Bilateral Information Disclosure in Adverse Selection Markets with Nonexclusive Competition*

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August 19, 2021

Abstract

We study insurance markets with nonexclusive contracts, introducing bilateral endogenous information disclosure about insurance sales and purchases by firms and consumers. We show that a competitive equilibrium exists under remarkably mild conditions, and characterize the unique equilibrium allocation. With two types of consumers the allocation consists of a pooling contract which maximizes the well-being of the low risk type (along the zero profit pooling line) plus a supplemental (undisclosed and nonexclusive) contract that brings the high risk type to full insurance (at his own odds). We show that this outcome is extremely robust and constrained Pareto efficient. Consumer disclosure and asymmetric equilibrium information flows are critical in supporting the equilibrium.

JEL Classification: D43, D82, D86.

Keywords: Adverse selection, nonexclusivity, nonexclusive competition, Rothschild-Stiglitz, asymmetric information, information disclosure, verifiable disclosure, information design.

*We are grateful to several anonymous referees whose comments significantly improved the paper, to the editors, to the participants in the seminar at Sciences Po at which an earlier version of this paper was presented, to Gerry Jaynes for helpful comments on an earlier draft, to Michael Rothschild and Richard Arnott, long time collaborators, to Byoung Heon Jun, Debarati Ghosh, Andrea Gurwitt, and Lim Nayeon for research and editing, and to the Institute for New Economic Thinking and the Ford Foundation and Fulbright Foundation for financial support. The companion paper "Characterization, Existence, and Pareto Optimality in Insurance Markets with Asymmetric Information with Endogenous and Asymmetric Disclosures; Revisiting Rothschild-Stiglitz" (Stiglitz et al. (2018), NBER Working Paper 24711) contains more results.

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1 Introduction

In 1976 Rothschild and Stiglitz characterized equilibrium in a competitive market with exogenous information asymmetries in which market participants had full knowledge of insurance purchases. Self-selection constraints affected individual choices; but unlike the monopoly equilibrium (Stiglitz (1977)), no single firm framed the set of contracts among which individuals chose. There never existed a pooling equilibrium (in which the two types bought the same policy); if there existed an equilibrium, it entailed the high risk getting full insurance, and the low risk individual only getting partial insurance; and under plausible conditions - e.g. if the two types were not too different - a competitive equilibrium did not exist; finally, the sufficient condition relied on to establish the existence of a competitive equilibrium when the accident probabilities are different enough, the single crossing condition, was very restrictive. The results were disquieting, as in reality equilibrium seemed to exist, and often entailed pooling.

A vast literature has applied the Rothschild and Stiglitz (1976) model (henceforth RS), to labor, capital, and product markets in a variety of contexts, with many empirical applications. A smaller literature focused on remedying the deficiencies in the underlying framework by formalizing the insurance "game", by changing the information assumptions, and by changing the equilibrium concept.

This paper introduces bilateral endogenous information disclosure about insurance purchases: each firm and each consumer make a decision about what information to disclose to whom. Thus, information about contract purchases is both endogenous, and potentially asymmetric - a firm may disclose information about a consumer to some firms, but not others, depending on what the consumer discloses to it. We are motivated by the following observations: the outcomes with full information disclosure (exclusivity is enforceable, so the RS model applies, and a pooling equilibrium is impossible), and with no information disclosure (in which case pooling again cannot be an equilibrium), are known. Without consumer disclosure, any disclosure that firms make has to be symmetric, since they have no basis for differentiation; as we show later symmetric disclosure cannot underpin an equilibrium. The question of what happens if disclosure is bilateral, endogenous, and thus potentially asymmetric, is thus natural.

That asymmetries in information about insurance purchases turn out to be important is perhaps not surprising. What is surprising is that: (i) equilibrium exists under mild assumptions (notably, the single crossing property need not hold); and (ii) equilibrium always entails a pooling component. The unique insurance allocation (an allocation describes the sum of benefits and premia over all insurance companies for each individual) consists of the pooling contract which maximizes the well-being of the low risk individual subject to the zero-profit constraint, plus, for the high risk individual, a supplemental contract that brings him to full insurance at his own odds. While the equilibrium allocation is unique, it can be supported by alternative information disclosure strategies.

The allocation we focus on has been the subject of much study in various guises, beginning with Jaynes (1978), and most recently, by Attar, Mariotti, and Salanié ((2020a), (2020b)). Under certain conditions, this allocation has been shown to emerge as the equilibrium in non-competitive models without endogenous information.

One part of our contribution is showing that this allocation can, in fact, be the outcome of a competitive equilibrium with endogenous bilateral disclosure, and that many standard but restrictive assumptions are not needed for this result. Previous work has studied some of the efficiency properties of this allocation; we extend that work in showing the context in which the allocation is constrained Pareto efficient, and link that property, in turn, to the set of possible equilibrium outcomes. We explicitly decentralize a constrained Pareto efficient allocation as a competitive equilibrium in an insurance setting with adverse selection, nonexclusivity, and endogenous information disclosure.

Crucially, we argue that decentralization via a competitive market in a setting with any fixed information structure is impossible. (Thus, Jaynes, and much of the other literature generating the allocation upon which we focus, are explicitly set in a context of imperfect competition.) Any competitive-like equilibrium requires endogenous and asymmetric information sharing. In RS, Akerlof, and much of the existing related literature, information is endogenous, but only affected by actions (e.g. directly observable purchases of insurance). In the analysis here, more in the spirit of much of the mechanism and information design literature that flourished after RS, individuals can disclose (truthful) information, which affects behavior, including others' disclosures. In the equilibrium we construct, firms do not always find out the type of agent they are trading with (as is the case in the RS separating equilibrium), but there are nontrivial information exchanges (between firms, and from consumers to firms) associated with insurance purchases.

We begin by characterizing the set of constrained Pareto efficient (CPE) allocations in the presence of a secret contract. We then show that the CPE allocation which maximizes the well-being of the low risk individual is the unique equilibrium allocation and can be supported by simple yet illuminating information disclosure strategies. As in RS, firms offer insurance contracts, but now they have an option to reveal (possibly partial) information about insurance purchases to other firms. In RS, it was assumed that contracts were exclusive, e.g. implicitly, that if a firm discovered a purchaser had violated the exclusivity restriction, the coverage would be cancelled. Here we consider a broader range of possible restrictions and under a broader set of assumptions concerning the information available to insurance firms. Obviously, the enforceability of any conditions imposed is dependent on information available to the insurance firm. Consumers, too, have a more complicated life: they have to decide which policies to buy, aware of the restrictions in place and the information that the firm may have to enforce those restrictions. They also have to decide on what information to reveal to which firms. A competitive equilibrium in this model is a set of insurance contracts, such that no one can offer an alternative contract or set of contracts and make positive profits. A contract is defined by the benefit, the premium, any restrictions associated with the contract, and the firm's disclosure policy. And in assessing the consequences of offering an alternative contract, each firm takes into account the consumers' response to the set of contracts on offer, both with respect to insurance purchases and disclosures.

The intuition behind our result is this: in RS a pooling equilibrium can always be broken by a deviant policy which will be purchased only by low risk individuals, and as a result, is profitable. But that deviant contract will be purchased only by low risk individuals because the deviant firm can enforce exclusivity. If high risk individuals can supplement the deviant contract (one breaking the putative pooling equilibrium) with secret insurance at their own odds, that policy may be purchased by high risk individuals, and thus make a loss. Hence, the deviant policy will not be offered and the pooling contract can be sustained. The trick is to find an information disclosure strategy which ensures that a deviant firm can't enforce exclusivity, but which also ensures that the firms selling insurance at the pooling odds (which we refer to as "established" firms) don't "oversell": high risk individuals would like to buy more insurance at the pooling odds than low risk individuals. If they did so, the pooling contract would lose money. Accordingly, there has to be information disclosure among the established firms to prevent the high risk individuals from doing so. Thus, supporting the equilibrium allocation requires an *intermediate* amount of disclosure: one needs some information sharing (enough to prevent overselling), but not too much (not enough to enforce exclusivity). Furthermore, disclosure has to be asymmetric in that established firms must have sufficient information, but deviant firms (which, of course, deviate secretly) must not. But firms by themselves have no basis for such asymmetric disclosures: without further information, they only know whether they themselves have sold insurance to an individual.[1]

This is where consumer disclosure - an essential feature distinguishing our paper from other work in this area - becomes critical: firms base the asymmetries in disclosure on consumer-revealed information. The equilibrium firm information disclosure strategy that we analyze induces truth-telling by consumers to established firms, and this in turn enables asymmetries in firm disclosures of information about insurance purchases. Thus, endogenizing consumer disclosure is not just a natural modeling postulate - it is neces-

[1]This is essentially the point that Hellwig (1988) makes in criticism of Jaynes (1978) argument that with endogenous information, there always exists an equilibrium. In contrast, he emphasizes that "...there does not exist a sequential equilibrium for the RS-type specification of the game." He shows that Jaynes' equilibrium requires that each firm's communication strategy be conditioned on the set of contracts that are offered by other firms, making the equilibrium a reactive equilibrium, like that of Wilson (1977), not a competitive equilibrium as in RS.

sary for the outcome we characterize. To the best of our knowledge, these disclosure strategies (expressed simply by saying that "once a consumer makes insurance purchases and freely reveals some subset of his insurers to each firm, each firm reveals information to those firms that the consumer did not reveal as his insurers") have not been identified previously.

One can obtain this result by formalizing this setting as a game with appropriately defined strategy spaces, and focusing on the outcome in a perfect Bayesian equilibrium; however, doing so would introduce unnecessary complexity,[2] thus detracting attention from the basic insight of our analysis. For this reason, we pursue the route of the original RS paper, positing only the elements that are absolutely necessary to make the point in the simplest possible setting that nevertheless has all of the features we are interested in, using an equilibrium concept that is in the spirit of competitive equilibrium.

We view this work as a contribution to the pure theory of competitive equilibrium with asymmetric information; but we believe that some of its main insights - the robustness of a pooling-plus-separating allocation, and the simple, nontrivial, and illuminating information disclosure strategies which can support such an equilibrium - are of significant relevance for real-world markets with adverse selection and nonexclusivity, and towards the end of the paper we illustrate this by considering its implications for government provision of insurance under the "public option". We hope that our results provide an impetus for further policy and empirical applications, with insights into why certain markets take the form they do, and how one might improve the design of markets with asymmetric information.

The paper is organized in nine sections. Section two lays out the basic features of our model of insurance with nonexclusive contracts, while section three provides a discussion of nonexclusive ("secret") contracts in workhorse RS and Akerlof models. We characterize the set of CPE contracts in the presence of secret insurance in section four. Section five provides a formal discussion of contracts, information disclosure strategies, and the equilibrium concept, and shows that there is a unique allocation that an equilibrium if it exists, has to implement. The explicit construction of the equilibrium is in section six. Section seven considers a generalization to the case of many types, while section eight considers the impact of a public insurance option. Section nine relates our results to previous literature. For brevity, we forego a concluding section, and discuss an extension where firms offer menus of contracts in the appendix B. Appendix A contains proofs and derivations omitted in the text.

[2]We work through a model along these lines in Stiglitz and Yun (2016).

2 Model

We employ the standard insurance model with adverse selection. An individual, indexed by $i \in [0,1]$, is faced with the risk of an accident. The two types of individuals - high risk ($t = H$) and low risk ($t = L$) - differ only in the probability of accident, P_t, with $P_H > P_L$. The type is privately known to the individual, while the proportion θ of high risk types is common knowledge. The average probability of accident for an individual is $\overline{P}$, where

$$\overline{P} = \theta P_H + (1-\theta)P_L \tag{1}$$

An accident involves damages, the cost of repairing which in full is d. An insurance firm pays a part of the repair cost, $\alpha \leq d$ (we disallow negative insurance). The benefit is paid in the event of accident, whereas the insurer is paid an insurance premium β when no accident occurs. Define $q \triangleq \frac{\beta}{\alpha}$ as the price of insurance (payment in the "good state" per dollar received in the case of an accident). We assume the utility of the individual of a given risk type is a function of his consumption if he has an accident $(w - d + \alpha)$, where w is the starting wealth level, and if he does not, $(w - \beta)$:

$$V_t(\alpha, \beta) \triangleq U_i(w - d + \alpha, w - \beta) \tag{2}$$

We assume that V is bounded, continuously differentiable, increasing in α, and decreasing in β.[3] A special case of eq. 2 is standard expected utility, with $U_t'' < 0$:

$$V_t(\alpha, \beta) = P_t U_t(w - d + \alpha) + (1 - P_t)U_t(w - \beta) \tag{3}$$

While it is a useful special case, we do not rely on the expected utility formulation for the results, and note only that it is a useful special case. If in eq. 3 the utility functions are the same for high and low risk individuals, the single crossing property that was essential in the RS analysis is satisfied. But if, for instance, individuals with a higher probability of an accident are also more risk averse, then the single crossing property will never be satisfied.[4] None of the results below depend on the single crossing property being satisfied, so we do not invoke it. We refer to a policy $A = \{\alpha, \beta\}$, and to the expected utility generated by that policy as $V_t\{A\}$. A policy A, with insurance level α and price q can also be described by the vector $\{\alpha, \alpha q\}$. We do not require the preferences to be convex for our results on the existence of equilibrium. The key property of $V_t(\alpha, \beta)$, which we assume is satisfied throughout the paper is that the income consumption curve at the

[3] For much of our analysis, we do not even require that, but the exposition is simplified by making this assumption. The assumption plays a role in Proposition 2.

[4] There are other reasons to be concerned about the single crossing property: in models with moral hazard and adverse selection, where P_t is endogenous, there is also the presumption that the single crossing property will not be satisfied (see Stiglitz and Yun (2016)).

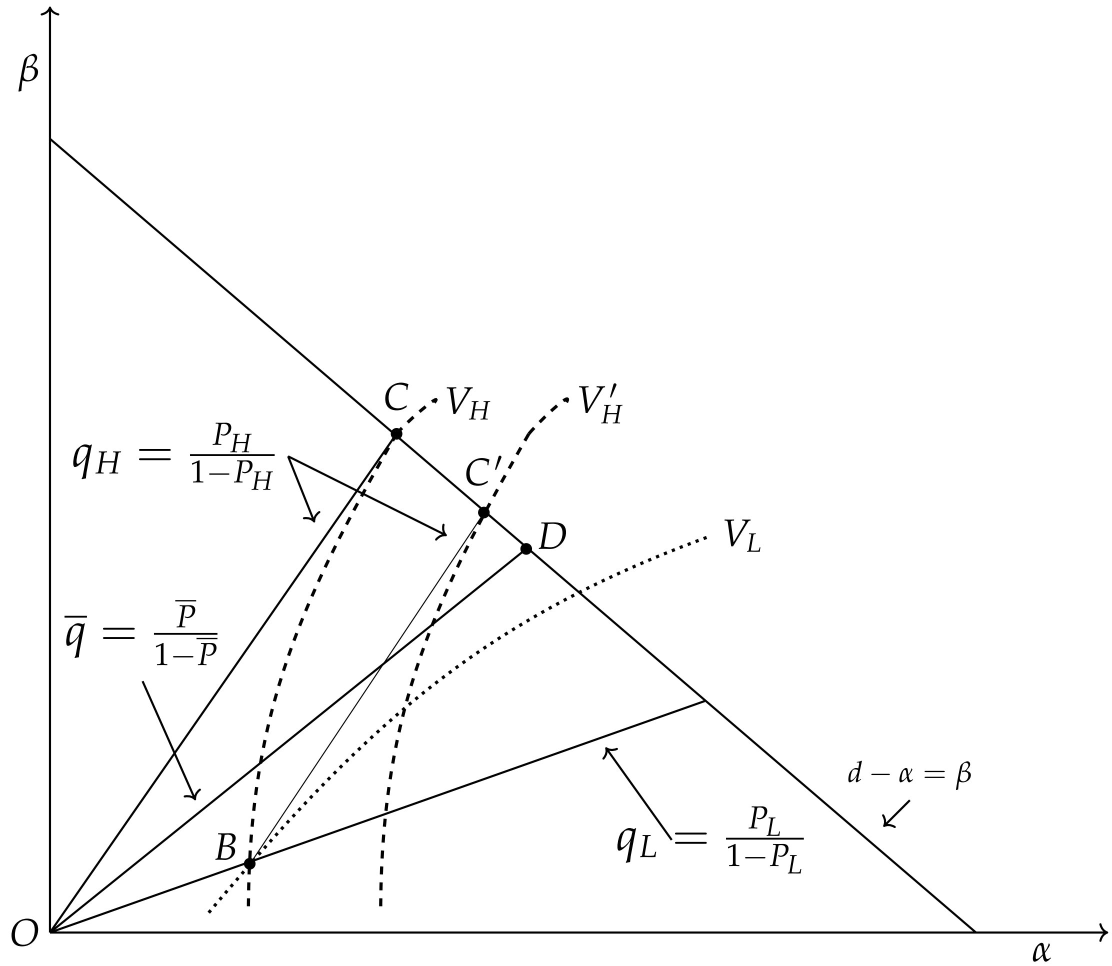

Figure 1: Breaking the RS separating equilibrium (B, C) in the presence of undisclosed contracts at high risk odds. V_H is an indifference curve of type H, V_L is an indifference curve of type L, the line $d - \alpha = \beta$ is the full insurance line.

insurance price $\frac{P_t}{1-P_t}$ is the full insurance line,[5] implying that at full insurance, the slope of the indifference curve equals the relative probabilities,

$$-\frac{\frac{\partial V_t(\alpha,\beta)}{\partial \alpha}}{\frac{\partial V_t(\alpha,\beta)}{\partial \beta}} = \frac{P_t}{1-P_t} \triangleq q_t \tag{4}$$

so that will full information, assuming convexity, equilibrium would entail full insurance for each type at their own odds.

The profit π_t of a contract (α, β) that is chosen by type t is $\pi_t(\alpha, \beta) = (1 - P_t)\beta - P_t\alpha$. $\pi_t(\alpha, \beta) = 0$ is defined as the t-type's zero profit locus (the line along which firms selling to type t make zero profit). We sometimes write the profits associated with policy A purchased by type t as $\pi_t\{A\}$. Figure 1 illustrates the zero-profit locus for a firm selling

[5] This, together with convexity, is the critical property needed for proving the main result of the paper.

insurance to a t-type (OB and OC, respectively) or both types of individuals (OD) by a line from the origin with the slope being $q_t(\triangleq \frac{P_t}{1-P_t})$ or $\overline{q}\left(\triangleq \frac{\overline{P}}{1-\overline{P}}\right)$, respectively. The latter is referred to as the zero profit pooling line. There are $N \geq 3$ firms and the identity of a firm is represented by j, with $j \in \mathcal{M}(\triangleq 1, \ldots, N)$. We also assume that these N firms are exogenously sorted into one of two groups: a set $\mathcal{M}^{\mathcal{E}}$ of "established" firms with $|\mathcal{M}^{\mathcal{E}}| = n \geq 2$, and a set $\mathcal{M}^{\mathcal{S}}$ of "secret" firms with $|\mathcal{M}^{\mathcal{S}}| = N - n \geq 1$; this assumption can be endogenized.[6] Of course, $\mathcal{M}^{\mathcal{E}} \cup \mathcal{M}^{\mathcal{S}} = \mathcal{M}$, and thus $|\mathcal{M}^{\mathcal{E}}| + |\mathcal{M}^{\mathcal{S}}| = N$. An individual may purchase multiple policies from any subset of firms. A set of benefits and premiums of the insurance policies purchased in the aggregate by each type of individual, denoted by $E = \{(\alpha_t, \beta_t)_{t=L,H}\}$ is called an *allocation*, with $\alpha_t = \sum_j \alpha_{t,j}$.

3 RS and Akerlof without Exclusivity

Central to RS was the assumption that there was sufficient information to enforce exclusivity; an individual could not buy insurance from more than one firm. As Rothschild and Stiglitz realized, once we introduce into the analysis unobservable contracts in addition to observable ones, the whole RS framework collapses, because exclusivity cannot be enforced. We first discuss why introducing secret contracts results in there being no equilibrium in any competitive model with fixed information, such as RS or Akerlof, before proceeding to analyze the implications for Pareto efficient allocations and competitive equilibrium with secret contracts and endogenous disclosure.

Exclusivity in RS implied, *inter alia*, the existence of contracts that break a putative pooling equilibrium. Without exclusivity some of these contracts no longer do so, because they will be taken up by not only low risk individuals, but also high risk individuals who will supplement the given contract with undisclosed insurance at price q_H. Such secret insurance (at the price reflecting the odds of a high risk individual) at least breaks even and so will always be on offer. Yet it turns out that there always exists some contract that even with secret insurance breaks a pooling equilibrium. But without exclusivity, the separating contracts from RS are also not equilibrium contracts, as illustrated in figure 1. The RS separating contracts are $\{B, C\}$, where C provides full insurance for the high risk individual at his own odds; and B is the contract at the low risk individual's odds which just satisfies the self-selection constraints, i.e. will not be purchased by the high risk individual. Clearly, if the high risk individual *can* supplement B with secret insurance at the high risk odds, he will purchase B. But if high and low risk individuals both purchase B, it makes a loss. It can easily be shown that there exists no separating equilibrium. Since there can not exist a pooling or a separating (nor a hybrid) equilibrium it follows that with

[6]If there is only one firm, it can trivially impose exclusivity, being a monopolist, so we assume there are at least two firms. As we discuss below, the same firm can, in fact, be both an established and a secret firm, via-a-vis different customers.

a fixed information structure, where firms that disclose their sales always do so and those that don't never do so, there never exists a RS competitive equilibrium.

The problems of non-existence of equilibrium with a fixed information structure are worse. The Akerlof equilibrium, where insurance firms offer insurance at a fixed (competitive) price also does not exist, provided only that the firms know the identity of the individual to whom they are selling, so that they can ascertain the total quantity of insurance they have sold.[7] This is a natural assumption for individuals purchasing insurance against risks which directly only affect them, such as against an accident or their death, but not necessarily for insurance against a market event, like the decrease in the price of oil. The standard Akerlof equilibrium in the context of insurance entails a price of insurance $q_c > \overline{q}$, reflecting the fact that high risk individuals buy more insurance than low risk individuals. The Akerlof equilibrium can be broken by a firm offering a quantity contract along the pooling line that the is most preferred by the low risk individual, since every individual will buy it, and it will break even. But that contract, supplemented by secret insurance at q_H (purchased only by high risk individuals) can't be an equilibrium, by the reasoning just given for the RS model. Thus, in this natural extension of the fixed information structure posited by Akerlof there also does not exist a (what we will refer to as the Akerlof price-quantity) equilibrium.[8]

Summarizing these observations, we have

Proposition 1. *With secret insurance contracts, there never exists either a RS or an Akerlof price-quantity equilibrium.*

Thus, if an equilibrium in the competitive spirit is to exist in the presence of secret contracts, the model has to be changed. There are at least two directions. One is departing from a competitive framework, e.g. assuming a reactive equilibrium (where incumbent firms respond to offers of an entrant), which may make sense when there are a relatively small number of firms.[9] Another is the approach taken here: endogenizing the information structure.

[7] To see this, observe that in an Akerlof price equilibrium, since high risk individuals buy more insurance than low insurance individuals, $q > \overline{q}$. But if a firm were to sell a fixed quantity insurance policy, at a price slightly above $\overline{q}$, everyone would buy one such policy, and the firm would make a profit. The result also holds even if the insurance firm cannot identify the identity of the purchaser, provided only that the two types of individuals are not too different and that individuals cannot buy "negative" insurance. The analysis below shows that the Akerlof equilibrium can be broken under quite general conditions with endogenous disclosure.

[8] While we couch our analysis in terms of insurance, the literature following RS made clear that an analogous analysis applied in many other markets, including credit markets. Enforcing exclusivity in that market has proven to be particularly problematic, with many high risk individuals over-borrowing. One proposed solution has been a public registry, incentivized by a legal provision that only loans registered would be recognized by courts. But our analysis has made clear the problems with this "solution".

[9] This is the approach taken by Wilson (1977), Riley (1979), Jaynes (1978) and a number of other papers discussed in section nine.

4 Pareto Efficiency with Secret Contracts

We begin by characterizing the set of constrained Pareto Efficient (CPE) allocations under the premise that there exists a secret (i.e. available for purchase to the consumers, but entailing no disclosure whatsoever) policy, where the constraint is that the government cannot proscribe the secret provision of insurance. This analysis is interesting in its own right, clarifying how the existence of secret contracts constrains the set of feasible allocations. But it will also be useful in the next section when we analyze potential equilibrium allocations in the presence of secret policies, showing, importantly though not surprisingly, that any allocation that is not CPE is not entry-proof. Formally, we use the following ex-interim variant[10] of constrained Pareto efficiency:

Definition 1. *An allocation $E = \{(\alpha_t, \beta_t)_{t=L,H}\}$ is constrained Pareto-efficient (CPE) if the government cannot force disclosure, and there does not exist another allocation that at least breaks even, and leaves each type of consumer as well off and at least one type strictly better off.*

We begin by assuming that there is always a secret contract offering unlimited insurance at the price q_H. Such insurance must at least break even, and so will be offered. Later, we show that no other secret insurance is viable. In the presence of such secret insurance a high risk individual with a less-than-full insurance policy (say, A or B in figure 2) would always supplement it by purchasing additional insurance at q_H to reach a full-insurance policy C or C' in figure 2. Thus, the only CPE pooling allocation entails full insurance for the high risk individual.

The set of CPE can now be easily described: it consists of a pooling contract, i.e. a contract along the pooling line OA^*D in figure 3) plus a supplemental contract, for the high risk individual only, bringing him to full insurance at the high risk odds. $\{A', C'\}$ is a typical CPE. A^* is the pooling contract that maximizes the utility of the low risk individual. The set of CPE entails equal or more insurance than A^*, i.e. the pooling policy lies between A^* and D. Later, we will show that $\{A^*, C\}$ is the unique *competitive* equilibrium allocation.

In characterizing CPE allocations, we will first provide an analytic representation of the possible set of allocations for a high risk individual, given that he can purchase secret insurance, and prove a general result concerning his utility level. Next, we narrow down the set of allocations that satisfy the zero-profit constraint to those satisfying the self-selection constraints in the presence of the undisclosed contracts, which enables us to fully characterize the set of CPE.

[10] See also Prescott and Townsend (1984), Hammond (1987), Bisin and Gottardi (2006), and Attar, Mariotti, and Salanié (2019) for important discussions of Pareto efficiency in related contexts; earlier versions of the present paper were the first to explicitly consider the "constrained Pareto efficiency" concept introduced here, allowing for secret contracts as well as disclosed contracts.

If a high risk individual can purchase a policy $(\alpha, \alpha q)$, he will supplement it by purchasing additional insurance at q_H to reach a full-insurance policy $\{\gamma(\alpha, \alpha q), \delta(\alpha, \alpha q)\}$, where

$$\gamma(\alpha, \alpha q) = \frac{1}{1+q_H}[d + \alpha(q_H - q)] \tag{5}$$

and

$$\delta(\alpha, \alpha q) = d - \gamma(\alpha, \alpha q) = \frac{1}{1+q_H}[q_H d - \alpha(q_H - q)] \tag{6}$$

The derivations of eqs. 5 and 6 can be found in the appendix. Denoting by $H(\alpha, q) \triangleq V_H(\gamma(\alpha, \alpha q), \delta(\alpha, \alpha q))$ the utility that a high risk individual with a less-than-full insurance policy $(\alpha, \alpha q)$ (where $\alpha(1+q) < d$) can obtain by supplementing it with the desired amount of insurance at a price q_H, we show the following lemma:

Lemma 4.1. *$H(\alpha, q)$ is decreasing in q while it is increasing (respectively, decreasing) in α if q <(respectively, >) q_H.*

Proof. The level of utility is a function of consumption in each of the two states, but along the full insurance line, utility is just a function of the level of consumption; thus let $H(\alpha, q) \triangleq v_H(\gamma(\alpha, \alpha q))$ along the full insurance line. Using eq. (5) and eq. (6), we have

$$H(\alpha, q) = v_H(w + \frac{1}{1+q_H}[(q_H - q)\alpha - q_H d]) \tag{7}$$

from which lemma 4.1 follows by inspection. □

Lemma 4.1, which plays a critical role for the results in this paper, implies that a high risk individual would always like to purchase more insurance (up to full insurance) at a price lower than q_H in the presence of the undisclosed supplemental purchase of insurance at q_H.

Lemma 4.2 provides a characterization of all zero profit allocations.

Lemma 4.2. *Any allocation $\{(\alpha_t, \beta_t)_{t=L,H}\}$ yielding zero profit can without loss of generality be represented as a sum of a pooling allocation $(\hat{\alpha}, \hat{\beta})$ and a set (α_t^S, β_t^S) of type-specific supplemental allocations:*

$$\alpha_t = \hat{\alpha} + \alpha_t^S, \beta_t = \hat{\beta} + \beta_t^S \text{ where } \hat{\beta} = \overline{q}\hat{\alpha}, \beta_t^S = q_t \alpha_t^S, \text{ and } \alpha_t^S = \alpha_t - \hat{\alpha} \tag{8}$$

The proof can be found in the appendix. Lemma 4.2 is illustrated in figure 2, which shows how an allocation (B, C) (that yields zero profit) can be decomposed into a pooling allocation A and the two supplemental allocations (AC, AB). When (α_t, β_t) is a pooling full insurance allocation, $\hat{\alpha} = \frac{1}{1+\overline{q}}d$, $\hat{\beta} = \frac{\overline{q}}{1+\overline{q}}d$, $\alpha_t^S = \beta_t^S = 0$. Lemma 4.2 implies that an allocation yielding zero profit can be characterized by the three parameters $\hat{\alpha}, \alpha_H^S$ and α_L^S.

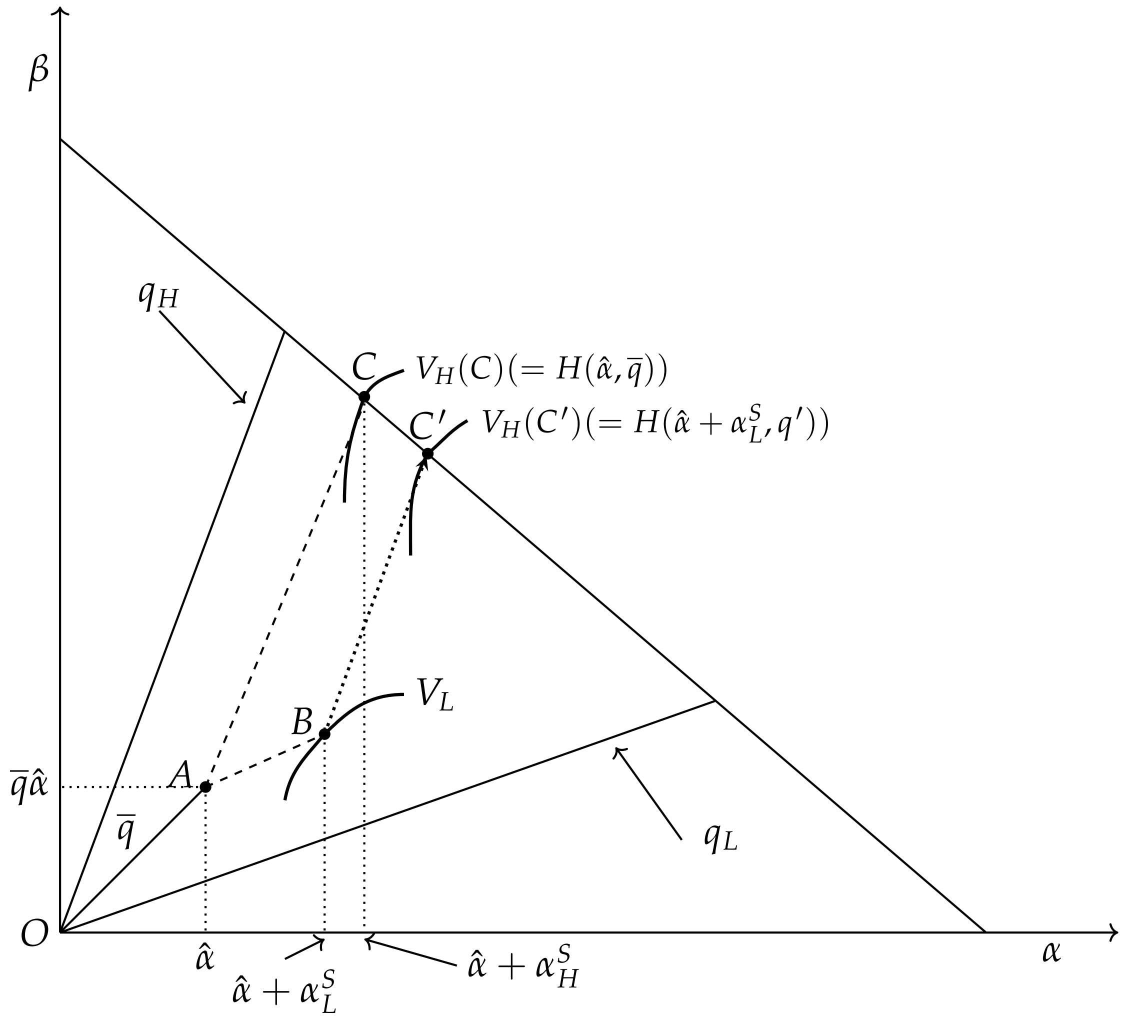

Figure 2: An allocation (B, C) can be decomposed as A (a pooling contract) and (AC, AB) (supplemental insurance at the individual's odds). Allocation (B, C) is not feasible as it does not satisfy the self-selection constraints in the presence of undisclosed policies at high risk odds q_H, while (A, C) is feasible; $q' =$ slope of $OB(< \overline{q})$.

An allocation is said to be *feasible* if it satisfies self-selection constraints - i.e. if given that allocation, neither type will choose to deviate to another allocation, given the presence of undisclosed insurance at price q_H. Lemma 4.3 shows that the self-selection constraints in the presence of the undisclosed policies result in the following restrictions on allocations:

Lemma 4.3. *Any feasible allocation must satisfy*

i) $\alpha_H^S = \frac{1}{1+q_H}(d - \hat{\alpha}(1+\overline{q}))$ *for* $\hat{\alpha} \leq \frac{1}{1+\overline{q}}d$.

ii) $\alpha_L^S = 0$.

Lemma 4.3 *i*) follows directly from eq. 5. To see lemma 4.3 *ii*), note that if $\alpha_L^S > 0$ the allocation would not satisfy the self-selection constraint, since then high risk individuals,

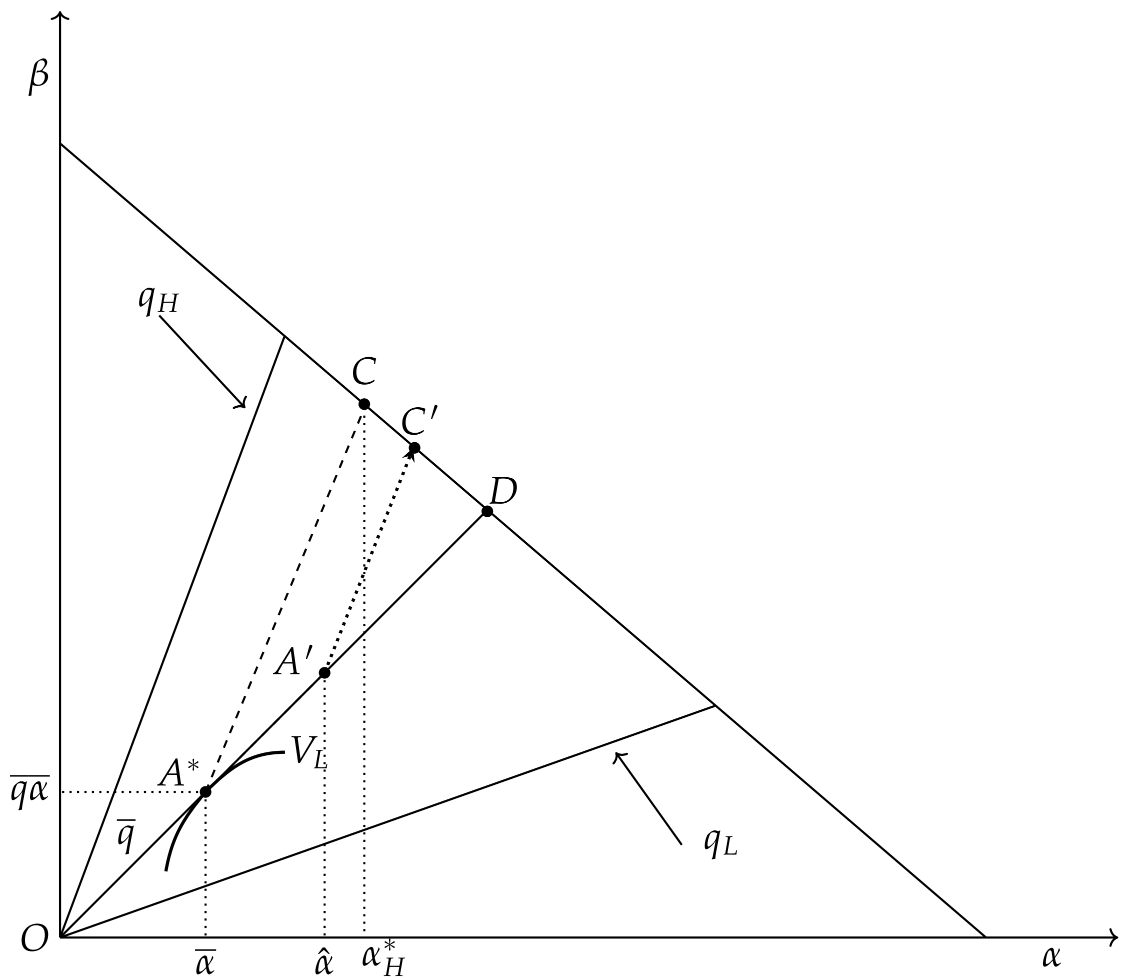

Figure 3: Pareto Efficient Allocations $\{Z_t(\hat{\alpha})\}_t$, denoted by (A', C'), and the equilibrium allocation $\{Z_t(\overline{\alpha})\}$ denoted by (A^*, C)

after choosing $(\hat{\alpha}, \hat{\alpha}\overline{q})$ (A in figure 2), would supplement it by choosing supplemental policy AB $(= \alpha_L^S, \alpha_L^S q_L)$, and supplement that with secret insurance BC', bringing them to full insurance which yields $H(\hat{\alpha} + \alpha_L^S, q')(= V_H(C')$ in figure 2), where $q' = \frac{1}{\hat{\alpha}+\alpha_L^S}(\overline{q}\hat{\alpha} + q_L\alpha_L^S) < \overline{q}$. By lemma 4.1, $H(\hat{\alpha} + \alpha_L^S, q') > H(\hat{\alpha}, \overline{q})$, because $q' < \overline{q}$ and $\hat{\alpha} + \alpha_L^S > \hat{\alpha}$. Of course, lemma 4.3 *ii*) implies that $(\alpha_L, \beta_L) = (\hat{\alpha}, \hat{\beta})$.

A feasible allocation for type t is denoted by $Z_t(\hat{\alpha})$, which is completely characterized by the parameter $\hat{\alpha}$ by lemma 4.3:

$$Z_H(\hat{\alpha}) = (\gamma(\hat{\alpha}, \hat{\alpha}\overline{q}), \delta(\hat{\alpha}, \hat{\alpha}\overline{q})) \tag{9}$$

$$Z_L(\hat{\alpha}) = (\hat{\alpha}, \hat{\alpha}\overline{q}) \tag{10}$$

where $\hat{\alpha} \in \left[0, \frac{1}{1+\overline{q}}d\right]$. The feasible allocation $\{Z_t(\hat{\alpha})\}_t$ with $\hat{\alpha} = \frac{1}{1+\overline{q}}d$ is the full-insurance allocation, whereas that with $\hat{\alpha} = 0$ implies that the low risk individual gets no insurance.

Only a subset of the feasible allocations are CPE. Define $\overline{\alpha}$ as

$$\overline{\alpha} \triangleq \max_{\alpha} \left[\arg\max_{\hat{\alpha}} V_L(\hat{\alpha}, \hat{\alpha}\overline{q}) \right] \tag{11}$$

$\overline{\alpha}$ is the amount of insurance that is the most preferred by low risk individuals[11] given a price $\overline{q}$, as illustrated by A^* in figure 3. Given our assumptions on V, and since the constraint set is compact, convex, and bounded, the maximization problem in eq. 11 has a solution. Lemma 4.1 implies that $V_H(Z_H(\hat{\alpha}))$ or $H(\hat{\alpha}, \overline{q})$ is increasing in $\hat{\alpha}$ because $\overline{q} < q_H$. It also implies that a feasible allocation $\{Z_t(\hat{\alpha})\}_t$ is not CPE if $\hat{\alpha} < \overline{\alpha}$, because both types of individuals could be made better off as $\hat{\alpha}$ increases to $\overline{\alpha}$. Also, a feasible allocation $\{Z_t(\hat{\alpha})\}$ with $d \geq \hat{\alpha} > \overline{\alpha}$ does not Pareto-dominate the allocation $\{Z_t(\overline{\alpha})\}$, implying that $\{Z_t(\hat{\alpha})\}$ is also CPE. Under the assumption of convex preferences, a feasible allocation $\{Z_t(\hat{\alpha})\}_t$ is also CPE if $\hat{\alpha} > \overline{\alpha}$ because $V_H(Z_H(\hat{\alpha})) > V_H(Z_H(\overline{\alpha}))$ while $V_L(Z_L(\hat{\alpha}))$ is decreasing in $\hat{\alpha}$ for $\hat{\alpha} > \overline{\alpha}$ (see figure 3).

With convexity, given a CPE allocation in the presence of undisclosed policies offered at q_H, no low risk type would like to purchase additional insurance at any price higher than $\overline{q}$. implying that no secret policy would be offered at a price $q, \overline{q} \leq q < q_H$. Moreover, any policy offered at a price $q < \overline{q}$ will be purchased by all individuals, and hence will lose money. Any insurance policy offered at a price above q_H will not be purchased by anyone. Hence, the set of CPE just described are the set consistent with any non-money losing secret contracts, not just secret policies at price q_H.[12]

This establishes the following:

Proposition 2. *With convex preferences, the set of CPE allocations is $\{Z_t(\hat{\alpha})\}_t$, with $\hat{\alpha} \geq \overline{\alpha}$.*

Among the CPE allocations, there is one on which we will focus, the allocation corresponding to the policy $(\overline{\alpha}, \overline{\alpha q})$ for the low risk type, and that policy supplemented by additional insurance at price q_H bringing the high risk individual up to full insurance. It is the CPE which maximizes the utility of the low risk individual.

[11] If preferences are not convex (and therefore the utility function not quasi-concave), there may be multiple values of $\hat{\alpha}$ which maximize V_L. While the low risk individual is indifferent among them, the high risk individual is not. Thus, the $\overline{\alpha}$ relevant for the CPE is the highest value of α maximizes V_L along the pooling line. However, for the purposes of Proposition 2 below, we do assume convexity, in which case a unique $\overline{\alpha}$ is assured, one can dispense with the first maximization operator in the definition, and the definition simplifies: $\overline{\alpha} = \arg\max_{\hat{\alpha}} V_L(\hat{\alpha}, \hat{\alpha}\overline{q})$

[12] This includes as well any quantity-rationed contracts, and any secret packages of contracts, where, e.g. the secret insurance firm offers to sell one policy conditional on the insured buying another, or where he offers a pair of policies, allowing those of different types to self-select among the policies.

5 Contracts and Equilibrium

Individuals are allowed to purchase one or more policies from one or more firms. An individual or his insurer may disclose to other firms all or some information about the set of policies purchased or sold, respectively. Information revealed must be truthful, but individuals or firms may choose not to reveal some or all information. What is critical about the information disclosure in the model is that they cannot reveal the fact that they have not purchased a particular policy.[13]

An insurance contract consists of two components: (i) a policy, defined by a price, q, a benefit, α, and a set of restrictions that have to be satisfied (as far as the insurer knows) if the policy is to go into effect; and (ii) an information disclosure rule. The set of conceivable contracts is quite rich; all that is required is that firms can only disclose a subset of what they know, and can impose restrictions that can only be implemented based on the knowledge of the insurer. But we show that there exists[14] an equilibrium with a simple set of contracts and disclosure rules.

5.1 Setting

We employ a two-stage framework consistent with the conventional setting of a screening model.

- First stage: each firm offers a set of insurance contracts. Any contract has two parts. The first is a "policy" P, specifying the quantity of insurance (the benefit), with an associated price (that is, the premium) and a set of restrictions on what insurance individuals can buy from other insurers. The second part is an information disclosure rule D, specifying what information the firm will disclose to whom. The implementation of any restrictions depends on the information available to the insurer, which depends on the disclosures of firms and individuals.

- Second stage: consumers purchase policies and disclose information about them (possibly selectively) to their potential insurers and others, following which each firm executes its contract for its consumer as announced in the first stage by disclosing information as specified by its disclosure rule. Consumers whose insurance purchases are found to be inconsistent with the policy restrictions have their policies cancelled.

[13] In terms of the literature on strategic communication, this is a setting of verifiable disclosure, or hard information (Milgrom (1981), Grossman (1981)). If such disclosure were possible, exclusivity would be enforceable, and we would be back in the RS world.

[14] We emphasize that the result that an equilibrium exists is not trivial: recall our earlier result that if disclosure is not complete but symmetric, there never exists an equilibrium.

Firms disclose their information simultaneously in the second stage, implying the disclosure rule of a firm may be made conditional only upon consumer-revealed information, in particular, on information about which firm(s) individuals have purchased insurance from.[15] As a policy offer is subject to cancellation once a firm receives information from other firms and from consumers, so the enforcement of the restrictions imposed within the policy offers can rely on information disclosed by consumers and firms. After the second stage, there is no further disclosure of information between firms, or further purchases of insurance by consumers.

Consumer disclosure is absolutely essential to our analysis, because it enables the asymmetric disclosure of information which is critical for the existence of equilibrium, as we have noted. Another critical aspect of this setting is that a contract offered by a firm does not depend upon those offered by other firms: it is non-reactive, keeping this paper in the spirit of competitive analysis, as in RS. As we have noted, there are other important strands of research in the theory of adverse selection focusing on imperfect competition and reactive equilibria.

5.2 A Simple Illustration of the Equilibrium Contract

Before conducting a formal analysis of an equilibrium we will describe how the set of equilibrium contracts we propose works in a simple context to highlight the core logic of the main argument on the existence of an equilibrium. The equilibrium we propose involves two kinds of firms: a given number of "established" firms selling insurance at the pooling odds, and many other "secret" firms, offering unlimited amount of insurance at price q_H without disclosure. An established firm sells a consumer insurance at the price $\overline{q}$ with the following restriction on additional insurance purchases, and with the following disclosure rule:

- Restriction: the total amount of revealed purchases is not greater than $\overline{\alpha}$ (the amount most preferred by the low risk consumers at price $\overline{q}$).
- Disclosure Rule: disclose its sale to all the other firms but those who are revealed by the consumer to be his insurers.

We denote the equilibrium contract offers for the secret and established firms by C_s^* and C_e^* respectively. For these contracts to sustain the equilibrium allocation E^*, they should be able to do the two things: 1) prevent over-purchases by high risk individuals and 2) deter a cream-skimming deviant contract from breaking an equilibrium, i.e. undermining the pooling contract by offering a contract that would be just be purchased by the low risk

[15] We explore the implications of alternative formulations, including sequential disclosure, in Stiglitz and Yun (2016) and Stiglitz et al. (2019). The present results appear to be quite robust.

individuals. The central result of this paper is to show that the allocation $\{Z_t(\overline{\alpha})\}_t$ is the only possible equilibrium allocation and can be sustained by the above set of contracts.

In section 5.3 we show that the equilibrium in this simple example with firms offering policies within a restricted set and with disclosure rules within a restricted set is in fact general; that is, if insurance firms are allowed to offer insurance policies and to engage in disclosure rules that were not so restricted, there is still an equilibrium of the form described for this restricted set.[16] Similarly, we show that the assumption dividing firms into established and secret firms is without loss of generality. Any firm could deviate in any way from its offers and disclosures in any way; but in the equilibrium we depict, no firm would want to.

The basic logic of the equilibrium is simple. Assume, for ease of exposition, that individuals honestly reveal to all the firms from whom they have purchased insurance all of their purchases from other firms if they do not lose anything by the revelations.[17] In particular, low risk individuals only buy $\overline{\alpha}$, and have no reason not to disclose it. As will be proven more formally later in lemma 6.1, the above disclosure rule adopted by the established firms leads to at least one firm knowing all the purchases by an individual.[18] This implies that in the equilibrium proposed above, high risk individuals would not be able to over- purchase insurance from the established firms by withholding some information from them. Thus, this limited version of honesty directly prevents anyone (that is, the high risk individuals) from overpurchasing the pooling contract.

More subtle is how the asymmetric disclosure rule prevents a deviant contract from breaking the pooling contract. Whenever a deviant firm, say A, offers a quantity of insurance equal to or less than $\overline{\alpha}$, charging a price lower than $\overline{q}$, the policy offered by A is always purchased, regardless of the restriction imposed by A, by both types of consumers, yielding losses for the firm A. This is because any consumer could always purchase the same amount $\overline{\alpha}$ in total from the deviant firm A and another established firm B, hence at an average price lower than $\overline{q}$. (The high risk individual will always then want to supplement it with secret insurance.) The deviant firm know this, so has to impose a restriction that the consumer not purchase any supplemental policy from an established firm (a firm selling a pooling contract). But the consumer will always disclose his purchase from A to B, knowing (under B's disclosure policy) that if he doesn't, B will disclose his sales to the consumer to all firms, including the deviant. That means that B does not disclose

[16]The same allocation may be sustained by different disclosure strategies, but all equilibria are allocation-equivalent, modulo information disclosure strategies.

[17]In the later analysis, we both show that this is the case and that our disclosure rule supports the equilibrium.

[18]It can easily be seen why a high risk individual would not lie. Assume he tried to over-purchase, say by purchasing $\frac{1}{2}\overline{\alpha}$ from 3 different firms, firms A, B, and C; but disclosed only one of his other purchases to each, say only his purchase from B to A, but not that from C to A, and symmetrically for the other firms. Then firm A discloses his sales to C. But then C knows that the individual's total purchases are $\frac{3}{2}\overline{\alpha}$ and his insurance is cancelled.

to A its sale (to that consumer) so that any restriction imposed by firm A can't be implemented. Thus, the asymmetric disclosure rule of the established firms can deter any cream-skimming deviant contract from upsetting an equilibrium while preventing over-purchases of the pooling contract by high risk individuals.

We note the importance of asymmetric disclosure by firms based on the consumer disclosure - which is only possible, as we have noted, because of consumer disclosure. Without consumer disclosure, there would be no basis for the asymmetry of the firm disclosure in a non-reactive framework where a disclosure rule of a firm does not depend upon the offers (policies or disclosure rules) of another firm. On the other hand, if the firm disclosure is symmetric and complete, we would obtain RS results, where a pooling allocation cannot be sustained in equilibrium.[19]

5.3 Contracts

Now we formalize these intuitions.[20] To simplify the notation and exposition, we begin by assuming all firms offer insurance with a single price,[21] while possibly imposing a constraint on aggregate purchases, and then in section 6.1 show that, given the equilibrium contracts described, no firm would want to offer other contracts with any other set of prices, restrictions or disclosure rules.[22]

1. Policies: (α, β) is given by $(\alpha, \alpha q) \in \mathbb{R}^2_+$. With the index i being suppressed for simplicity, a policy purchased by individual i from firm j is represented by $x(j) \in \mathbb{R}^2_+$:
$$x(j) \triangleq (\alpha_j, \alpha_j q_j) \tag{12}$$
while the set of policies purchased from all of the established insurers is denoted by $\hat{X} \triangleq \{x(j)\}_{j \in K}$ where $K \subset \mathcal{M}^{\mathcal{E}}$ is the set of the established insurers from which individual i purchases insurance. The amount[23] α_j of insurance offered by a firm j may be required to satisfy a restriction, which can be in general be represented by a set of insurance amounts allowed, denoted $\psi_j(X_j^T)$, where $X_j^T \subseteq \hat{X}$, as defined by eq. 18 below, is the total information (about the individual's purchases) available

[19] We have already established that if disclosure is not complete but symmetric, there exists no equilibrium.

[20] The formal notation employed in this section is employed only to a limited extent in the subsequent sections. Readers not interested in these formalities may, in a first reading, proceed directly to section 5.4.

[21] See appendix B and Stiglitz et al. (2017) for a generalization to the case with multiple prices and cross-subsidization. The equilibrium outcome is unchanged.

[22] Similarly, while we allow disclosure of any information available to firms, the equilibrium entails only disclosure of information revealed to it by its consumers plus what it knows from its own sales. Moreover, the only information disclosed is the quantity of insurance purchases. In more general models with sequential revelation of information (that is, firms can reveal information that they have from other firms to still other firms), disclosure rules can be more complex.

[23] In effect, the firms are offering contracts with linear prices; we show in section 6.1 and in appendix B that this involves no loss of generality.

to the firm j. That is, the individual (knowing what information the firm will have available to enforce whatever restrictions it imposes) will only choose to purchase an amount of insurance consistent with those restraints (for otherwise, the insurance will be cancelled):

$$\alpha_j \in \psi_j(X_j^T) \tag{13}$$

A policy offer by a firm j may thus be represented by $(q_j, \psi_j(X_j^T))$.

For the contracts discussed in section 5.2 the policy offers take simple forms.

a) The established firms' price is $\overline{q}$, the offer set is given by

$$\psi_j(X_j^T) = \{[0, \overline{\alpha}] | T(X^T) \leq \overline{\alpha}\} \tag{14}$$

with $T(X^T) = \sum_{x(k) \in X} \alpha_k$, the total amount of insurance known to have been purchased. A policy is cancelled is $T(X^T) > \alpha$.

b) For the secret firms, $q_j^* = q_H$ with $\psi_j^*(X^T) = \mathbb{R}_+$ (i.e. offering unlimited insurance with no restrictions).

2. Disclosure Rules: an information disclosure rule by a firm j, denoted DIS_j, specifies a set $RE_j(\subseteq M)$ of firms receiving information from j about a particular individual, and information $INF_{jk}(\subseteq X_j)$ to be disclosed to a firm $k(\in RE_j)$, where X_j (defined by eq. 15 below) combines the information the firm has directly about j with the information disclosed by a consumer to firm j about his purchases (including the purchase from j). The information disclosed is obviously a subset of X_j.

   The information disclosed by an individual i to his insurer j about purchases from others is denoted by $X_j^o(\subseteq \hat{X})$, indicating that an individual cannot disclose a policy that he does not purchase[24] although he may withhold from his insurer information about some policies purchased. Thus, the information set of firm j about individual i, X_j, before receiving information from other firms is

$$X_j = x(j) \cup X_j^o \tag{15}$$

   We suppose that whenever a policy $x(j)$ is disclosed, the identity j of the insurer is also disclosed. Thus, the set $I(\subset M)$ of firms (including firm j) disclosed as providing insurance by the consumer is given by:

$$I(X_j) \triangleq \{k \in \mathcal{M} | x(k) \in X_j\} \tag{16}$$

[24] That is, the individual cannot lie about purchases he has not made; in our model, so long as there is no negative insurance, individuals would have no incentive to make such lies.

Now a disclosure rule DIS_j of firm j may be represented as follows:

$$DIS_j(X_j) = (RE_j(X_j), INF_{jk}(X_j)) \tag{17}$$

specifying what firms will be disclosed to, and, given that there is some disclosure to firm k, what information is disclosed. Given the disclosure rules $\{DIS_j\}_j$ of all the firms, the aggregate of them will determine the information disclosed to firm j by all the other firms, denoted by X_j^{-j}. Thus, all the information X_j^T available to a firm is that disclosed to firm j by the consumer, by other firms, and what it knows directly from its own sales:

$$X_j^T \triangleq X_j \cup X_j^{-j} \tag{18}$$

For the contracts discussed in section 5.2 the disclosure rule can be simply described: for the established firms,

(a) Disclose to all of the firms that have not been disclosed by the consumers as insurers, i.e.

$$RE_j^*(X_j) = M \setminus I(X_j) \tag{19}$$

(b) All the information that a firm j has about a consumer that he has obtained from the consumer plus his own sales:

$$INF_{jk}^*(X_j) \triangleq X_j, \forall k \in RE_j \tag{20}$$

The secret firms disclose nothing:

$$RE_j^*(X_j) = INF_{jk}^*(X_j) = \emptyset \tag{21}$$

A contract C_j offered by a firm j is thus represented by a policy, characterized in turn by a price and a possible constraint on quantities purchased, and a disclosure rule:[25]

$$C_j = \{q_j; \psi_j(X_j^T); DIS_j(X_j)\} \tag{22}$$

5.4 Consumer Response

We now analyze consumers' responses to the set of offers. Consumer i chooses the mix of available contracts and a disclosure policy to maximize his utility, aware of the restrictions

[25] An attractive aspect of this disclosure rule is that the amount of information disclosed is very limited, in particular, insurance firms are not disclosing any information about the prices they are selling insurance to particular individuals. Such limited disclosures do not impose any significant anti-trust issues.

and the disclosure rules which may affect the implementation of those restrictions. Formally, given a set $\{C_j\}_{j\in M}$ of contracts offered by firms, a consumer i optimally chooses a set K of established insurers from which to purchase insurance, the set $\hat{X}(=\{x(j)\}_{j\in K})$ of policies to be purchased from them, $\{X_j^o\}_{j\in M}$ specifying which information about his purchases to disclose to whom, and amounts (if any) of insurance to purchase from other (the non-established) firms. If indifferent across multiple contracts, the consumers randomly chooses one.[26] Further, we assume that consumers tell the truth (disclose information) unless it is in their interests not to do so, which we refer to as the assumption of predilection for truth. It is important to emphasize that we do not assume that consumers are always truthful – we only assume that if they are indifferent between truth telling any anything else, they tell the truth. In other words, this is a tie-breaking rule, not an assumption requiring truth-telling.[27]

We can formalize the optimization problem for the consumers: each type t chooses a set $K\subseteq\mathcal{M}^{\mathcal{E}}$ (and $K'\subseteq\mathcal{M}^{\mathcal{S}}$) of established (and secret) firms, a set of policies $x(k)(=(\alpha_k,\beta_k))$ to purchase from them, and their disclosure rules $\{X_k^o\}_{k\in K\cup K'}$ to solve

$$\max_{\{\alpha_k,\beta_k\}_{k\in K\cup K'}}\sum_k V_t\left(\alpha_k,\beta_k\right) \tag{23}$$

$$s.t.\ \alpha_k\in\psi_k(X_k^T),\forall k\in K\cup K' \tag{24}$$

We say that a consumer's choice $\{\{x(j)\}_{j\in M},\{X_j^o\}_{j\in M}\}$ and disclosure rule is optimal if given $\{x(j)\}_{j\in\mathcal{M}},X_j^o,$ and$\{DIS_j(X_j)\}_{j\in\mathcal{M}}$ no policy is ever cancelled, and $\{x(j)\}_{j\in\mathcal{M}}$ solves the above problem.

5.5 Equilibrium Allocations

An equilibrium is defined as follows:

Definition 2. *An equilibrium is a set $\{C_j^*\}_j$ of contracts offered by firms such that, given the contracts offered by other firms $\{C_j^*\}_j$, there does not exist any other contract that a firm can offer to make positive profits given consumers' optimal responses to firms' announced contracts.*

We allow a deviant firm to offer any policy $x(j)$ with any restriction and to choose any disclosure rule based on any or all the information available to the firm. In this section, we show that the only possible equilibrium allocation is $\{Z_t(\overline{\alpha})_t\}$, the CPE allocation in the presence of undisclosed insurance which maximizes the well-being of the low risk individual, denoted by E^*.

[26] This tie-breaking specification is without loss of generality – the same equilibrium exists under other specifications.

[27] This tie-breaking specification will be used in the proof of lemma 6.1. If consumers pursue another action when indifferent, our equilibrium construction will have to change – in other words, this is a consequential tie-breaking assumption.

$$E^* \triangleq \{\{\alpha_t^*, \beta_t^*\}_{t=L,H}\} = \{\{Z_t(\overline{\alpha})\}_{t=L,H}\} \tag{25}$$

For any other posited equilibrium allocation, it is possible for an entrant to attract all of the low risk consumers and make a profit; hence that allocation could not be an equilibrium allocation.

While a formal proof is in the appendix, the result is almost trivial: assume that there were some other equilibrium allocation, generated by any set of contracts purchased from any array of insurance firms, that was not E^*. It cannot be preferred to the contract E^* by the low risk individual, for if it were it would have been purchased by high risk individuals as well, unless the contracts purchased by the high risk individuals make them even better off. But there cannot exist such a set of contracts that make both the high and low risk individuals better off than E^*, because we know that E^* is CPE. And it should be obvious that it cannot generate a lower level of utility for the low risk individuals, because an insurance firm that offered E^* would then attract all the low risk individuals, and at least break even. The low risk individuals would purchase that contract regardless of its information disclosure and cancellation provisions, since they will not purchase supplemental insurance and will not be affected by these provisions. The putative equilibrium can thus be broken. This establishes:

Theorem 5.1. *Suppose an equilibrium exists. The unique equilibrium outcome is the allocation E^*, the CPE which maximizes the welfare of the low risk individual.*

Attar, Mariotti, Salanié (2020b) and Jaynes (1978, 2011) have argued that (in somewhat different contexts) the unique equilibrium allocation with secret contracts must be the allocation that we have labelled as E^*. The reason (which they have not noted) that we should not be surprised is simple: E^* is the CPE, and any allocation that is not E^* is not entry proof in a Nash or competitive putative equilibrium, for the reasons that our proof makes clear. The only condition under which another allocation might not be upset by E^* is if incumbent firms react to the offer of E^* in ways which make E^* unprofitable, as might conceivably be the case in some reactive equilibria. Of course, if the reactions are not *too* strong, then any allocation that is not the CPE that maximizes the well-being of the low risk individual would be overturned. Here, however, we focus on competitive equilibria, where it seems inappropriate to assume that existing firms would respond to the entrant of a small firm offering an alternative policy or set of policies.

6 Proof of Equilibrium Existence

In this section we show that the contracts described in section 5 support the allocation E^* as an equilibrium. There may, of course, exist other equilibrium contracts that differ in the information disclosure elements, or in the number and identity of established and secret firms, but nonetheless result in the same allocation. Our objective is simply to demonstrate the existence of an equilibrium with endogenous information that is implemented using simple, interesting, and illuminating contracts.

In showing that the equilibrium set of contracts C_j^* implements E^*, we first prove the following lemma:[28]

Lemma 6.1. *Given the set of contracts C_j^*, no individual purchases more than $\overline{\alpha}$ from the established firms.*

This in turn implies that all purchase just $\overline{\alpha}$. While a formal proof is given in the appendix, the intuition is clear. Assume an individual purchased more than $\overline{\alpha}$ in the aggregate from the established firms. Given C_j^* he cannot disclose that he has purchased more than $\overline{\alpha}$ (to any of his insurers) because were he to do so, the policy would be cancelled. So there must not be full disclosure. If the consumer does not disclose one of his insurers, say purchases from firm j, then all the other insurers disclose to the firm j what they know about the consumer's purchases (i.e., their sales to the consumer, and what the consumer reveals to them), and then the firm knows that the individuals has purchased more than $\overline{\alpha}$, so j cancels its policy. But the individual would have known that, and so would not have purchased a policy from j.

There is one important corollary of lemma 6.1: all individuals reveal their purchases from established firms to all established firms, since they have no reason not to (using the assumption of predilection for truth). We now prove the main theorem of the paper:

Theorem 6.2. *Suppose that the income consumption curve for insurance for type t individual at price $\frac{P_t}{1-P_t}$ is the full insurance line, and let $C_j^* = \{q_j^*; \psi_j^*(X_j^*); DIS_j^*(X_j)\}$ be defined as follows:*

1. *For established firms $(j = 2, 3, \ldots, n)$, $q_j^* = \overline{q}$, with $\psi_j^*(X_j^T)$ given by*

$$\psi_j^*(X_j^T) = \{[0, \overline{\alpha}] | T(X_j^T) \leq \overline{\alpha}\} \tag{26}$$

where $T(X_j^T) = \sum_{x(k) \in X_j^T} \alpha_k$. The information disclosure rule of established firms is given by

$$DIS_j^*(X_j) = (RE_j^*(X_j), INF_{jk}^*(X_j)) \tag{27}$$

[28]Note that, in equilibrium, no established firm sells more than one contract to an individual. It would be only the high risk individuals that may be interested in purchasing multiple contracts from a firm, because by so doing they might underreport their purchases from the insurer to another established firm (disclosing one policy but not another) to be able to purchase more than $\overline{\alpha}$ at $\overline{q}$. Knowing that the only individual who would wish to buy multiple policies is a high risk individual, an established firm would not sell multiple contracts to an individual without charging a price equal to or higher than q_H, which, however, would not be taken by any individual.

a) *Disclose to all of the firms that have not been disclosed by the consumers as insurers, i.e. to*

$$RE_j^*(X_j) = M \setminus I(X_j) \tag{28}$$

b) *All the information that a firm j has about a consumer that he has obtained from the consumer plus his own sales:*

$$INF_{jk}^*(X_j) \triangleq X_j, \forall k \in RE_j \tag{29}$$

A policy is cancelled if

$$T(X_j^T) > \overline{\alpha} \tag{30}$$

2. *For secret firms* $(j = n+1, n+2, \ldots, N), q_j^* = q_H$ *with no restrictions i.e., offering unlimited insurance at* $q_H : \psi_j^*(X_j^T) = \mathbb{R}_+$, *and make no disclosure:*

$$RE_j^*(X_j) = INF_{jk}^*(X_j) = \emptyset \tag{31}$$

Then C_j^* *implements allocation* E^*.

The equilibrium contract, identical to the one described informally in section 5.2, imposes incentive-compatible and enforceable restrictions on the amounts of insurance which an individual can purchase from other firms.

Proof. It is obvious that by lemma 6.1, the set of contracts C_j^* generates the equilibrium allocation E^*. Because of lemma 6.1 and its corollary, every established firm has the information required to effectively implement the allocation. There is no over-insurance by high risk individuals. They just purchase $\overline{\alpha}$ from the established firms and supplement it with undisclosed insurance at price q_H, bringing them to full insurance.

We now show the set of contracts C_j^* sustains E^* against any deviant contract, offered by a new entrant or by one of the existing established or secret firms. Note first that a deviant firm, indexed by d, cannot make profits by attracting only high risk individuals in the presence of firms offering secretly any amount of insurance at q_H. This is because then no individual would pay a price higher than q_H since a deviant firm cannot induce the established firms (with C_j^*) to sell more than $\overline{\alpha}$ at $\overline{q}$ under any circumstance. If the deviant firm d attracts both high and low risk individuals, its policy would have to charge a price q_d equal to or lower than $\overline{q}$, yielding zero profit at best. A deviant firm d can thus make positive profits only by attracting low risk types only, i.e., only by a cream-skimming contract C_d. We will now show that in the presence of undisclosed insurance at price q_H, the contract C_d always attracts high risk individuals. To attract low risk individuals, we must have $q_d < q_H$. It is obvious from lemma 4.1 that, the high risk individual, if he could, would purchase the contract OD in figure 4 plus additional pooling insurance DB

in figure 4 up to $\overline{\alpha}$ plus supplemental undisclosed insurance (BF in figure 4) at q_H, rendering the deviant contract unprofitable. The deviant firm knows this, and hence must put a restriction on the amount of supplemental pooling insurance that the individual can purchase. The problem is that no such restriction can be enforced. The high risk individual obviously will not disclose directly that he has made the supplemental pooling purchases. If the high risk individual discloses his purchase of the deviant contract to the established firms and limits his total purchases (combining what he has purchased from the deviant firm and amounts purchased from other established firms) to $\overline{\alpha}$, no established firm will cancel insurance that it has sold, and, by its disclosure rule of C_j^*, no established firm will reveal to the deviant firm its sales to the individual. Thus, firm d cannot enforce any restriction entailing total purchases from itself, plus from the established firms, being less than or equal to $\overline{\alpha}$. Accordingly, high risk individuals will purchase the deviant contract, and it loses money because the deviant firm could not enforce any restriction on such purchases and without such restrictions, the high risk individual will purchase the contract. Similarly, the deviant firm cannot make its disclosure rule effectively depend on such supplemental insurance, and so can't use the disclosure rule to deter purchases. That is, a deviant contract would not be able to upset the proposed equilibrium regardless of *any* restriction or any disclosure rule it may take. □

6.1 Generalizations

We began the analysis by assuming all firms offer insurance with a single price while possibly imposing a constraint on aggregate purchases. But the proof showed that, given the equilibrium contracts described, no firm would want to offer any other contract(s), so that the equilibrium established in theorem 6.2 holds when firms are not so constrained.[29] This includes an established firm offering only fixed quantity contracts or offering both price and quantity contracts. It could offer a contract at a price higher than $\overline{q}$ but lower than q_H, but such a contract would only be purchased by the high risk individuals, and thus lose money. It could offer a contract at a price lower than $\overline{q}$, and such a contract will attract both all high and low risk individuals, and thus lose money (we've already discussed the impossibility of cream skimming in the proof of theorem 6.2). By the same token, we began our analysis assuming a fixed number $|\mathcal{M}^{\mathcal{E}}|$ of established firms and $|\mathcal{M}^{\mathcal{S}}|$ secret firms. But since there is no incentive either to enter or to change from being an established firm to a secret firm, $|\mathcal{M}^{\mathcal{E}}|$ and $|\mathcal{M}^{\mathcal{S}}|$ are perfectly arbitrary (so long as

[29]The proof also makes it clear that no firm would want to change its disclosure rule (or to change simultaneously its contract and its disclosure rule.) It should be obvious that the results do not depend on convexity of preferences. All that is required is that the income consumption curve for insurance for the high risk individual at the q_H is the full insurance line, i.e. the high risk individual's indifference curve is tangent to the price line along the full insurance line, and any other point of tangency generates a lower level of utility.

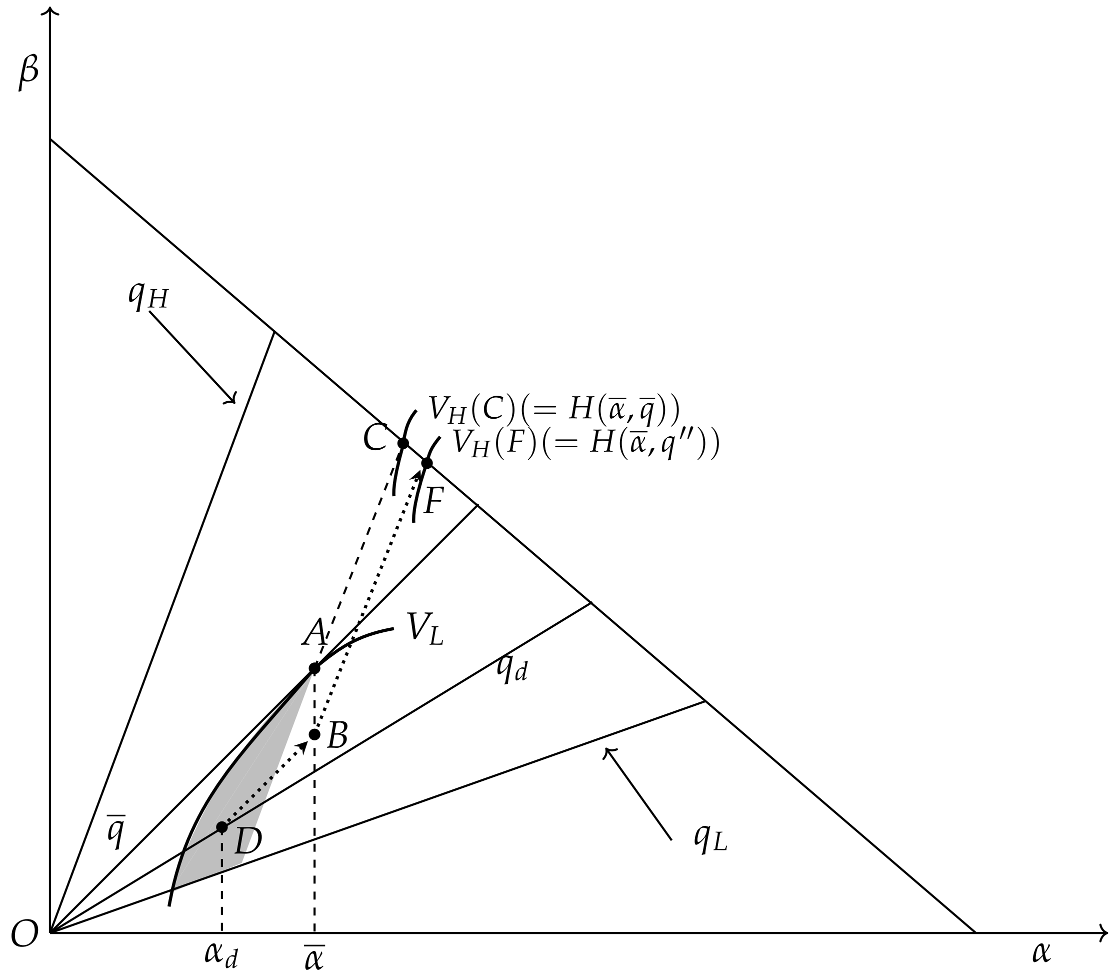

Figure 4: Sustaining an equilibrium against C_d (offering D); $q'' =$ slope of $OB(< \overline{q})$. High risk individuals would always supplement D with pooling insurance (DB) (disclosed only to non-deviant established firms) and secret insurance (BF). As a result $V_H(F) > V_H(C)$, and accordingly, the deviant contract would make losses.

they are large enough that the assumption that any individual firm's action has no effect on the behavior of others is plausible): for any $|\mathcal{M}^{\mathcal{E}}|$ and $|\mathcal{M}^{\mathcal{S}}|'$ there is an equilibrium with the same allocations and the same contracts (policies and disclosure policies).

There is one extension which is not so simple: what happens if the deviant firm offers a menu of policies, in particular one purchased by high risk individuals, the other by low risk individuals? Is it possible that such a pair of policies - with cross-subsidization - could break the equilibrium?[30] In the appendix, we show that, even when a deviant firm offers

[30]Without subsidization, the analysis is unchanged. As we have already noted, any firm can offer any array of policies; in effect, it can offer every individual a menu of options. Of course, no high risk individual would ever buy a secret policy (at the high risk odds) from any firm from which it bought a pooling contract, for the firm would then know that the individual was a high risk individual, and (in a slight extension of our analysis) it would accordingly cancel the insurance. (That is, a condition for the purchase of a pooling contract is that the individual not buy a policy at any price higher than the pooling price - that it knows about. In

multiple contracts at different prices, there still exists an equilibrium.[31]

7 Extension to Cases with Many Types

The result on existence of equilibrium can be extended to the case with many types. An equilibrium strategy in a case with the three types, for example, can be described in a similar way to the case with two-types. As illustrated in figure 5, there is a pooling contract with all three types, contract A, the most preferred by the lowest risk type; and a partial pooling contract AB, with additional insurance entailing partial pooling, bringing together the two riskiest types, where B is the most preferred along the zero profit line for partial pooling for the middle risk individual; and finally, a contract BC, at the high risk individual's odd, leading to full insurance to the highest risk type. A, B, and C represent the total individual allocations for the three types. In equilibrium, the lowest risk type consumers purchase A only, the "middle" risk consumers purchase A and the supplemental policy AB, and the high risk individuals purchase A and both supplemental policies AB and BC.

There are three types of firms, those selling the full pooling contract, those selling the partial pooling contract, and those selling the secret insurance (the price contract at the price q_H) contract to the high risk individuals. They adopt analogous information disclosure rule as in the case of two types of individuals.[32] Consumers truthfully fully reveal to the other insurers their information about their purchases of the fully pooling contract A (since all purchase the same amount, such information in equilibrium reveals no information about who they are).[33] Consumers reveal information about their purchases of the partial pooling policies B only to firms not selling them the fully pooling policy (for if they disclosed that, the firm would know that they were of one of the two riskier types). By

the context of our model, where firms either sell secret insurance or pooling insurance, the firm would never know that an individual purchased the secret insurance.) Within the menu of policies that each firm can offer are price-quantity contracts (e.g. of the kind that RS analyzed). Our disclosure strategies both for firms and consumers remain unchanged. Consumers still disclose the amount of insurance that they have purchased and from whom they have purchased it; and firms disclose information about j's purchases to those firms that have not (been disclosed to have) sold insurance to j. The assumptions of our simplified model are chosen to highlight one of the key issues we focus on, the problem of high risk individuals buying more than $\overline{\alpha}$ of insurance in the context where the individual can purchase small amounts from many providers, and there is not automatic disclosure of information about purchases.

[31] We can also allow firms to offer multiple policies, allowing individuals to purchase more than one, and disclosing only some of the purchases. But the only individuals who would not want full disclosure of the information concerning their purchases are high risk individuals, and so the firm would have no incentive not to disclose all the purchases of an individual, if it discloses any information about the individual.

[32] That is, revealing information only to firms not revealed to be sellers to individuals.

[33] Accordingly, no individual is worse off revealing his purchases of the full pooling contract A than not fully revealing his purchases. In fact, in the three-type case, an individual buying insurance from other than a fully pooling seller has an incentive to disclose his purchase from a fully pooling seller, because otherwise that insurer discloses to his fully pooling insurer his sales, and then the seller of that policy would cancel the contract it sold to him.

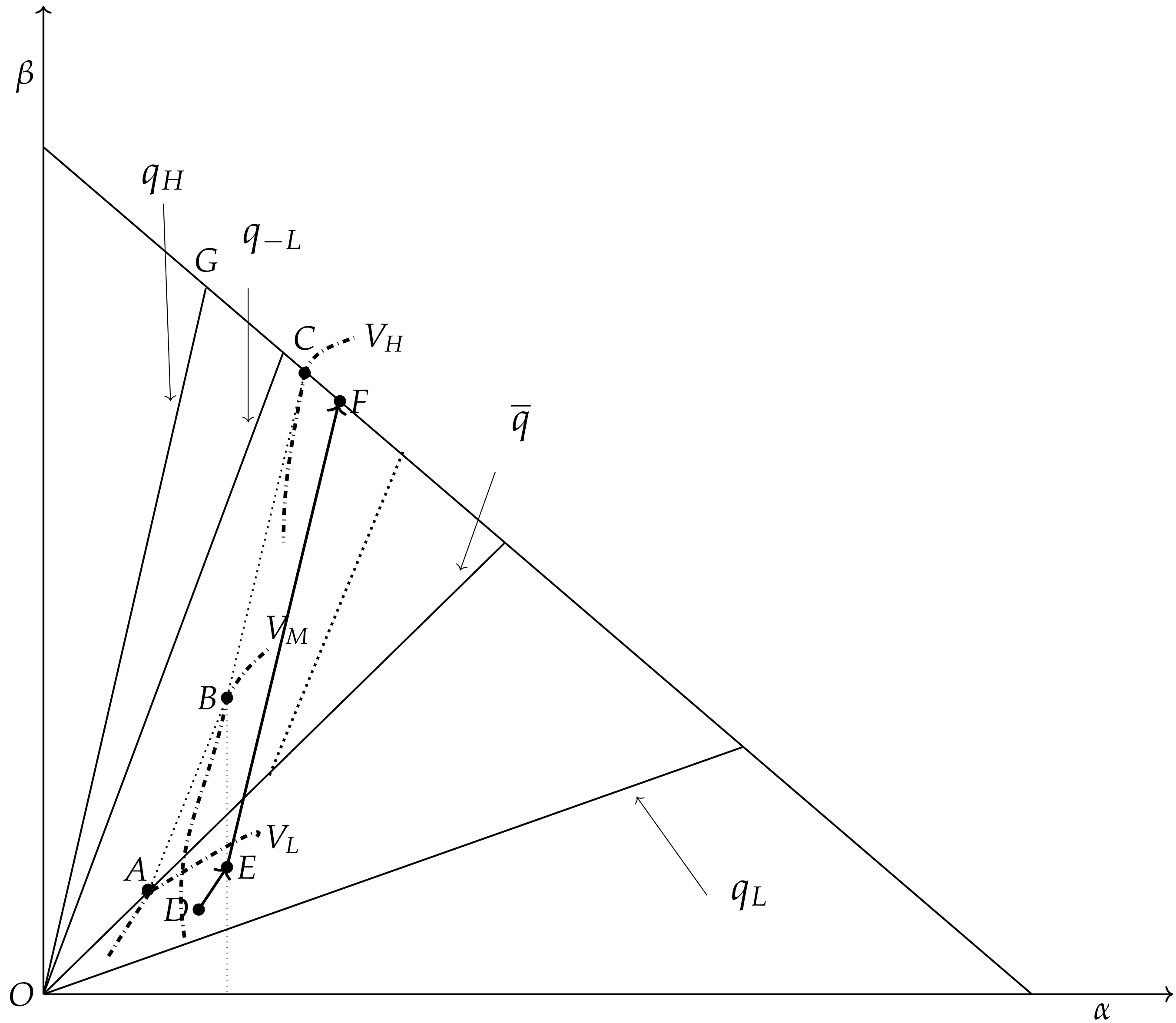

Figure 5: Equilibrium allocation (A, B, C) with three types, which cannot be broken by D as individuals of higher-risk type supplement it by additional pooling insurance (along the arrow) without being disclosed to the deviant firm. The slopes of the various lines are indicated by the q's. P_{-L} denotes the average probability of accident for the two highest risk types (and $q_{-L} = \frac{P_{-L}}{1-p_{-L}}$), while V_i indicates an indifference curve for i-risk type ($i = H, M, L$). The lines AB and DE are parallel to OC; the lines EF and BC are parallel to OG.

the same reasoning as in the two-type case, there is no possibility of B or C buying more of the fully pooling contract, or C buying more of the partially pooling contract and no room for a cream-skimming deviant contract. Figure 5 illustrates with a deviant contract offering D that attempts to profitably attract only low and medium types, as riskier types are also induced to choose D as they can purchase additional insurance along the lines DE and EF; and by the same argument the partially pooling contract B can be sustained. This reasoning allows us to extend the analysis not only to any finite number of types, but to a continuum of types.

8 The Public Option

Our equilibrium analysis highlights the importance of asymmetric disclosure of information about insurance purchases. While there is some complexity in the formal description of the market equilibrium that we describe it is actually simple and intuitive. But that raises the question, is there some other way of implementing a robust CPE equilibrium? Could government regulation, such as a public register, help?

Our analysis has provided some caution to such approaches. Recall that the "trick" in sustaining the pooling contract - at the heart of the constrained Pareto equilibrium and which has distinct welfare benefits, with those who are fortunate in having a low probability of the occurrence of the insured against event subsidizing those with a high probability - is creating asymmetries in information about insurance purchase, as a response to the natural asymmetries of information about risk types. If the government required disclosure of information and made that information public and could enforce such a requirement for all firms, we would be back in the RS world, with no pooling and with the problems delineated in the beginning of the paper. More likely, while the government can force disclosure of "established" firms, it cannot force disclosure of informal insurance and insurance purchased from small firms. With disclosure of established firms accompanied by secret insurance, we are in the world of section 3, where no equilibrium exists, either in quantity constrained world or in the price cum quantity constrained world (the Akerlof quantity-price equilibrium).

An alternative is a well-designed public option, accompanied by appropriate disclosure policies. The government offers insurance at the market odds, requiring individuals to disclose whether they have purchased insurance at that price from anyone else, and restricting (as in our model) total purchases to be less than or equal to $\overline{\alpha}$. But the government discloses its information only to those firms that have not been disclosed to been sellers of insurance to the individual. Then our reasoning shows that the public option cannot be undercut. It is sustainable. Without such disclosure, there will be over insurance - the problem with the Akerlof equilibrium; with full disclosure, there is cream skimming *à la* RS. Our disclosure rule under the public option prevents both over insurance from established firms and cream skimming.

One of the intents of the public option is to encourage competition in lowering transactions costs - though there is little evidence that the private sector can come anywhere near the costs of government. But we can allow for this. The government can still prevent cream skimming. If the entrant is truly more efficient, it will still constrain its own sales to $\overline{\alpha}$, and displace government sales. If it is not more efficient, it will lose money, because if it attempted to charge a price equal to or greater than its costs (greater than $\overline{q}$ but less than or equal to q_H) it would only attract high risk individuals and so couldn't make a profit.

9 Previous Literature

In the more than four decades since RS appeared, its disquieting results have given rise to several large literatures.

Varying Equilibrium Concepts and Game Forms

The first strand looked for alternative equilibrium concepts or game forms, under which equilibrium might always exist, or under which a pooling equilibrium might exist. Rothschild and Stiglitz (1997) and Mimra and Wambach (2014) reviewed the literature as it existed to those points, with Rothschild and Stiglitz (1997) suggesting that proposed seeming resolution of their non-existence result contravened plausible specifications of what a competitive market equilibrium should look like in the presence of information asymmetries. For instance, in the "reactive" equilibrium of Riley (1979) contracts are added in response to out of equilibrium offers, while in "anticipatory" equilibrium (Wilson (1977)), the entry of even a very small firm induces all firms to withdraw their pooling contracts, making the deviant contract unprofitable, and enabling the pooling equilibrium to be sustained. Miyazaki (1977) (in the case of two types) and Spence (1978) (in the case of n types) extend this reactive equilibrium concept to allow for menus of contracts; the Miyazaki-Wilson-Spence (MWS) outcome entails separating, jointly zero-profit contracts with cross-subsidization.

The formal game theory literature since then taken two tracks. One has supported the MWS equilibrium under various conditions, with Mimra and Wambach (2017) endogenizing capital level choice before playing the RS game, Mimra and Wambach (2019) relying on latent contracts, and Netzer and Scheuer (2014) showing that the MWS outcome is a "robust" equilibrium when there are small costs associate with withdrawing from the market. The second approach has focused on a mixed strategy equilibrium (Dasgupta and Maskin (1986)); Farinha Luz (2017) provides a full characterization of equilibria in this setting.

Consequences of Different Information Structures

In one important strand of research attention is focused not on different equilibrium constructs but on the consequences of different information structures, allowing for nondisclosed contracts, but *not endogenizing disclosure*. Most notable are the series of papers by Attar, Mariotti, Salanié (2011, 2014, 2016), employing a variety of assumptions about consumer preferences, firm behavior, and market structure. While a complete explication of the differences would take us beyond this paper, we note some key salient differences.

The 2014 model, employing strictly convex preferences, provides necessary conditions for the existence of an equilibrium in a much more general setting than discussed here. Applied to the insurance market, the equilibrium (when it exists) turns out to be the allocation where no one but the highest-risk individuals purchase insurance. The difference between their results and ours, where we have focused on endogeneity of information disclosure, are marked and obvious. In their (2016) model, they allow individuals to buy insurance from multiple insurers, without disclosure.[34] Within their equilibrium construct (distinct from ours), they prove a parallel result, that any competitive equilibrium must entail the allocation E^*. Our analysis in section 3 has explained why this should not be a surprise: E^* is the CPE allocation which maximizes the welfare of the lowest risk individuals. They are able to establish the existence of equilibrium, using latent contracts, but only under a very restrictive set of preferences, more restrictive even than the single crossing property.

Attar, Mariotti, and Salanié (2020b) construct an equilibrium employing a sophisticated auction scheme under the assumption of a fixed information structure. Though the equilibrium is Nash, the structure of the model has an important reactive element - firms can observe and punish deviators. The resulting allocation is again E^*, but that allocation is sustainable for very different reasons. Ours is a *competitive* theory, where the market does not respond, say, the entry of a new firm or a deviation in the contract offers of a single deviant firm. Theirs is in the long tradition of trying to build in the possibility of such large responses (so, for example, the entry of a new firm does not alter the response function), and it takes those large responses which enable the equilibrium to be sustained, e.g. entry to be deterred.

Furthermore, the key issue posed in our analysis of nonexclusivity is that firms can offer contracts that are not observed by other firms, and that in the presence of such contracts, overinsurance by the high risk types is difficult to deter. Thus, in our model markets are nonexclusive not only in the sense that buyers can trade with multiple sellers (which is the sense of "nonexclusivity" employed by Attar, Mariotti, and Salanié (2020b)) but also in a much stronger sense - sellers cannot *observe* purchases made by consumers

[34] See also Ales and Maziero (2014).

from others.[35] Attar, Mariotti, and Salanié (2020b) prevent such overinsurance by a combination of elements – utilizing a sequential auction where offers cannot be withdrawn, using a rationing rule for sales, leveraging the fact that deviations by sellers are observable (and therefore can be punished) - which together obviate the issue of unobservable offers. Thus, the most important difference between our analysis and theirs is how equilibrium is sustained - in ours, endogenous and asymmetric information disclosure is crucial in establishing the competitive equilibrium; in theirs, it is the combination of elements just described, which result in their non-competitive equilibrium. Given the markedly different settings of the analyses, it is not surprising that we require different assumptions to ensure the existence of equilibrium: Attar, Mariotti, and Salanié (2020b) do not focus on information revelation, market equilibrium, and employ strong assumption on preferences (strict convexity, differentiability, and a condition stronger than single crossing). By contrast, our analysis has focused on equilibrium and how the CPE allocation we have identified can be decentralized, and our results do not even require the single crossing property to be satisfied. Of course, the most important different is that the endogeneity of information is at the heart of our analysis.

There are therefore, several senses in which our contributions differ: 1) our notion of nonexclusivity is stronger, 2) in our setting firms cannot observe the offers of other firms (and therefore, cannot directly detect cream-skimming deviations) whereas in their setting they can, 3) they decentralize the allocation using an auction scheme, whereas we use information disclosure in a way that seems perhaps more illuminating, 4) the convexity of the price schedule is a consequence of the auction rules assumed, whereas we obtain it without requiring the schedule to have any properties in advance, and 5) we have a simultaneous, competitive setting with endogenous information revelation, whereas they have a sequential setting of almost perfect information, with a specific, fixed, information structure which generates far more complete information than that which arises in our model. Both of our models decentralize the allocation E^*, but in markedly different ways. The key difference, however, is the reactive nature of their equilibrium. As they write (italics added): the allocation E^* "...emerges as the essentially unique outcome of competition *when each seller can quickly react to his competitors' offers.*" Which formulation is more natural depends, of course, on applications.

Endogenous Information

The closest works to our paper within the adverse selection literature are Pauly (1974), and especially Jaynes (1978, 2011) and Hellwig (1988), who analyze a model with a kind of strategic communication *among firms* about customers' contract information. Jaynes

[35]This usage is also consistent with that of Arnott and Stiglitz (1990) who examined the welfare economics of nonexclusivity in much simpler settings.

(1978) analyzes the same allocation E^* that we do. However, as Hellwig (1988) clarified, in Jaynes (1978)'s two-stage framework, the strategy of firms, including the associated strategic communication, is not a Nash but a reactive equilibrium, with firms responding to the presence of particular deviant contracts, and thus Jaynes (1978)'s formulation was subject to the same objections to reactive equilibria raised earlier. Hellwig (1988) formulated a four-stage game, in which E^* emerges as the sequential equilibrium, but as he emphasizes, it has the unattractive property that firm behavior (in the final two stages) is conditioned on knowing the offers of all firms, including the deviant firm. Thus, in contrast to our model, a firm cannot offer a contract in secret. Moreover, as Hellwig (1988) observes, "...it is not the endogenous treatment of interfirm communication that solves the existence problem of Rothschild, Stiglitz, and Wilson. Instead the existence problem is solved by the sequential specification of firm behavior which allows each firm to react to the other firms' contract offers."[36]

While our work differs from that of Jaynes (1978, 2011) and Hellwig (1988) in several ways, perhaps most important is that we consider information revelation by consumers as well as firms.[37] This allows the creation of asymmetries of information about insurance purchases between established firms and deviant firms, which, in turn, enables the pooling contract to be sustained *in the context of a competitive non-reactive equilibrium*. As we have noted, there is a delicate balance: on one hand, one has to prevent overinsurance by high risk individuals purchasing pooling contracts (which requires established firms to know certain information), and on the other, one has to prevent a deviant firm from having enough information to enforce an exclusive contract that would break the pooling equilibrium. The consumer and firm information strategies which we describe achieve this. In contrast, at least in a simple setting, models relying on just firm information strategies cannot do this, because they do not have any basis on which to engage in

[36]More generally, changing the sequence of behavior, e.g. which side of the market moves first, can have a significant effect on the market equilibrium. While in the context of insurance markets, it is natural to have firms move first (making offers), in the context of other adverse selection models, e.g. the labor market where individuals have to choose a level of education and firms have to decide the wages to pay to those with different levels of education, there is more ambiguity. See Stiglitz and Weiss (2009).

[37]By the same token, while the equilibrium we identify shares some features with the equilibrium of the limit-order book studied by Glosten (1994), the context is different, most importantly, there is not the endogenous determination of information sharing between consumers and firms that is the central feature of our analysis. So too, Jaynes (1978, 2011) posited competitive equilibrium allocation - which, as we have noted, turned out not to be a competitive equilibrium in the usual sense but a reactive equilibrium disguised within a multi-stage sequential equilibrium - is identical to what we have established to be the competitive equilibrium with endogenous information. Of course, with endogenous information, the equilibrium itself requires a specification of disclosure rules, something that is not part of any equilibrium where there is not endogenous disclosure. Our paper is to our knowledge the first to incorporate consumer disclosure - indeed, as we have emphasized, such disclosure is essential to creating the asymmetries of information (disclosure) that, in turn, are essential to establishing equilibrium; accordingly, earlier literature had nothing to say about consumer disclosure rules in or out of equilibrium. It is important not to confuse the similarity of the allocations in these different settings with the marked differences in the settings and the specification of the full equilibrium.

this necessary kind of selective disclosure. Moreover, it is natural to allow consumer revelation of information: such revelation is a standard feature in markets with asymmetric information, and especially when dishonest disclosure can be punished, it occurs naturally in such markets, as less risky individuals attempt to distinguish themselves from the more risky. Furthermore, the distinction which is central to our paper, and much of the literature, between a requirement to "tell the truth and nothing but the truth," and the requirement to "tell the whole truth" is also a natural one. Individuals can be punished for lying (e.g. when the accident occurs, the individual cannot collect on the benefits if it ascertained he lied); but there may be implicit insurance, e.g. from one's family, which is not disclosed.

Welfare Economics of Adverse Selection

While the main focus of our analysis has been the analysis of equilibrium in markets with adverse selection, section 4 addressed issues of the welfare economics of adverse selection, introducing the concept of constrained Pareto optimality. Our work can thus be viewed as a continuation of that of Bisin and Gottardi (2006) who prove versions of the first and second welfare theorems in a setting with adverse selection, but with exclusivity; our result in effect shows that a limited analogue of the second welfare theorem with nonexclusivity (in the way we use the term) also holds: the (constrained) Pareto efficient equilibrium that maximizes the utility of the low risk individual can be implemented by a market mechanism; and we show explicitly how it can be done. Moreover, in parallel to the first welfare theorem, we have established that if there exists a market equilibrium, it must be CPE.

Attar, Mariotti, Salanié (2020a) take a normative point of view, similar to our analysis in section 4. The constraint on which they focus is that the planner is unable to prevent consumers from *trading* with a third firm. This is related to, but not the same as, the constraint that we focus upon: our definition of constrained Pareto optimality here and in our earlier papers (2016, 2018) centers on the inability of government to force disclosure, though it may (as in our equilibrium construct) induce disclosure.[38] Thus, while the allocation they identify is the same, the logic is different.[39] Still, it is not surprising that the results are parallel.

[38] One could, for instance, force disclosure, but not restrict trade. It is, of course, difficult to restrict trade without information about what trades occur. Thus, there is some presumption that the constraint restricting trade is a more binding constraint.

[39] Attar, Mariotti, and Salanié (2020a) in discussing an earlier version of this paper, observe: "...the logic of our approach is entirely different. First, these authors allow firms to react to the information disclosed by their competitors by possibly enforcing exclusivity clauses, which is at odds with the very notion of side trading that we emphasize. Second, [...] we are interested in the normative implications of side trading and not in characterizing the equilibrium of a given extensive-form game."

Concluding Remark

Rothschild and Stigliltz (1976) and Akerlof (1970) showed that market equilibrium with information asymmetries look markedly different from those without such asymmetries. The difference in the characterization of the equilibrium in the two papers highlighted the importance of information about purchases (quantities). Indeed, a corollary of theorem 5.1 is that using a slight generalization of an Akerlof equilibrium, in which firms know about their insurance sales to any individual but not about the sales of others, is that generically an Akerlof equilibrium does not exist (Stiglitz, Yun, and Kosenko (2018)). Models which assume full disclosure to everyone, as RS and its decedents do, and models which assume no disclosure to anyone, as Akerlof and the decedents in that tradition do, do not create asymmetries of information about the amounts of insurance purchased. It is hardly a surprise that such asymmetries matter. What is perhaps a surprise how much they matter: with appropriately structured asymmetries, which arise *endogenously* as part of a natural definition of equilibrium in competitive markets with endogenous disclosure, equilibrium always exists, even without the single crossing property being satisfied, and entails partial pooling. Finally, it is extremely surprising (at least to us) that among all possible information revelation structures in our setting, there exists a very simple one that, without restricting what firms and consumers share with whom, endogenously implements the equilibrium outcome.

Appendix A: Omitted Proofs

Derivations of equations 5 and 6:

Assume an individual with a policy $(\alpha, q\alpha)$ supplements it by purchasing $\alpha^S (> 0)$ at q_H to reach full insurance, $(\alpha + \alpha^S, q\alpha + q_H\alpha^S)$, where $q\alpha + q_H\alpha^S = d - (\alpha + \alpha^S)$. Note that $\alpha^S > 0$ if $\alpha + q\alpha < d$. Then

$$\gamma(\alpha, q\alpha) = \alpha + \alpha^S = \frac{1}{1 + q_H}[d + (q_H - q)\alpha] \tag{32}$$

which determines $\delta(\alpha, \alpha q)$ as $B(\alpha, \alpha q) = d - \gamma(\alpha, \alpha q)$.

Proof. **(Lemma 4.2)**: Consider any set $\{(\alpha_t, \beta_t)_{t=L,H}\}$ of policies that yield zero profit, i.e.,

$$\theta\pi_H(\alpha_H, \beta_H) + (1-\theta)\pi_L(\alpha_L, \beta_L) = \theta\{(1-P_H)\beta_H - P_H\alpha_H\} + (1-\theta)\{(1-P_L)\beta_L - P_L\alpha_L\} = 0 \tag{33}$$

Let each policy (α_t, β_t) be represented as the sum of $(\hat{\alpha}_t, \hat{\beta}_t)$ and (α_t^S, β_t^S), where

$$\alpha_t = \hat{\alpha}_t + \alpha_t^S, \beta_t = \hat{\beta}_t + \beta_t^S \text{ while } \hat{\beta}_t = \overline{q}\hat{\alpha}_t, \beta_t^S = q_t\alpha_t^S. \tag{34}$$

It will then suffice to show that $\hat{\alpha}_H = \hat{\alpha}_L$. Since $\pi_t(\alpha_t^S, \beta_t^S) = 0$ for $t = H, L$ we have

$$\theta\pi_H(\hat{\alpha}_H, \hat{\beta}_H) + (1-\theta)\pi_L(\hat{\alpha}_L,)\hat{\beta}_L = 0. \tag{35}$$

Using $\overline{\beta}_t = \overline{q\alpha}_t$ and rearranging the terms, we have

$$\theta[\overline{P} - P_H]\hat{\alpha}_H + (1-\theta)[\overline{P} - P_L]\hat{\alpha}_L = 0. \tag{36}$$

Using the result that $\theta[\overline{P} - P_H] + (1-\theta)[\overline{P} - P_L] = 0$, this implies that

$$\theta[\overline{P} - P_H](\hat{\alpha}_H - \hat{\alpha}_L) = 0 \tag{37}$$

i.e., $\hat{\alpha}_H = \hat{\alpha}_L$. □

Proof. **(Theorem 5.1)**: Suppose, to the contrary, there were another equilibrium allocation $\tilde{E} \neq E^*$, implemented by an arbitrary set of contracts, and let $\tilde{E} = \{(\tilde{\alpha}_L, \tilde{q}_L), (\tilde{\alpha}_H, \tilde{q}_H)\}$. $\tilde{E}$ may be pooling, fully separating, or hybrid; let $\tilde{q}_t$ be the average price for type t.

Decompose $\tilde{E}$ using lemma 4.2:

$$\tilde{\alpha}_L = \hat{\alpha} + \tilde{\alpha}_L^S \tag{38}$$

$$\tilde{\alpha}_H = \hat{\alpha} + \tilde{\alpha}_H^S \tag{39}$$

If $\tilde{\alpha}_L \neq \overline{\alpha}$, by lemma 4.3, the allocation is infeasible. So it must be that $\tilde{\alpha}_L = \overline{\alpha}$ (or equivalently, $\tilde{\alpha}_L^S = 0$). Suppose that $\tilde{q}_L > \overline{q}$. Then a firm could enter, offer a contract $(\overline{\alpha}, \overline{q})$, selling at most one policy for each individual. $V_L(\overline{\alpha}, \overline{q}) > V_L(\overline{\alpha}, \tilde{q}_L)$, and so the low risk types prefer this to their putative equilibrium allocation, purchase it, so $\pi(\overline{\alpha}, \overline{q}) \geq 0$. Thus, $\tilde{E}$ cannot be an equilibrium.

Suppose now that $\tilde{q}_L < \overline{q}$. By lemma 4.2, $\tilde{\alpha}_H = \overline{\alpha} + \tilde{\alpha}_H^S$, by lemma 4.1, $\tilde{\alpha}_H^S > 0$, and thus both types purchase $\overline{\alpha}_L$. Then $\pi(\tilde{\alpha}_L, \tilde{q}_L) < 0$, and $\tilde{E}$ cannot be an equilibrium, since firms selling this policy are making a loss.

So we must have $\tilde{q}_L = \overline{q}$. But then by lemma 4.1, $\tilde{E}$ coincides with E^*, which concludes the proof. □

Proof. **(Lemma 6.1)**: Given the equilibrium contract, a consumer purchasing more than $\overline{\alpha}$ must not reveal his full purchases to any firm from whom he has purchased insurance. We first prove the following result: given the equilibrium disclosure rules, in spite of this non-disclosure by consumers, there is at least one firm that knows all the firms from whom the individual has purchased insurance. Assume a consumer purchases more than $\overline{\alpha}$ from K firms, and suppose the consumer makes any set of disclosures. Pick up first the firm that is the most informed (by the consumer) , say firm $j_1 (< K)$, who knows about the consumer's purchases from firms $1, \ldots, j_1$ (including his own sales) and does not know about his purchases from firms $j_1 + 1, \ldots, K$, a group of firms undisclosed to j_1. (When there is a tie in which firm is the most informed, choose any of those;$j_1 = 1$ if a consumer does not disclose anything to any firm). Focus then upon the firms $(j_1 + 1, \ldots, K)$ undisclosed to j_1, and consider a firm who is the most informed of the purchases from those firms, say j_2, who knows about the purchases from $j_1 + 1, \ldots, j_2$. Similarly, we consider the most informed of the firms undisclosed to j_2 and j_1, say j_3. We can continue until we get j_k, where $k = K$. Then, clearly, the purchase from firm j_k is undisclosed to firms $j_1, j_2, \ldots, j_{k-1}$. Now consider the disclosures by firms. As a firm discloses to any other firm that is undisclosed by the consumer as his insurer, all the firms $j_1, j_2, \ldots, j_{k-1}$ (at least) will disclose to the firm j_k their own sales and information received from the consumer, implying that the firm j_k knows all of the K purchases.

The result of lemma 6.1 is now immediate: since that firm knows all of the individual's purchases, it knows that the individual has purchased more than $\overline{\alpha}$, and so cancels the policy. But the individual would not make those purchases, knowing that they would be cancelled. □

Appendix B: Multiple Contracts and Cross-subsidization

In this appendix, we show that our results hold even when firms are allowed to sell multiple contracts at different price. The central issue is whether this allows a deviant firm to break our putative equilibrium. A deviant firm does so to induce self-selection among the applicants - with the self-selection process reducing the costs of the high risk individuals buying insurance from the deviant.

We first discuss why the analysis presented in the text no longer works. (A^*, C^*) in figure 6 represent graphically the equilibrium allocation described earlier. Now consider the deviant pair of policies $\{A^*B, G\}$, where A^*B is offered at $\frac{\overline{P}}{1-\overline{P}}$ without disclosure and G is offered at a price lower than $\frac{\overline{P}}{1-\overline{P}}$ but greater than $\frac{\overline{P_L}}{1-\overline{P_L}}$ with with disclosure and with G being offered conditional on no additional insurance being purchased. There always exists a set $\{A^*B, G\}$ such that G is chosen by all the low risk individuals while A^*B is chosen by all the high risk who simultaneously buy OA^*, that is, the high risk individuals supplement A^*B with the pooling insurance OA^*. Because the price of insurance is greater than $\frac{P_L}{1-P_L}$ the deviant firm makes a profit on G even though it makes a loss on the contract purchased by the high risk individuals. By carefully choosing $\{A^*B, G\}$, the deviant firm can make overall positive profits. In particular, this will be so if A^*B is small. While there are large total losses associated with the purchase of insurance by high risk individuals, most of those losses are borne by the established firms, who now sell their pooling contract only to the high risk individual. The deviant firm gets all the low risk individuals for all of their insurance, and the high risk people only for the supplemental amount A^*B.

To prevent this type of a deviation, we need to make the choice of G more attractive to high risk types by providing more additional insurance at $\frac{\overline{P}}{1-\overline{P}}$ than the original equilibrium does, while limiting the total provision by all the firms to $\overline{\alpha}$ in equilibrium. One way of doing this is to have a latent contract,[40] which offers an individual sufficient amount of extra insurance at $\frac{\overline{P}}{1-\overline{P}}$ in the presence of a deviant contract G that the high risk individual purchases G. The established firms announce that they will sell to anyone who purchases insurance at a price lower than $\frac{\overline{P}}{1-\overline{P}}$ additional insurance in fixed quantity $\overline{\alpha}$, which they will not disclose. An individual i choosing G would not reveal to the deviant firm d his purchases of pooling insurance from other firms, but has an incentive to reveal to the established firms his purchase of low price insurance, for that triggers the offer of supplemental insurance. But that means that the established firms don't disclose their sales to the deviant, which ensures that the exclusivity provision associated with G cannot be enforced.

[40]The equilibrium allocation may be supported in other ways, but investigating that (both policy offers and disclosure rules) would take us beyond the scope of this paper.

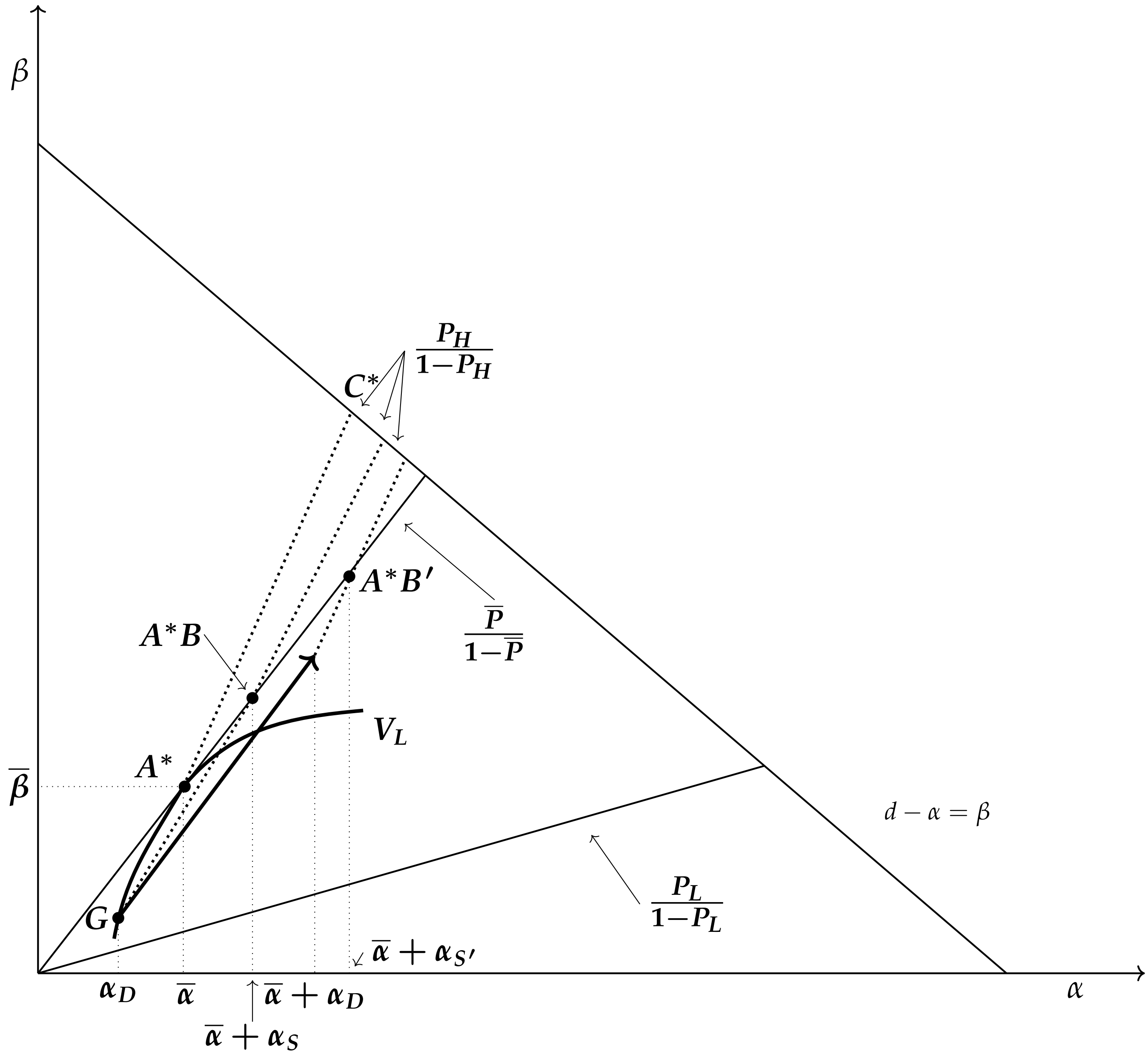

Figure 6: Equilibrium can be sustained against multiple deviant contracts $\{A^*B, G\}$ or (A^*B', G) offered at different prices as high risk individuals also choose G (over A^*B) or as (A^*B', G) yields losses for the deviant firm (while inducing self-selection).

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