E_0\left[\min\left\{s^{max} - s, s^{max}\right\}\right]\right)$, where $p$ is the price paid for the unit of cash, and $\frac{1}{s^{max}} E_0\left[\min\left\{s^{max} - s, s^{max}\right\}\right]$ is the amount raised from selling the CDS to the pessimist who values it the most.

   (d) Summing over the two agents' budget constraints (A.1), and imposing the market clearing conditions in equilibrium (holdings of CDS contracts must cancel out and the sum of total cash holdings is normalized to 1) yields:

$$a_0 + a_1 + \hat{p} \times 1 = n_0 + n_1$$

[28] Note that even though $\frac{1}{\gamma}\left[E_0\left[\min\left\{s^{max} - s, \gamma\right\}\right] - E_1\left[\min\left\{s^{max} - s, \gamma\right\}\right]\right]$ is maximized at $\gamma = s^{max}$. This is does not mean the optimist will always want to sell the fully collateralised CDS at any $p$. The $p^{opt}(\bar{s})$ curve plots the optimal collateral level $\bar{s}$ for the optimist at any given price $p$. (Derived from the interior solution to $\max_\gamma R$, where $R := \frac{1 - \frac{1}{\gamma} E_1\left[\min\left\{s^{max} - s, \gamma\right\}\right]}{p - \frac{1}{\gamma} E_0\left[\min\left\{s^{max} - s, \gamma\right\}\right]}$).

Recall that by Assumption A1 $n_0 + n_1 > \frac{E_0[s^{max}-s]}{E_1[s^{max}-s]} = 1 + \frac{E_1[s]-E_0[s]}{E_1[s^{max}-s]} > 1 + \frac{[E_1[s]-E_0[s]]}{s^{max}}$, so $\hat{p} \in [1, 1 + \frac{[E_1[s]-E_0[s]]}{s^{max}}]$ implies that $a_0 + a_1 > 0$ (i.e. one or more agents must hold the illiquid asset in equilibrium).

(e) Since $p < 1 + \frac{[E_1[s]-E_0[s]]}{s^{max}}$, it follows from equation (B.7) that $v_1 > 1$. Therefore, the pessimist must be the one holding the illiquid asset in the collateral equilibrium, which gives us $v_0 = 1$.

(f) Lastly, without loss of generality, CDS contracts with $\gamma > s^{max}$ will not be used in equilibrium (such contracts tie down a larger amount of collateral without a compensating increase in price).

2. Agents' bid and ask prices for CDS contracts:

(a) An agent's bid price is the price that would make her indifferent between buying the CDS contract and simply receiving the equilibrium value per net worth $v_i$ , so:

$$q_0^{bid}(\gamma) = \frac{E_0[\min\{s^{max}-s,\gamma\}]}{v_0} = E_0[\min\{s^{max}-s,\gamma\}]$$
$$> q_1^{bid}(\gamma) = \frac{E_1[\min\{s^{max}-s,\gamma\}]}{v_1} \tag{B.8}$$

(b) An agent's ask price for the CDS contract $\gamma$ is the price that would make the trader indifferent between taking a negative position in the CDS $\gamma$ and simply receiving the equilibrium value $v_i$, so:

$$v_0 = \frac{1-\frac{1}{\gamma}E_0[\min\{s^{max}-s,\gamma\}]}{p-\frac{1}{\gamma}q_0^{ask}(\gamma)} = 1$$
$$v_1 = \frac{1-\frac{1}{\gamma}E_1[\min\{s^{max}-s,\gamma\}]}{p-\frac{1}{\gamma}q_1^{ask}(\gamma)} > 1 \tag{B.9}$$

(c) Market clearing for CDS contracts $(\hat{\mu}_1^+ + \hat{\mu}_0^+ = \hat{\mu}_1^- + \hat{\mu}_0^-)$ implies:

$$\min_i q_i^{ask}(\gamma) \geq q(\gamma) \geq \max_i q_i^{bid}(\gamma) \quad \forall\gamma$$

(Suppose $\max_i q_i^{bid}(\gamma) > q(\gamma)$, then some buyer wants to buy an infinite amount, but seller can only sell a finite amount due to the collateral constraint. It also cannot occur that $q(\gamma) > \max\left\{\min_i q_i^{ask}(\gamma), \max_i q_i^{bid}(\gamma)\right\}$, so one must have $\min_i q_i^{ask}(\gamma) \geq q(\gamma)$)

(d) A CDS contract is traded in positive quantities only if:

$$q_i^{ask}(\hat{\gamma}) = q(\hat{\gamma}) = q_j^{bid}(\hat{\gamma}) \quad \text{for some } \{i,j\} = \{0,1\}$$

(e) Claim the pessimist's ask prices are always higher than optimist's bid prices:

$$\begin{aligned} q_0^{ask}(\gamma) &= \gamma(p-1) + E_0[\min\{s^{max} - s, \gamma\}] \quad \text{from eqn (B.9)} \\ &> E_0[\min\{s^{max} - s, \gamma\}] \quad \text{since } p > 1 \\ &= q_0^{bid}(\gamma) > q_1^{bid}(\gamma) \quad \text{from eqn (B.8)} \end{aligned}$$

so there are no traded CDS contracts in which the optimist buy and the pessimist sell.

(f) The equilibrium prices of CDS contracts are therefore:

$$\begin{cases} q(\hat{\gamma}) = q_0^{bid}(\hat{\gamma}) = q_1^{ask}(\hat{\gamma}) & \text{for each } \hat{\gamma} \text{ with positive trade} \\ q(\gamma) \in \left[\max_i q^{bid}(\gamma), \min_i q^{ask}(\gamma)\right] & \text{for each } \gamma \end{cases}$$

3. Characterize the equilibrium in CDS markets for a given price for cash $p \in [1, \frac{E_0[s^{max}-s]}{E_1[s^{max}-s]})$

- The optimist faces quasi-equilibrium prices for all CDS contracts (even those that are not positively traded in equilibrium):

$$\tilde{q}(\gamma) = q_0^{bid}(\gamma) = E_0[\min\{s^{max} - s, \gamma\}]$$

- Given these quasi-equilibrium prices, optimists solve the following optimization problem:

$$\begin{aligned} v_1 n_1 = \max_{c_1 \geq 0, \mu_1^-} \; & c_1 - \int_{\gamma \in B^{CDS}} \frac{1}{\gamma} E_1[\min\{s^{max} - s, \gamma\}] \, d\mu_1^- \\ \text{s.t. } & pc_1 - \int_{\gamma \in B^{CDS}} \frac{1}{\gamma} E_0[\min\{s^{max} - s, \gamma\}] \, d\mu_1^- = n_1 \quad \text{[budget constraint]} \\ & \int_{\gamma \in B^{CDS}} \frac{1}{\gamma} d\mu_1^- \leq c_1 \quad \text{[collateral constraint]} \end{aligned}$$

- Since $v_1 > 1$, the collateral constraint binds in equilibrium. Let $\lambda_1$ denote the Lagrangian multiplier for the collateral constraint. $v_1$ will correspond to the multiplier for the budget constraint

- The FOCs for $c_1$ and $\mu_1^-$ yields:

$$\begin{aligned} 1 + \lambda_1 &= v_1 p \\ v_1 \frac{1}{\gamma} E_0[\min\{s^{max} - s, \gamma\}] &\leq \frac{1}{\gamma} E_1[\min\{s^{max} - s, \gamma\}] + \lambda_1 \quad \text{with equality only if } \gamma \in supp\left(\mu_1^-\right) \end{aligned}$$

- Combining the FOCs yield:

$$\begin{aligned} v_1 p &= 1 + \lambda_1 \\ &\geq 1 + v_1 \frac{1}{\gamma} E_0 \left[\min\left\{s^{max} - s, \gamma\right\}\right] - \frac{1}{\gamma} E_1 \left[\min\left\{s^{max} - s, \gamma\right\}\right] \\ \Rightarrow v_1 &\geq \frac{1 - \frac{1}{\gamma} E_1 \left[\min\left\{s^{max} - s, \gamma\right\}\right]}{p - \frac{1}{\gamma} E_0 \left[\min\left\{s^{max} - s, \gamma\right\}\right]} =: R_1^{CDS}(\gamma) \quad \text{with equality only in } \gamma \in \text{supp}\left(\mu_1^-\right) \end{aligned}$$

- As per the proof of Proposition 2, $R_1^{CDS}(\gamma)$ has a unique maximum characterized by $p^{opt}(\gamma = \tilde{s})$. So again the unique collateral-GE is pinned down by the intersection between $p^{opt}(\bar{s})$ and the market clearing condition for cash: $p^{mc}(\tilde{s}) = n_1 + \frac{1}{\tilde{s}} E_0 \left[\min\left\{s^{max} - s, \tilde{s}\right\}\right]$, s.t. the equilibrium collateral level $\hat{\gamma}$ and the price of cash $\hat{p}$ satisfies:

$$\hat{p} = p^{opt}(\hat{\gamma}) = p^{mc}(\hat{\gamma})$$

4. It follows that the unique general equilibrium is equivalent to the principal-agent equilibrium.

## B.5 Proof of Proposition 4

Let us consider the case where the optimist becomes 'more pessimistic'. Let $g_1(s) := \frac{f_1(s)}{1 - F_1(s^{max} - \bar{s})} \quad \forall s \in [s^{max} - \bar{s}, s^{max}]$, $\forall \bar{s} \in [s^{min}, s^{max})$ and $\tilde{g}_1(s) := \frac{\tilde{f}_1(s)}{1 - \tilde{F}_1(s^{max} - \bar{s})} \quad \forall s \in [s^{max} - \bar{s}, s^{max})$ . Then by Assumption A2 $g(\cdot)$ and $\tilde{g}(\cdot)$ must also satisfy the monotone likelihood ratio condition: $\frac{d}{ds}\left(\frac{f_1}{\tilde{f}_1}\right) > 0 \quad \forall s \in S \Rightarrow \frac{d}{ds}\left(\frac{g_1}{\tilde{g}_1}\right) > 0 \quad \forall s \geq (s^{max} - \bar{s})$. This in turn implies $\tilde{E}_1\left[s | s > s^{max} - \bar{s}\right] < E_1\left[s | s \in s^{max} - \bar{s}\right]$ $\forall \bar{s} \in \left(s^{min}, s^{max}\right)$, so the upward sloping $p^{opt}$ curve shifts down when the optimist becomes 'more pessimistic'. The converse of the proposition follows from the same logic.

## B.6 Proof of Proposition 5

The objective is to show that a sufficient (but not necessary) condition for the equilibrium collateral level to increase is for the pessimist to attach sufficiently larger probability weights to the default states.

Let $F_0(s)$ and $\tilde{F}_0(s)$ denote the initial and the new beliefs of the pessimist respectively. For brevity, let $\tilde{E}_0[x] := E_{\tilde{F}_0}[x]$; $\tilde{p}^{opt}(\bar{s}) := \tilde{F}_0(s^{max} - \bar{s}) + \left(1 - \tilde{F}_0(s^{max} - \bar{s})\right) \frac{\tilde{E}_0\left[s^{max} - s | s > s^{max} - \bar{s}\right]}{E_1\left[s^{max} - s | s > s^{max} - \bar{s}\right]}$; and $\tilde{p}^{mc}(\bar{s}) := n_1 + \frac{1}{\bar{s}} \tilde{E}_0\left[\min\left\{s^{max} - s, \bar{s}\right\}\right]$. Define, respectively, the initial and the new equilibrium collateral levels $\gamma^*$ and $\gamma^{**}$ implicitly as:

$$\begin{aligned} p^{opt}(\gamma^*) &= p^{mc}(\gamma^*) \\ \tilde{p}^{opt}(\gamma^{**}) &= \tilde{p}^{mc}(\gamma^{**}) \end{aligned}$$

Then given $\tilde{p}^{mc}$ is strictly decreasing, $p^{opt}$ is strictly increasing, and the two curves intersects within

$\left(s^{min}, s^{max}\right)$ (see proofs for Propositions 2 and 3), it holds that $\gamma^{**} > \gamma^{*}$ iff:

$$\tilde{p}^{mc}\left(\gamma^{*}\right) > \tilde{p}^{opt}\left(\gamma^{*}\right)$$
$$\Leftrightarrow \tilde{p}^{mc}\left(\gamma^{*}\right) - p^{mc}\left(\gamma^{*}\right) > \tilde{p}^{opt}\left(\gamma^{*}\right) - p^{opt}\left(\gamma^{*}\right)$$

With a little bit of algebra, one can show that:

$$\begin{aligned}
&\tilde{p}^{mc}\left(\gamma^{*}\right) - p^{mc}\left(\gamma^{*}\right) \\
=&\frac{1}{\gamma^{*}}\left[\tilde{E}_0\left[\min\left\{s^{max} - s, \gamma^{*}\right\}\right] - E_0\left[\min\left\{s^{max} - s, \gamma^{*}\right\}\right]\right] \\
=&\frac{1}{\gamma^{*}}\left[\gamma^{*}\tilde{F}_0 + \int_{s^{max}-\gamma^{*}}^{s^{max}}\left(s^{max} - s\right)\tilde{f}_0\left(s\right)ds - \gamma^{*}F_0 - \int_{s^{max}-\gamma^{*}}^{s^{max}}\left(s^{max} - s\right)f_0\left(s\right)ds\right] \\
=&\left(\tilde{F}_0 - F_0\right) + \frac{1}{\gamma^{*}}\left[\int_{s^{max}-\gamma^{*}}^{s^{max}}\left(s^{max} - s\right)\tilde{f}_0\left(s\right)ds - \int_{s^{max}-\gamma^{*}}^{s^{max}}\left(s^{max} - s\right)f_0\left(s\right)ds\right]
\end{aligned}$$

and

$$\begin{aligned}
&\tilde{p}^{opt}\left(\gamma^{*}\right) - p^{opt}\left(\gamma^{*}\right) \\
=&\tilde{F}_0 + \left(1 - \tilde{F}_0\right)\frac{\tilde{E}_0\left[s^{max} - s|s > s^{max} - \gamma^{*}\right]}{E_1\left[s^{max} - s|s > s^{max} - \gamma^{*}\right]} \\
&\cdots - \left[F_0 + \left(1 - F_0\right)\frac{E_0\left[s^{max} - s|s > s^{max} - \gamma^{*}\right]}{E_1\left[s^{max} - s|s > s^{max} - \gamma^{*}\right]}\right] \\
=&\left(\tilde{F}_0 - F_0\right) + \frac{\left[\int_{s^{max}-\gamma^{*}}^{s^{max}}\left(s^{max} - s\right)\tilde{f}_0\left(s\right)ds - \int_{s^{max}-\gamma^{*}}^{s^{max}}\left(s^{max} - s\right)f_0\left(s\right)ds\right]}{E_1\left[s^{max} - s|s > s^{max} - \gamma^{*}\right]}
\end{aligned}$$

Taken together, one gets:

$$\begin{aligned}
&\left[\tilde{p}^{mc}\left(\gamma^{*}\right) - p^{mc}\left(\gamma^{*}\right)\right] - \left[\tilde{p}^{opt}\left(\gamma^{*}\right) - p^{opt}\left(\gamma^{*}\right)\right] \\
=&\left[\frac{1}{\gamma^{*}} - \frac{1}{E_1\left[s^{max} - s|s > s^{max} - \gamma^{*}\right]}\right]\left[\int_{s^{max}-\gamma^{*}}^{s^{max}}\left(s^{max} - s\right)\tilde{f}_0\left(s\right)ds - \int_{s^{max}-\gamma^{*}}^{s^{max}}\left(s^{max} - s\right)f_0\left(s\right)ds\right] \\
=&\underbrace{\left[\frac{1}{E_1\left[s^{max} - s|s > s^{max} - \gamma^{*}\right]} - \frac{1}{\gamma^{*}}\right]}_{>0}\left[\int_{s^{max}-\gamma^{*}}^{s^{max}}\left(s^{max} - s\right)\left(f_0\left(s\right) - \tilde{f}_0\left(s\right)\right)ds\right]
\end{aligned}$$

Therefore, whether the collateral requirement in the new equilibrium increases ($\gamma^{**} > \gamma^{*}$) depends on whether the second term $\int_{s^{max}-\hat{\gamma}}^{s^{max}}\left(s^{max} - s\right)\left(f_0\left(s\right) - \tilde{f}_0\left(s\right)\right)ds$ is positive, for which a sufficient condition is that $f_0\left(s\right) > \tilde{f}_0\left(s\right) \quad \forall s \in \left[s^{max} - \gamma^{*}, s^{max}\right)$.

## C Part 39 CDS prices

End-of-day (EOD) prices within the Part 39 data set are provided by ICC in terms of points upfront. CDS prices historically have been quoted in terms of conventional or "break-even" spreads, defined as the annualized quarterly spread payment per unit of purchased protection that makes the market value of the position zero at initiation. Contracts thus were negotiated bilaterally over the counter and, depending on when they were traded, carried different spreads. The push for standardized CDS contracts, however, has drastically changed the landscape of CDS price quotes and traded contracts. In particular, the 2011 "CDS Big Bang" resulted in standardized CDSs having fixed coupons (usually 100 or 500 basis points). Thus, contract market values are often non-zero at outset. When trading standardized CDSs, the protection buyer makes an upfront payment to the protection seller at initiation (or vice versa). Price quotes are then in "points upfront" instead of break-even spreads. For instance, if a CDS contract were quoted at 0.97, the protection buyer would pay $1 - 0.97 = 3\%$ of the notional to the seller at contract initiation. Notice that this quote convention is analogous to bond quotes, where a higher price quote represents a lower payment for the buyer. Some data providers, such as Bloomberg, convert the quoted prices using a standardized model provided by the International Swaps and Derivatives Association (ISDA) and, by convention, record break-even spreads.

We note that quoted prices are model prices. Since CDSs trade relatively thinly, EOD transaction prices are not always available. ICC and Markit have a specific price discovery process tailored to the CDS market. Participants submit price quotes at the end of every business day and the clearinghouse creates periodic trade executions among participants via an auction process. The resulting prices are used for daily mark-to-market purposes.

## D Market events during our sample period

In this section, we briefly review the main world events that affected, directly or indirectly, CDS markets during our sample period May 2014-February 2019. In particular, during this period we find: (i) the plummet of oil prices in November 2014, when Saudi Arabia blocked OPEC from cutting oil production; (ii) the plunge in the Euro when the ECB chief Mario Draghi expressed unexpectedly dovish outlooks on monetary policy in January 2015; (iii) the 2015–2016 stock market sell-off, starting with the Chinese stock market burst ("Black Monday" ), and followed by an unexpected devaluation in the Renminbi, which was further fueled by Greek Debt default; (iv) the unexpected negative interest rate policy announced by the Bank of Japan in January 2016; (v) the volatility spike when the Brexit referendum was announced in February 2016; (vi) Donald Trump's election in November 2016 which, immediately following the announcement, created extreme volatility spikes in global markets and led the total trading volumes in CDS markets to double on the election night; (vii) OPEC's decision to cut oil production, followed by non-OPEC countries, led to hikes in oil prices in November 2016, especially because this was the first time after financial crisis; (viii) the Venezuela's delayed payments on its sovereign debt and bonds issued by state oil giant Petroleos de Venezuela in November 2017 constituted a failure to pay "credit event", and led to extreme volatility in CDS prices which sky-rocketed in a period of 6-7 days; (ix) the current COVID 19 pandemic, which caused the US stock market to hit the circuit breaker mechanism four times in ten days in March 2020, led to the announcement of a zero-percent interest rate policy and a $700 billion quantitative easing (QE) program, and led to a spike in unmployment rates which exceeded levels of 15%.

Our sample period also covers the (widely expected) interest rate hike by the Federal Reserve in December 2015, the first increase in nearly a decade, the Bitcoin's record price surge in the year 2017, and the (expected) Bank of England's decisions to raise interest in November 2017 and August 2018, respectively first and second time after the global crisis, despite the ongoing uncertainty over the future of the UK economy.

# E Time-series test of the $VaR$ rule using realized returns

We consider the $Z$ statistic:

$$Z := \frac{1}{NT} \sum_{t=1}^{T} \sum_{n=1}^{N} \mathbb{I}\{\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\},$$

where $\mathbb{I}\{\cdot\}$ is the indicator function. The indicator takes value 1 when realized $M-$day losses exceed the initial margin requirement; this is typically referred to as an *exceedance*. The statistic $Z$ is the empirical frequency at which exceedances occur, averaged over time and across market participants. We have, for quite general correlation structures:

$$Z \xrightarrow{\mathbb{P}} \alpha,$$

by the law of large numbers. For $M$ and $\alpha$ specified in the null hypothesis, we can test $H_0$ using $Z$ as the test statistic.

We compute standard errors for the test using binomial probabilities. To proceed, we assume that exceedances are perfectly correlated when underlying losses overlap (for robustness against autocorrelation), and also assume that exceedances are uncorrelated across accounts. In particular, standard errors are computed as

$$S.E. = \sqrt{\frac{\alpha(1-\alpha)\zeta_M}{NT}}.$$

In the above equation, the term $\zeta_M := 2M - 1$ adjusts for our assumption that exceedances are perfectly correlated when underlying losses overlap. For $\alpha = 1\%$, $NT = 23,310$ and $M = 5$, we obtain a standard error of 0.19%. We remark that our standard errors are likely to be overly conservative. As a robustness check, we also compute one-day returns and autocorrelation estimates. For each account, we find autocorrelation estimates on the orders of $10^{-4}$ for the first five lags. Thus, autocorrelation would likely have a smaller impact on actual standard errors compared to our assumption of perfect correlation.

Finally, to explicitly account for potential cross-sectional correlation in the returns on margins, we perform the test separately account by account, finding that the null hypothesis is rejected in every case.

## F Procedure to compute counterfactual Returns

Following Duffie et al. (2015), we group together all $I$ contracts written on the $K$ underlying reference entities, and denote the net position in that reference entity by $Y_k$. Precisely, let $\Omega_k$ denote the collection of contracts referencing name $k$, then

$$Y_{t,k}^n := \sum_{i \in \Omega_k} X_{t,i}^n.$$

For each reference entity, therefore, $Y_k$ indicates the net exposure to reference entity $k$, aggregating together the CDS contracts on that reference entity across maturities, seniority level, and doc clause. We then collect historical on-the-run 5-year credit spread series for each reference entity, $\mathbf{S}_t \in \mathbb{R}^K$, where the spread is the coupon payment that equates the value of premium and protection leg. Under standard assumptions on the loss rate, $\mathbf{S}_t$ can be converted to $\mathbf{P}_t$. We do not report the exact conversion formula here, but observe that, given that the market value of a CDS position at $t$ is the notional of the position multiplied by $(1 - \mathbf{P}_t)$ (see also (1)), then $\mathbf{S}_t$ is increasing in $1 - \mathbf{P}_t$. Unlike $\mathbf{P}_t$, i.e., the quoting price following the Big Bang convention according to which the buyer makes an upfront payment of $1 - \mathbf{P}_t$ and then pays a running fixed spread premium throughout the life of the contract, $\mathbf{S}_t$ is not a price. Hence, $P\&L$ can only be approximated via the DV01 formula, given by

$$\Psi_{5,u}(\mathbf{X}_t^n) \approx d \times \mathbf{Y}_t^n \cdot (\mathbf{S}_{u+5} - \mathbf{S}_u),$$

where $d$ is the effective duration of the position. We use $d = 3$ as in Duffie et al. (2015), meaning that the average duration of CDS positions is 3 years (corresponding to the median maturity of the CDS market).

## G Time-series test of the $VaR$ rule using counterfactual returns

We compute our test statistic using an extended version of Eq. (E):

$$Z' = \frac{1}{NTU} \sum_{t=1}^{T} \sum_{u=1}^{U} \sum_{n=1}^{N} \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\},$$

where $\Psi_{5,u} MtM(\mathbf{X}_t^n)$ is constructed as in Duffie et al. (2015) (see Appendix F for additional details), and $U$ is the number of evaluation dates, that is, dates for which we observe the portfolio. For each portfolio $\mathbf{X}_t^n$, we estimate the frequency at which losses exceed portfolio margins. Under the null hypothesis of a 5-day 99% VaR margining rule, $Z'$ should converge to 1% in probability.

The test can be simply implemented as a regression of observed exceedances onto a constant, with double-clustering as in Petersen (2009) by time and by account (there is no need to use the binomial model as exceedances are observed in the data, so the variance of the residuals is nonzero).

# H Cross-sectional test of the $VaR$ rule

The margining rule $H_0$ implies $\mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)) = \alpha$ for all $n$, which further implies

$$H_0' : \mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)) = \mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^{n'}) < -IM_t(\mathbf{X}_t^{n'})),$$

for all $n \neq n'$. The statistics to consider are then

$$Z_n := \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}\{\Psi_{M,t}(\mathbf{X}_t^n) + IM_t(\mathbf{X}_t^n) < 0\} \xrightarrow{\mathbb{P}} \alpha.$$

We describe here how to implement a test for equality ($H_0'$). The most straightforward test for equality of the frequencies of exceedances across accounts is the $G-$test (i.e. the two-way likelihood ratio test). Because the confidence level is expected to be large (the expected frequency of exceedances is low), the typical $\chi^2-$test for homogeneity is not appropriate (Hoey (2012)). As exceedances are expected to occur with low probability, we instead use the $G-$test to test the null hypothesis.

The test statistic is computed as:

$$G := 2\sum_{n=1}^{N} O_n \log \frac{O_n}{E_n},$$

where $O_n$ is the observed number of exceedances for clearing member $n$, and $E_n$ is the expected number of exceedances for account $n$. The probability of observing an exceedance, needed for calculating $E_n$, is estimated by pooling observations across accounts. In particular,

$$E_n := TU \times Z = \frac{1}{N}\sum_{t=1}^{T}\sum_{u=1}^{U}\sum_{n'=1}^{N} \mathbb{I}\{\Psi_{5,u}MtM(\mathbf{X}_t^{n'}) < -IM_t(\mathbf{X}_t^{n'})\},$$

and

$$O_n := \sum_{t=1}^{T}\sum_{u=1}^{U} \mathbb{I}\{\Psi_{5,u}MtM(\mathbf{X}_t^{n}) < -IM_t(\mathbf{X}_t^{n})\}.$$

Under the null that frequencies are the same for each account, $G \xrightarrow{d} \chi^2_{N-1}$.

We also derive an extension of this test that explicitly accounts for potential autocorrelation of the exceedances. For a fixed portfolio, we first count the number of exceedances, and then divide it by the number of evaluation dates. This gives an estimate for the probability of an exceedance occurring for that portfolio. We then sum the exceedance probabilities for portfolios associated with each fixed account, and use the rounded up integer as the estimate of observed exceedances for that account. We enter this estimate into the contingency table used for the $G-$test. Formally, we estimate the probability of an exceedance for an account/day combination $(n, t)$ as

$$\hat{p}_{n,t} = \frac{1}{T} \sum_{u=1}^{T} \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\}.$$

The number of (estimated) observed exceedances is then

$$\hat{O}_n := \left\lceil \sum_{t=1}^{T} \hat{p}_{n,t} \right\rceil.$$

The estimate $\hat{O}_n$ replaces $O_n$ in our computation of the $G$ statistic (Eq. (H)).[29] The estimated observations are thus more robust to autocorrelation compared to treating each observation as an individual count, which may inflate the sample size.

# I Robustness: Details

In this section, we provide more details about the robustness tests of Section 4.4.

Value-at-Risk and maximum shortfall used in the panel analyses (Tables 5 and 4) were based on P&L generated from our entire sample of credit spreads (that is, on the entire historical distribution of returns for each portfolio held at time $t$ by member $n$). Because our data set covered the financial crisis, the risk measures captured extreme movements and may thus be viewed as overly conservative for estimating portfolio losses. In this section we consider using only the last 1000 days (approximately 4 years) of credit spreads data to generate P&L, as in Duffie et al. (2015). Using these newly estimated counterfactual P&L, we compute Value-at-Risk and maximum shortfall. We replicate our panel analyses and report the results in Tables A.3 and A.4.

Comparing Table A.3 to Table 5, we see there is a decrease in explanatory power of Value-at-Risk ($VaR$) (column (1)). This is likely due the exclusion of the financial crisis period in our simulation, resulting in both lower level and variability of Value-at-Risk. Aggregate short notional ($AS$) still retains its strong explanatory power (columns (3) and (4)), and columns (7) and (8) show that the DSV model still captures a significant portion of variation in initial margins, and outperforms Value-at-Risk in terms of explanatory power (columns (1) and (2)). Our conclusions remain consistent with our previous results.

Comparing Table A.4 to Table A.3, we observe again that there is a non-negligible increase in explanatory power compared to models that include only portfolio variables (Table A.3). This confirms that market variables can capture a dimension of initial margins not explained by portfolio variables, especially for the VIX.

[29] The ceiling operation is performed to ensure that the contingency table only contains integer entries. We also performed the test with unrounded data, yielding similar, if not stronger, results.

Table A.1: Initial margins and market variables summary statistics

| Summary Statistic | *Market variables, in basis points (bps)* | | | |
|---|---|---|---|---|
| | In. Margins ($IM_{n,t}$) | Overnight Index Swap Spread ($OIS_t$) | LIBOR-OIS spread ($LOIS_t$) | CBOE VIX ($VIX_t$) |
| Pooled mean (over all n and t): $\mu\left(x_{n,t}\right)$ | 654.0 | 100.6 | 25.5 | 1,606.1 |
| Std. deviation (over all n and t): $\sigma\left(x_{n,t}\right)$ | 427.4 | 81.8 | 16.6 | 750.0 |

| Summary Statistic | *Market variables, in basis points (bps)* | | |
|---|---|---|---|
| | In. Margins ($IM_{n,t}$) | CDS spread ($CDS_{n,t}$) | Avg. CDS spread ($ACDS_t$) |
| Pooled mean (over all n and t): $\mu\left(x_{n,t}\right)$ | 654.0 | 73.6 | 73.6 |
| Std. deviation (over all n and t): $\sigma\left(x_{n,t}\right)$ | 427.4 | 33.8 | 19.8 |
| Time-series variation of cross-sectional averages: $\sigma\left(\bar{x}_t\right)$ | 108.4 | 19.8 | |
| Mean cross-sectional dispersion: $\mu\left(\sigma_t\left(x_{n,t}\right)\right)$ | 421.0 | 26.4 | |
| Cross-sectional dispersion of time-series averages: $\sigma\left(\bar{x}_n\right)$ | 360.7 | 21.4 | |
| Mean time-series dispersion: $\mu\left(\sigma_n\left(x_{n,t}\right)\right)$ | 225.4 | 24.4 | |

**Note:** The table displays summary statistics of our key market variables and initial margins, in basis points and millions of USD, respectively. Definitions of market variables are reported in Table 2. In addition to the overall mean and standard deviations (dispersions), we report panel statistics that describe properties of variables both across accounts and time, the calculations of which are reviewed in Table 3. Panel summaries are not reported for market variables that do not vary across accounts.

Table A.2: Check for multicollinearity

| Estimates ($R^2$) | *Dependent variable:* | |
|---|---|---|
| | $SD$ | $ES$ |
| $VaR$ | 0.306$^{***}$ | 1.422$^{***}$ |
| (OLS) | (97.1%) | (95.0%) |
| $VaR$ | 0.295$^{***}$ | 1.364$^{***}$ |
| (Two-way Panel) | (97.7%) | (96.8%) |
| Observations | 23,310 | 23,310 |

**Note:** We regress both expected shortfall and standard deviation on Value-at-Risk, and report the results. The first row corresponds to estimates from (pooled) OLS regression, and the second row corresponds to estimates after accounting for time and account fixed effects. Coefficient estimates are all significant at the 1% level. $R^2$s are in parentheses.

## Table A.3: Regression results for explaining initial margins with portfolio variables

| | *Dependent variable:* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Initial margins (IM) - Daily Frequency | | | | | | | | | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| Value-at-Risk (VaR) | 4.284*** | 2.394*** | 1.917** | 1.529*** | | | | | 0.037 | 0.285 | 0.946 | 0.919*** |
| | (0.738) | (0.467) | (0.769) | (0.332) | | | | | (0.588) | (0.499) | (0.607) | (0.316) |
| Maximum shortfall (MS) | | | −0.072 | −0.036 | 0.420*** | 0.333*** | | | | | | |
| | | | (0.142) | (0.079) | (0.132) | (0.063) | | | | | | |
| Aggregate short notional (AS) | | | 0.030*** | 0.020*** | 0.034*** | 0.022*** | | | | | | |
| | | | (0.003) | (0.004) | (0.004) | (0.004) | | | | | | |
| Duffie et al. model (DSV) | | | | | | | 1.085*** | 0.685*** | 1.079*** | 0.641*** | | |
| | | | | | | | (0.131) | (0.089) | (0.144) | (0.147) | | |
| Modified DSV model (MDSV) | | | | | | | | | | | 0.880*** | 0.563*** |
| | | | | | | | | | | | (0.107) | (0.108) |
| Number of Observations | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 |
| Adjusted $R^2$ | 0.400 | 0.772 | 0.700 | 0.847 | 0.678 | 0.837 | 0.595 | 0.817 | 0.595 | 0.818 | 0.689 | 0.844 |
| Account Fixed Effect | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* |
| Time Fixed Effect | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* | *N* | *Y* |

*Note:* *p<0.1; **p<0.05; ***p<0.01
Two-Way Clustered Standard Errors (by Time and Account)

**Note:** Same as Table 5, but computing risk measures only using last 1000 days of simulated return on margins

Table A.4: Regression results for initial margins using portfolio and market variables with last 1000 days of P&L

| | *Dependent variable:* | | | | | |
|---|---|---|---|---|---|---|
| | Initial margins (IM) - Daily Frequency | | | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Value-at-Risk (VaR) 1% | 1.040* | 0.733** | 1.014* | 0.622** | 2.193*** | 1.442*** |
| | (0.577) | (0.331) | (0.607) | (0.303) | (0.689) | (0.303) |
| Modified DSV Model (MDSV) | 0.861*** | 0.614*** | 0.864*** | 0.631*** | | |
| | (0.102) | (0.094) | (0.102) | (0.087) | | |
| Maximum Shortfall (MS) | | | | | −0.171 | −0.051 |
| | | | | | (0.135) | (0.071) |
| Aggregate Short Notional (AS) | | | | | 0.029*** | 0.022*** |
| | | | | | (0.003) | (0.003) |
| CBOE Volatility Index (VIX) | 0.013 | 0.040** | 0.006 | 0.020 | 0.037** | 0.052*** |
| | (0.016) | (0.018) | (0.017) | (0.014) | (0.015) | (0.017) |
| Member CDS Spread (DCDS) | 2.683** | 0.824 | 2.680** | 0.830 | 2.713** | 0.987 |
| | (1.229) | (0.765) | (1.206) | (0.739) | (1.228) | (0.756) |
| Average CDS Spread (ACDS) | −0.111 | −0.699 | | | −0.424 | −0.836 |
| | (1.210) | (1.158) | | | (1.207) | (1.147) |
| LIBOR-OIS Spread (LOIS) | | | 0.487 | 1.004 | | |
| | | | (0.994) | (1.114) | | |
| Number of Observations | 23310 | 23310 | 23310 | 23310 | 23310 | 23310 |
| Adjusted $R^2$ | 0.719 | 0.837 | 0.719 | 0.837 | 0.733 | 0.842 |
| Account Fixed Effect | *N* | *Y* | *N* | *Y* | *N* | *Y* |
| Time Fixed Effect | *N* | *N* | *N* | *N* | *N* | *N* |
| *Note:* | *p<0.1; **p<0.05; ***p<0.01<br>Two-Way Clustered Standard Errors (by Time and Account) | | | | | |

**Note:** Same as Table 4, but computing risk measures only using last 1000 days of simulated return on margins