The Collateral Rule: Evidence from the Credit Default Swap Market*

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Abstract

We explore a novel dataset of daily cleared credit default swap (CDS) positions along with the posted margins to study how collateral vary with portfolio risks and market conditions. Contrary to many theoretical models, which assume that collateral constraints follow Value-at-Risk rules, we find strong evidence that collateral requirements are set *an order of magnitude larger* than what standard Value-at-Risk rules imply. The panel variation in collateralization rates of CDS portfolios (over time and across participants) is well explained by measures of extreme tail risks, related to the maximal potential loss of the portfolio.

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1 Introduction

Collateral plays a central role in sustaining risk sharing in the economy. However, as highlighted during the financial crisis of 2008-2009, it can also amplify fundamental shocks and create self-reinforcing death spirals that can affect the entire economy.

The role of the "collateral channel" in amplifying fundamental shocks has been studied by a vast theoretical literature. At the core of models in this literature is the *collateral rule*, which determines how margins are set and how they respond to changes in economic conditions. Different models have employed different assumptions about the collateral rule. Some studies have specified the rule exogenously (e.g., Brunnermeier and Pedersen (2009) assume a Value-at-Risk (VaR) criterion; Gromb and Vayanos (2002) assume a maximum-loss constraint). Other work has proposed an endogenous rule, and determined the collateral levels via general equilibrium models (e.g., Geanakoplos (1997) and Fostel and Geanakoplos (2015)).

Understanding how collateral is set, and how it responds to changes in market or portfolio conditions, is central for understanding the mechanisms through which shocks propagate and are amplified. Yet, empirical evidence on the collateral rule is scarce.

In this paper, we aim to fill this gap by studying the collateral rule in depth, first empirically and then theoretically, in a large market that was at the very center of the financial crisis: the credit default swap (CDS) market. Using a novel dataset of positions and corresponding margins in the cleared CDS market, we document several novel facts about the way collateral is determined, and the way it is adjusted in response to changes in individual and macroeconomic risks. Du et al. (2020) construct a theoretical model of the endogenous collateral rule in derivatives markets, building on the work of Geanakoplos (1997), Fostel and Geanakoplos (2015), and Simsek (2013), that allows us to interpret our empirical findings and connect them to the existing literature.

The CDS market is a trillion-dollar market for credit risk transfer. It experienced remarkable growth in the years before the global financial crisis, and has been at the center of many policy debates during the financial crisis. Over the past decade, the CDS market has transitioned towards mandatory clearing: after two parties enter a bilateral CDS contract, all counterparty obligations are transferred to a clearinghouse. That is, the clearinghouse becomes the counterparty to each the original trading parties, referred to as *clearing members*. Operating as a *central counterparty* (CCP), the clearinghouse insulates members from default risk, but requires them to post daily-settled collateral (margin). Our dataset, collected and maintained by the U.S. Commodity Futures Trading Commission (CFTC), is built from the universe of CDS trades cleared by Ice Clear Credit (ICC), the largest clearinghouse for these contracts. In our data, we can observe each CDS position of every member in this market, covering a total of more than 24,000 contracts. More importantly, we also observe the amount of collateral posted each day by each member to the clearinghouse. Our sample covers the period 2014-2019 at the daily frequency.

Using this data set, we document several new empirical findings on collateral setting in the cleared CDS market. First, we show that there is a large amount of heterogeneity (both across clearing members and over time) in the collateralization rate – the value of collateral posted as a fraction of the total size of the members' portfolios. This indicates that clearing members trade portfolios with very different risk characteristics, which command different levels of collateralization.

Next, we study what determines the amount of collateral posted by clearing members. The natural benchmark to evaluate the level of collateralization of a portfolio is the widely-used Value-at-Risk (VaR) rule, under which collateral levels should be set to cover a certain fraction of losses occurring over a limited period of time. The VaR rule is especially important in this setting, not only because it is the rule assumed by many theoretical models,[1] but also because most clearinghouses (including ICC itself) explicitly use it when they describe their collateral requirements (Ivanov and Underwood (2011)). The benchmark rule in this context is a 5-day 99%-level VaR, according to which collateral should be sufficient to cover 99% of the 5-day loss distribution of each member's portfolio. The VaR rule tries to strike a balance between the ability to recover payments in case of counterparty default and the cost of keeping cash immobilized as collateral. Under this rule, we should therefore observe losses from individual member portfolios that exceed the posted collateral in about 1% of all 5-day periods.

The second finding of our empirical analysis is that, in the cleared CDS market, collateral levels *far exceed* those implied by the benchmark VaR. In fact, for our entire sample period, 2014-2019, the largest realized 5-day losses are *at most* only around 40% of the posted collateral: exceedance (i.e., losses above the posted collateral) *never* occurred for any member in any 5-day window, despite significant market events such as the Chinese stock market panic of early 2015, the Brexit referendum in 2016, and the Venezuelan default in late 2017.

More strikingly, this conservativeness in collateral levels holds true even when we incorporate in our analysis the history of CDS price movements since 2004, therefore including the large shocks occurred to the CDS market during the financial crisis. To perform this analysis, we collect historical CDS prices since 2004, and for each portfolio observed in our sample we build the time series of returns that the portfolio would have realized over that much longer time period. We can therefore ask, for each portfolio, how that portfolio would have performed on any 5-day interval since 2004, including during the financial crisis. While in this case a small number of exceedances would have been observed, they still represent a fraction that is two orders of magnitudes smaller than the one implied by the standard 99% VaR. To sum up, our second finding is that collateral levels are set orders of magnitude more conservatively than standard benchmarks imply.

Motivated by this result, and guided by the theoretical literature, we then explore in our data what factors determine margins in our panel.[2] We begin by estimating a panel model relating margins to VaR as well as other portfolio risk measures that have been proposed in the theoretical literature:

[1]For example, see Figlewski (1984), Brunnermeier and Pedersen (2009), Hull (2012) and Glasserman et al. (2016).

[2]As we describe in greater detail in the next sections, ICC follows a complex set of procedures to determine collateral levels that include calibration of different scenarios and simulations – together with a discretionary component – to determine the amount of collateral each member needs to post. The goal of this paper is not to reverse-engineer this complex procedure, but rather to identify and quantify the main economic determinants of the variation in collateral over time and across members. We therefore study both the direct effect of variables that the clearinghouse takes explicitly into account (like portfolio risks) as well as the effect of market variables (like aggregate volatility) that only affect margins indirectly, for example through the choice of models or their parameters.

expected shortfall (expected loss conditional on exceeding the VaR), maximum shortfall (maximum potential loss obtained from historical simulations), aggregate short notional (total notional of short positions only), and aggregate net notional (across long and short positions). We show that these variables explain a significant portion of the panel variation with an overall R^2 of 74% (and 84% if member fixed effects are added), and they significantly improve over standard VaR, which falls short to explain not only the average level of margins, but also the panel variation.

The two variables that stand out empirically in terms of explanatory power are the *maximum shortfall* and the *aggregate short notional.* Interestingly, Duffie et al. (2015) assume a theoretical collateral rule based precisely on a combination of these two variables; our empirical analysis strongly supports that assumption. These two variables have an interesting economic interpretation: they are much more related to extreme tail risks than VaR. In comparison, VaR focuses on less extreme losses, and is empirically highly correlated with simple volatility (.99 correlation).

Given that the maximum shortfall of a portfolio is based on its historical performance, it captures the *experienced* maximal loss in the data (which, of course, is more severe than the 99th loss quantile used by VaR). Aggregate short notional, on the other hand, captures the fact that in the CDS market the biggest tail losses occur on the short side: if a default occurs suddenly ("jump-to-default") the liability of the short side can jump to the notional value of the CDS (less the recovery). Hence, the aggregate short notional represents the maximum *potential* loss of a portfolio if *all* short positions default simultaneously (and the recovery rate of the underlying bond is zero). In this sense, aggregate short notional captures the most extreme form of tail risk, and provides a measure that is also less sensitive to the exact specification of the portfolio loss distribution compared to other tail risk measures.

To sum up, the third finding of our empirical analysis is that maximal shortfall and aggregate short notional – two extreme tail risk measures – outperform VaR in describing the collateral rule in this market. To be clear, our results do not imply that the collateral rule covers *every* possible loss in practice; they do show however that it is the very extreme tail risks that the clearinghouse is concerned about, much farther out the tails than standard VaR (which instead is closely related to simple volatility).

Finally, we explore how *market variables* enter the collateral rule: because collateral rules adapt to market conditions, we expect collateral levels to respond to variables that capture the state of the economy. We incorporate in our panel analysis measures of aggregate risk and risk premia, such as VIX and the average CDS spread of all dealers, and measures of individual member funding opportunity costs. We find that margins increase for all members when risk in the economy increases, suggesting that the collateral rule adapts to the state of the economy.

Our empirical results have direct implications for models of financial markets and intermediation, in which the collateral constraint plays an important role. For models in which the collateral constraint is specified *exogenously*, our findings provide support for defining it as a function of extreme tail risks, such as a maximum-loss constraint (Gromb and Vayanos (2002)), instead of a standard, less conservative VaR (as in Brunnermeier and Pedersen (2009)). The difference is significant: empirically, VaRs are very closely related to volatility, and do not fully capture the extreme tail risks; in models that assume VaR rules, the dynamics of collateral requirements will mostly reflect movements in portfolio volatility. Instead, as we have shown, collateral levels are driven by extreme tail events, that capture nonlinear effects beyond volatility (for example, jumps): collateral demand may not be very sensitive to small changes in portfolio or market risks, but may change significantly if large shocks occur.

Our results also have implications for models of the *endogenous* collateral rule, like Fostel and Geanakoplos (2015). A key prediction of Fostel and Geanakoplos (2015) is that in a binomial economy (i.e., when there are two states of nature only), any collateral equilibrium is equivalent to one in which there is no default – that is, where collateral covers the most extreme losses. Intuitively, therefore, our results support the key intuition of this model, which argues for very conservative collateral rules. However, it is difficult to link tightly our empirical results on the CDS markets with this model, because (1) the conclusions only hold if there are two states of nature, and (2) defaults on CDS obligations do sometimes arise in practice. Most recently, Du et al. (2020) overcome these two limitations by developing a new equilibrium model, which explains the conservativeness of collateral levels through disagreement of market participants about the extreme states of the world, in which CDSs pay off and counterparties default.

Our paper relates to a large theoretical literature on the relation between margin requirements and asset prices, and the collateral equilibrium. Noticeable contributions include Brunnermeier and Pedersen (2009), Gromb and Vayanos (2002), Coen-Pirani (2005), and Chabakauri (2013), in which the collateral rule is exogenous; and Geanakoplos (1997), Holmström and Tirole (1997), Brunnermeier and Sannikov (2014), and Simsek (2013) in which collateral requirements arise endogenously, and depend both on market conditions and specific characteristics of market participants. A recent theoretical literature has focused specifically on cleared derivative markets, like Koeppl et al. (2012) and Biais et al. (2016). The impact of central clearing reforms on the collateral demand for derivatives transactions is investigated in Heller and Vause (2012), Sidanius and Zikes (2012), and Loon and Zhong (2014), assuming exogenously specified margin requirements based on VaR, expected shortfall, or a mix of the two.[3]

Empirical work on the *determinants* of collateral is scarce, mostly for the difficulty of obtaining data on positions and collateral.[4] The papers in this area mostly focus on margining in the futures market (Figlewski (1984), Gay et al. (1986), Fenn and Kupiec (1993), Hedegaard (2014)). Our study of collateral in the cleared CDS space enriches the existing empirical evidence on collateral rules along several dimensions. First, while prior studies have looked at headline margin requirements for individual securities, their approaches are less applicable in the modern setting of portfolio margining, where margins are set at the portfolio level rather than for individual contracts (as in the case of CDS

[3] Other work (Cruz Lopez et al. (2017), Menkveld (2017)) has studied the determination of collateral requirements accounting for systemic interdependencies.

[4] Collateral data for non-cleared OTC markets is often scattered among a variety of participants, with no centralized data sets available. Clearinghouse data contain proprietary information of large market participants and are often disclosed only under strict confidentiality and anonymity arrangements. Due to such data limitations, there is little empirical work focusing on portfolio-level margins (as opposed to individual-security collateral requirements), and on how well conventional risk measures relate to the required collateral levels.

clearinghouses). Our disaggregated, granular CDS data provide a valuable source of information for analyzing portfolio-level collateral requirements and the associated systemic risk implications. Second, we consider a market where payoffs are highly skewed (default probabilities can jump upward suddenly, and defaults can occur instantaneously), which implies that collateral plays a crucial role in allowing this market to function properly. Third, while existing studies focus mostly on the cross-sectional dimension of margins, we focus on both the cross-sectional and time-series variation. Fourth, we consider not only portfolio-specific risk measures, but also aggregate risk and funding measures as potential determinants of collateral – all factors that can play an important role in the amplification of aggregate shocks via the collateral-feedback channel. Fifth, we document that margins are best captured by using a combination of tail risk measures (maximum shortfall and short notional). As the short notional does not depend on historical probabilities and correlations nor on the state of the market, this shows that clearinghouses use a rule that is robust to the exact specification of the model for tail events.

2 Institutional Details and Data

In this section, we introduce the main institutional details of our setting. We also describe our data, and show our first finding: collateralization rates vary substantially across clearinghouse members and over time.

2.1 Clearinghouse Margining in Practice

Clearinghouses have significant discretion over modeling assumptions and parameters used to generate and justify margin requirements. They set them taking into account market conditions, the demand for trading, and collateral quality. In practice, margining rules involve a wide range of scenarios and simulations to arrive at a portfolio loss distribution, requiring the clearinghouses to make various modeling and statistical assumptions.

Most clearinghouses, *including ICC* (e.g., Ivanov and Underwood (2011)), state that their margins are broadly "set to cover five days of adverse price/credit spread movements for the portfolio positions with a confidence level of 99%", which we refer to as a 5-day 99% Value-at-Risk (VaR) margining rule. However, this is only a simplified description of their actual margining rules. Scenario-specific add-ons are often applied to produce the final margin requirement (CME Group (2010), ICE Clear US (2015)).[5] In particular, the margin requirement set by ICC is the sum of seven components. In addition to considering (i) losses due to credit quality (changing credit spreads), the methodology also considers losses due to (ii) changing recovery rates and (iii) interest rates. There are additional charges capturing (iv) bid-offer spreads, (v) large, concentrated positions, (vi) basis risk arising from different trading behavior of indices and their constituents. Finally, there is (vii) an additional jump-to-default requirement due to the potential large payouts associated with selling credit protection on

[5] These rule are not described in detail in publicly available documents, but they are available to CFTC officials. For ICC's public disclosure, see https://www.theice.com/publicdocs/clear_credit/ICE_CDS_Margin_Calculator_Presentation.pdf.

single name contracts. Similar to the Basel capital requirements, the ICC margin framework follows a bucket approach. It first calculates each of the seven components ("buckets"), and the final collateral requirement is simply the sum of these components. Importantly, even if clearinghouses were restricted to using VaR based margining rules, the confidence level, margin period of risk, and the distributional assumption of losses are inputs that give the clearinghouse significant freedom in setting the actual margin levels.

Overall, clearinghouses employ complex rules to determine the amount of required margins. These rules make it difficult to understand what the main economic determinants of collateral requirements are, partly because they depend on the interactions of several variables and calibration choices, and partly because they do not explicitly take into account variables that may still matter indirectly: for example, aggregate volatility or default risks do not directly enter into the calculations, but may still affect the collateral rule because they affect the scenarios used by the clearinghouse to simulate portfolio losses, or affect the choice of discretionary parameters. The goal of this paper is not to reverse-engineer this complicated procedure, but rather to identify and quantify the main economic determinants of collateral both in the level and in its panel variation.

2.2 Data and Summary Statistics

In this section, we provide an overview of our data and present descriptive statistics for the key variables. We construct a database of the entire universe of CDS positions cleared by ICE Clear Credit (ICC), for the period between May, 1 2014 and February, 20, 2019. ICC managed a significant fraction of the U.S. cleared CDS market, totaling 56% in 2015 and 2016, 52% in 2017, and 70% in 2018 and 2019. The absolute value of the cleared amount increased from nearly 9 billion in 2015, 2016, and 2017 to about 12 billion in 2018 and 2019.[6] Hence, it has always remained the largest CDS clearinghouse throughout our sample.

Clearinghouse collateral data: the Part 39 data set. The Dodd–Frank Wall Street Reform and Consumer Protection Act grants the U.S. Commodity Futures Trading Commission (CFTC) authority over Derivative Clearing Organizations (DCOs). As a result, major clearinghouses recognized as DCOs are required to report confidential swap trade data to CFTC on a daily basis. The data are collectively referred to as "Part 39 data," as the relevant rules and regulations are codified in Title 17, Chapter I, Part 39 of the Code of Federal Regulations. Part 39 data provides a complete overview of the centrally cleared swaps in the U.S..

We obtain clearing member data from the CFTC Part 39 database. Our data set consists of both positions data and account summary data for CDS trades cleared by ICE Clear Credit (ICC) (combined with ICEU, the European arm of ICE's CDS clearing, they account for over 90% of the cleared CDS market registered in the database). Our sample period covers nearly five years, from 05/01/2014 to 02/20/2019, for a total of 1203 business days.

[6]These percentages are computed from the quantitative disclosure statements of the three clearinghouses in the CDS market, i.e., ICE Europe ICC, and LCH CDSClear. These documents disclose the notional of cleared positions for the house accounts of their members.

CDS positions data. Credit default swaps (CDS) are credit derivatives used to trade the credit risk of a reference entity (a bond). The protection buyer (the long side of the contract) is obligated to pay a quarterly premium to the protection seller (the short side of the contract) up until contract maturity or the arrival of a credit event for the reference entity, whichever occurs earlier. Upon arrival of the credit event, the seller pays to the buyer the difference between the face value and the market value of the reference obligation. If the reference entity is a sovereign or corporate entity, the CDS is referred to as a single name CDS, and is uniquely identified by its coupon rate (the quarterly premium), maturity, reference bond seniority, and doc clause (which defines what constitutes a credit event), typically rolled out quarterly. If the reference entity is a weighted basket of bonds from various sovereign or corporate entities, it is referred to as an index CDS, typically rolled out semiannually. When components of the reference basket default, the protection seller pays a pro rata cash flow depending on the weights of the components. The index contract is then reversioned (i.e., the basket is updated), and coupon payments and the contract notional are reduced accordingly. A CDS index contract is identified by its notional, coupon rate, maturity, reference basket, version, and doc clause. Our data set includes both single name and index contracts.

The CDS position component of the Part 39 data set contains daily reports of each account's end-of-day (EOD) position in each cleared CDS contract, for each account used by a clearinghouse member (see below). For each day/account/contract combination, we observe long/short gross notional, EOD prices for the contract, the currency denomination and exchange rates, and the mark-to-market (MtM) value of the position.[7]

In the sample period considered, the most liquid CDS index contracts were mandatorily required to be cleared through a clearinghouse. Many other CDS contracts were cleared as well, but only voluntarily; as a result, our data presents only a partial view of the entire CDS market (which is inconsequential for the goals of this paper). Our data set includes CDS on 600 distinct reference entities, 590 single names and 10 indices. A total of 24,283 distinct contracts referencing these reference entities were cleared during our sample period (multiple CDS contracts are written on each underlying reference entity, since CDS contracts differ by seniority, doc clause, etc., as described above).

We adjust for changes in reference entities due to spin-offs, split-offs, or combined firms from mergers and acquisitions. After accounting for this, we are left with a total of 593 distinct reference entities.

Account and margin data. The cleared CDS market is dominated by a handful of *clearing members* who act as dealers to the outside market. Smaller clearing participants access the cleared market by becoming customers to clearing members. Each clearing member may have several accounts with ICC. The account is designated as a "customer account" if the account positions are taken on behalf of a customer, and designated as a "house" account if the positions are proprietary. Customer accounts are commingled; that is, they consist of multiple sub-accounts for many customers, and segregated customer specific data are not reported. We observe 45 accounts in total, each identified by a distinct

[7]We report in Appendix A a brief overview of standardized CDS price quote conventions.

clearing firm identification number. Of these accounts, 14 are designated as customer accounts and 31 are house accounts.

Many house accounts are set up to help with the processing of client trades, but have little open interest, as clearing members usually use one house account to hold the majority of their proprietary positions. We thus define a house account to be "auxiliary" if there are little to no positions associated with them. We refer to the remaining house accounts as "active" house accounts.[8]

For each clearing member account, the account summary portion of the Part 39 data set contains daily reports of EOD information. For each day/account combination, we observe the so-called *initial margin* requirement, the initial margin posted, the currency denomination and exchange rates, and the MtM value of the portfolio. The initial margin requirement is the level of collateral the clearinghouse demands from the account holders, whereas the margin posted is the actual amount that account holders supply; the two are almost always the same, or extremely close.[9]

It is important to emphasize that what is referred to as the initial margin in this market – the collateral requirement we study in this paper – is the collateral kept by the clearinghouse with the purpose of buffering against potential *future* losses in case clearing members default on their obligations. Despite the name "initial" margin, this margin is *not* just posted at initiation of a CDS position: instead, it is updated every day and it covers the entire portfolio of an account. It therefore corresponds directly to what is typically referred to as collateral requirement in standard theoretical models. We will use the terms initial margin, margin requirements, and collateral requirements interchangeably.[10]

We provide descriptive statistics for each of the three account categories (active house, customer, and auxiliary house) in Table 1. Table 1 reports, for each account, the pooled averages of the key variables over our sample period. Pooled averages are computed by averaging point observations within the account categories and across the sample time period. The table shows that we identify 15 active house accounts, which on average own more than 3,500 distinct contracts, for a total average notional position of \$120bn, and post an average amount of collateral to ICC of \$622m. Auxiliary accounts have activity levels that are one order of magnitude smaller.[11] Customer accounts have similarly small activity levels, but post higher amounts of collateral because of the lower diversification. In the remainder of the paper, we will focus exclusively on on the active house accounts. This is because customer accounts are commingled and margins information aggregated. Therefore, the observed margins are not associated with a specific institution's portfolio in our data set, so that we cannot

[8]To be precise, a house account is auxiliary if (i) the average gross notional is less than \$15 billion USD, (ii) the average number of distinct CDS contracts traded is less than 500, or (iii) the number of distinct CDS reference entities traded is less than 100. Our empirical conclusions are robust to changes in these thresholds.

[9]Margin requirements are reported separately in USD and Euro; we combine them using the appropriate exchange rate and express the total initial margin requirement of the entire portfolio in USD. The actual collateral posted is often reported entirely in USD, and covers both the USD and Euro margin requirements.

[10]All cleared contracts are marked to market daily, so that the change in *current* value of the portfolio is transferred to the clearinghouse by the next day. This transfer is referred to as *variation margin*, and is distinct from the initial margin as it does not represent a stock of collateral meant to cover for future changes in the value of the portfolio, but rather a cash flow reflecting the mark-to-market process. So the variation margin will play no role in our analysis, as it is different from what we typically refer to as collateral.

[11]In fact, five auxiliary house accounts had zero margins throughout, indicating no trading activity at all. We excluded these accounts when calculating descriptive statistics of auxiliary house accounts.

study the relationship between collateral posted and portfolio characteristics. We also exclude auxiliary house accounts because there are little to no positions associated with them.

To gain further insight into active house account margins, we compute the level of collateralization for cleared portfolios. We measure this with the *margin to net notional* ratio, which accounts for varying sizes of cleared portfolios; it is computed by taking the ratio of the initial margins requirement and aggregate net notional. The results are reported as a histogram in Figure 1. The figure shows that there is substantial heterogeneity in collateralization rate across time and accounts, from a low close to zero to a high above 20% (the observations that cluster around 15% all belong to one specific clearing member). In the remainder of this paper, we analyze what characteristics of the members' portfolios drive this significant heterogeneity in collateralization rates.

Market events during our sample period. Mandatory clearing of standardized CDS contracts was imposed only after the financial crisis. Thus, our data set does not include the years of the crisis in which the financial system (and the CDS market) underwent significant stress, which are particularly interesting times for understanding collateral requirements and their interaction with the broader economy. However, several important events happened during this period, among which commodity market events (e.g., the drop in oil prices in November 2014), currency market events (the plunge in the Euro in 2015), political events (the Brexit referendum and Trump's election in 2016), and credit events (Venezuela's default in 2017). Appendix B reviews in detail the most important events that occurred during our sample period.

We also remark that, while we do not have *positions* data going back to the financial crisis, we do observe CDS spreads going back to 2004. This allows us to do counterfactual simulations of portfolio returns, and to assess how well current collateral buffers can absorb shocks of magnitudes as large as those observed in 2008–2009.

3 Collateral requirements and the *Value-at-Risk* rule

In this section, we test whether the standard VaR margining rule is a good description of the collateral rule in the cleared CDS market. Using two different approaches, we show that that is not the case: actual collateral levels are orders of magnitude more conservative than predicted by the standard VaR rule.

3.1 Notation

Consider a set of dates $\mathcal{T} := \{1, \ldots, T\}$, a set of contracts $\mathcal{I} := \{1, \ldots, I\}$, and a set of market participants (clearing members) $\mathcal{N} := \{1, \ldots, N\}$. The portfolio held by participant n at time t is a vector $\mathbf{X}_t^n \in \mathbb{R}^I$, whose i-th component $X_{i,t}^n$ is the portfolio's *notional position* in contract i. $X_{i,t}^n$ can be positive or negative, depending on whether n has a long or a short position in the contract i.

We denote the end-of-day (EOD) quoted prices of cleared contracts at time t by $\mathbf{P}_t$, whose i-th component, $P_{i,t}$, is the EOD quoted price of contract i. As explained in detail in Appendix A, the market value of a position with one dollar notional and quoted price $P_{i,t}$ is simply $(1 - P_{i,t})$; we follow this conventional notation here, and express all quantities in terms of quoted prices $P_{i,t}$.

The mark-to-market value of the portfolio $\mathbf{X}_t^n$ held by market participant n at time t, denoted by $MtM_t(\mathbf{X}_t^n)$, can be computed as[12]

$$MtM_t(\mathbf{X}_t^n) = \sum_i X_{i,t}^n \, (1 - P_{i,t}) \tag{1}$$

The profit and loss ($P\&L$) between times t and $t + M$ (for a given time-t portfolio $\mathbf{X}_t^n$) is given by

$$\begin{aligned} \Psi_{M,t}(\mathbf{X}_t^n) &:= MtM_{t+M}(\mathbf{X}_t^n) - MtM_t(\mathbf{X}_t^n) \\ &= \mathbf{X}_t^n \cdot (\mathbf{P}_t - \mathbf{P}_{t+M}) \,. \end{aligned}$$

We use $VaR_t^{M,\alpha}(\cdot)$ to denote the $\alpha-$th quantile of the profit-and-loss (P&L) distribution of the portfolio $\mathbf{X}_t^n$ held by market participant n at time t over an $M-$day period starting at t.[13] Hence, Value-at-Risk (VaR) is defined by

$$\mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -VaR_t^{M,\alpha}(\mathbf{X}_t^n) | \mathcal{F}_t) = \alpha,$$

where $\mathcal{F}_t$ represents the information set available at time t. M is commonly referred to as the margin period of risk (or liquidation period), and $1 - \alpha$ is the confidence level.

3.2 Testing the *Value-at-Risk* rule

The standard VaR collateral rule assumed in the literature stipulates that collateral requirements (initial margins) at time t are set equal to $VaR_t^{M,\alpha}(\cdot)$, for a certain confidence level α and margin period of risk M. That is, under the VaR rule, initial margins are set as

$$H_0 : IM_t(\mathbf{X}_t^n) = VaR_t^{M,\alpha}(\mathbf{X}_t^n),$$

where $IM_t(\mathbf{X}_t^n)$ is the margin required by the clearinghouse at time t for holding portfolio $\mathbf{X}_t^n$.

We test the VaR hypothesis H_0 using two different approaches. The first approach can be applied if α and M are known. For instance, CDS clearinghouses typically claim that initial margins are set to cover 5-day losses with 99% confidence (Ivanov and Underwood (2011)), so $\alpha = 1\%$ and $M = 5$. If initial margins are set to be a certain conditional quantile of the returns distribution (say the 1% quantile), the fraction of times the portfolio loss exceeds the posted collateral is expected to be on average equal to that quantile (1% of the time). We refer to this approach as the *time-series* test of the VaR hypothesis; this is in fact equivalent to the "backtesting" procedure advocated by the Basel Accords (Hull (2012)).

[12]There is an additional adjustment factor for CDS indices that have been reversioned after the default of a component, which we omit here for ease of exposition but account for in our empirical analysis. The adjustment factor is smaller than one and accounts for a proportional decrease in effective notional due to the contract payout.

[13]By definition, this assumes that margins cannot be increased *during* the M days between t and $t + M$.

A second approach can be considered in the cases that α and M are unknown. Rather than testing the rule jointly across all counterparties, this test looks at whether the *same* VaR rule is applied to all counterparties, similar to the approach implemented by Gay et al. (1986). That is, no matter what α and M are, under VaR margining we would expect the same margining rule to be applied to all counterparties. This test reveals whether the proposed rule (VaR) is able to capture all portfolio- and counterparty-specific factors that are relevant for determining margin requirements. We refer to this as the *cross-sectional* test of the VaR hypothesis.

3.3 Time-series test of the VaR hypothesis

We start with the time-series test of the VaR hypothesis, using the null H_0 described by the clearinghouse: $M = 5$ days and $\alpha = 1\%$. Since $IM_t(\mathbf{X}_t^n)$ is observable in our data set, the main step of our analysis is to estimate the empirical distribution of the P&L of the portfolio over M days $(\Psi_{M,t})$. We consider two different approaches, one using the *realized* P&Ls during our sample period, 2014-2019; the other using *simulated* P&Ls for the period 2004-2019.

Using realized returns. The first tests asks the question: how often do we see a 5-day portfolio P&L $\Psi_{5,t}(\mathbf{X}_t^n)$ negative enough to exceed the collateral that had been posted against it (we refer to this as an *exceedance*)? If we define the *return on margin* as the ratio of the realized 5-day P&L to the initial margin, we can restate the question as: how often do we see a 5-day return on margin below -100%? Under the null of a VaR rule, we should see these exceedances approximately 1% of the time; we can the simply test the null by looking at the empirical proportion of exceedances in our data. Note that this test does not require us to observe or specify the information set $\mathcal{F}_t$ – since under the null, the initial margin fully takes it into account.

Figure 2 reports the empirical distribution of realized 5-day-ahead returns on margins. We compute returns on margins for each account/day in our sample and obtain $18,606$ observations. A few interesting patterns emerge from the figure. First, the dispersion of returns in our sample period is quite small relative to the amount of collateral posted. Second, the distribution of returns on margins does not exhibit distinctly heavy tails, despite the fact that several important events occurred during our sample period (see Appendix B). Third, and most relevant for the analysis of margining and losses, even the *most negative* return observed during our sample period is only around 40% of the collateral posted. That is, while under a 5-day 99% VaR rule we would expect to see exceedances in 1% of our sample (or about 66 account/days), no exceedances were actually observed. On average, posted margins were about *8 times larger* than the 99th percentile of losses experienced in this sample.

In order to formally test the hypothesis H_0, we perform a statistical test comparing the observed empirical frequency (0) to the one predicted by the model (α). The distribution of the test statistic (the difference between the empirical frequency and α) is derived in Appendix C. Not surprisingly, our statistical test strongly rejects the VaR null (p-value<0.001). Overall, this first test provides a strong indication that margins are set more conservatively than the standard VaR rule.

Counterfactual return estimates (historical simulation). Next, we use historical simulation methods to estimate the distribution of returns on margins over a longer time period. Such a time period covers the financial crisis, the most significant period of market distress since the Great Depression. More specifically, the idea of our counterfactual simulation is as follows. First, we collect CDS price data on all the CDS in our Part 39 sample, going back to 2004, from Markit and Bloomberg. For each day in our sample period (2014-2019), we observe the portfolio held by each member, $\mathbf{X}_t^n$. By looking at the history of (joint) price movements for all the constituents of those portfolios, we can then ask what the historical distribution of returns of that specific portfolio *would have been,* starting from 2004 and up until $t+5$, that is, including the realized 5-day return that begins on date t. The resulting distribution of P&L therefore includes the large price changes that occurred during the financial crisis, and incorporates the dramatic increase in correlations observed in those years. In other words, we backtest the ability of current (2014-2019) initial margins to prevent exceedances due to CDS price movements similar to those observed since 2004.

Implementing this test involves a few additional steps. Since there are new contracts issued and old contracts expiring every quarter, historical prices for a currently traded contract are not always available. To deal with this practical obstacle, we follow exactly the methodology of Duffie et al. (2015), designed specifically for this purpose. We first aggregate net exposures by name (reference entity), and then use the historical 5-year CDS spread on those names (for which we have accurate spreads data) to compute counterfactual returns for all days for which CDS spreads are available, starting from 2004/01/01. We review the details of the methodology in Appendix D.[14]

Using this approach, for each observed portfolio held by account n on each day t, we compute all returns on margins that could have occurred to that portfolio in each 5-day window since 2004 and up to $t+5$. This procedure yields a total of $61,532,415$ simulated 5-day returns on margins.

We report the distribution of counterfactual returns on margins in Figure 3. Due to the large number of observations clustering around zero, we only display the histogram in the range $[-50\%, +50\%]$ in Figure 3a, and zoom in on the left tail of the histogram in Figure 3b. We see that the distribution is sharply peaked, and that most returns on margins lie between $\pm 20\%$. The frequency of returns decreases rapidly as we move away from the mean.

When we consider counterfactual returns, we observe a very small number of margin exceedances: the portfolios held during 2014-2019 would have sometimes experienced losses larger than the posted collateral if prices moved as they did during the financial crisis, but only in 0.004% of all 5-day periods. This fraction is two orders of magnitude smaller than the 1% predicted by the standard VaR. On average, posted margins were 7 times higher than the empirical 99th percentile of 5-day simulated losses starting in 2004.

We also extend the formal statistical test of the previous section to include all the historical counterfactual returns for each portfolio held in each day by each account. As before, the test compares the empirical frequency of margin exceedances (0.004%) with that predicted by the VaR rule (1%). To

[14] A day is included in our data analysis only if prices are observed for at least 250 out of the 593 reference entities; this filter excludes few days in the early part of the sample for which price information was not uniformly available across contracts.

fully account for potential time-series and cross-sectional correlation of the errors, we double-cluster the standard errors of the test statistic at both the day and the account levels (as described in Petersen (2009)). Appendix E reports the details of the test statistic and its distribution. Not surprisingly, the test again strongly rejects the null of VaR (p-value<0.001).

To sum up, both versions of the time-series test strongly reject the null that collateral is set by a 5-day, 99% VaR rule. We observe no exceedances in the period 2014-2019, and historical simulations that include the financial crisis imply exceedances only in 0.004% of cases. Instead, we find that posted collateral levels are one order of magnitude (700%-800%) higher than the 99% percentile of realized and simulated losses. The collateral rule in this market appears *very* conservative.

3.4 Cross-sectional test of the VaR hypothesis

We conclude by performing the cross-sectional test of the VaR hypothesis, that requires no assumptions about the confidence level (α).[15] Recall that the margining rule requires that

$$\mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)) = \alpha \quad \text{for all}\, n,$$

that is, the exceedance ratios should be the same across clearing members. We analyze the validity of this rule by testing that the empirical frequencies of exceedances across accounts are the same, using a G-test described in Appendix F.

The test strongly rejects the null (p-value <0.001) that exceedance ratios are the same across clearing members. There is therefore direct evidence against equality of exceedance probabilities, and thus against the null hypothesis that there exists a VaR rule which can explain observed initial margins for all counterparties.

4 The Determinants of the Collateral Rule: Portfolio Risk and Market Risk

The previous section has shown that a simple VaR rule fails to capture observed margins along different dimensions. This suggests that other variables (both at the level of the individual member's portfolio and marketwide) might enter the collateral rule.

In this section, we consider two groups of potential explanatory variables. *Portfolio variables* are those that are specific to the portfolio that an account holds with the clearinghouse, and are conventionally used to measure the risk of positions. These include, in addition to VaR, expected shortfall (ES), maximum shortfall (MS), aggregate net notional (AN), aggregate short notional (AS), and the volatility of the portfolio (SD). We describe these in detail below. *Market variables* are those that are determined by market forces, and include the clearing members' CDS spreads, the LIBOR-

[15] We perform the test using the counterfactual returns over the period 2004-2019. The results obtained using only the period 2014-2019 are trivial: there were no exceedances for any account, so a VaR with 100% confidence level fits the cross-section perfectly.

OIS spread, the average clearing member CDS spread, and aggregate volatility as measured by VIX. Table 2 summarizes the full list of variables for convenience.

4.1 Description of portfolio and market variables

Portfolio variables. Let Ω_k denote the set of CDS contracts with reference entity k (that differ by maturity, doc clauses, etc...). For each reference entity k, net notionals are defined by

$$Y^n_{k,t} := \sum_{i \in \Omega_k} X^n_{t,i}.$$

The aggregate net notional AN^n_t is then defined as

$$AN^n_t := \sum_{k \in K} |Y^n_{k,t}|,$$

i.e., as the sum of absolute net notional values across reference entities. The aggregate *short* notional, AS^n_t, is instead defined as

$$AS^n_t := \sum_{Y^n_{k,t} < 0} |Y^n_{k,t}|.$$

The aggregate short notional plays an important role because of the highly asymmetric nature of CDS payoffs. While the premium leg makes fixed payments, the protection leg (i.e. the short side of the CDS position) is exposed to jump-to-default risk. Such an asymmetry induces strong left skewness in the payoff function of a short position, which is why it is typically the seller of protection that posts the most collateral.

Duffie et al. (2015) propose an initial margin model alternative to VaR that focuses on extreme tail risks, by combining different portfolio variables:

$$DSV^n_t = MS_5(\mathbf{X}^n_t) + 0.02 \times AS(\mathbf{X}^n_t), \tag{2}$$

where $MS_M(\cdot)$ represents the maximum shortfall of the portfolio for a $M-$day margin period of risk, computed using historical simulations. We refer to this as the DSV model.[16] This margin model incorporates both the maximum historical loss, and a 2% "short charge". We evaluate here the DSV model (which was not estimated, but assumed exogenously) together with the other variables.

Finally, we also consider a modified version of the DSV model, MDSV:

$$MDSV^n_t = w_1 \times MS_5(\mathbf{X}^n_t) + w_2 \times AS(\mathbf{X}^n_t), \tag{3}$$

[16] While Duffie et al. (2015) compute maximum shortfall for a fixed look-back period of 1000 days, we use a longer price series starting from the year 2004. As both ours and their time series data cover the years of the crisis, when the largest losses occurred, the difference between the initial margins computed by the two approaches is negligible. We also explore robustness with respect to this choice in Section 4.4.

that estimates the weights w_1 and w_2 from the data instead of using the ones calibrated by Duffie et al. (2015). The corresponding estimates are reported in Table 4, and given by $w_1 = 1.121$ and $w_2 = 0.025$.

We estimate the empirical distribution of the simulated series $\psi := \left\{\hat{\Psi}_{5,\mathrm{t}}(\mathbf{X}_t^n)\right\}_{t=1}^T$ of 5-day ahead P&L via the historical simulation approach discussed in Section 3.3. Using the empirical distribution, we form estimates of volatility (SD, standard deviation), 99% Value-at-Risk (VaR), expected shortfall (ES, the expected loss conditional on exceeding the VaR), and maximum shortfall of the portfolio (MS, the maximum loss experienced in simulations). All portfolio variables are in millions of USD to conform with the level of initial margins.

Market variables. We collect from Bloomberg time series data of the 3-month Overnight Index Swap (OIS) spread, the 3-month USD LIBOR rates, clearing member 5-year CDS spreads, and aggregate volatility as measured by VIX. CDS spreads on the members themselves can be interpreted as a measure of individual member counterparty risk, or potentially as a measure of funding cost for a clearing member, because higher spreads make it more costly for a member to borrow funds. We include in our analysis both individual CDS spreads and average CDS spreads of the members. As an alternative to the average credit spread $ACDS_t$, we also consider the LIBOR-OIS spread to control for market distress. The LIBOR-OIS spread

$$LOIS_t := LIBOR_t - OIS_t.$$

is typically viewed as a measure of financial sector stress, capturing mainly the interest rate differential between uncollateralized and collateralized loans. All market variables are recorded in basis points (bps) to conform with market convention.

Summary statistics. Table 3 displays summary statistics of our key portfolio variables and initial margins, in millions of USD. Note that each portfolio variable (like SD, VaR, etc) is computed separately for each time t and each member n; in the table, in addition to the pooled mean and standard deviations (which we also refer to as dispersions) across all n and t, we also describe other measures of portfolio dispersion in the time series and in the cross-section.

We observe that all measures of dispersion increase in the order of extreme tail risk captured. That is, as more weight is put into the tail of the distribution, there is more variability in the computed measures both across time and across accounts. The measure with the smallest value is the standard deviation (SD), followed in order by Value-at-Risk (VaR), expected shortfall (ES), and maximum shortfall (MS).

Consistent with the results of the previous section, Table 3 shows that VaR is about one order of magnitude smaller than initial margins on average. Interestingly, even ES and MS, that do capture more extreme tails, are much smaller than the posted margins – suggesting that these variables alone should not be able to explain the observed level of margins either. On the other hand, the table also shows that the DSV model matches well not only the level of margins, but also all the dispersion measures; the modified DSV model does even better.

Table A.1 in the Appendix reports summary statistics of our key market variables and initial margins, in basis points and millions of USD.

4.2 Margins and Portfolio-specific Risks

In this section we perform a panel analysis relating observed margins to portfolio variables. In particular, we estimate the following panel regression model with time and account fixed effects:

$$IM_t^n = \alpha^n + \eta_t + \sum_{v \in PV} \beta_v v_t^n + u_t^n, \tag{4}$$

where PV is the set of portfolio variables included in the panel regression. Note that in the model specification of Eq. (4), the regression coefficients do not depend on the specific clearing member n, a necessary condition if margining rules are implemented uniformly across accounts.

We start by examining the set of portfolio variables to include in the regression. First, we note that aggregate net notional (AN) serves primarily as a measure of portfolio size. As portfolio size is already accounted for by risk measures such as VaR, MS and AS, all expressed in dollar units, we drop this variable from our regression.[17] Second, we perform a check for multicollinearity, reported in Appendix Table A.2. The table shows that VaR explains more than 96% of the variation of both expected shortfall and standard deviation. This strongly points to multicollinearity issues, and thus we leave out standard deviation and expected shortfall in our panel model specification.

Our final set of portfolio variables includes Value-at-Risk, maximum shortfall, aggregate short notional, and the DSV model given in Eq. (2); we also construct the MDSV variable by adjusting the DSV weights (0.5 on MS, 0.02 on AS) to maximize the in-sample fit (see (3)). We report all the results in Table 4. We use double-clustered standard errors (by time and account) as in Petersen (2009), thus accounting for potential correlation in the errors, both within each account over time, and across accounts within each day. The signs of all the coefficients are in line with intuition: because larger values for each of the explanatory variables point to a riskier portfolio, all coefficients are expected to be positive.

Columns (1) and (2) of Table 4 show that Value-at-Risk alone can explain 58% of the variation in initial margins, and 79% of the variation if fixed effects are added to the regression. The estimated slope coefficient, however, is much higher than unity in either case. In particular, a multiplier of at least 400% is needed for the regression fit, again showing that collateral requirements are set much more conservatively than what would be implied by the conventional 5-day 99% VaR rule. Columns (3) and (4) introduce maximum shortfall (MS) and aggregate short notional (AS) as explanatory variables in conjunction with Value-at-Risk. The addition of these variables brings the R^2 to 74%, even without fixed effects. Moreover, the magnitude of the VaR slope coefficients are much closer to unity once these variables are included. Our results therefore show that initial margins depend on risk

[17] We have conducted a regression analysis including AN as an explanatory variable, and found that, qualitatively, our results are largely unaffected.

characteristics which cannot be captured only by VaR, and in particular more extreme tail risks.

It is worth remarking that while both MS and AS relate to the extreme tails of the distribution, they differ significantly in their nature. MS represents the maximum *experienced* loss in historical simulations. As such, it depends on the experienced realization of shocks and historical correlations across portfolio components. AS, instead, captures the maximal *potential* loss if all short CDS positions jump to default simultaneously, and recovery rates are zero; in this sense, it represents a theoretical worst-case scenario for the potential loss, and represents a measure of tail risk that is less sensitive to the specification of the loss distribution (as well as of the correlations between portfolio components).

Motivated by the DSV model that only features MS and AS, we drop Value-at-Risk as an explanatory variable in columns (5) and (6), and find that there is little loss in explanatory power compared to columns (3) and (4). Maximum shortfall is positively correlated with Value-at-Risk, and dropping Value-at-Risk increases the statistical significance of the maximum shortfall loading.[18] Interestingly, the aggregate short notional coefficient estimate remains very stable (in the 1.5–2.5% range) and highly significant for all the estimated models.

Columns (7)–(12) investigate the usefulness of the DSV initial margin model in explaining empirically observed margins. Columns (7) and (8) show that the DSV model captures a significant portion of the variation in initial margins; it strongly outperforms Value-at-Risk in terms of explanatory power (columns (1) and (2)). The significance of the DSV slope coefficient persists when we introduce Value-at-Risk, and the explanatory power remains roughly the same, showing again that Value-at-Risk has little explanatory power beyond that already captured by DSV. The overall explanatory power improves marginally when we consider MDSV (columns (11) and (12)), whose coefficients are based on the estimates reported in columns (5) and (6) (so we should expect a similar explanatory power). Again, Value-at-Risk still has little explanatory power beyond that already captured by our modified DSV model.

To sum up, our empirical results provide strong support for the DSV model (in which the parameters were calibrated, not estimated), against alternatives such as VaR. More generally, the results show that tail risk variables like maximum shortfall and aggregate net notional work significantly better than VaR in explaining the observed collateral rule.

While our proposed measures capture significant variation in initial margins, the explanatory power is obviously not 100%. On the one hand, that's to be expected, since we are estimating a simple approximation of the true collateral rule, which is far more complex. On the other hand, it leaves open the possibility that other factors might help explain the margin setting, which are not captured by standard portfolio risk measures.

4.3 Funding Cost, Collateral Rates and other Market Variables

In this section we incorporate market variables into our panel analysis and assess their ability to explain margin requirements. The included market variables are chosen to capture variation in margins that

[18] Given that maximum shortfall corresponds to the realized maximal loss, it is not surprising that its estimate can be noisy, which can sometimes affect its statistical significance.

is due to changes in default risk and funding costs. We consider the following panel regression model:

$$IM_t^n = \alpha^n + \eta_t \sum_{v \in PV} \beta_v v_t^n + \sum_{v \in MV} \beta_v v_t^n + u_t^n, \tag{5}$$

where $PV = \{VaR, MS, AS, DSV, MDSV\}$ and $MV = \{LOIS, CDS, ACDS, VIX\}$ are, respectively, the portfolio and market variables included in the panel regression. Because market variables are often not account-specific (e.g. the LIBOR-OIS spread), time fixed effects cannot be included in the regression. Thus, in this section we only consider time fixed effects when non-account-specific variables are excluded.

We estimate the model in Eq. (5) using least squares regressions, choosing initial margins as the dependent variable and portfolio and market variables as explanatory variables. The results with double-clustered (by account and time) standard errors are reported in Table 5.

Columns (1) and (2) report the results when member CDS spreads, average CDS spreads, and the VIX are included, and the portfolio variables VaR and MDSV are controlled for. There is a small but significant increase in explanatory power compared to models that include only portfolio variables (Table 4). The VIX appears to have a strong effect on margins, whereas average CDS spreads appear not significant. There is weak evidence that funding costs, proxied by the individual members' CDS spread, matter. Overall, while market variables do seem to influence margin levels, their effects seem to be much smaller than that of portfolio variables.

Columns (3) and (4) report our results when we replace the average CDS spread with the LIBOR-OIS spread. There is almost no change in explanatory power and the loadings on the portfolio variables remains very similar; the LIBOR-OIS spread appears insignificant. Columns (5) and (6) report the results when maximum shortfall and aggregate short notional are used together instead of MDSV. The increase in explanatory power is small when compared to the results in columns (1) and (2). Interestingly, the estimated coefficient for aggregate short notional is significant and in the range of 1-2%, again showing the robustness of our previous results in Table 4 (MS is borderline insignificant, with a p-value of 0.106 and a magnitude comparable with that of the previous table).

Across columns, we find that among all market measures, the VIX is the only one that robustly seems to affect the collateral rule. The magnitude of the VIX's effect, however, is substantial. In our estimates, a one-point increase in the VIX increases required margins by $6.2 million. During crisis episodes movements of the VIX of even 50 points are possible, these estimates imply large potential effects on prices and systemic stability through the collateral channel, consistent the model of Brunnermeier and Pedersen (2009).[19]

4.4 Robustness

Two of the portfolio variables (Value-at-Risk and maximum shortfall) used in our analysis were based on P&L generated from our entire sample of credit spreads. Because our data set covered the financial

[19] Of course, these estimates are obtained in a relatively calm period, so it is hard to extrapolate the estimates to times of crisis; however, they give a sense of the magnitude of these effects.

crisis, the risk measures captured extreme movements and may thus be viewed as overly conservative for estimating portfolio losses. In Appendix G, we consider using only the last 1000 days (approximately 4 years) of credit spreads data to generate P&L, as in Duffie et al. (2015). The results of our analysis presented in the appendix show that the results remain qualitatively similar to those reported in Tables 4 and 5.

5 Conclusions

We study the empirical determinants of collateral requirements in a large market in which counterparty risk plays an important role – the cleared CDS market. Our analysis exploits the availability of a unique data set on clearing members' portfolio exposures and associated margin levels. Margins in this market are set at the portfolio level rather than at the individual security level; this allows us to study how risk measures like VaR and other portfolio characteristics affect margins. We also study how market variables – in addition to portfolio variables – affect collateral requirements, highlighting the implications of our findings for models of the collateral feedback channel.

We document four novel empirical results on the collateral rule in this market. First, we show that there is large variability in the collateralization rate across accounts and over time, suggesting corresponding variability in the risk characteristics of the clearinghouse members' portfolios.

Second, we show that collateral in this market is set much more conservatively than what would be implied by a standard VaR rule – a rule that is at the core of many theoretical models with collateral constraints and that clearinghouses themselves state they use. In fact, the amount of collateral appears about 7-8 times larger than what would be needed to cover 99% of 5-day losses (which is the standard for VaR in this context).

Third, we find that other portfolio variables dominate VaR in explaining the time-series and cross-sectional variation in margins. In particular, two measures of extreme tail risk suggested by Duffie et al. (2015), maximum shortfall (the largest experienced portfolio loss) and aggregate short notional (the notional amount held by a member in short CDS positions, representing the theoretical maximal loss on all CDS positions simultaneously) dominate VaR in explaining the panel variation of required collateral. These two variables alone account for almost 75% of the entire variation in margins in our panel. Whereas empirically VaR is strongly related to simple portfolio volatility, these measures capture much more extreme tail risks, that are more strongly related to jump-to-default risk and less related to volatility. These measures are conceptually (and empirically) quite different from VaR.

Finally, we find that shocks to some market variables (particularly the VIX) increase the total amount of required collateral, even after controlling for portfolio-level risks, suggesting that average margin levels vary with aggregate market conditions beyond what individual portfolio measures capture.

Our findings have several implications for theoretical models of the collateral feedback channel. First, the fact that extreme tail risk measures explain margins better than VaR indicates that the clearinghouse is worried about more extreme losses than what the standard VaR captures. Given that standard VaR is highly correlated with volatility, it suggests that the margin spiral mechanisms examined in many theoretical models (in which the collateral rule is exogenously specified) could operate through the direct effects of shocks on the extreme tail of the distribution, rather than through changes in volatility. In other words, our results suggest that collateral levels may respond little to small changes in risks (like an increase in the variance of the portfolio), but may spike if the probability of an extreme event increases or the worst-case-scenario worsens. This nonlinearity can potentially play an important role in general equilibrium models, amplifying the largest shocks but dampening moderate-sized shocks.

Second, our empirical results are consistent with some of the key results of the theoretical literature on endogenous collateral, like Geanakoplos (1997) and Fostel and Geanakoplos (2015). These models cannot directly be mapped to our data (because they counterfactually assume a binomial world and feature no default in equilibrium); nevertheless Du et al. (2020) have recently developed a new model of the endogenous collateral equilibrium, based on Simsek (2013), that is specifically tailored to the CDS market. In this model, trade is motivated by difference in beliefs, that also determine the equilibrium collateralization rate. The model can rationalize the observed empirical patterns of extremely high collateralization rates with disagreement about the extreme events in which members would default on their obligations to the clearinghouse.

Finally, collateral requirements are directly affected by market conditions: increases in aggregate risks directly induce an increase in collateral requirements, holding the portfolios fixed. Our empirical analysis therefore documents the existence of two channels for the amplification of fundamental shocks (studied, for example, in Brunnermeier and Pedersen (2009)): at the portfolio level, where an increase in perceived tail risk following a shock may affect the member's margin requirement; and at the macro level, where an increase in aggregate risk can increase the collateral requirements of all members.

Taken together, our empirical findings and theoretical results can provide guidance for building empirically grounded models of the collateral feedback channel.

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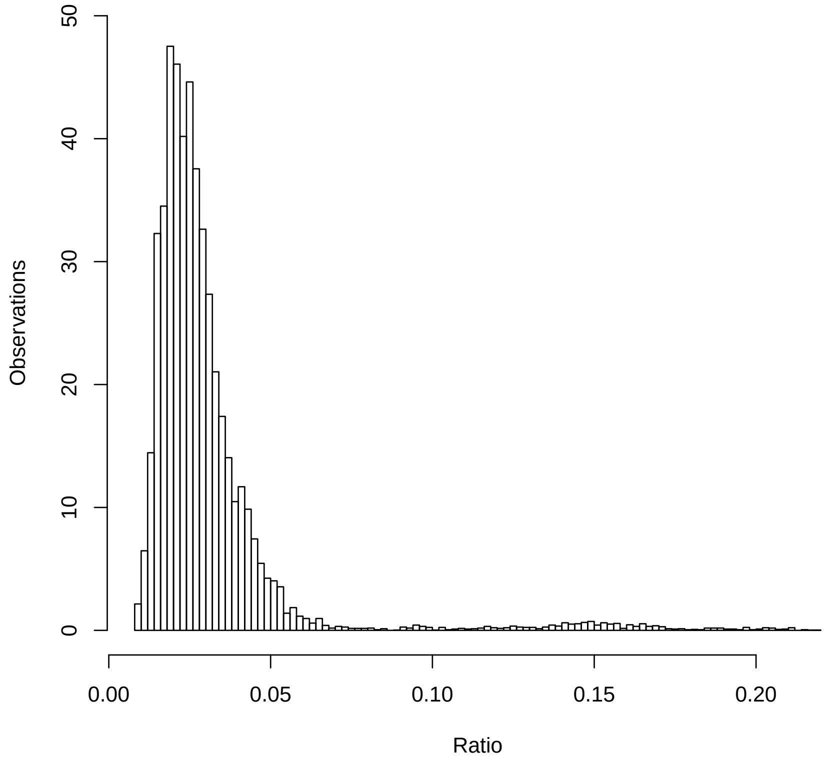
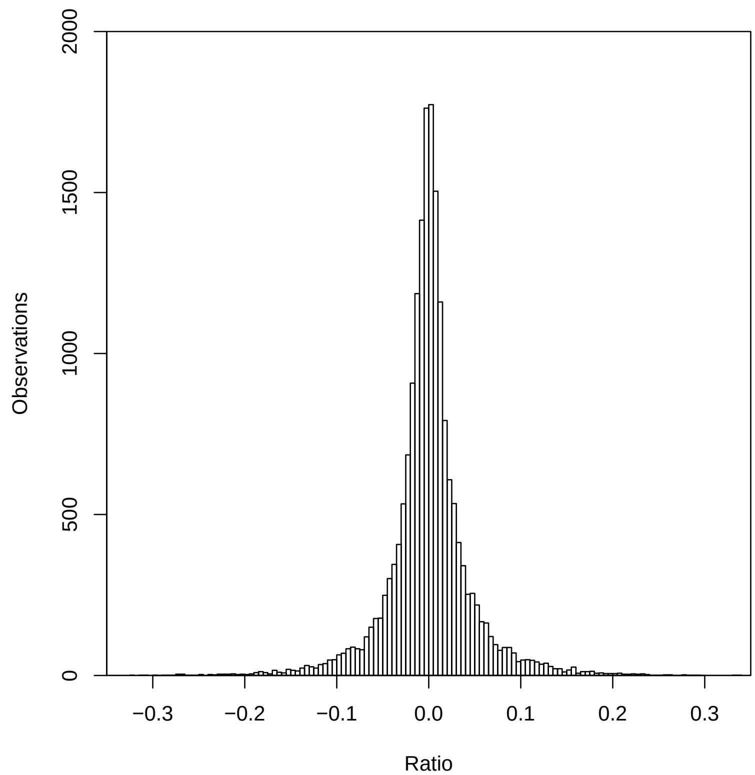
Figure 1: Histogram of margin/notional ratio observations.



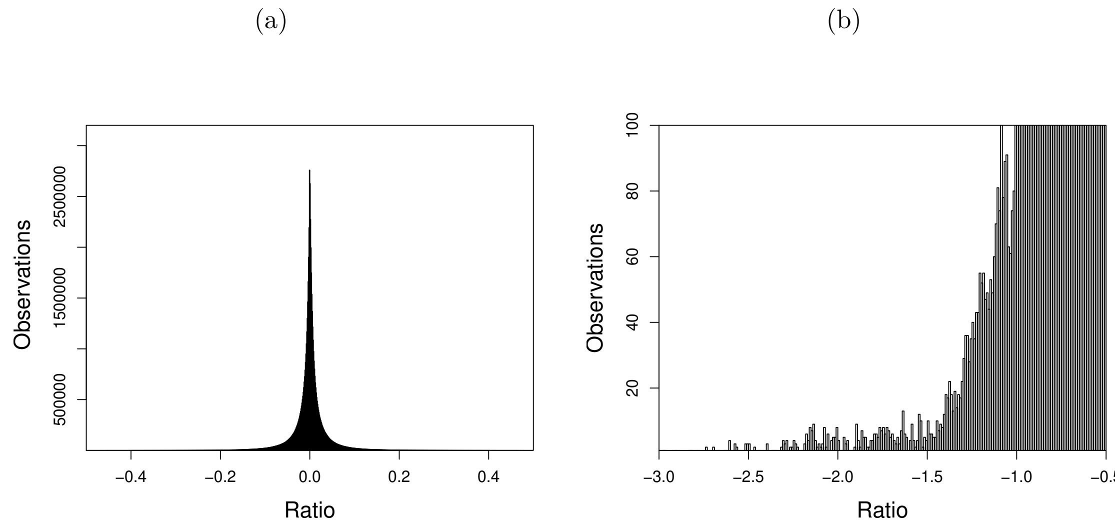
Note: For each active house account/day combination, we compute the margin to notional ratio by dividing initial margins with aggregate net notional. We obtain the aggregate net notional by computing the net notional for each reference name and then summing the absolute net notional values across names. The figure reports the histogram of margin/notional ratio across all 18,615 account/day observations.

Figure 2: Histogram of realized return on margins for cleared portfolios.



Note: We compute the realized 5-day ahead returns on margins as the 5-day ahead $P\&L$ divided by posted margins. We compute this for each account/day and obtain 18,540 observations. The figure plots the histogram of the return on margins.

Figure 3: Histogram for historically simulated return on margins (left), with zoom on left tail (right)



Note: The figure shows the histogram of simulated returns on margins. Due to the large number of observations clustering around zero, we only display observations between ±50% in Figure 3a, and report the left tail of the histogram in Figure 3b. We use the DV01 formula to approximate the 5-day ahead $P\&L$ with the product of net exposures to a reference name and the change in 5-year credit spreads for rolling 5-day windows from 2004/01/01 up to each position date $t + 5$ days (for all business days from 2014/05/01 to 2019/02/20), adjusted for an average duration of $d = 3$. We compute this for each account/day/historical 5-day window and obtain 61,532,415 observations.

Table 1: Descriptive statistics for different account categories.

	Active House	Customer	Auxiliary House
Number of accounts	15	14	11(16)†
Number of contracts	3587.3	287.8	165.4
Number of names	238.8	86	57.8
Gross notional (billions $)	119	26.4	8.9
Initial margins (millions $)	622.2	865.7	54.5

Note: The table reports the pooled averages of key variables within our data set depending on account type over our sample period. The number of contracts/names for each account counts those contracts/names for which the account has a non-zero position. Gross notional is computed by summing the absolute notional exposure for all contracts in the account. Margins are computed by summing the USD margin requirement and the Euro margin requirement, after adjusting for the historical exchange rates.

†Five auxiliary house had zero margins throughout, indicating no trading activity at all. We excluded these accounts when calculating auxiliary house account descriptive statistics.

Table 2: Portfolio and market variables.

Notation	Units	Definition
ψ	millions \$	Empirical 5-day distribution of profit and losses for a portfolio
IM	millions \$	Observed initial margins posted for a portfolio
Y	millions \$	Net notional aggregated over reference names for a portfolio
Portfolio Variables		
VaR	millions \$	1 percent quantile of ψ
ES	millions \$	Average of profit and losses less than equal to VaR
MS	millions \$	Minimum of ψ
SD	millions \$	Sample standard deviation of ψ
AN	millions \$	Aggregate net notional (by reference entity) of portfolio
AS	millions \$	Aggregate short notional (by reference entity) of portfolio
DSV	millions \$	Initial margin estimate used by Duffie et al. (2015), equal to $MS + 0.02 \times AS$
$MDSV$	millions \$	Adjusted initial margin from DSV, with estimated weights
Market Variables		
OIS	bps	End of day 3-month Overnight Index Swap spreads
$LOIS$	bps	End of day 3 month USD LIBOR-OIS spreads
CDS	bps	End of day market quote for clearing member specific 5-year CDS spread
$ACDS$	bps	Average end of day clearing member 5-year CDS spread
$DCDS$	bps	Deviation of end of day 5-year CDS spreads from the average, equal to $CDS - ACDS$
VIX	bps	End of day CBOE Volatility Index

Note: This table displays the key variables and notation we use in our regression analyses. Portfolio variables are those that are specific to the portfolio that an account holds with the clearinghouse, and are conventionally used to measure the risk of positions. Market variables are those that are determined by market forces. Portfolio variables estimated from the empirical distribution via the historical simulation method outlined in Section 3.3 include Value-at-Risk, expected shortfall, maximum shortfall, and standard deviation. Portfolio variables estimated directly from positions include aggregate net notional and aggregate short notional. We record portfolio variables in millions USD. Market variables include the Overnight Index Swap (OIS) spread, the LIBOR-OIS spread, clearing member CDS spreads, the average clearing member CDS spread, and the aggregate volatility as measured by VIX. We record market variables in basis points.

Table 3: Initial margins and portfolio variables summary statistics

Summary Statistic	*Portfolio variables, in millions \$* In. Margins ($IM_{n,t}$)	Portfolio SD ($SD_{n,t}$)	VaR ($VaR_{n,t}$)	Exp. Shortfall ($ES_{n,t}$)	Max Shortfall ($MS_{n,t}$)
Pooled mean (over all n and t): $\mu(x_{n,t})$	622.2	23.0	71.1	103.4	198.9
Std. deviation (over all n and t): $\sigma(x_{n,t})$	408.2	15.7	49.8	70.1	142.8
Time-series variation of cross-sectional averages: $\sigma(\bar{x}_t)$	83.5	2.1	5.9	10.4	38.6
Mean cross-sectional dispersion: $\mu(\sigma_t(x_{n,t}))$	407.0	16.0	50.9	71.1	137.7
Cross-sectional dispersion of time-series averages: $\sigma(\bar{x}_n)$	346.3	13.0	40.9	57.2	109.9
Mean time-series dispersion: $\mu(\sigma_n(x_{n,t}))$	208.4	8.2	26.2	37.6	84.6

Summary Statistic	*Portfolio variables, in millions \$* Aggr. Notional ($AN_{n,t}$)	Aggr. Short Notional ($AS_{n,t}$)	Duffie et al. ($DSV_{n,t}$)	Modified DSV ($MDSV_{n,t}$)
Pooled mean (over all n and t): $\mu(x_{n,t})$	24,070.1	11,663.6	432.2	514.5
Std. deviation (over all n and t): $\sigma(x_{n,t})$	15,223.1	8,864.4	292.4	349.2
Time-series variation of cross-sectional averages: $\sigma(\bar{x}_t)$	1,991.8	1,786.6	66.4	78.6
Mean cross-sectional dispersion: $\mu(\sigma_t(x_{n,t}))$	15,588.3	8,774.5	290.6	347.2
Cross-sectional dispersion of time-series averages: $\sigma(\bar{x}_n)$	14,529.3	6,653.8	235.9	281.2
Mean time-series dispersion: $\mu(\sigma_n(x_{n,t}))$	5,335.7	4,977.6	157.8	188.2

Note: Table 3 displays summary statistics of our key portfolio variables and initial margins, in millions of USD. Definitions of portfolio variables are reported in Table 2. In addition to the pooled mean and standard deviations (dispersions), we report panel statistics that describe properties of panel variables both across accounts and time. In particular, for panel data $x_{n,t}$, we define

$$\bar{x}_t := \frac{1}{N}\sum_{n=1}^{N} x_{n,t}, \quad \bar{x}_n := \frac{1}{T}\sum_{t=1}^{T} x_{n,t}, \quad \sigma_t^2(x) := \frac{1}{N-1}\sum_{n=1}^{N}(x_{n,t}-\bar{x}_t)^2, \quad \sigma_n^2(x) := \frac{1}{T-1}\sum_{t=1}^{T}(x_{n,t}-\bar{x}_n)^2.$$

Above, we refer to $\sigma(\bar{x}_t)$ as the *time-series variation of cross-sectional averages*, $\bar{\sigma}_t(x_{n,t})$ as the *mean cross-sectional dispersion*, $\sigma(\bar{x}_n)$ as the *cross-sectional dispersion of time-series averages*, and $\bar{\sigma}_n(x_{n,t})$ as the *mean time-series dispersion*.

Table 4: Regression results for explaining initial margins with portfolio variables

	Dependent variable:											
	Initial margins (IM) - Daily Frequency											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Value-at-Risk (VaR)	6.237***	4.115***	1.530*	1.654**					1.129	1.109*	1.218*	1.189**
	(1.065)	(1.081)	(0.899)	(0.689)					(0.733)	(0.589)	(0.732)	(0.553)
Maximum shortfall (MS)			0.751*	0.431	1.121***	0.824***						
			(0.430)	(0.322)	(0.369)	(0.274)						
Aggregate short notional (AS)			0.023***	0.017***	0.025***	0.019***						
			(0.003)	(0.003)	(0.004)	(0.004)						
Duffie et al. model (DSV)							1.198***	0.892***	1.036***	0.758***		
							(0.095)	(0.084)	(0.078)	(0.124)		
Modified DSV model (MDSV)											0.858***	0.627***
											(0.060)	(0.099)
Number of Observations	18615	18615	18615	18615	18615	18615	18615	18615	18615	18615	18615	18615
Adjusted R^2	0.580	0.790	0.745	0.849	0.737	0.843	0.736	0.842	0.742	0.846	0.744	0.847
Account Fixed Effect	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*
Time Fixed Effect	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*

*p<0.1; **p<0.05; ***p<0.01
Two-Way Clustered Standard Errors (by Time and Account)

Note: We perform least squares regressions using initial margins as the dependent variable and portfolio variables as explanatory variables. Two-way clustered (by time and account) standard errors are reported in parentheses and used for the significance tests. We consider both the case with and without (time and account) fixed effects.

Table 5: Regression results for explaining initial margins with portfolio and market variables

	Dependent variable:					
	Initial margins (IM) - Daily Frequency					
	(1)	(2)	(3)	(4)	(5)	(6)
Value-at-Risk (VaR) 1%	1.507**	1.059*	1.544**	1.024**	1.939***	1.487**
	(0.637)	(0.577)	(0.625)	(0.497)	(0.750)	(0.624)
Modified DSV Model (MDSV)	0.825***	0.658***	0.818***	0.661***		
	(0.047)	(0.086)	(0.039)	(0.067)		
Maximum Shortfall (MS)					0.651	0.486
					(0.403)	(0.341)
Aggregate Short Notional (AS)					0.023***	0.018***
					(0.003)	(0.003)
CBOE Volatility Index (VIX)	0.062***	0.068***	0.062***	0.062***	0.060***	0.067***
	(0.017)	(0.016)	(0.017)	(0.015)	(0.017)	(0.016)
Member CDS Spread (DCDS)	1.880*	0.290	1.888*	0.286	2.002**	0.321
	(1.053)	(0.855)	(1.030)	(0.881)	(0.962)	(0.767)
Average CDS Spread (ACDS)	0.186	−0.204			0.093	−0.291
	(1.035)	(0.976)			(1.103)	(0.981)
LIBOR-OIS Spread (LOIS)			0.458	0.785		
			(1.184)	(1.435)		
Number of Observations	18615	18615	18615	18615	18615	18615
Adjusted R^2	0.761	0.844	0.761	0.844	0.764	0.846
Account Fixed Effect	*N*	*Y*	*N*	*Y*	*N*	*Y*
Time Fixed Effect	*N*	*N*	*N*	*N*	*N*	*N*

*p<0.1; **p<0.05; ***p<0.01
Two-Way Clustered Standard Errors (by Time and Account)

Note: We perform least squares regressions using initial margins as the dependent variable and portfolio and market variables as explanatory variables. Two-way clustered (by time and account) standard errors in parentheses are reported and used for the significance tests. We consider both the case of with and without fixed effects. Because market variables are often dependent only on time, we consider only account fixed effects when such variables are introduced.

Appendices

A Part 39 CDS prices

End-of-day (EOD) prices within the Part 39 data set are provided by ICC in terms of points upfront. CDS prices historically have been quoted in terms of conventional or "break-even" spreads, defined as the annualized quarterly spread payment per unit of purchased protection that makes the market value of the position zero at initiation. Contracts thus were negotiated bilaterally over the counter and, depending on when they were traded, carried different spreads. The push for standardized CDS contracts, however, has drastically changed the landscape of CDS price quotes and traded contracts. In particular, the 2011 "CDS Big Bang" resulted in standardized CDSs having fixed coupons (usually 100 or 500 basis points). Thus, contract market values are often non-zero at outset. When trading standardized CDSs, the protection buyer makes an upfront payment to the protection seller at initiation (or vice versa). Price quotes are then in "points upfront" instead of break-even spreads. For instance, if a CDS contract were quoted at 0.97, the protection buyer would pay $1 - 0.97 = 3\%$ of the notional to the seller at contract initiation. Notice that this quote convention is analogous to bond quotes, where a higher price quote represents a lower payment for the buyer. Some data providers, such as Bloomberg, convert the quoted prices using a standardized model provided by the International Swaps and Derivatives Association (ISDA) and, by convention, record break-even spreads.

We note that quoted prices are model prices. Since CDSs trade relatively thinly, EOD transaction prices are not always available. ICC and Markit have a specific price discovery process tailored to the CDS market. Participants submit price quotes at the end of every business day and the clearinghouse creates periodic trade executions among participants via an auction process. The resulting prices are used for daily mark-to-market purposes.

B Market events during our sample period

In this section, we briefly review the main world events that affected, directly or indirectly, CDS markets during our sample period May 2014-February 2019. In particular, during this period we find: (i) the plummet of oil prices in November 2014, when Saudi Arabia blocked OPEC from cutting oil production; (ii) the plunge in the Euro when the ECB chief Mario Draghi expressed unexpectedly dovish outlooks on monetary policy in January 2015; (iii) the 2015–2016 stock market sell-off, starting with the Chinese stock market burst ("Black Monday"), and followed by an unexpected devaluation in the Renminbi, which was further fueled by Greek Debt default; (iv) the unexpected negative interest rate policy announced by the Bank of Japan in January 2016; (v) the volatility spike when the Brexit referendum was announced in February 2016; (vi) Donald Trump's election in November 2016 which, immediately following the announcement, created extreme volatility spikes in global markets and led the total trading volumes in CDS markets to double on the election night; (vii) OPEC's decision to cut oil production, followed by non-OPEC countries, led to hikes in oil prices in November 2016, especially because this was the first time after financial crisis; (viii) the Venezuela's delayed payments on its sovereign debt and bonds issued by state oil giant Petroleos de Venezuela in November 2017 constituted a failure to pay "credit event", and led to extreme volatility in CDS prices which sky-rocketed in a period of 6-7 days. Our sample period also covers the (widely expected) interest rate hike by the Federal Reserve in December 2015, the first increase in nearly a decade, the Bitcoin's record price surge in the year 2017, and the (expected) Bank of England's decisions to raise interest in November 2017 and August 2018, respectively first and second time after the global crisis, despite the ongoing uncertainty over the future of the UK economy.

C Time-series test of the VaR rule using realized returns

We consider the Z statistic:

$$Z := \frac{1}{NT}\sum_{t=1}^{T}\sum_{n=1}^{N}\mathbb{I}\{\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\},$$

where $\mathbb{I}\{\cdot\}$ is the indicator function. The indicator takes value 1 when realized $M-$day losses exceed the initial margin requirement; this is typically referred to as an *exceedance*. The statistic Z is the empirical frequency at which exceedances occur, averaged over time and across market participants. We have, for quite general correlation structures:

$$Z \xrightarrow{\mathbb{P}} \alpha,$$

by the law of large numbers. For M and α specified in the null hypothesis, we can test H_0 using Z as the test statistic.

We compute standard errors for the test using binomial probabilities. While we would ideally compute cluster-robust standard errors for our test, having observed no exceedances means residuals are all zero. To proceed, we assume that exceedances are perfectly correlated when underlying losses overlap (for robustness against autocorrelation), and also assume that exceedances are uncorrelated across accounts. In particular, standard errors are computed as

$$S.E. = \sqrt{\frac{\alpha(1-\alpha)\zeta_M}{NT}}.$$

In the above equation, the term $\zeta_M := 2M - 1$ adjusts for our assumption that exceedances are perfectly correlated when underlying losses overlap. For $\alpha = 1\%$, $NT = 18,615$ and $M = 5$, we obtain a standard error of 0.22%.

We remark that our standard errors are likely to be overly conservative. As a robustness check, we also compute one-day returns and autocorrelation estimates. For each account, we find autocorrelation estimates on the orders of 10^{-4} for the first five lags. Thus, autocorrelation would likely have a smaller impact on actual standard errors compared to our assumption of perfect correlation.

Finally, to explicitly account for potential cross-sectional correlation in the returns on margins, we perform the test separately account by account, finding that the null hypothesis is rejected in every case.

D Procedure to compute counterfactual Returns

Following Duffie et al. (2015), we group together all I contracts written on the K underlying reference entities, and denote the net position in that reference entity by Y_k. Precisely, let Ω_k denote the collection of contracts referencing name k, then

$$Y_{t,k}^n := \sum_{i \in \Omega_k} X_{t,i}^n.$$

For each reference entity, therefore, Y_k indicates the net exposure to reference entity k, aggregating together the CDS contracts on that reference entity across maturities, seniority level, and doc clause. We then collect historical on-the-run 5-year credit spread series for each reference entity, $\mathbf{S}_t \in \mathbb{R}^K$, where the spread is the coupon payment that equates the value of premium and protection leg. Under standard assumptions on the loss rate, $\mathbf{S}_t$ can be converted to $\mathbf{P}_t$. We do not report the exact conversion formula here, but observe that, given that the market value of a CDS position at t is the notional of the position multiplied by $(1 - \mathbf{P}_t)$ (see also (1)), then $\mathbf{S}_t$ is increasing in $1 - \mathbf{P}_t$. Unlike $\mathbf{P}_t$, i.e., the quoting price following the Big Bang convention according to which the buyer makes an upfront payment of $1 - \mathbf{P}_t$ and then pays a running fixed spread premium throughout the life of the contract, $\mathbf{S}_t$ is not a price. Hence, $P\&L$ can only be approximated via the DV01 formula, given by

$$\Psi_{5,u}(\mathbf{X}_t^n) \approx d \times \mathbf{Y}_t^n \cdot (\mathbf{S}_{u+5} - \mathbf{S}_u),$$

where d is the effective duration of the position. We use $d = 3$ as in Duffie et al. (2015), meaning that the average duration of CDS positions is 3 years (corresponding to the median maturity of the CDS market).

E Time-series test of the VaR rule using counterfactual returns

We compute our test statistic using an extended version of Eq. (C):

$$Z' = \frac{1}{NTU} \sum_{t=1}^{T} \sum_{u=1}^{U} \sum_{n=1}^{N} \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\},$$

where $\Psi_{5,u} MtM(\mathbf{X}_t^n)$ is constructed as in Duffie et al. (2015) (see Appendix D for additional details), and U is the number of evaluation dates, that is, dates for which we observe the portfolio. For each portfolio $\mathbf{X}_t^n$, we estimate the frequency at which losses exceed portfolio margins. Under the null hypothesis of a 5-day 99% VaR margining rule, Z' should converge to 1% in probability.

The test can be simply implemented as a regression of observed exceedances onto a constant, with double-clustering as in Petersen (2009) by time and by account (there is no need to use the binomial model as exceedances are observed in the data, so the variance of the residuals is nonzero).

F Cross-sectional test of the VaR rule

The margining rule H_0 implies $\mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)) = \alpha$ for all n, which further implies

$$H_0' : \mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)) = \mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^{n'}) < -IM_t(\mathbf{X}_t^{n'})),$$

for all $n \neq n'$. The statistics to consider are then

$$Z_n := \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}\{\Psi_{M,t}(\mathbf{X}_t^n) + IM_t(\mathbf{X}_t^n) < 0\} \xrightarrow{\mathbb{P}} \alpha.$$

We describe here how to implement a test for equality (H_0'). The most straightforward test for equality of the frequencies of exceedances across accounts is the $G-$test (i.e. the two-way likelihood ratio test). Because the confidence level is expected to be large (the expected frequency of exceedances is low), the typical χ^2-test for homogeneity is not appropriate (Hoey (2012)). As exceedances are expected to occur with low probability, we instead use the $G-$test to test the null hypothesis.

The test statistic is computed as:

$$G := 2\sum_{n=1}^{N} O_n \log \frac{O_n}{E_n},$$

where O_n is the observed number of exceedances for clearing member n, and E_n is the expected number of exceedances for account n. The probability of observing an exceedance, needed for calculating E_n, is estimated by pooling observations across accounts. In particular,

$$E_n := TU \times Z = \frac{1}{N}\sum_{t=1}^{T}\sum_{u=1}^{U}\sum_{n'=1}^{N} \mathbb{I}\{\Psi_{5,u}MtM(\mathbf{X}_t^{n'}) < -IM_t(\mathbf{X}_t^{n'})\},$$

and

$$O_n := \sum_{t=1}^{T}\sum_{u=1}^{U} \mathbb{I}\{\Psi_{5,u}MtM(\mathbf{X}_t^{n}) < -IM_t(\mathbf{X}_t^{n})\}.$$

Under the null that frequencies are the same for each account, $G \xrightarrow{d} \chi^2_{N-1}$.

We also derive an extension of this test that explicitly accounts for potential autocorrelation of the exceedances. For a fixed portfolio, we first count the number of exceedances, and then divide it by the number of evaluation dates. This gives an estimate for the probability of an exceedance occurring for that portfolio. We then sum the exceedance probabilities for portfolios associated with each fixed account, and use the rounded up integer as the estimate of observed exceedances for that account.

We enter this estimate into the contingency table used for the $G-$test. Formally, we estimate the probability of an exceedance for an account/day combination (n, t) as

$$\hat{p}_{n,t} = \frac{1}{T} \sum_{u=1}^{T} \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\}.$$

The number of (estimated) observed exceedances is then

$$\hat{O}_n := \left\lceil \sum_{t=1}^{T} \hat{p}_{n,t} \right\rceil.$$

The estimate $\hat{O}_n$ replaces O_n in our computation of the G statistic (Eq. (F)).[20] The estimated observations are thus more robust to autocorrelation compared to treating each observation as an individual count, which may inflate the sample size.

G Robustness: Details

In this section, we provide more details about the robustness tests of Section 4.4.

Value-at-Risk and maximum shortfall used in the panel analyses (Tables 4 and 5) were based on P&L generated from our entire sample of credit spreads (that is, on the entire historical distribution of returns for each portfolio held at time t by member n). Because our data set covered the financial crisis, the risk measures captured extreme movements and may thus be viewed as overly conservative for estimating portfolio losses. In this section we consider using only the last 1000 days (approximately 4 years) of credit spreads data to generate P&L, as in Duffie et al. (2015). Using these newly estimated counterfactual P&L, we compute Value-at-Risk and maximum shortfall. We replicate our panel analyses and report the results in Tables A.3 and A.4.

Comparing Table A.3 to Table 4, we see there is a decrease in explanatory power of Value-at-Risk (VaR) (column (1)). This is likely due the exclusion of the financial crisis period in our simulation, resulting in both lower level and variability of Value-at-Risk. Aggregate short notional (AS) still retains its strong explanatory power (columns (3) and (4)), and columns (7) and (8) show that the DSV model still captures a significant portion of variation in initial margins, and outperforms Value-at-Risk in terms of explanatory power (columns (1) and (2)). Our conclusions remain consistent with our previous results.

Comparing Table A.4 to Table A.3, we observe again that there is a non-negligible increase in explanatory power compared to models that include only portfolio variables (Table A.3). This confirms that market variables can capture a dimension of initial margins not explained by portfolio variables, especially for the VIX.

[20] The ceiling operation is performed to ensure that the contingency table only contains integer entries. We also performed the test with unrounded data, yielding similar, if not stronger, results.

Table A.1: Initial margins and market variables summary statistics

Summary Statistic	In. Margins ($IM_{n,t}$)	Market variables, in basis points (bps) Overnight Index Swap Spread (OIS_t)	LIBOR-OIS spread ($LOIS_t$)	CBOE VIX (VIX_t)
Pooled mean (over all n and t): $\mu(x_{n,t})$	622.2	84.4	23.1	1,498.8
Std. deviation (over all n and t): $\sigma(x_{n,t})$	408.2	75.7	11.5	437.9

Summary Statistic	In. Margins ($IM_{n,t}$)	Market variables, in basis points (bps) CDS spread ($CDS_{n,t}$)	Avg. CDS spread ($ACDS_t$)
Pooled mean (over all n and t): $\mu(x_{n,t})$	622.2	78.3	78.3
Std. deviation (over all n and t): $\sigma(x_{n,t})$	408.2	30.9	18.7
Time-series variation of cross-sectional averages: $\sigma(\bar{x}_t)$	83.5	18.8	
Mean cross-sectional dispersion: $\mu(\sigma_t(x_{n,t}))$	407.0	23.9	
Cross-sectional dispersion of time-series averages: $\sigma(\bar{x}_n)$	346.3	18.1	
Mean time-series dispersion: $\mu(\sigma_n(x_{n,t}))$	208.4	23.0	

Note: The table displays summary statistics of our key market variables and initial margins, in basis points and millions of USD, respectively. Definitions of market variables are reported in Table 2. In addition to the overall mean and standard deviations (dispersions), we report panel statistics that describe properties of variables both across accounts and time, the calculations of which are reviewed in Table 3. Panel summaries are not reported for market variables that do not vary across accounts.

Table A.2: Check for multicollinearity

Estimates (R^2)	*Dependent variable:*	
	SD	ES
VaR (OLS)	0.313*** (98.0%)	1.379*** (96.3%)
VaR (with FE)	0.306*** (98.5%)	1.358*** (97.7%)
Observations	18,615	18,615

Note: We regress both expected shortfall and standard deviation on Value-at-Risk, and report the results. The first row corresponds to estimates from (pooled) OLS regression, and the second row corresponds to estimates after accounting for time and account fixed effects. Coefficient estimates are all significant at the 1% level. R^2s are in parentheses.

Table A.3: Regression results for explaining initial margins with portfolio variables

	Dependent variable:											
	Initial margins (IM) - Daily Frequency											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Value-at-Risk (VaR)	5.113***	2.994***	2.140*	1.554**					−0.298	0.258	0.825	0.944*
	(1.007)	(0.600)	(1.177)	(0.677)					(1.078)	(0.638)	(0.999)	(0.557)
Maximum shortfall (MS)			−0.189	−0.003	0.617*	0.544**						
			(0.242)	(0.219)	(0.372)	(0.238)						
Aggregate short notional (AS)			0.031***	0.020***	0.033***	0.021***						
			(0.004)	(0.004)	(0.004)	(0.004)						
Duffie et al. model (DSV)							1.307***	0.872***	1.356***	0.833***		
							(0.151)	(0.090)	(0.185)	(0.153)		
Modified DSV model (MDSV)											0.913***	0.577***
											(0.123)	(0.116)
Number of Observations	18615	18615	18615	18615	18615	18615	18615	18615	18615	18615	18615	18615
Adjusted R^2	0.410	0.762	0.683	0.829	0.669	0.824	0.644	0.820	0.644	0.820	0.674	0.828
Account Fixed Effect	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*
Time Fixed Effect	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*

*p<0.1; **p<0.05; ***p<0.01
Two-Way Clustered Standard Errors (by Time and Account)

Note: Same as Table 4, but computing risk measures only using last 1000 days of simulated return on margins

Table A.4: Regression results for initial margins using portfolio and market variables with last 1000 days of P&L

	Dependent variable:					
	Initial margins (IM) - Daily Frequency					
	(1)	(2)	(3)	(4)	(5)	(6)
Value-at-Risk (VaR) 1%	1.220	0.609	1.052	0.398	2.587**	1.230*
	(0.942)	(0.624)	(0.985)	(0.598)	(1.069)	(0.645)
Modified DSV Model (MDSV)	0.867***	0.628***	0.889***	0.654***		
	(0.115)	(0.109)	(0.120)	(0.105)		
Maximum Shortfall (MS)					−0.245	0.038
					(0.225)	(0.181)
Aggregate Short Notional (AS)					0.029***	0.021***
					(0.004)	(0.004)
CBOE Volatility Index (VIX)	0.081***	0.081***	0.063***	0.059***	0.080***	0.080***
	(0.016)	(0.019)	(0.017)	(0.016)	(0.016)	(0.019)
Member CDS Spread (DCDS)	2.038*	−0.136	2.013*	−0.148	2.092*	−0.039
	(1.160)	(0.784)	(1.125)	(0.824)	(1.093)	(0.750)
Average CDS Spread (ACDS)	−0.892	−0.996			−0.909	−0.987
	(1.089)	(1.076)			(1.162)	(1.098)
LIBOR-OIS Spread (LOIS)			1.171	1.421		
			(1.416)	(1.626)		
Number of Observations	18615	18615	18615	18615	18615	18615
Adjusted R^2	0.695	0.821	0.695	0.821	0.704	0.823
Account Fixed Effect	*N*	*Y*	*N*	*Y*	*N*	*Y*
Time Fixed Effect	*N*	*N*	*N*	*N*	*N*	*N*

*p<0.1; **p<0.05; ***p<0.01
Two-Way Clustered Standard Errors (by Time and Account)

Note: Same as Table 5, but computing risk measures only using last 1000 days of simulated return on margins