NBER WORKING PAPER SERIES

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Working Paper 23556
http://www.nber.org/papers/w23556

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2017

They are grateful to Debarati Ghosh and Lim Nayeon for research and editorial assistance and to the Institute for New Economic Thinking and the Ford Foundation and Fulbright Foundation for financial support. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 23556
June 2017
JEL No. D82,D86

ABSTRACT

This paper investigates the existence and nature of equilibrium in a competitive insurance market under adverse selection with endogenously determined information structures.

Rothschild-Stiglitz (RS) characterized the self-selection equilibrium under the assumption of exclusivity, enforcement of which required full information about contracts purchased. By contrast, the Akerlof price equilibrium described a situation where the insurance firm has no information about sales to a particular individual.

We show that with more plausible information assumptions - no insurance firm has full information but at least knows how much he has sold to any particular individual - neither the RS quantity constrained equilibrium nor the Akerlof price equilibrium are sustainable.

But when the information structure itself is endogenous - firms and consumers decide what information about insurance purchases to reveal to whom - there always exists a Nash equilibrium. Strategies for firms consist of insurance contracts to offer and information-revelation strategies; for customers - buying as well as information revelation strategies. The equilibrium set of insurance contracts is unique: the low risk individual obtains insurance corresponding to the pooling contract most preferred by him; the high risk individual, that plus (undisclosed) supplemental insurance at his own actuarial odds resulting in his being fully insured. Equilibrium information revelation strategies of firms entail some but not complete information sharing. However, in equilibrium all individuals are induced to tell the truth.

The paper shows how the analysis extends to cases where there are more than two groups of individuals and where firms can offer multiple insurance contracts.

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1. Introduction

Understanding the existence and nature of market equilibria in the presence of information imperfections (asymmetries) has been one of the most challenging topics in economic theory over the past half century. Neither of the two prevalent models, that due to Rothschild-Stiglitz (1976) of a self-selection equilibrium, nor that of Akerlof (1970), a price equilibrium, are fully satisfying. Equilibrium may not exist, when it exists, there may be no trade, or no trade for a subset of the population for whom trade would seemingly be beneficial.

Moreover, one of the most important implications of adverse selection models is that self-selection equilibria are associated with distortions (relative to the full-information equilibrium). In insurance markets, low-risk individuals purchase too little insurance - with perfect information, they would have obtained full insurance. When the distortion associated with self-selection is too large, there is always a pooling contract (purchased by high and low risk individuals) that will be preferred, in which case the "separating" equilibrium cannot be sustained. In this case, there exists no competitive equilibrium.

In addition, both the Akerlof and Rothschild-Stiglitz equilibria are dependent on unrealistic information assumptions. Rothschild-Stiglitz (1976) denoted by RS hereafter, for example, described a model where every contract purchased by an individual is fully known to each firm. This enables a firm to sell contracts to its customers exclusively. On the other hand, in Akerlof's model firms engage in price competition but each insurer is uninformed of trades (amounts of insurance purchased) with other insurers.[1] Rothschild-Stiglitz assumes too much information, Akerlof too little. If it is possible for a new insurance firm (or even an existing firm) to hide some information about the insurance it has provided, the RS equilibrium will be broken, as we will show. Intuitively, this is obvious: the low risk individual identifies himself by rationing the amount of insurance that

[1] Akerlof focused on the market for used cars. What we describe as the "Akerlof model" is the natural extension of that model to insurance. (Individuals typically buy only one used car; a critical question in the insurance market is how much insurance does each individual purchase at the market price). Moreover, in Akerlof's model, the seller was the more informed agent; here it is the buyer. This distinction is not important. It is important that in the game theoretic framework set out below, the uninformed party (the seller) moves first. See Stiglitz and Weiss (1990, 2009).

he purchases, and whenever there is a rationing constraint, there is an incentive to circumvent the constraint.

This paper explores equilibria in which market participants decide on what information to share with others. Individual insurers may choose to reveal to or hide from other firms contract information - contracts that they have sold. Individuals may choose to reveal to or hide from any firm the contracts that they have bought from others.

The fact that individuals and firms can hide information has profound implications for self-selection equilibria, because it means that the insurance firm may not be able to extract the information he would otherwise have been able to obtain from the choices an individual makes. In a market with hidden knowledge, the amount of trade undertaken by an informed agent (the size of the insurance policy purchased) conveys valuable information about his type. Based on this insight, Rothschild-Stiglitz (1976) characterized the competitive equilibrium in an insurance market under adverse selection while presenting the possibility of the non-existence of equilibrium. They did so in an environment where firms can offer individuals contracts exclusively[2].

But if the individual has simultaneously some hidden contracts, the insurer may not be able to make the same inferences. High risk as well as low risk individuals may purchase the quantity-constrained policy. At the same time, allocations - like the pooling equilibrium preferred by the low risk individual - may not be “broken” in the way that such equilibria are in the standard RS analysis: RS showed that there was a policy, at an implicit price lower than the pooling contract, which would be purchased only by low risk individuals, ensuring that the pooling equilibrium couldn’t be sustained. But if there are other hidden policies, high risk individuals might purchase the putative breaking contract, making that contract unprofitable, so in fact it would not be offered - in

[2] Exclusivity means that if an individual purchases a policy from one firm, he cannot purchase any additional insurance from any other firm. In the absence of information about what insurance individuals have purchased, it may be impossible to enforce an equilibrium relying on exclusivity. In the case of a market with moral hazard and *identical* individuals, it may be possible to offer a large enough policy such that no one wishes to buy any additional policy that could *profitably* be offered. Determining the conditions under which this is true is the central question in Arnott-Stiglitz (1987, 1991a).

which case it would not have broken the pooling equilibrium.[3] In short, once there is not full disclosure, the entire RS analysis breaks down: it is conceivable that an allocation that RS showed could not be an equilibrium might be; and it is clear that the RS allocation itself is *not* an equilibrium.[4]

Similarly, the Akerlof price equilibrium has to be rethought in situations where firms have *some* information about quantities - they at least have information about the quantities of insurance that they have sold to a particular customer.

Endogenous information equilibria

This paper thus considers equilibria in the context of endogenously determined information. An insurance firm chooses not only a set of contracts to offer, but also decides on what information to share with which firms. An individual, similarly, not only chooses a set of contracts among those offered by firms but also decides what information to reveal to which firm. We analyze the full equilibrium - in contracts and in information sharing.

In this context of *endogenous information,* we ask i) whether an equilibrium exists, ii) if an equilibrium exists, what an equilibrium set of contracts and information-revealing strategies looks like, and iii) is the equilibrium Pareto efficient?

We first establish two results, with important consequences for the large literature which has developed since Rothschild-Stiglitz and Akerlof, based on their models: with endogenous information structures, neither the Akerlof nor the RS equilibrium exists. Both are "broken," i.e. assuming the putative Akerlof or RS equilibrium, there exists a set of contracts, within the endogenously determined information structure, which can be offered and make a profit.

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[3] Recall the definition of as RS equilibrium, requiring that there does not exist another insurance policy *which could profitably be offered,* which would be purchased by someone.

[4] See also Pauly (1974).

But while the Akerlof and RS contracts do not constitute an equilibrium, there always exists an equilibrium (unlike in RS) in which both high and low risk individuals buy insurance (unlike Akerlof), and the equilibrium *includes* a pooling contract (which can never happen in RS).

Characterizing an equilibrium for this model is not as complex as it looks. First, we identify a unique set of combinations of insurance coverage and premium for each type of individuals *that might* be sustained in equilibrium, which is what we call an equilibrium allocation.[5] We identify the *only* allocation which is sustainable in the presence of incentives on the part of firms and individuals to reveal or hide their contract information: the pooling allocation most preferred by the low-risk type plus supplemental full-insurance for the high-risk type. Second, we show that the equilibrium allocation can be sustained as a Nash equilibrium; *there always exists an equilibrium* – a set of contracts and set of information-revealing strategies.

Thus the market equilibrium is markedly different from either that analyzed by Akerlof[6] or RS. There is a pooling contract - the low risk subsidize the high risk. But the high risk buy a (here, supplemental) contract at their own "odds" to bring them to full insurance. In many ways this looks like some insurance markets observed in practice. Individuals are offered a pooling contract (by the government, or their employer) and then *some* purchase additional insurance, presumably at odds reflecting their own risks.[7] Moreover, the informational structure endogenously determined both by firms and by their customers resolves the problem of the non-existence of equilibrium under adverse selection noted by R-S.

In the following section we discuss some of the related literature. Section 3 introduces the basic model. We describe the game structure, the institutional framework for information-revelation, define the (Nash)

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[5] Thus, when we refer to the "allocation," we are referring to the insurance policies bought and sold. The equilibrium allocation describes the amount of coverage of each type. Since all insurance firms are identical, it makes no difference which insurance firm offers which policy to which individual. As we note below, the kinds of insurance policies offered (e.g. a fixed sized contract at the pooling price) does make a difference.

[6] Trivially, the equilibrium allocation coincides with Akerlof's when the amount of pooling allocation preferred by the low-risk type is zero.

[7] Often, the supplemental purchases are by rich individuals, not by high risk individuals. Individuals differ in respects other than the accident probability, as assumed here.

equilibrium, and propose a candidate allocation - called the equilibrium allocation - *that might* be sustained as an equilibrium under some feasible informational structure. It is shown that there is at most one possible equilibrium allocation. Section 4 analyzes a set of equilibrium information strategies that can always sustain the equilibrium allocation. Section 5 discusses some other related issues, such as extension to the case with many types or a continuum of types. Some welfare properties of the equilibrium outcome, together with some concluding remarks, are given in Sections 6 and 7.

2. Previous Literature

We cannot summarize here the vast literature which developed following the early work of Akerlof and RS. This literature explored the application of the model to different contexts (markets), different equilibria concepts[8], and, to a limited extent, the consequences of different information structures.

The importance of assumptions about information should be obvious: as we noted, whether or not an insurance firm can offer insurance exclusively is critical in determining whether an equilibrium exists and characterizing the equilibrium allocation in a competitive market.[9] [10]

Several recent papers have addressed the nature of equilibrium under adverse selection when firms cannot

[8] Different equilibrium concepts may seem more appropriate in different contexts (see, e.g. (Stiglitz [1976, 1992, 2009]) just as whether a market is best described by a screening equilibrium or a signaling equilibrium may differ according to context (see Stiglitz and Weiss [1983, 1990, 2009].)

[9] For instance, as we also noted, RS showed that an equilibrium may not exist under exclusivity and that if an equilibrium existed, it was a separating equilibrium, that is the different groups did not buy the same insurance policy. There could not be a pooling equilibrium. The concept of a pooling equilibrium had been introduced somewhat earlier (Stiglitz, 1975). In a model where resources could be allocated by identifying characteristics (i.e. screening), there could exist a pooling equilibrium, a separating equilibrium, or multiple equilibria, one of which entailed pooling, another of which separating.

[10] Some strategic equilibrium concepts other than Nash have been employed (Wilson [1977], Riley [1979], Miyazaki [1977], Spence [1978]) to characterize a market outcome under adverse selection. Later in the paper we comment on these reactive equilibria. Hellwig (1986, 1987) has shown the sensitivity of the standard results to the precise game-theoretic formulation. Others (e.g. Engers-Fernandez [1987]) have formulated dynamic extensions of the adverse selection model in a Nash equilibrium game-theoretic framework. In these models, rather than there being no equilibrium there are a multiplicity of equilibria. Dubey-Geanakoplos (2002), on the other hand, formulated a type of a signaling model describing how a pool, characterized by a specific level of insurance it offers, can be formed in a perfectly competitive environment when an individual is constrained to choose a single contract, thus implicitly again assuming exclusivity. Using an equilibrium refinement concept, they showed there always exists a unique equilibrium, which involves separating allocations served by different pools. This work has been extended by Bisin and Gottardi (1999) to a context of non-exclusivity, but in a rich model incorporating both adverse selection and moral hazard in which non-linear pricing plays a key role. See also Bisin and Guaitoli (2004).

sell contracts exclusively. (See also the earlier papers of Pauly [1974] and Jaynes [1978].) Ales-Maziero (2012) and Attar-Mariotti-Salanie (2014, 2016) establish that the problem of underinsurance by low risk types or of equilibrium non-existence could become even more severe: under non-exclusivity, an equilibrium, if it exists, entails *no* insurance for low-risk types, and the chance for equilibrium existence is even lower.

By contrast, under somewhat different assumptions, Attar-Mariotti-Salanie (2011) explores the possibility that under non-exclusivity an equilibrium may entail some pooling and thus some insurance for low-risk types. In particular, in a more general model of adverse selection with non-exclusivity they characterize a pooling equilibrium as well as a separating one with no insurance for the low risk type. But while in their model trade is welfare-enhancing for both sellers and buyers, they set the maximum amount of trade exogenously. This implies they do not have to worry about the incentive for over-trading (purchasing “too much” insurance) on the part of the high risk type, which turns out to be one of the crucial issues that we have to deal with in this paper. More recently, Attar-Mariotti-Salanie (2016) analyze conditions for existence of equilibrium, which turned out be restrictive, mainly due to excessive demands for insurance by high-risk consumers.[11]

As we noted in the introduction, this paper not only explores the consequences of different information structures, but more importantly, the implications of endogenizing the information structure - allowing firms and individuals to decide what information to disclose to whom. The closest works to our paper within the adverse selection literature are Jaynes (1978, 2011) and Hellwig (1988), who analyze a model with a certain type of strategic communication about customers’ contract information. Jaynes (1978) characterizes an equilibrium outcome that involves a pooling allocation plus supplemental provision at the high-risk price, sometimes referred to as the Jaynes’ allocation, which is the allocation upon which our analysis focuses.[12] However, as Hellwig (1988) clarified, the pooling contract equilibrium and the associated strategic communication presented by Jaynes (1978) is not a Nash equilibrium but a reactive equilibrium, responding to

[11] The insurance allocation upon which they focus is in fact the allocation which we show is the unique equilibrium allocation with endogenous information.

[12] This equilibrium allocation upon which we focus also appears elsewhere in the literature, in particular, in Beaudry and Poitwevin (1993, 1995) and in Gale (1991).

the presence of particular deviant contracts.[13] While our work differs from that of Jaynes and Hellwig in several ways, perhaps most important is that we consider information revelation strategies by consumers as well as firms. This turns out to be critical in the analysis of the existence of a Nash equilibrium.[14]

Moral hazard

The consequences of imperfect information concerning insurance purchases and the consequent non-exclusivity of insurance have also received some attention in the insurance-with-moral hazard literature. In that case, the market price-equilibrium is particularly inefficient - individuals buy excessive insurance; and it is easy to show that it is possible that the only price equilibrium entails no insurance, a result parallel to the no-trade result of Akerlof (1970) in the context of adverse selection.[15] Arnott-Stiglitz (1987, 1991a, 1991b, 2013)[16] observe, however, that a firm can make use of the quantity information of his own sales - even if it has no information about that of others. They refer to the resulting equilibrium as a “quantity” equilibrium, show that it always exists, and that it never coincides with the exclusive contract equilibrium, may not coincide with the price-equilibrium, may occur at a point where the individual is just indifferent between buying one or more units of insurance, and may entail positive profits.[17] Further, they introduce the concept of a latent policy - a

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[13] Hellwig (1988) argued that Jaynes' equilibrium was not a sequential equilibrium of a two stage game with firms simultaneously choosing policy offers and communication strategies at stage one, and therefore was not a Nash equilibrium, though he showed it was sequential equilibrium for a four-stage game with firms choosing communication strategies after observing competitors' contracts. In response, Jaynes (2011), while keeping a two-stage framework, characterizes a Perfect Bayesian Equilibrium, which requires communication strategies as well as contract strategies to be sequentially rational based upon the beliefs derived from Bayes rule (conditional upon those strategies). He shows that for any given deviant contract of a particular type, there can be a large enough number of firms such that the deviant contract is unprofitable. The appropriate question, though, is *given* a particular value of N, is there *some* deviant contract that will break the posited equilibrium. The answer is that there is. That is why we have introduced not just communication between firms, but also communication between customers and firms.

[14] The role of information structure in determining an equilibrium outcome, which is critical for the existence of an equilibrium in this paper, has also been recognized in the recent literature on so-called information design and Bayes correlated equilibrium for a game with incomplete information. Characterizing Bayes-correlated equilibrium for given prior information, Bergemann-Morris (2013, 2016) showed that increasing prior information reduces the set of the equilibria. Taneva (2016) and Kamenica-Gentzkow (2011), on the other hand, solve for the information structure that can lead to the optimal outcome. But in contrast to this literature, here, uninformed firms move first to compete with each other for contracts to offer and the information structure emerges as part of the market equilibrium. This paper highlights another role of the endogenously information structure, in supporting an equilibrium that would otherwise be unsustainable. There are a number of other studies, such as Gossner (2000), that examined how different information structures lead to different outcomes using Bayes Nash equilibrium or alternative solution concepts.

[15] Unlike RS, an equilibrium always exists (under the given information assumptions), though it may entail positive profits.

[16] See also Stiglitz (2013a). Of course, it is not just purchases of other insurance policies covering the same risk that affect relevant behavior; risk taking can also be affected by consumption of other goods. As Greenwald and Stiglitz (1986) emphasize, there are fundamental pecuniary externalities which arise whenever there is moral hazard. See also Arnott and Stiglitz (1990).

[17] The essential insight is that even though actions may be continuous in the amount of insurance purchased, the amount of insurance purchased may be discontinuous in insurance offerings: a small supplementary (deviant) policy may induce discontinuous purchases

policy not purchased in equilibrium, but which would be purchased were an entrant to enter; and with that purchase, the entrant would lose money. The latent policy serves to deter entry, enabling even a positive profit equilibrium to be sustained.

Verifiable disclosure

There is a literature dating back to Stiglitz (1975), Milgrom (1981), and Grossman (1981) on verifiable information disclosure which is linked to the analysis here. The central result, which Stiglitz has dubbed the Walras' law of screening, is that if there is verifiable disclosure of types, there can be no pooling equilibrium, as each type finds it worthwhile (if the costs of verification are low enough) to have itself identified. Here, it will turn out, individuals reveal the amounts of insurance they purchase - information that may be relevant for implementing a self-selection equilibrium - but then whether they are honest in their statements will be revealed subsequently, i.e. it will be effectively verified. The verification, though, is not done through a test (a screening mechanism), but through disclosures on the part of the firm, or subsequent disclosures (possibly to other firms) by the individual himself. It turns out, though, since there is no formal verification mechanism, the standard unraveling argument does not apply, but the rational expectations that there would be such an unravelling has behavioral consequences for the high risk individuals plays a critical role in shaping the equilibrium.

3. Model

We employ the standard insurance model with adverse selection. An individual is faced with the risk of an accident with some probability, P_i. P_i depends upon the type *i* of the individual. There are two types of individuals – high risk (H-type) and low-risk (L-type), who differ from each other only in the probability of accident. The type is privately known to the individual, while the portion θ of H-type is common knowledge. The average probability of accident for an individual is $\bar{P}$, where $\bar{P} \equiv \theta P_H + (1-\theta) P_L$.

of insurance, leading to discontinuous changes in behavior, so that even if there are profits in equilibrium, a small entrant would make losses. This is the consequence of the fundamental non-convexities which arise in the presence of moral hazard. See Arnott and Stiglitz (1988).

An accident involves damages. The cost of repairing the damage in full is d. An insurance firm pays a part of the repair cost, α. The benefit is paid in the event of accident, whereas the insurer is paid insurance premium β when no accident occurs.[18] The price of insurance, p, is defined by β/α. (In market equilibrium, the amount of insurance that an individual can buy may be limited.) The expected utility for an individual with a contract (α, β) is

$$V_i\left(\alpha, \beta\right) = P_i U\left(w - d + \alpha\right) + (1 - P_i) U\left(w - \beta\right). \qquad (1)$$

where U" < 0 (individuals are risk averse.) The profit π_i of insurance firm offering a contract $\left(\alpha, \beta\right)$ that is chosen by i-type (i=H,L) is

$$\pi_i\left(\alpha, \beta\right) = (1 - P_i)\beta - P_i\alpha \qquad (2)$$

There are N firms and the identity of a firm *j* is represented by j, where j = 1, --, N.

3-1. The standard framework

Before analyzing an equilibrium with endogenous information, we review the standard RS and Akerlof models to see more precisely why endogenous information matters so much and to identify the challenges that have to be addressed in constructing an endogenous information equilibrium.

Akerlof equilibrium

We begin with the competitive price-equilibrium, which we also refer to as the no-information price equilibrium, because no insurer has any information about the purchases of insurance by any individual. In the price equilibrium, different individuals choose the amount of insurance at a fixed price. In the competitive equilibrium, the price of insurance will reflect the weighted average accident probability. Figure 1a illustrates.

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[18] This has become the standard formulation since RS. In practice, customers pay β the period before the (potential) accident, receiving back $\alpha + \beta$ in the event the accident occurs, i.e. a net receipt of α.

α is on the horizontal axis, β on the vertical axis. At $\alpha = d - \beta$ there is full insurance. From (2), the break-even premium for each type of individual is $\beta = \frac{P_i}{1-P_i}\alpha$ for $i = H.L$. These break-even loci are depicted in the figure, as is the break-even pooling line, where the average accident probability reflects the different amounts of insurance bought by the different types.[19] We denote the purchase by a high risk individual at a price corresponding to an accident probability P as $\alpha_H(P)$, and similarly for the low risk as $\alpha_L(P)$. The weighted average accident probability is then

$$\hat{P}(\mathrm{P}) \equiv P_H\theta\frac{\alpha_H(P)}{\bar{\alpha}^{\mathrm{e}}(P)} + P_L(1-\theta)\frac{\alpha_L(P)}{\bar{\alpha}^{\mathrm{e}}(P)}, \qquad (2a)$$

where

$$\bar{\alpha}^e(P) = \theta\alpha_H(P) + (1-\theta)\alpha_L(P),$$

and

$$\alpha_L(P) = Argmax\ V_L(\alpha,\beta) \quad \text{s.t. } \beta = \frac{P}{1-P}\alpha$$
$$\alpha_H(P) = Argmax\ V_H(\alpha,\beta) \quad \text{s.t. } \beta = \frac{P}{1-P}\alpha$$

Since the high risk buy more insurance, i.e., since $\alpha_H(P) > \alpha_L(P)$, the weighted accident probability $\hat{P}(\mathrm{P})$ is higher than the population weighted average $\bar{P}$ (in Figure 1a), where

$$\bar{P} = P_H\theta + P_L(1-\theta). \qquad (2b)$$

That is, $\hat{P}(\mathrm{P}) > \bar{P}$.

Now we define a (competitive) price equilibrium as P^e satisfying the following conditions: (a) (uninformed) sellers have rational expectations P^e about the quality or the accident probability of the buyers; (b) with those rational expectations, prices are set to generate zero profits (equal to $\frac{P^e}{1-P^e}$); and (c) at those prices consumers buy the quantities that they wish.[20] Thus, a price equilibrium P^e satisfies

[19] Throughout the paper, when we say a "price reflecting an accident probability P" we mean (from 2) a price $\frac{P}{1-P}$.

[20] The latter conditions are equivalent to the standard conditions of demand equaling supply, for this particular model.

$$P^e = \hat{P}(P^e) \tag{2c}$$

A price equilibrium P^e is depicted in Figures 1a-1e. In Figure 1a, the high risk individual's purchase of insurance is denoted by B, the point of the tangency of their indifference curve V^H to the break-even line with the slope $\frac{P^e}{1-P^e}$; that of the low risk individual is denoted by A. Figures 1b, 1c and 1d plot the RHS of (2c) as a function of P^e over the relevant range $[P_L, P_H]$. There is an interior equilibrium when the RHS crosses the 45^o line. The "normal" case (on which we focus) entails the low risk individual buying some insurance even at a price corresponding to P_H. Thus, $\hat{P}(P_H) \leq P_H$, with $\hat{P}(P_H) < P_H$ so long as the low risk type buys some insurance at P_H. Moreover, $\hat{P}(P_L) > P_L$. Finally, it is easy to establish that $\hat{P}(P)$ is a continuous function of P. Hence, there exists at least one value of P, P^e, for which equation (2c) holds. We call this the no-information price equilibrium. Figures 1c-1d show that there may be multiple no-information price equilibria. $\hat{P}(P)$ is normally upward sloping, as the low risk individuals diminish their purchases of insurance at the high price. This is the normal (and original) *adverse selection* effect. But the slope depends on the elasticities of demand of the two groups as well as their relative proportions.

Zero insurance for the low risk

There is one special case, that where there is zero insurance for low-risk types in the price equilibrium. This arises if[21]

$$\alpha_L(P_H) = 0 \quad \text{or} \quad (0{,}0) = Argmax\ V_L(\alpha, \beta) \quad \text{s.t. } \beta = \frac{P_H}{1-P_H}\alpha. \tag{3}$$

Condition (3) says that at the price corresponding to the high risk individuals, low risk individuals do not buy any insurance, so that $\hat{P}(P) = P_H$ for $P = P_H$ (and typically for some values of P less than P_H). Condition (3) will hold when P_H is so high relative to the low risk type's marginal rate of substitution at zero level of insurance that their demand for insurance is zero.[22] An Akerlof equilibrium, which is defined to be a

[21] (3) implies $\frac{P_L}{1-P_L}\frac{U'(W-d)}{U'(W)} \leq \frac{P_H}{1-P_H}$.

[22] The equilibrium associated with (3) is analogous to the no-trade equilibrium in Akerlof, which is why we have referred to it as the Akerlof equilibrium. The low risk individuals would obviously like to get insurance, but because of adverse selection, they choose, in

price equilibrium where only the high-risk individuals purchase insurance, is depicted as a corner solution in Figures 1d and 1e. The Akerlof equilibrium is the unique price equilibrium in Figure 1e while it is one of the multiple price equilibria in Figure 1d.

We can summarize these results in

Proposition 1a) There exists at least one no information price equilibrium. If (3) is satisfied, there exists a boundary equilibrium- an Akerlof equilibrium - in which only high risk individuals purchases insurance. If (3) is not satisfied, there exists at least one interior equilibrium with $P_L < P^e < P_H$*. There may exist multiple equilibria.*

Nash equilibria and non-existence of a partial information price equilibrium

In the no-information price equilibrium, the insurance firms simply take the price as given, but have rational expectations about the risk of *those* who buy insurance at that price. There is no strategic interaction among firms. We could define a price equilibrium in a Nash-Bertrand fashion by adding another condition that each firm, taking the prices of others as given, chooses the price which maximizes its profits. In this case, it can be shown that there exists a unique price equilibrium, the lowest price at which (2c) is satisfied.[23]

More interesting is the Nash equilibrium with partial information. While a firm doesn't know the size of the policies taken up by an individual from other firms, he knows the size of his own policy. An insurance firm can offer a large policy - he knows to whom he sells, so he knows he wouldn't sell a second policy to the same individual.[24]

We define a *partial-information (Nash) price equilibrium* as an equilibrium where the insurance firm knows

equilibrium, not to. This no-trade result is different from that of Stiglitz (1982) and Milgrom and Stokey (1982) where though there are asymmetries of information, buyers and sellers effectively have the same utility function—one side of the market is not trying to share inherent risk with another.

[23] This should be can be contrasted with the multiplicity result in Proposition 1a.

[24] In the context of moral hazard, the implication of this simple observation were explored in Arnott-Stiglitz (1991a, 1987). See also Jaynes (1978).

at least information about the amount of insurance it sells: a partial information price equilibrium is a set of contracts such that (a) each contract at least breaks even; (b) each individuals buys as much insurance at the price offered at he wishes; and (c) there does not exist any policy which (given the information structure) can be offered which will be purchased and make a profit.

Any policy proposing to break a price equilibrium must satisfy two conditions: to be purchased, it has to have a lower price than the market price, but to make a profit, it must have a higher price than that corresponding to the actual pool of people buying the policy.

Consider a deviant firm that secretly offered a quantity policy, say the policy which maximizes the utility of the low risk individuals at a price corresponding to P', with $P^e > P' > \bar{P}$ (such as (α', β') in Figure 1a). It sells only one unit of the policy to each individual, and restricts the purchases of all to the fixed quantity policy. Then everyone will buy the policy, and it will make an (expected) profit. It thus breaks the price-equilibrium.

The one case where this argument doesn't work is that where the following condition (3') is satisfied:[25]

$$\alpha_L(\bar{P}) = 0 \quad \text{or} \quad (0,0) = Argmax\ V_L(\alpha, \beta) \quad \text{s.t. } \beta = \frac{\bar{P}}{1-\bar{P}}\alpha \qquad (3')$$

Note that the condition (3') is stricter than (3), i.e. (3) can be satisfied, and yet (3') may not be. This means that even if there exists an Akerlof price equilibrium (where only the high risk buy insurance), the Akerlof equilibrium is not a partial information Nash equilibrium. A quantity-constrained contract (α', β') can break it. (See figure 1f.)

We have thus established

Proposition 1b[26]

[25] (3') implies that $\frac{P_L}{1-P_L}\frac{U'(W-d)}{U'(W)} \leq \frac{\bar{P}}{1-\bar{P}}$.

[26] In a somewhat different set-up, Jaynes (1978) presents a set of results similar to Proposition 1b.The condition for the existence of a partial information price equilibrium, (3'), is stricter than that specified by Jaynes (1978), which would be equivalent to (3).

There is no partial information price equilibrium where both types of individuals buy insurance, that is, a price equilibrium where firms can offer an undisclosed quantity contract and ration the sale, say one policy to a customer. There is a Nash partial information price equilibrium where only the high risk individuals buy insurance if and only if condition (3') is satisfied.

What is remarkable about Proposition 1 is how little information is required to break the price equilibrium (and even the "corner" Akerlof equilibrium): the firm just uses its own contract information to implement the quantity constraint.

It is natural to ask, if there is not a price equilibrium, is there some analogous equilibrium, with say fixed quantity contracts? Consider a case where the two groups are quite similar. Each insurance firm sells insurance in fixed units, say $(\bar{\alpha}, \bar{\beta})$, say the policy which is most preferred by the low risk individual along the break-even pooling line. The high risk individual would not want to buy two units of that insurance. But he would supplement his purchase with the undisclosed insurance at his own price, in an amount that brings him to full insurance. Below, we show that this kind of pooling contract cannot be an equilibrium: there is always a deviant policy that could be offered that would be taken up only by the low risk individuals, given the posited information structure.

In other words, given this partial information structure, there is no equilibrium, ever, where both groups buy insurance. By contrast, with the more complex *endogenous* information structure, to be described later in the paper, there is *always* an equilibrium.

Rothschild Stiglitz equilibrium[27]

Central to the analysis of Rothschild and Stiglitz was the assumption that there was sufficient information to enforce exclusivity; the individual could not buy insurance from more than one firm. Once we introduce into the

[27] This section follows along the framework of Rothschild-Stiglitz, analyzing separately pooling and separating equilibria. The following subsections formalize the analysis, describing the Nash equilibrium in information and contracting.

RS analysis unobservable contracts, in addition to the observable ones, the whole RS framework collapses. Exclusivity cannot be enforced. We assume that undisclosed contracts can be offered, and will be offered if they at least break-even. We ask, given the existence of such contracts, and given that the deviant contract that might break any putative equilibrium itself may not be disclosed, can the RS analysis be sustained? That is, will it be the case, as RS argued, that any possible pooling equilibrium be broken (i.e. there is no pooling equilibrium), and will it still be the case that, provided the two groups are different enough in accident probabilities, that a separating equilibrium exists?

Breaking a separating equilibrium

It is easy to show that the standard separating contracts - the policies that would have separated had there been no hidden contracts, so exclusivity could have been enforced - no longer separate.

Figure 2a shows the standard separating pair of contracts. C is the full insurance contract for the high risk individual assuming he was not subsidized or taxed and A is the contract on the low risk individual's break-even curve that just separates, i.e. is not purchased by the high risk individual. In RS, the pair of contracts {A, C} constitutes the equilibrium so long as A is preferred to the contract on the pooling line which is most preferred by the low risk individual.[28] But {A, C} can *never* be an equilibrium if there can be undisclosed contracts, because if there were a secret offer of a supplemental contract at a price reflecting the "odds" of the high risk individual, such as AC' in Figure 2a, then both the high and low risk individuals would buy A and it would not separate. And it would obviously be profitable to offer such a secret contract.

Breaking a pooling equilibrium with no disclosure of deviant policy

RS showed that there could be no pooling equilibrium by showing that because of the single crossing property, there always exists a contract which is preferred by the low risk individual and not by the high risk, and lies below the pooling zero profit line and above the low risk zero profit line. But the ability to supplement

[28] If this is not true, there exists no equilibrium.

the breaking contract *may* make the contracts which broke the pooling equilibrium, under the assumption of no hidden contracts, attractive to the high risk individual. If that is the case, that contract cannot break the pooling equilibrium: there is no contract which can be offered which attracts only the low risk individuals.

Figure 2b provides an illustration. The pooling contract A is the most preferred policy of the low risk type along the pooling line with slope $\frac{\bar{P}}{1-\bar{P}}$,[29] the only possible pooling equilibrium. Consider the high-risk price line through A. The high risk individual also purchases the insurance contract A, thereby obtaining a subsidy from the low risk individual, and supplements it with secret insurance at the high risk odds (represented in figure 2b by AC', where C' is the full insurance point along the line through A with slope $\frac{P_H}{1-P_H}$.[30] Consider a policy D_o below the low risk individual's indifference curve through A, above that for the high risk individual, and which also lies below the zero profit line for high risk individuals through A. D_o would be purchased by the high risk individual. In the RS analysis, *with exclusivity,* D_o would have broken the pooling equilibrium A. Now, it does not, because the high risk individuals would buy D_o and the (secret) supplemental insurance.[31] And if they do so, then D_o makes a loss, and so D_o could not break the pooling equilibrium.

But the question is, are there *any* policies which could be offered that would break the pooling equilibrium, that would be taken up by the low risk individuals, but not by the high risk individuals *even if they could supplement the contract with a secret contract breaking even.* The answer is yes. There are policies which lie below the zero profit pooling line and above the zero profit line for low risk individuals (that is, would make a profit if purchased only by low risk individuals), below the low risk individual's indifference curve (i.e. are preferred by low risk individuals), and lie *above* the high-risk zero profit line through A (i.e. even if the high risk individual could have secretly supplemented his purchases with insurance at his actuarial fair odds, he

[29] Sometimes referred to as the Wilson equilibrium.

[30] Recall that at full insurance, the slope of the indifference curve of the high risk individual is just $\frac{P_H}{1-P_H}$, and full insurance entails $\alpha = d - \beta$.

[31] This is different from the way that the matter was framed by Wilson and Riley, who described the policy A as being withdrawn when a policy such as D_o is offered (which is why their equilibrium concepts are typically described as *reactive*). Here, policy offers are made conditional on certain (observable) actions. See the fuller discussion in the next sections.

would be worse off than simply purchasing A, the pooling contract). These policies break the pooling contract. In figure 2b, any point (such as D_1) in the shaded area in the figure, which we denote by z, can thus break the pooling equilibrium. It should be obvious that the set z is not empty.

Formally, for any point such as D_1, $V_L\{D_1\} > V_L\{A\}$, while $V_H\{D_1 + S_H\} < V_H\{A\}$ where in the obvious notation $V_H\{D_1 + S_H\}$ is the maximized value of the high risk individual's utility, when he purchases policy D_1 plus supplemental insurance at the actuarial odds, S_H, bringing him to full insurance.[32]

We collect the results together in

Proposition 2.

(a) The RS Separating Contracts do not constitute an equilibrium, if firms can offer non-loss making undisclosed contracts.

(b) The pooling equilibrium may always be "broken" if there exists undisclosed supplemental insurance and if a deviant firm can choose to keep his offers secret.

(c) Some of the contracts that broke the pooling equilibrium in the standard RS equilibrium with exclusivity no longer do so.

The analysis so far has assumed the information structure, i.e. that the deviant contract and the supplementary policy at the price corresponding to P^H are not disclosed. The remaining sections focus on the core issue of an endogenous information structure, with the simultaneous determination of contract offers and information strategies of firms and with contract purchases and information disclosure by individual customers.

3-2. **Strategies of firms and customers, Game Structure, and Equilibrium**

We model the insurance market as a game with two sets of players, the "informed" customers, who know

[32] Note that the utility V_i of an individual is a function of total amount of insurance α and and the total premium paid β, which can be expressed in terms of the policies purchased (such as A or D_1). The notation $D_1 + S_H$ refers to the $\{\alpha, \beta\}$ associated with the purchase of D_1 plus the optimized value of secret insurance along the price line associated with the high risk individual.

their accident probabilities, and the uninformed insurance firms (all of whom are identical), who do not know the characteristics of those who might buy their insurance. They have no way of directly ascertaining their potential customers' accident probabilities. They know that if they ask their customers, they won't necessarily tell the truth (the high risk customers have an incentive to claim to be low risk), but they may try to infer their type from the choices they make. In the Stiglitz' analysis of monopoly insurance markets (1977), the insurance firm structures their choices so as to make those inferences "efficiently" (with the least loss of profits to the insurance firm); in the Rothschild Stiglitz analysis of competitive insurance markets, firms know what other policies individuals purchased, even if they cannot control the choice set, and thus they can enforce exclusivity. Here, what each firm knows is being endogenously determined not only through inter-firm communication but also through information disclosure by its customers, and as a result exclusivity may not (and in equilibrium will not) be enforced.

Thus, the key difference between the analysis below and that of RS is that they assumed that any insurance firm knew about *all* of the insurance purchases of any individual (in which case, the equilibrium allocation is *as if* an individual purchased insurance exclusively from one insurance firm); while here, we assume that in the equilibrium allocation, some insurance purchased may not be observable to other insurance firms and, most importantly, additional insurance policies can be offered unbeknownst to those offering insurance in the putative equilibrium.

The game structure has three stages:

Stage 1: firms announce their strategies simultaneously. Strategies consist of a set of insurance contracts that are offered, possibly with conditions, and information policies about what information it will reveal to which firms. These are commitments, e.g., the firm cannot renege on its offer, if the conditions are satisfied, and if the firm has information that the conditions of offer have not been satisfied, then it follows through and the insurance policy is cancelled.[33] The firm may impose conditions of exclusivity (no insurance may be purchased from another firm), limitations on aggregate purchases from other firms, or even minimum levels of insurance purchased from other firms. Information strategies as well as contract strategies of a firm towards an individual *i* may be conditional upon the contract information available about that individual, the endogenous revelation of which is itself part of the game. The firm may reveal information not only about his own sales, but also information that has been revealed to it from others. Customers know that if the firm finds out that the individual has violated the conditions of the contract, the contract is voided.[34]

Stage 2: each customer optimally responds by purchasing a contract or contracts while revealing whatever information about the contracts he purchased to whomever he wants. The individual can purchase insurance from more than one firm.[35] The strategy of an individual is thus a choice of insurance policies and a set of decisions about what information about these purchases to reveal to which firms.

Stage 3: Each firm discloses (or receives) contract information about its customer to (or from) other firms, but only as announced in Stage 1. Those announcements may depend on information that has been revealed to the firm in stage 2. Any purchase of a contract or contracts by an individual is cancelled if the contract information revealed about him does not meet the conditions required by the contract.

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[33] This can be compared to Jaynes (2011) and Hellwig (1988), who formulate a multi-stage sequential game where firms are allowed not to execute in the later stage the exclusivity announced in the earlier stage.

We focus on Nash equilibrium, so that each firm (in equilibrium) takes the announced strategies of other firms as given. This means that the strategy of a firm in stage 1 is not conditional upon the strategies announced by the other firms in stage 1. (When that is not the case, e.g. when a firm announces that it will not sell a particular policy if some other firm offers a policy belonging to a particular set of policies, we call the resulting equilibrium, if it exists, a reactive equilibrium. Such behavior is not consistent with the spirit of competitive analysis, since any firm is sufficiently important that it can alter the behavior of any other firm.)

[34] The game may be formulated in a number of different ways, e.g. if the individual has purchased more than policies he has purchased allow, then there may be a scaling back of the sales by each firm (according to some rule) to bring the individual into conformity with the conditionality. The approach we take simplifies the analysis and provides strong incentives for truth telling on the part of consumers.

[35] When an individual purchases multiple contracts from different insurers, he is assumed to make those purchases simultaneously, bearing in mind the conditions associated with each policy. As will be shown later, in equilibrium the individual fully discloses all contracts purchased and the individual knows that accordingly, he cannot violate the conditions associated with any contract purchased.

The key strategic decisions occur in the first two stages, while the final stage, which does not entail any additional strategic element in the structure, is simply a working out of the actions and commitments previously undertaken.[36]

Now we define a Nash equilibrium as follows:

A Nash equilibrium is defined as a set $\{S_j^*\}_j$ *of strategies by firms, such that given* $\{S_j^*\}_j$*, there does not exist any other strategy that any firm can adopt and make higher profits than it is currently making, given the optimal responses of consumers.*[37] When there exists such a strategy, we say that it "breaks" the putative equilibrium.

This is a natural extension of the equilibrium defined by Rothschild-Stiglitz (1976), who focus on *competitive* markets, where no firm believed that its actions will alter that of others. A strategy S_j^* of a firm we consider in the above game structure is not a reactive one, as it does not condition its contract offers and disclosure policies upon the strategies of the other firms.[38]

We define an *equilibrium allocation* $\{(\alpha_i, \beta_i)\}_i$ to be a set of insurance policies purchased by each type of

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[36] Thus, the last stage does not generate any reactive feature to a firm's strategy. This is important, because earlier analyses of the problem at hand were rightly criticized for being "reactive equilibrium," not Nash equilibrium, and as we noted in an earlier footnote, contrary to the spirit of competitive equilibrium. See, e.g. Hellwig's (1988) criticism of Jaynes (1978).

[37] More formally, we should say, *given the (equilibrium) strategies of consumers.* Consumers have strategies relating to their purchases of insurance and disclosure of information. Each consumer is assumed to be small, and his actions have no effect on the behavior of firms, and to know this. Thus, the consumer's choice of contract(s) is straightforward: he simply maximizes his expected utility. The analysis to this point is parallel to that of RS, which focused solely on firm strategies, simply because the consumer strategies were so simple and straightforward. Consumers' information strategy is slightly more complicated, but is shown below to entail truth telling about purchases of pooling contracts, but not about purchases of other contracts.

[38] To the extent that a firm may condition its contract offers upon the information disclosed by its consumer, however, the strategy S_j^* embeds a response to the contract offers by another firm j', when a consumer purchases insurance from both of the two firms, when his purchase from j' is revealed to j, and when, as a result of the contract violation, the policy is cancelled. As will be clear from the analysis below, it is not this seemingly potential reactive feature itself that leads to the existence of an equilibrium: existence requires that strategies have to be designed to deter entry by a deviant firm. In RS, entry is easy, because the entrant (deviant firm) can make use of exclusivity, which follows from the assumed information structure. Here, the information structure is endogenous, so that whether a firm can deter entry by impairing the deviant firm's ability to enforce exclusivity is a more complicated matter. In earlier analyses, the entry deterrence required to establish equilibria depended on reactive equilibria (putting the analysis outside of the usual Nash equilibrium framework). See Wilson (1977) and Hellwig's (1988) discussion of Jaynes (1978). We will show instead that the information revelation made as a part of the optimal response by consumers, interacting with that by firms, in a Nash equilibrium enables S_j^* to deter a deviant firm from exercising the exclusivity necessary to break the equilibrium.

individual in equilibrium. It is easy to show that what matters is the *total* insurance coverage α_i and insurance premium β_i, and when we refer to the equilibrium allocation, we refer to these totals. The full equilibrium is characterized by an equilibrium allocation and an equilibrium information structure.

Normally, we would expect equilibrium allocations to have zero profits[39], for if a firm were offering a set of contracts making positive profits, some other (new) firm could offer a more attractive set of contracts and still make positive profits, upsetting the original equilibrium. While RS showed that to be the case in their model which *assumed* full information about contract purchases by all individuals, the analysis in section 3.2 establishes the corresponding result more generally, and identifies the *unique* possible equilibrium allocation.

We proceed as follows: first we discuss in greater detail the information revelation strategies and the contract strategies. We then (in the next sub-section) analyze the set of possible allocations, showing that there is a unique possible allocation. Section 4 then uses this to construct the equilibrium strategies.

Information revelation

An insurer can, if he wants to, reveal information on particular contracts purchased by a particular individual; similarly, an individual can reveal information on particular contracts purchased from particular firms. Consumers and insurance firms may choose either to reveal or to hide information. The contract information which is relevant for our analysis consists of two elements – the revealed amount of insurance purchased elsewhere and the revealed identities of those providing the insurance.

Information revelation by consumers

We make the following further assumption concerning consumers' information revelation:

[39] The non-convexities associated with incentive constraints imply that in competitive equilibria with moral hazard there may be positive profits. That is, a policy with slightly better terms than the putative equilibrium policy that makes a profit, induces a discrete change in behavior, resulting in a discontinuous change in profitability. See Arnott and Stiglitz (1991, 1993) and Stiglitz (2013a).

Assumption (A)

An individual cannot prove that he has no contract information to reveal. But any contract information revealed by an individual is truthful.

In other words, what individuals reveal is the truth, nothing but the truth, but not necessarily the whole truth. An individual can, for instance, reveal his contract information on the contract(s) that he has with other firm(s) simply by showing his contract.[40] When an individual has contracts with more than one firm (or even multiple contracts with one firm), the contract information for the individual would not be fully revealed if he withholds information on some or all of them. We assume there is no legal framework that can force a customer to reveal all of his contracts.[41] Thus what each firm knows about other insurance contracts of each insurance applicant is truthful, but possibly incomplete.

Since an individual's particular contract with a firm can affect what he can get from other firms an individual might ask a firm to "manipulate" the contract for him, facilitating the non-full disclosure, whenever it is in the interest of a firm and/or of an individual. A firm may divide its true contract into two[42] so that its customer may choose to reveal only part of what he has purchased. An individual is not able to have non-full disclosure about the insurance bought from a particular firm without contract manipulation by the firm of the form just described. We assume that the firm chooses to engage in such manipulation if and only when it is profitable. Contract manipulation (CM) is a part of a strategy of a firm. We establish below that it will not be profitable in equilibrium.

[40] This assumes all insurance contracts are written. In the absence of such a written contract, it would be difficult to enforce (except through "reputational mechanisms," which are not relevant in the simplified context analyzed here). We ignore the possibility here of outright fraud, e.g. that the individual simply makes up a contract that does not exist.

[41] In practice, this is not always the case. In his insurance application, he can be asked to reveal all of his existing contracts, and if it should turn out later that the individual has not been fully truthful, the insurers' obligation to honor the contract is negated.

[42] The division can be perfectly arbitrary: all that is required is that the sum of the benefits of the two parts equal the total benefit, and that the sum of the premiums equal the total premiums. We assume, in other words, that regardless of what has been revealed, the customer pays the full premium, and if an accident occurs, he receives the full benefit.

That this is so is far from obvious. While it should be obvious that non-full disclosure of his insurance contracts may be in the interest of the customer - it might enable him to get more insurance at more favorable terms than he might otherwise get - such non-disclosure may also be in the interests of the insurance firm. In fact, as we shall see, non-disclosure by a firm plays a critical role in sustaining an equilibrium. For the insurance firm may want to deter sales by others (since such sales undermine the ability of the firm to make inferences about who is buying its policies), and non-disclosure of information can serve as an entry deterrent. It may result in the entrant getting an adverse set of applicants. Nonetheless, we show below that contract manipulation is not part of the equilibrium strategies.

Information revelation by firms

Each insurance firm may also choose which contract information it has with a particular customer to reveal to which firms. A firm may get contract information about its customer *i* not just from the individual himself, but also through information revelation by his insurers. A firm j may disclose to each of the other firms some or all of the contract information Ω_j^i for its customer *i* that the firm j knows about.[43] Thus a firm may not be able to know full contract information for each of its consumers. We make the further natural assumption that a firm's strategy for an individual *i* can only be conditioned upon contract information revealed about *that* individual, not upon any contract information revealed for other individuals.[44]

An information strategy of firm j specifies a subset of the contract information Ω_j^i of an individual *i* to reveal and a set of firms to disclose to, where Ω_j^i includes not only information on a contract that j sells to i but also information on contracts revealed to j to have been purchased elsewhere.[45] Formally, an *information*

[43] In earlier versions of this paper, we assumed that there were information sharing networks of firms. Firms within the network agreed to share information—but while they agreed to do so, they could cheat. The results presented here show that nothing depends on these institutional arrangements.

[44] It is a natural assumption in this context because there are no interaction effects and no reason that information about individual k would be of any relevance for the sale of insurance to individual i. If a contract for an individual were conditional upon contract information of others, the optimal response by individuals to strategies of firms would be quite complicated.

[45] As discussed in Section 5, there are alternative formulations of the model where contract information Ω_j^i to be revealed consists only of information on a contract sold by j. Similarly, if the individual purchases from only two firms, the only information that is

strategy I_j *of a firm j* specifies whether or not it reveals to the other insurers (actual and potential) of its customer *i the information on the identities of customer i's insurers (including j)*[46] revealed to *j* . For the sake of expositional simplicity, with the information on the identities of insurers being suppressed, an information strategy I_j is represented by an N-dimensional vector with each element being 1 or 0, indicating whether or not the contract information for an individual *i* is revealed to each of the N firms, respectively[47] [48]. The k-th element of I_j represents the information policy of a firm *j* toward a firm k about the identities of insurers (including j) revealed to j by an individual i. Importantly, the information strategy of a firm *j* for an individual *i* can be (and in the equilibrium strategy will be) conditional upon D_j^i (a set of identities of insurers for *i* that are revealed to j by the individual *i* himself) as well. Thus, we will represent the information strategy of a firm *j* by $I_j(D_j^i)$. Note, by contrast, that the information strategy of a firm in RS is exogenously given by $I_j(D_j^i) = \mathbf{1}$ for all D_j^i; in the partial information price equilibrium, by $I_j = \mathbf{0}_{-j}$ for all D_j^i, where $\mathbf{0}_{-j}$ is an N-dimensional vector with all but the *j*-th element being equal to 0.[49]

Contract Strategies

One critical piece of information that firm *j* may use in conditioning its sales to individual *i* is the total revealed amount α_{-j}^i of insurance *i* purchased elsewhere, which may be revealed by an individual *i* [50]:

$$\alpha_{-j}^i = \sum_{d \in D_j^i} \alpha_d^i \qquad (4)$$

relevant is that on a contract sold by j.

46 Obviously, formally, since the firm knows the contracts it itself has issued.

47 There is no information strategy of a firm in RS, where any contract information for an individual is revealed to all the firms.

48 As formulated, the firm either transmits all information about a particular individual to a particular firm or none, but clearly, the information strategy can be generalized—firm *j* can transmit some information but not others (say about the amount of insurance but not the premium charged). This will be important in the analysis of some of the alternative equilibrium strategies below. If a firm sells more than one policy to an individual i, then the information strategy would specify which information it will reveal about each policy. But as we already noted, the only reason to divide the policy into more than one component is to avoid full disclosure, with the presumption is that it would not disclose the "second" part; but it still has to make a decision about the first. In the interest of generality, we can think of the kth element of I_i as itself being a vector of 0's and 1, describing whether it reveals information about each of the parameters of each of the policies to the *i*th individual to the *k*th firm.

49 It is obvious that one can think of the firm as revealing to itself its sales to customer i.

50 As we note below, while there is a unique allocation, it can be implemented through multiple alternative information strategies. Some of these use information beyond that in (4).

where α_d^i is the revealed amount of insurance to firm *j* an individual *i* purchased from an insurer d. If α_j^i is i's purchase of insurance from j, then $\alpha^i = \alpha_j^i + \alpha_{-j}^i$ is i's total purchases of insurance, as *perceived* by j.

Define $\widetilde{D}_j^i$ as the set of firms which are revealed to j by other firms to be i's insurers while not being revealed by i himself to j, i.e., while not being in a set D_j^i.[51] A *truthful* contract requires that the individual fully reveal his contract purchases. If an individual does not reveal all of his insurers to the firm j, $\widetilde{D}_j^i \neq \emptyset$, in which case a cancellation occurs in stage 3.

The (truthful) *contract strategy* $C_j(\alpha_{-j}^i; \widetilde{D}_j^i)$ *of a firm j* specifies a set of contracts to offer:

$$\begin{aligned} C_j(\alpha_{-j}^i; \widetilde{D}_j^i) &= \mu(1,\ q), \mu \in \mathrm{A}(\alpha_{-j}^i) \qquad \text{for} \quad \widetilde{D}_j^i = \emptyset \\ &= \quad 0 \qquad\qquad\qquad\qquad \text{for} \quad \widetilde{D}_j^i \neq \emptyset, \end{aligned} \tag{5}$$

where $\mathrm{A}(\alpha_{-j}^i)(\subseteq R_{>0})$ indicates a set of the levels μ's of insurance the firm is willing to sell at price q, which is conditional upon the sum α_{-j}^i of insurance an individual *i* is revealed to have purchased elsewhere if $\widetilde{D}_j^i = \emptyset$. Equation (5) is a simple way of representing a general conditional price or quantity contract. A contract $\mu(1,\ q)$ is a policy with benefit α equal to μ and premium equal to μq. μ is constrained; the constraints are, in general, a function of α_{-j}^i . We assume that firms are committed to fulfilling their contract offer (including to cancelling policies when the firm obtains information that the contract conditions have been violated).

Thus, below we will consider several special cases. (i) In one, α is either zero or $\bar{\alpha}$ (where $\bar{\alpha}$ is the most preferred level of insurance by the low risk individual at the pooling price) if α_{-j}^i = 0, and 0 otherwise. That is, the firm only offers insurance to an individual that, according to its information, has not purchased insurance elsewhere, and then either sells him the policy $\bar{\alpha}$ or zero. (ii) In another case, μ is constrained so that $\mu + \alpha_{-j}^i \leq \bar{\alpha}$.[52] That is, it only sells insurance up to an amount where, based on the information it has, the

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[51] Recall that D_j^i is defined as the set of identities of insurers for *i* that are revealed to j by the individual *i* himself.

[52] $\mu(1,\ q)$ can alternatively be written as the scalar μ times the vector (1,q)', where the "prime" denotes the transpose of a vector. There are many alternative ways of representing the set of contracts, e.g. as (potentially non-linear) functions of what he has

individual is purchasing less than or equal to $\bar{\alpha}$. (iii) In the standard price contract μ is constrained only by the amount of damage the individual has suffered, so that the set of available contracts are all policies $(\mu, \mu q)$, $0 \leq \mu \leq d$, and there is no conditioning; the contract is made available to all. (iv) The exclusive quantity contract in RS is one which says that if $\alpha^i_{-j} > 0$, $\mu = 0$: the firm sells the individual nothing if the firm knows that the customer has bought anything elsewhere. But if the individual has not bought anything elsewhere (to his knowledge), the firm offers the separating contracts, two distinct contracts each with a particular μ and price.

Thus, the concept of a contract strategy which is relevant in an environment with incomplete contract information is different and more general than that used in RS in three respects. First, a contract strategy in RS is represented by a contract with a particular value $\mu \in R_{>0}$ and has a specific conditionality on contract information revealed: here, the firm may offer a price contract and/or a multiplicity of quantity contracts, with the offerings conditional on the knowledge it has about individual's other purchases.

Secondly, contract manipulation (CM) is allowed: A firm with a contract strategy $C_j(\alpha^i_{-j})$ engages in CM if, for instance, it sells an individual two contracts with different values (μ_1, μ_2) simultaneously, instead of one contract with $\mu\ (= \mu_1 + \mu_2)$[53].

Thirdly, if the firm discovers that the individual has not been truthful in the information he has revealed, the contract may get cancelled. Whether the firm finds out such information is, of course, endogenous; and in equilibrium, no individual will have his insurance cancelled. But the knowledge that the policy would be canceled plays an important role in inducing truth-telling and in deterring entry.[54]

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purchased elsewhere (possibly a function of all the parameters of those policies) and of the amount he purchases from the given firm. (Note that restricting the amount purchased is equivalent to imposing an infinite price on purchases beyond the given amount.)

[53] As we noted, this might be important, because it opens up the possibility of partial revelation of insurance purchased.

[54] A set of contract offers by a firm is conditional upon the information α^i_{-j} revealed by its consumer i, while it is affected only indirectly by information revelation by other firms, which, as will be shown below, just induces the consumer to reveal truthfully.

We can now represent a strategy S_j of a firm *j* as consisting of two parts, a contract strategy and an information strategy:

$$S_j = \{C_j(\alpha^i_{-j}; \widetilde{D}^i_j), I_j(D^i_j)\} \quad \text{for } j = 1, \text{--}, \text{N}. \tag{6}$$

Our proof strategy is to first analyze equilibrium allocations, showing that under very weak informational assumptions concerning disclosure and non-disclosure, the set of allocations that can be sustained is very limited. Focusing on this limited set of possible equilibrium allocations, we then analyze potential *full* equilibrium, identifying both the contract offerings/take-ups and information disclosures that can be sustained by a Nash equilibrium.

3-3. Equilibrium Allocation

Any allocation must be of one of four types: (a) different quantities of insurance at different prices for the two groups; (b) the same quantity at the same price; (c) different quantities at the same price; and (d) the same quantities at different prices. An allocation (a) is a separating one while (b) is a pooling one. An allocation (a), which satisfies self-selection constraints will be called as an RS allocation, as it is a candidate for an equilibrium allocation under complete disclosure (as in the RS model). An allocation (c), on the other hand, is the standard price equilibrium under complete non-disclosure discussed in Section 3-1, where we showed that it could never be an equilibrium allocation. It is obvious that an allocation of the form (d) with the same quantity at the different prices cannot be an equilibrium allocation since no one would purchase the higher price policy (policies). Now we will see if either (a) or (b) (the non-price equilibria) can be an equilibrium when the informational structure is endogenously determined.

First, we establish the following Proposition.

Proposition 3

Any allocation $(\alpha_i, \beta_i)_i$ *with insurance purchased by both groups satisfying self-selection constraints that* *satisfies the zero-profit condition can be represented by a pooling contract* (α^P, β^P) *(where* $\beta^P = \frac{\bar{P}}{1-\bar{P}}\alpha^P$*)* *plus a set* $(\alpha_i^S, \beta_i^S)_i$ *of supplementary contracts for the two types such that*

$$\alpha_i = \alpha^P + \alpha_i^S \;,\; \beta_i = \beta^P + \beta_i^S,\; \beta_i^S = \frac{P_i}{1-P_i}\alpha_i^S.$$

The pooling contract and each of the supplementary contracts break-even. Figure 3 decomposes any pair of contracts into a pooling allocation and a set of supplementary policies. Project back from the contract (α_L, β_L) along a line with slope $\frac{P_L}{1-P_L}$ and similarly from the high risk type's contract. That line intersects the zero profit pooling line at (α^P, β^P), the pooling contract. The addition - going from (α^P, β^P) to (α_L, β_L) - are zero profit supplementary contracts. Similarly, for the high risk type. Proposition 3 says that for the allocation, the projections back to the pooling line intersect the pooling line at the same point, the pooling contract (α^P, β^P). The supplementary contract is only the addition (the arrow from (α^P, β^P) to (α_L, β_L)). Similarly, for high-risk types.[55] Note also that in order for $(\alpha_i, \beta_i)_i$ to be an equilibrium allocation, the pooling contract in conjunction with the set $(\alpha_i^S, \beta_i^S)_i$ of supplementary policies should satisfy the self-selection constraint.[56] Note too that the RS equilibrium allocation under complete disclosure has a zero pooling allocation and positive supplementary one for each of the two types.

Now we can prove the following Proposition.

Proposition 4

Any allocation with incomplete supplementary insurance for high-risk types or with positive supplementary insurance for low-risk types (including an RS equilibrium allocation) is not an equilibrium one when the informational structure is endogenously determined.

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[55] Assume the projections from A and C to the zero profit line hit the zero profit line at two different points, Z and Z' respectively. Since profits are zero along AZ, $\pi_L\{Z\} = \pi_L\{A\}$, and similarly for C and Z'. But $\pi_L\{A\} = -\pi_H\{C\}$, but along the zero profit line $\pi_L\{Z\} = -\pi_H\{Z\}$. Thus Z and Z' must coincide.

[56] An equilibrium in Akerlof (1970), Ales-Maziero (2011) and Attar-Mariote-Salanie (2014), if exists (entailing zero insurance for the low risk individuals), is the one in which both pooling allocation and the supplemental one for the low-risk type are zero.

The intuition for Proposition 4 is simple. Figure 4 demonstrates that an equilibrium allocation cannot entail supplemental insurance for the low risk type. Assume {A, C} is an equilibrium allocation satisfying the self-selection constraints. The high risk individual prefers C, the low risk A. But with potential secrecy, this won't work: at A, the slope of V_H is greater than $\frac{P_H}{1-P_H}$, so that high risk individuals strictly prefer A plus supplemental insurance that breaks even (at the high risk probability of accident), i.e. using our earlier notation, $V_H(A+S_H) > V_H(C)$, so {A,C} are no longer a separating set of contracts. The key role of secrecy should be noted. In RS, the original insurance firm would have observed the supplemental policies originally purchased, and his contract would have been offered conditional on such policies not being purchased.

Equilibrium Allocation

Define $(\bar{\alpha}, \bar{\beta})$ as a pooling allocation that is the most preferred by the low-risk type subject to the zero-profit condition:

$(\bar{\alpha}, \bar{\beta}) = Argmax\ V_L(\alpha, \beta)$ s.t. $\beta = \frac{\bar{P}}{1-\bar{P}}\alpha$

$\bar{\alpha} = 0$ if (3') holds.

We can then establish the following Theorem on the equilibrium allocation.

Theorem 1

There exists a unique allocation that can be an outcome of a Nash equilibrium. It is a combination of a pooling allocation $(\bar{\alpha}, \bar{\beta})$, which is the most preferred by the low risk individual, and supplemental insurance for high-risk type which leads him to be fully insured. The low-risk type buys zero supplemental insurance.

More formally, the equilibrium allocation is $E^* \equiv \{(\alpha_H^*, \beta_H^*), (\alpha_L^*, \beta_L^*)\}$, where

$$\alpha_H^* + \beta_H^* = \text{d}, \quad \text{and} \quad \beta_H^* - \bar{\beta} = \frac{P_H}{1-P_H}(\alpha_H^* - \bar{\alpha})$$
$$\alpha_L^* = \bar{\alpha},\ \beta_L^* = \bar{\beta}.$$

Note that the allocation E^*, which is shown diagrammatically in Figure 5, satisfies the self-selection and the zero profit constraints. The intuition for Theorem 1 is clear, given Proposition 4. Any allocation with a pooling contract (α', β') other than $(\bar{\alpha},\bar{\beta})$ will be upset by a contract involving $(\bar{\alpha},\bar{\beta})$, since the low type would prefer $(\bar{\alpha},\bar{\beta})$ to (α', β') if (α', β') entails more insurance than $(\bar{\alpha},\bar{\beta})$, while everyone would prefer $(\bar{\alpha},\bar{\beta})$ if (α', β') entails less insurance. Thus, $(\bar{\alpha},\bar{\beta})$ will be preferred by the low type individuals in any case and would "break" the putative equilibrium.[57] Proposition 4 establishes that there cannot be an equilibrium allocation with positive supplementary insurance for the low risk type, since that would entail a pooling contract with (α', β') providing less insurance than $(\bar{\alpha},\bar{\beta})$; or with less than full insurance for the high risk type, since clearly the high risk individual would want to buy secret insurance at his own odds to get him to full insurance. E^* is thus the only possible equilibrium allocation.

4. Equilibrium Strategy

In this section we will present a set of strategies of firms and a corresponding set of strategies of individuals that lead to the equilibrium allocation E^*, consisting of the pooling contract most preferred by the low risk individual, and supplemental contracts at the high risk individual's odds, bringing high risk individuals to full insurance, proving that there exists an equilibrium in a competitive insurance market under adverse selection when firms and individuals can choose their information strategies.

Our analysis focuses on the strategies of firms (for reasons that will be clear shortly). We construct the set of equilibrium strategies – contract and information strategies – that can lead to E^* in two steps: 1) We first present a *simple* set of strategies that entails inter-firm communication and no information revelation by customers, which directly yields E^* in the absence of any deviant strategy. 2) After showing that the proposed strategies are not an equilibrium - there is a possible profitable deviation - we show that a slight modification of the strategy with information revelation by customers being taken into account can sustain E^* as an equilibrium allocation.

[57] If it entails less insurance, a firm could offer a policy which made profits, being purchased only by the low types.

As an equilibrium entails more than one strategy being pursued in equilibrium, that is, different firms pursue different strategies. We will simplify the analysis, assuming that each firm offers a single insurance contract, with an associated information strategy; that is, any given firm *either* offers the pooling contract with some disclosure, or offers the price contract corresponding to the high risk individuals secretly.[58]

The hardest part of the proof is establishing that the equilibrium allocation can be sustained. This requires that it is not possible to "break" the allocation by offering a policy that would be purchased just by the low risk individual. This, in turn, requires that any individual purchasing the "deviant" policy be able to buy sufficient insurance at the pooling price *and* that the deviant firm not know whether the (high risk) individual has done so. Thus, this information cannot be disclosed to the deviant firm (though, of course, the other firms may not know who is deviant). But a policy of general non-disclosure will not work, for with no disclosure at all, we have already established that there is no equilibrium. Correspondingly, if all information is disclosed, we are in the world of RS, where we know that a pooling equilibrium cannot be sustained.

Thus, any equilibrium must have some form of *selective* disclosure based upon information revelation by customers. The trick is determining what information should be disclosed to whom. The answer turns out to be somewhat surprising: the insurance firm discloses its sales to individual *j* only to the firms who have not been disclosed by the individual *j* as sellers of insurance to him.

Simple Strategies with Inter-firm Communication and with No Revelation by Customers

We first consider an equilibrium which is the natural generalization of the RS equilibrium, where there are two sets of firms. One subset of firms discloses all of its information (amounts of insurance sold as well as their identities) to all other firms and sells the pooling contract to anyone who has been revealed not to have insurance, but zero to anyone who has been revealed to have purchased insurance. Formally, we represent this as[59]:

[58] We discuss later in this paper how the results would change if we allow a deviant firm to offer more than one insurance contract.
[59] Formally, the value of n makes no difference (since the firms are identical and nothing depends on the number of firms selling any particular contract). For there to be plausible competition, N – 1 > n > 1.

$$S_j = S1_j = \{C1_j(\alpha^i_{-j}), \mathbf{1}\} \quad \text{for } j = 1, \text{--}, n, \ (1 < n < N) \qquad (7)$$

where α^i_{-j} is the amount of insurance revealed by other firms to have been purchased elsewhere and $\mathbf{1}$ is an N-dimensional vector with all the elements being equal to 1)[60]

$$C1_j(\alpha^i_{-j}) = \bar{\alpha}(1, \tfrac{\bar{P}}{1-\bar{P}}) \quad \text{for } \alpha^i_{-j} = 0$$
$$= 0 \quad \text{for } \alpha^i_{-j} > 0,$$

The second set of firms offers a price contract at the price of the high risk individual and does not reveal any information:

$$S_j = S2_j = \{C2_j(\alpha^i_{-j}), \mathbf{0}_{-j}\}, \qquad \text{for } j = n+1, \text{--}, N, \qquad (8)$$

where $\mathbf{0}_{-j}$ is an N-dimensional vector with all but the j-th elements being equal to 0, and

$$C2_j(\alpha^i_{-j}) = \delta\left(1, \frac{P_H}{1-P_H}\right) \quad \text{with } \delta \in R_{>0} \quad \text{for all } \alpha^i_{-j} \geq 0.$$

It is clear that the above set of strategies implements the equilibrium allocation. The question is, is it a Nash equilibrium? Does it pay any firm to deviate?

Sustainability of $\{S_j\}_j$ as an Equilibrium

RS showed that one could break a pooling equilibrium by "cream skimming," offering a contract that would be purchased only by low risk type individuals; but our discussion of Proposition 1 explained that the presence of undisclosed contracts changed the analysis.

Consider a strategy S_d of a deviant firm:

$$S_d = \{C_d(\alpha^i_{-j}), \mathbf{0}\}, \qquad (9)$$

where

[60] Recall from our earlier discussion that $\bar{\alpha}(1, \frac{\bar{P}}{1-\bar{P}})$ just describes the insurance policy with benefit equal to $\alpha = \bar{\alpha}$ and premium $\beta = \bar{\alpha}\frac{\bar{P}}{1-\bar{P}}$.

$$C_d(\alpha^i_{-j}) = \bar{\alpha}(1, \frac{P_d}{1-P_d}) \text{ with } P_L < P_d < \bar{P} \quad \text{for } \alpha^i_{-j} = 0$$
$$= 0 \qquad \text{otherwise.}$$

The contract $C_d(\alpha^i_{-j})$ is illustrated by B in Figure 6, a point above the high risk individual's indifference curve but below the low risk individual's.[61] The strategy S_d secretly[62] offers anyone with no contract information revealed the amount $\bar{\alpha}$ of insurance at a price lower than $\frac{\bar{P}}{1-\bar{P}}$. Without $S2_j$, the strategy S_d would profitably attract only low-risk individuals, making $S1_j$ unsustainable. The possibility of non-exclusive provision of insurance with non-disclosure, such as when some firms employ the strategy $S2_j$, however, reduces the scope of a profitable cream-skimming strategy. High-risk individuals as well as low-risk ones would choose B, because when B is supplemented by the (secret) insurance at the high risk odds, the high risk individual is better off than purchasing the pooling contract A; and with both high and low risk individuals purchasing B, contract B makes a loss.[63]

But the strategy $S2_j$ is not able to completely eliminate the possibility of a profitable cream-skimming strategy. Consider, for example, another deviant strategy $S_d{}'$ that is the same as S_d except that it offers contract B in Figure 7 for those with no contract information revealed, a contract with less than $\bar{\alpha}$ of insurance. This contract will be chosen by low-risk individuals only, because it does not involve an amount of insurance that is large enough to attract the high-risk. In other words, even with $S2_j$, $(\bar{\alpha}, \bar{\beta})$ is better than B for high-risk individuals. In general any contract in the shaded area of Figure 7, the set we defined earlier as Z in Figure 2b[64], will always be bought by the low risk individuals but not by the high risk. Since the shaded area always exists whenever $\bar{\alpha} > 0$, the set $\{S_j\}_j$ of strategies alone cannot constitute an equilibrium in the model

[61] The conditionality of the offer A is necessary to prevent high-risk individuals from choosing it and S_j.
[62] In fact, S_d may not necessarily be offered secretly. If it is disclosed to all the other firms, an individual customer would not have to reveal his contract information to a non-deviant firm.
[63] This point was also emphasized by Jaynes (1978, 2011).
[64] This is defined as the set of policies below V_L, the equilibrium indifference curve of the low risk, but above the line through A, the equilibrium pooling contract, with slope $\frac{P_H}{1-P_H}$.

unless (3) holds.[65]

Toward an Equilibrium Strategy

The strategies need to be modified so that *any* contract, such as B in Figure 7, will be chosen by the high-risk individuals as well as the low-risk ones. To do this, we must be sure that the high risk individual can supplement the policy {B} with enough insurance, say, at the pooling price, that he prefers {B} to {A}. The trick is to do this in such a way that the additional contracts are not purchased in equilibrium - for if they were, since the high risk individuals would then be buying more pooling insurance than the low risk, the pooling contract loses money. The {contract, information} strategies we are about to describe do this.

α_{-j}^{i} is given by (4) while a contract strategy $C_j(\alpha_{-j}^{i}; \widetilde{D}_j^i)$ is given by (5). $S2_j$ remain unchanged, but strategy $S1_j^*$ has to be modified to offer a consumer *i* with the revealed contract information about purchases from others (α_{-j}^{i}) additional insurance to bring the total amount of insurance up to at least $\bar{\alpha}$ [66]; while disclosing information on the identities it has of the individual's insurers (including its own identity) to all the firms *other* than the insurers revealed by the consumer.[67]

More formally, we consider the following set $\{S_j^*\}_j$ of strategies

$$S_j^* = S1_j^* = \{C1_j^*(\alpha_{-j}^{i}; \widetilde{D}_j^i), I_j^*(D_j^i)\} \quad \text{for } j = 1, \text{--}, n, \ (1 < n < N) \tag{10}$$

where

$$\begin{aligned} C1_j^*(\alpha_{-j}^{i}; \widetilde{D}_j^i) &= \mu\left(1, \frac{\bar{P}}{1-\bar{P}}\right), \ \mu \in [0, Max\left\{\bar{\alpha} - \alpha_{-j}^{i}, 0\right\} \qquad \text{for} \quad \widetilde{D}_j^i = \emptyset \\ &= 0 \qquad \text{for} \quad \widetilde{D}_j^i \neq \emptyset \ , \end{aligned} \tag{11}$$

[65] When (3) holds so that $\bar{\alpha} = 0$, the equilibrium strategy would be just $S2$, since the contract $(\bar{\alpha}, \bar{\beta})$ becomes {0,0}. In this case, the equilibrium strategy and outcome would be the same as those in Akerlof (1970), Ales-Maziero (2011) and Attar-Mariote-Salanie (2014).

[66] If the individual has already purchased more than $\bar{\alpha}$ from others, then nothing is sold to him.

[67] With a slightly different information structure, it is even easier under some circumstances to sustain the pooling contract, even without the modifications we are about to describe. Consider the case noted earlier where firms offer only 0 or the full pooling insurance. Then if the two individuals are very similar, so A and C are near each other, it may be that $V_H(A + B) < V_H(C)$, so that B separates even without disclosure of purchases of pooling contracts.

and $I_j^*(D_j^i)$ is an N-dimensional vector with all the d-th elements (where $d \in D_j^i$), being equal to 0 and with all the others being equal to 1.[68] The contract strategy brings the individual's (known) total purchases of insurance up to (at least) $\bar{\alpha}$.

$I_j^*(D_j^i)$ is critically different from that of earlier analysis (7), and consists of two parts: disclosing information on the identities of firms from which a customer *i* revealed purchasing insurance at the pooling price to all the firms other than the insurer(s) revealed by the individual *i* himself, i.e., to all $j' \ (\notin D_j^i)$, and not disclosing the identities of insurers for *i* to any of the insurer(s) revealed by *i* himself, i.e., to any $j' \in D_j^i$. There are two features of the information strategy $I_j^*(D_j^i)$ that should be noted. First, it is consumer's revelation (in Stage 2), that determines the information disclosure of a firm (in Stage 3).[69] Second, the information strategy $I_j^*(D_j^i)$ entails the precise kind of disclosure that can implement the kind of exclusivity necessary to support the equilibrium allocation E^*.[70]

For simplicity we will hereafter omit the subscript *j* (not the subscript (-j)) in specifying strategies of a firm *j* by denoting $S_j^*(\equiv \{S1_j^*, S2_j^*\}$ by $S^*(\equiv \{S1^*, S2^*\})$, for example.

Truthful Revelation of Information by Individuals

We now assess an individual's incentive to reveal his contract information to potential insurers, given the strategy S^*. We first prove the following Proposition.

Proposition 5.

Given the information revelation strategies of the firms, there exists at least one firm that will have complete information about any consumer's purchases of pooling insurance.

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[68] Recall that D_j^i is a set of identities of insurers of *i*, who are revealed by *i* himself.

[69] Not another firm's disclosure in Stage 3—commitments about which were made in stage 1.

[70] We will show below that this type of a selective disclosure policy is a dominant information strategy for a firm.

Proof: Fix a particular consumer. There are *N* firms; suppose that the consumer buys insurance from $K \leq N$ of them. The rest $N - K$ firms (if any) do not matter since they do not share any information and simply passively receive information. Thus it is without loss to focus on the K firms that have sold a positive amount of insurance to the consumer.

A firm can get information in three ways: first of all, by assumption, each firm has information about its own sales. In addition, the customer may reveal information about his purchases from other firms. Finally, given the equilibrium strategy other firms will disclose information about this consumer to those firms who have not been revealed to be its insurers.

We can represent this by a K-by-K matrix A with only zeros and ones as entries. An entry of 1 in the a_{ij} place means that firm i has information about firm j's sales to a particular (fixed) consumer, while an entry of 0 means that i doesn't have information about j's sales.

The matrix starts out with ones on the diagonal (that is. $a_{ii} = 1\ \forall i = 1,2,\dots,K$ since by assumption each firm knows its own sales to this consumer), plus perhaps some other nonzero entries, where that consumer revealed any information. The information sharing/updating algorithm then takes the following form:

Step 1: entries are generated according to the consumer's information revelation strategy.

Step n: if $a_{ij}^{n} = 0$, then $a_{jk}^{n+1} = \max\{a_{ik}^{n}, a_{jk}^{n}\}$ for $k = 1,2,\dots,K$.

where the superscript refers to the step in the algorithm. We denote by A^n the information matrix at the end of step n. The updating step is reflecting the fact that if any firm i doesn't have information that j has been revealed as a seller of insurance for this consumer, then at the next step it will reveal everything it knows to j, so that j, in turn, will know everything it already knew, as well as anything i knew for all other firms for this particular consumer.

We now prove the proposition by contradiction – suppose that the algorithm has stopped updating[71], namely, that $A^n = A^{n+1}$ and yet there is no firm with a complete knowledge of that consumer's purchases. In

[71] Clearly, once the algorithm stops updating, it will never start updating again. Moreover, the algorithm will, in fact converge, since once an entry changes from a 0 to a 1, it will never change back and there are a finite number of entries.

the language of this algorithm this means that there is a zero in every row. Also, since the entries are not changing at this point, we do not have to keep track of the superscript. In particular, say that $a_{ij} = 0$ for firm/row i with $i \neq j$ so that i doesn't know that j is a seller of insurance for this consumer. Then we must have $a_{ji} = 1$ since i would reveal everything it knows to j. By definition, for firm/row j then, for all $k = 1,2, \dots, K$ we have that $a_{jk} = \max\{a_{jk}, a_{ik}\}$. In other words, since j has i's information, it knows everything is ever knew itself, plus whatever i knows. Note that we're using the fact that the algorithm has converged here in asserting this; this would not necessarily be true if the algorithm is still operating. If the j'th row does not contain any zeros, we are done, since then j has complete information about this consumer. Suppose to the contrary, then, that there is a zero for some $k \neq i, j$ $a_{jk} = 0$ (k cannot be the same as j by assumption of own knowledge of sales, and it cannot be i by the deduction above). Thus there is some firm k for whom j doesn't have information and so $a_{kj} = 1$. Note that this also means that $a_{ik} = 0$ since otherwise i would have known about k's sales, which it would have revealed to j. The latter fact implies that $a_{ki} = 1$. By the updating rule, for $l = 1,2, \dots, K$, $a_{kl} = \max\{a_{kl}, a_{il}\}$ and $a_{kl} = \max\{a_{kl}, a_{jl}\}$. Thus we have that $a_{ki} = a_{kj} = a_{kk} = 1$. If $K = 3$ we are done. If not, suppose that $a_{kl} = 0$ for some $l \neq i, j, k$. Then $a_{lk} = 1$ and for $m = 1,2, \dots, K$ we have that $a_{lm} = \max\{a_{lm}, a_{km}\}$. Thus we have that $a_{lk} = a_{li} = a_{lj} = a_{lk} = 1$, using the previous steps. In other words, this row has four ones, which means that firm l has information about its own sales, plus those of three other firms. If $K = 4$ we are done. If not, we proceed in the same way, at each step constructing a firm that knows strictly more than the others. Since K is finite, this second algorithm also converges and will produce a row of ones of length K which contradicts our original assumption that there is no firm that has complete information. This completes the proof.

Since there will be some firm with full information for any consumer, that firm will cancel its insurance contract if the total purchases of insurance for that consumer exceed $\bar{\alpha}$ (recall that by Theorem 1 there is a unique outcome that can be sustained in equilibrium, so $\bar{\alpha}$ is indeed the threshold for cancellation). In this event the consumer will be left with less than $\bar{\alpha}$ of insurance bought at the pooling price and therefore worse off than if he purchased just $\bar{\alpha}$. Thus, under the given strategies, the consumer—even the high risk individual--has a strict incentive to truthfully reveal all purchases of insurance at the pooling price and to purchase no more than $\bar{\alpha}$.

Under-reporting by CM cannot occur in equilibrium. No firm would not agree to it without charging a price at least equal to $\frac{P_H}{1-P_H}$ (since it knows for sure that any individual requesting CM is high-risk). But high risk individuals are at least as well of purchasing secret insurance at the price $\frac{P_H}{1-P_H}$.

Of course, high risk individuals (or their insurance firms) do not reveal their purchases of the supplemental policies at the high risk price, because if they did so (truthfully), then all those selling pooling contracts would condition their sales on such supplemental policies not being bought (for such purchases reveal that the individual is high risk).[72]

Equilibrium Existence

We now establish the main Theorem on the existence of equilibrium.

We now prove[73].

Theorem 2

Under Assumptions A , the set S^* $(\equiv \{S1^*, S2^*\})$ *of strategies described above constitutes an equilibrium, and supports the equilibrium allocation* $E^* \equiv \{(\alpha_H^*, \beta_H^*), (\alpha_L^*, \beta_L^*)\}$.

Proof:

[72] More formally, our strategies S1 entail the individual's insurers sales of the pooling contract being reduced by the amount of (disclosed) purchases of supplemental insurance. Hence no high risk individual would make such a disclosure. Later, we analyze incentives for disclosure of purchases of out of equilibrium offers of insurance.

[73] Throughout we make use of the following assumption: if any firm offers a contract that yields the same utility for a consumer relative to the entire profile of strategies (and in particular, relative to the equilibrium profile) then no consumer will purchase that contract. This is essentially a tie-breaking assumption (if a consumer is indifferent, he follows the prescribed strategy instead of deviating and choosing this other contract) and in equilibrium this can be viewed as focusing on an equilibrium in weakly dominant strategies.

In purchasing $\bar{\alpha}$ at $\frac{\bar{P}}{1-\bar{P}}$ from $S1^*$, an individual may either choose to purchase the whole $\bar{\alpha}$ from one insurer or choose to purchase partitions of $\bar{\alpha}$ from multiple insurers. In the former case $S1^*$ coincides with S1 defined in (7). The strategy $S1^*$ plays a critical role in the determination of E^* by limiting the total amount purchased at $\frac{\bar{P}}{1-\bar{P}}$ from all the firms as a whole to $\bar{\alpha}$, which is made possible by the information strategy $I^*(D^i)$ (firms' disclosure policy, in particular) together with Proposition 5 ensuring that at least one firm that has sold j a pooling policy is fully informed about purchases of pooling insurance.

We prove the theorem by noting what restrictions on the possible outcome have been made by the preceding results, and then show that the profile of strategies for firms and consumers satisfies these restrictions, and in addition, is robust to deviations.

First, we recall the fact that Theorem 1 and the preceding discussion showed that there is a unique outcome/allocation that can even potentially be an equilibrium. Secondly, Proposition 5 showed that consumers who try to manipulate their disclosure policy (relative to what is prescribed for them by S^*) will have some or all of their contracts cancelled. Thus, in any putative equilibrium, given the unique possible outcome, if consumers manipulate their strategies, they will be left on a lower indifference curve relative to the candidate equilibrium allocation. Therefore, they will, in fact, reveal truthfully. That is, no consumer can profitably deviate against any strategy of firms that a) results in the equilibrium allocation and b) utilizes the information disclosure policy as specified in the previous discussion.

Consider now the firms' side; clearly, no firm can do better by changing the price it charges for the secret supplemental insurance, so we only have to consider the pooling contracts and the associated disclosure policies. We consider separately the two possible types of deviant contract strategies: one, called S_d^h, which entails charging a price that is not lower than $\frac{\bar{P}}{1-\bar{P}}$, and the other, S_d^l, with a price that is lower than $\frac{\bar{P}}{1-\bar{P}}$.[74] and

[74] A deviant strategy is allowed to use all the contract information including price of insurance, while the equilibrium strategy uses a

finally consider a strategy that involves only deviations with regard to information revelation.

Suppose that a firm deviates to charging a price that is weakly higher than $\frac{\bar{P}}{1-\bar{P}}$ for the pooling contract, and adopts *any* information revelation strategy. The fact that this argument works for any information revelation strategy is crucial, but it is also obvious. Using the (tie-breaking/weakly dominant strategy) assumption that if a contract that just breaks even relative to the other candidate equilibrium contracts is offered, no player takes up this contract. Thus, no low-risk types purchase this contract and therefore it cannot make a profit, so this cannot be a profitable deviation.

Now consider a deviation to a contract policy S_d^l which charges strictly less for the pooling contract, and again use any information strategy for the firm. Recall that Nash equilibrium only requires robustness against unilateral deviations. Since this policy makes all consumers better off, they all purchase it. One key step here is that the consumers would not reveal to a deviant firm their pooling insurance purchased elsewhere particularly if its strategy S_d^l entails exclusivity, i.e., requiring no other policies to be purchased. Another key step is, given their information disclosure strategy, they would all reveal the purchase of a deviant policy to their non-deviant insurers. This will have two important consequences. First, the revelation by consumers induces their non-deviant insurers not to disclose their sales to the deviant firm (by their equilibrium information strategy $I^*(D^i)$), leaving the firm uninformed of the consumers' purchases from the non-deviant insurers. Second, given that they reveal the purchase of a deviant policy, if the total amount of their insurance exceeds $\bar{\alpha},$ some contracts are cancelled, and they end up worse off, by the same argument as above. Given firm strategies, all consumers have an incentive to buy at least $\bar{\alpha}$. If all consumers purchase $\bar{\alpha}$, there are no cancellations, and since the high risk individuals buy it as well as the low risk, the policy makes a loss and hence cannot be a profitable deviation. If a deviant strategy S_d^l does not entail exclusivity or if it entails CM (contract manipulation), on the other hand, it should attract high-risk individuals whenever the same deviant strategy but

subset of it – insurance amounts purchased and identities of insurers.

with exclusivity and without CM does. This is because a strategy with no exclusivity or with CM allows consumers to purchase more pooling insurance than they could under the same strategy but with exclusivity and no CM.

Suppose finally that a deviant firm does not alter its contract policy but instead changes only its information revelation policy. The only way it can make positive profits is if it engages in cream-skimming. There are only two ways it can alter its information policy: by not revealing purchases of the pooling contract to those who under the equilibrium strategies it reveals, or revealing information to those who under the equilibrium strategies it does not reveal. The former would attract all the high risk individuals (who might hope that thereby they could avoid detection of buying more than $\bar{\alpha}$), so obviously makes a loss. The latter will not deter high risk individuals (because their optimal strategies entails, in any case, truth telling). Thus, it does not increase profits.[75]

Hence, this is a Nash equilibrium.

A final observation: there is a simple way of seeing the difference between our equilibrium and the standard reactive equilibrium, in which responses to an out-of-equilibrium behavior of a deviant firm destroys the seemingly profitable opportunities for an entrant. In our analysis, a firm j responds in the same way to contract information about its customer with a firm j', regardless of whether the firm j' is a deviant or a non-deviant firm. (Firms base their behavior simply on the amount α_{-j}^{i} of insurance purchased and the identities D_j^i of the insurers revealed.)

5. Further Discussion

Uniqueness of Equilibrium

[75] Our definition of equilibrium requires that there be a strictly profitable deviation. Note that if all firms but one deviated by disclosure to other firms selling the pooling contract, exclusivity could be enforced by that firm, and it could engage in cream skimming against the rest. This disclosure policy would thus result in a loss of profits.

The equilibrium is not unique: there are other information strategies that can, together with the same contract strategy $C1^*(\alpha_{-j}^{i}; \widetilde{D}^{i})$ (defined in (11)), sustain the equilibrium outcome E^*. There are also other contract strategies that also can implement the equilibrium allocation E*. Suppose, for example, that there are two groups of firms: K firms in group 1 and (N-K) firms in group 2, in each of which a firm shares with the other members any contract information. Taking each of the two groups as an individual firm, we can think of the following information strategy that is essentially the same as the information strategy $I^*(D^i)$: 1) to reveal its contract information on $S1^*$ for its customer *i* to its own group, 2) to reveal contract information for *i* to the other group if the other group has no revealed insurer; but not to reveal the contract information for *i* to the other group if the other group has a revealed insurer.[76] This information strategy, together with the contract strategy $C1^*(\alpha_{-j}^{i}; \widetilde{D}^{i})$, can be shown to constitute an equilibrium.

Extensions to Cases with Many Types

The result on existence of equilibrium can be extended to the case with many types. (See Stiglitz-Yun [2016].) An equilibrium strategy in a case with the three types, for example, can be described in a similar way to the case with two-types. As indicated in Figure 8, there is a pooling contract with all three types, contract A, the most preferred by the lowest risk type; and a partial pooling contract B with additional insurance pooling together the two riskiest types, where B is the most preferred along the zero profit line for partial pooling; and finally, a contract C, providing full insurance to the highest risk type. In equilibrium consumers purchase A only or A and B or A, B and C, depending upon their types. Consumers truthfully fully reveal to the other insurers their information about their purchases of the fully pooling contract A (since all purchase the same amount, such information in equilibrium reveals no information about who they are). Consumers reveal information about their purchases of the partial pooling policies B only to firms not (revealed to be) selling the fully pooling policy[77]. There are three types of firms, those selling the full pooling contract, those selling the partial pooling

[76] The two groups can be thought of as "clubs," or information sharing associations. Our analysis shows that there can exist an equilibrium with more than one such association.

[77] As always in this paper, what matters is not firm offers, but firm sales to a particular individual.

contract, and those selling the price contract to the high risk individuals. They adopt the same information strategy as in the case of two types of individuals.[78] Proposition 5 can be generalized to the case with many types.[79]

By the same reasoning as in the two-type case, there is no room for a deviant contract offering $\tilde{\alpha}$ that profitably attracts low or medium types, as shown in Figure 8, as riskier types are also induced to choose $\tilde{\alpha}$ by the equilibrium strategy (as illustrated by the dotted arrow in Figure 8).[80]

This argument can also be applied to the case with continuum types as well. In a case with continuum types of individuals, where an individual of type θ, for whom the probability of accident is P_{θ}, is distributed over $[\theta_1, \theta_2]$ with an arbitrary but commonly known, atomless and differentiable distribution with a finite density, we can think of an equilibrium insurance-premium schedule as depicted by a bold curve in Figure 9. This schedule generates an equilibrium allocation $\hat{E}$, where, say, (a) B in Figure 9 is the optimal point along the locus for type θ'; and (b) at B (at the optimal insurance $\alpha(\theta')$ for the type θ'), $\frac{d\beta}{d\alpha} = \frac{P(\theta')}{1-P(\theta')}$ where $P(\theta')$ is the population weighted average probability of an accident for types θ higher than θ', i.e. the marginal cost for additional insurance is the break-even price among the riskier individuals buying that policy. Stiglitz-Yun (2016) shows that there exists an insurance-premium schedule yielding zero profits for each level of partial pooling insurance, and that with information strategies analogous to those described above, this constitutes an equilibrium.

Equilibrium in the Extended Strategy Space

Once we extend the strategy space of firms so that contract sales to an individual can be conditioned on the

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[78] That is, revealing information only to firms not revealed to be sellers to individuals, where a firm reveals information that he has sold insurance to a particular individual, but also information that he has about the sales of others.

[79] In fact, in the three-type case, an individual has an additional incentive to disclose his purchase from a fully pooling seller, because otherwise his potential insurer (or a partially pooling seller) discloses to his fully pooling insurer, who then would cancel (in Stage 3) the contract it sold to him.

[80] By the same token, there is no incentive for contract manipulation.

price as well as the amount of insurance purchased and the identity of the insurer, and information revelation strategies specify the revelation of not just to whom insurance has been sold (from whom the insurance has been bought) and the amounts of insurance, but also the price, we can formulate a slightly different {information, contract} strategy supporting the same equilibrium allocation E^*, as is shown in the Appendix A. In some ways the analysis of the equilibrium is simpler, but it requires using *latent* policies, policies which are *only* sold in response to out of equilibrium offers by other insurance firms but which are not purchased in equilibrium. Also, the information strategy of a firm requires a simpler set of contract information to be revealed: its own identity (as an insurer of an individual i) only, not the identities of other insurers revealed by its customer.[81]

Deviants who offer multiple contracts

In the Nash Equilibrium described above, each firm offers a single insurance contract. If we allow for the possibility that a deviant firm can offer *multiple policies*,[82] we can show that the main results still hold, but to do so, we need to modify our equilibrium concept - there can't exist *a set policies* which a deviant firm could offer which would be profitable. Appendix B shows that while the set S^* of strategies defined by (10) and (11) is not an equilibrium (it is not immune against profitable entry), a slight modification of the strategies constitutes an equilibrium.

6. Welfare Properties of Equilibrium

As always, the assessment of the Pareto efficiency of the market has to be conducted *relative to the information that is or might be available to the government and the feasible mechanisms for redistribution.* If the government knew who were the low and high risk individuals, it would clearly provide complete insurance to each, with the nature and magnitude of the redistributions dependent on the available set of redistributive mechanisms and the nature of the social welfare function. Stiglitz (2009) provides a fuller analysis of the Pareto frontier in the presence of asymmetric information, where the government controls fully the provision of

[81] We have also explored models in which insurance is purchased *sequentially,* with sales at any point being conditional on previous purchases. Preliminary analysis suggests that E* can be implemented using a simpler information strategy than that discussed in this paper.

[82] This possibility is also raised and discussed by Attar-Mariotti-Salanie (2016).

insurance and there are costless redistributions. The high risk individual will, as here, get full insurance, but the low risk individual will obtain a RS separating policy, *whether or not that policy is sustainable as part of a market equilibrium.* The RS equilibrium, when it exists, is Pareto efficient. When the RS equilibrium does not exist, the Pareto set of contracts entails a subsidy from the low risk individuals to the high risk individuals. In the equilibrium here, there is a subsidy from the low risk to the high risk through the pooling insurance policy. But the subsidy is not as efficient as we can obtain when the government fully controls information.

Any welfare differences between government and private provision arises not from any innate greater ability of government in the provision of insurance or in its information, but from differences in its incentives. In models of adverse selection, such as explored here, both the welfare losses and the complexities associated with insurance contracts and information sharing arise simply because of the incentives to cream skim, and the attempts by those providing pooling (or partially pooling) contracts to prevent cream skimming.[83] The government's objective is not to maximize its own profits (either in equilibrium or in response to out of equilibrium moves by others), but to ensure that the equilibrium that emerges, given its contract and information strategies, maximizes social welfare.

The RS equilibrium is Pareto efficient, *given* that the government can control the private secret provision of information (ensuring exclusivity can be enforced). The problem is that the government cannot *fully* control the provision of insurance or the information flows concerning its purchase. Insurance is embedded in a myriad of economic and social relations. (Arnott-Stiglitz, 1991b.. And the government cannot force fully the disclosure of all of these explicit and implicit insurance contracts.

This paper has made clear that the inability to control fully the provision of insurance and information about insurance purchased[84] leads to an equilibrium different from the RS allocation, and indeed different from any

[83] In addition, when firms have some monopoly power, they attempt to discriminate, including through the use of self-selection mechanisms. There are welfare costs associated with this price discrimination. See Stiglitz (1977).

[84] The problem is worsened by the fact that, as Arnott and Stiglitz emphasize, much insurance is informal, provided by families, or implicit, hidden inside employment or other contracts.

Pareto efficient allocation - *assuming that the private provision of insurance could be controlled.* Accordingly, the private provision of insurance can be thought of as welfare-decreasing. Matters would be better if the government could proscribe these other sources of insurance provision. But it cannot. Still, there is a presumption that there exist indirect controls (taxes) or other actions of the government that affect (limit) the provision of private insurance and increase information flows. But the theory of the second best provides an important warning: there are some interventions that seemingly improve information (or which limit insurance) which might have a more ambiguous effect on welfare. For instance, government could establish a national registry for all (contractual) insurance,[85] which would enable the economy to create the RS equilibrium *when it exists.* But in contrast to the model presented here, where there always exists an equilibrium, there are plausible circumstances in which equilibrium does not exist - indeed, it never exists in the standard insurance model when there exists a continuum of types.[86]

In comparing the equilibrium described here (where the government imposes no restrictions on the private provision of insurance and information flows), which for brevity we refer to as the SYK equilibrium, and that where it controls both fully (the welfare analysis relevant to RS), it is clear that the high risk individuals are better off. And whenever the RS equilibrium exists, the expected utility of the low risk individual is higher in the RS equilibrium than in the SYK equilibrium. For we know that the RS equilibrium exists if and only if the pooling contract most preferred by the low risk types yields a lower level of utility than the separating contract.[87] [88] Because when the RS equilibrium exists, one group is worse off in that equilibrium compared to the SYK equilibrium, one group better off, neither equilibrium Pareto dominates the other. Obviously, with a sufficiently inequality averse social welfare function, the SYK equilibrium yields a higher level of social

85 Obviously, non-contractual insurance, such as that provided within the family, would not be included in the registry.
86 See Stiglitz (2009).
87 The pooling contract is preferred (not preferred) to the separating contract provided that (for a given utility function) the differences are not too large (small). In particular, it is a standard result that, given the utility functions and say P_H, there exists a $\Delta\hat{p}$ (>0) , such that if $P_H - P_L > (<) \Delta\hat{p}$, there exists (does not exist) an equilibrium, i.e. the separating contract is preferred (not preferred) to the pooling contract.
88 More precisely, we know that in the RS equilibrium, the high risk individual is indifferent between the contract of the low risk individual and his full insurance; here the high risk individual strictly prefers his contract. Moreover, in RS, there is no subsidy from the low risk to the high risk individuals, here there is, through the pooling contract. Accordingly, the insurance obtained by the low risk individual here is unambiguously worse than in the RS equilibrium.

welfare than the RS equilibrium.[89]

This leads to the difficult question, which we cannot answer here: in the context of adverse selection, are there ways by which the government could preclude some insurance and/or force more disclosure which would be welfare increasing?[90]

7. Concluding Remarks

When contracts are traded with contract information being revealed by firms or by individuals, it is natural to suppose that firms condition their contracts for an individual upon contract information that is revealed. More specifically, the strategy of a firm includes what to reveal to whom, as well as which contracts to offer, conditional upon what (revealed) contract information. Individuals, on the other hand, respond to strategies of firms by choosing contracts to purchase as well as by deciding on which contract information to reveal to whom.

Expanding the equilibrium construct to include equilibrium information revelation strategies is complex, but in fact helps resolve some longstanding conundrums in information economics. There were two unsettling aspects of the Rothschild-Stiglitz analysis. The first was that equilibrium often did not exist. In their model with two types, equilibrium only existed if the two types were not too different in accident probabilities; but if there were a continuum of types, equilibrium never existed.[91]

The second unsettling aspect of the standard model was that if an equilibrium existed, it was never a pooling equilibrium. In real life there existed pooling contracts. This led to several attempts to formulate alternative

[89] Here, we are simply comparing the market equilibria that emerge with two different information structures. We have shown that the equilibrium in the "better" information structure (RS) does not Pareto dominate that in the worse information structure (SYK). But in the better information structure, there is a government policy (a "lump sum" tax on low priced insurance used to finance a lump sum subsidy on high priced insurance) which will lead to an equilibrium which Pareto dominates the SYK equilibrium.

[90] Arnott and Stiglitz (1991b) analyze the analogous problem in the context of moral hazard. The essential insight is that if non-disclosed insurance entails good monitoring, it can be welfare enhancing: public (disclosed) insurance free rides on the monitoring services of the non-disclosed insurance. Otherwise, it will be welfare-decreasing.

[91] The only possible equilibrium was a fully separating equilibrium; but there always existed a pooling contract that "broke" such an equilibrium, because the separating equilibrium separated types that were arbitrarily close to each other. That is, there was always a pooling contract that grouped all of the highest risk individuals together and would be preferred by all of the high risk individuals to the separating contract. See Stiglitz (2009).

equilibrium concepts, most famously by Wilson (1977) and Riley (1979). These were "reactive" equilibria, in which existing firms responded to the new firm by discontinuing to offer a policy that was no longer profitable once the new contract was offered. Rothschild and Stiglitz had deliberately formulated their analysis of *competitive* insurance markets as a Nash equilibrium, because they wanted to investigate what would happen if all firms were *very* small, sufficiently small that no firm by itself could upset an entire market equilibrium.

The equilibria analyzed here are not reactive equilibria, though market participants are allowed to condition offers upon observables. What is observable is, at least in part, endogenous. *Endogenizing* the information structure/revelation dramatically changes the standard results on the nature and existence of equilibrium. The earlier work of Akerlof and Rothschild-Stiglitz had, of course, shown the importance of the information structure: information about insurance purchased conveyed important information about the individual's type, and therefore, whether that information was available was central in determining the nature of the equilibrium. The RS equilibrium could not be sustained without sufficient information about insurance purchases to enforce exclusivity. And the Akerlof price-equilibrium could not be sustained in a world without full anonymity. Typically, firms at least know about the amount of insurance that they sell to any particular individual. Allowing undisclosed contracts and incorporating realistic assumptions about things that insurance firms know, that they know the identities of their customers and the quantities purchased destroys both the Rothschild-Stiglitz and the Akerlof equilibria.

When we endogenize information revelation, the unique equilibrium allocation is a partially disclosed pooling contract - the pooling contract most preferred by the low risk individual[92] - *plus* undisclosed supplemental insurance for the high risk individuals and no supplemental insurance for the low risk individuals.

There were two difficult parts of the analysis: determining the information structure - what to reveal to whom - that sustains equilibrium; and establishing the sustainability of the pooling contract. Here, the trick was

[92] That is, the pooling allocation *at the population weighted accident probabilities* most preferred by low-risk individuals. (This pooling contract is that upon which Wilson [1977] focused.)

to find information and contract strategies that ensured that a contract that attempted to cream skim the low risk individuals would fail - any such policy would be purchased by the high risk as well, as high risk individuals supplemented and/or changed in other ways what they would otherwise have purchased. We showed that one could always find such strategies.

The endogenously determined equilibrium informational structure[93] entails full non-disclosure of information on some of the contracts that are traded in equilibrium, with *selective* disclosure by firms to each other of pooling contract purchases (depending on whether a firm has been disclosed to be a seller to a particular individual). In equilibrium, consumers, on the other hand, truthfully reveal their insurance purchases to all firms. This information revelation by consumers was shown to be crucial in ensuring the existence of equilibrium.

We show that this information strategy can sustain the equilibrium against deviant policies - that is, no firm can profitably offer an alternative policy or pursue an alternative information strategy.

The insurance model has proven a useful tool for analyzing more generally markets with asymmetric information, and the papers analyzing imperfect and asymmetric information in that context have spawned a huge literature, applying the concepts to a rich variety of institutional structures[94]. The natural information assumptions, both concerning potentially hidden actions and potentially hidden characteristics, differ across markets. This paper has raised questions about both the Akerlof and RS analyses, in both of which information structures are assumed fixed (though they differ between them). It has shown the lack of generality of both the results concerning existence and the characterization of equilibrium. Accordingly, it raises questions about the results in the large literature based on them. In some ways, the results presented here are more consonant with

93 We showed that the equilibrium allocation can be supported by other information strategies as well.

94 These papers, as we have noted, include not only the Akerlof and RS models of adverse selection upon which we have focused, but also the models of moral hazard (including the early and canonical models of Arrow [1963], Arnott-Stiglitz [1988], and Pauly [1974]). It is important to recognize that, for the most part, these models were not intended to provide a good institutional analysis of the insurance market; rather, the insurance market provided the paradigm for studying behavior in, for example, labor, product, and capital markets because it seemed so simple to strip away these institutional details, and study markets unencumbered by them. It was for this reason that these paradigmic models proved so fruitful. The analysis of this paper should be taken in the same spirit.

what is observed than either those of Akerlof or RS. While RS showed that equilibrium might not exist - and an easy extension of RS shows that with a continuum of types, equilibrium never exists - we do see markets seemingly working[95]; and while RS showed that pooling contracts never exist, we do see pooling contracts.[96] In some ways, then, the equilibrium that arises with endogenous information looks much more like observed equilibria: it always exists, and always entails some degree of pooling.

We hope that this paper will, like the earlier RS and Akerlof analyses, spawn further research in the context of other markets in the analysis of market equilibrium with asymmetric information where contracts and the information structure/revelation are endogenously and simultaneously determined.

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[95] Though insurance markets often do not work well, and there may be other reasons that we do not see as much entry and exit (or the Dasgupta-Maskin (1986) mixed strategies) as the RS non-existence of equilibrium would suggest, e.g. imperfections of competition and costly search.

[96] There are, of course, other reasons that pooling equilibria may exist, related to problems of non-commitment and/or moral hazard. See, e.g. Roberts (1984), Stiglitz (2013b), Gale and Stiglitz (1989) and Stiglitz and Yun (2013).

Appendix A: Equilibrium in Extended Strategy Space

If we assume firms make use of price information (the terms at which insurance has been purchased), there is another way of supporting the equilibrium allocation. Let $\widehat{D}^i$ be the set of identities of firms that is revealed to sell an individual *i* some insurance at a price q lower than $\frac{\bar{P}}{1-\bar{P}}$. Consider then a strategy S^E ($\equiv \{S1^E, S2^E\}$), where $S2^E = S2^*$ and $S1^E$ is defined as follows.

$$S1^E = \{C1^E(\alpha^i_{-j}; \widetilde{D}^i), I^E(\widehat{D}^i)\}, \tag{12}$$

where $C1^E\left(\alpha^i_{-j}; \widetilde{D}^i\right) = C1^*(\alpha^i_{-j}; \widetilde{D}^i)$ (as defined in (11)) and $I^E(\widehat{D}^i)$ is an N-dimensional vector with all the k-th elements (k $\in \widehat{D}^i$) being equal to 0 and with all the others being equal to 1. In other words, the strategy S^E is the same as S^* except for the information strategy $I^E(\widehat{D}^i)$ which is to disclose to all but the revealed insurer(s) who sell an individual *i* a policy at a price lower than $\frac{\bar{P}}{1-\bar{P}}$. This information strategy is a *latent one*, unlike the information strategy $I^*(D^i)$ used in the main text, as it is implemented not in equilibrium but in out-of-equilibrium.[97] The non-disclosure policy against the deviant firm(s) offering insurance at a price lower than $\frac{\bar{P}}{1-\bar{P}}$, together with the contract strategy $C1^E(\alpha^i_{-j}; \widetilde{D}^i)$, provides us with the necessary (limited) non-exclusivity. We can establish the following Theorem.

Theorem 3

Under Assumption A, the set S^E of strategies described above constitutes an equilibrium, and supports the equilibrium allocation E^*.

The proof of Theorem 3 is similar to that of Theorem 2. First of all, given the strategy S^E adopted by all the firms, Proposition 5 holds as all the firms have complete information about the purchases of pooling insurance by all consumers, implying that the equilibrium allocation E^* will be realized. Next, as for the robustness of an equilibrium in response to any deviancy, the same argument used in the proof of Theorem 2 can be applied here, except for the two following. First,

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[97] Latent policies have been employed elsewhere in the literature in insurance with asymmetric information, e.g. Arnott and Stiglitz (1987, 1991a), and widely employed particularly in the literature on adverse selection with non-exclusivity, such as Jaynes (1978, 2011), Ales-Maziero (2012) and Attar-Mariotti-Salanie (2011, 2016). The term seems to have been first used by Arnott and Stiglitz. Here, we employ a latent *information* strategy, as the contemplated disclosures do not occur in equilibrium, since in equilibrium, no firm offers a policy at a price lower than $\frac{\bar{P}}{1-\bar{P}}$. In Appendix B, we support the equilibrium through the use of latent policies.

any cream-skimming deviant firm k (with any information strategy) who charges a price of the pooling insurance below $\frac{\bar{P}}{1-\bar{P}}$ cannot implement the necessary exclusivity because no consumer i would reveal to k his purchases from other firms while no non-deviant firm would reveal its sales to i to the deviant firm k. Thus, high-risk individuals will purchase the deviant contract from k, bringing losses to k. Second, as a deviant information strategy, any firm with an information strategy of not revealing to all but the firm k in $\widehat{D}^i$ will attract all the high-risk individuals to make losses, while any firm with an information strategy of revealing to the firm k in $\widehat{D}^i$ will not be able to deter the deviant firm k.

Appendix B: Deviants using Multiple Contracts

In this appendix, we consider the possibility of a deviant firm offering a pair of contracts. It does so to induce self-selection among the applicants - with the self-selection process reducing the costs of the high risk individuals buying insurance from the deviant. We first explain why the set of strategies considered earlier now doesn't "work"; we then describe intuitively the challenges involved in finding an equilibrium strategy. Next we provide the formal analysis, establishing the main theorem of this appendix. Finally, we note in particular the critical role played in the analysis here of *latent policies*, policies that are not taken up in equilibrium but would be taken up out of equilibrium, and which deter equilibrium-destroying deviations.

Proposition 6.

The set S^* of strategies defined by (10) and (11) does not constitute an equilibrium

Figure 10 gives a graphical illustration of why S^* cannot be sustained as a Nash equilibrium. {A, C} represents the equilibrium allocation described earlier. Consider, as earlier, S^* that leads to an allocation (A, C) where A and C are chosen by both types of individuals and by high-risk ones, respectively. Now consider the deviant pair of policies {AB, G}, where AB is offered without disclosure and G is offered with disclosure and with G being offered conditional on no additional insurance. There always exists a set (AB, G) such that G is chosen by all the low-risk individuals while AB is chosen by all the high-risk who simultaneously buy OA. (The high risk individuals supplement AB with pooling insurance OA). The deviant entrant firm offering (AB, G) may make positive profits because the positive profits from G can outweigh the losses from AB, since the deviant firm can share with the other firms any losses from $(\bar{\alpha}, \bar{\beta})$ (the pooling contract) that is chosen by high-risk individuals. (That is, now, only high risk individuals purchase the pooling contract, so it loses money. The deviant firm gets all the low risk individuals for all of their insurance, and the high risk people only for the supplemental amount AB).

Finding an equilibrium strategy

To prevent this type of a deviation, we need to make the choice of G more attractive to high-risk types by providing more additional insurance at $\frac{\bar{P}}{1-\bar{P}}$ than $S1^*$ does, while limiting the total provision by all the firms to $\bar{\alpha}$ *in equilibrium*. We need to have a *latent* strategy, which offers an individual sufficient amount of extra insurance at $\frac{\bar{P}}{1-\bar{P}}$ in the presence of a deviant contract G, that the high risk individual purchases G.

More formally, we will consider a strategy in the extended strategy space which is conditional upon price of insurance purchased, as well as its quantity. More specifically, let $\widehat{D}^i$ be the set of identities of firms that are revealed to sell an individual *i* some insurance at a price lower than $\frac{\bar{P}}{1-\bar{P}}$. We can then consider a strategy S^o ($\equiv \{S1^o, S2^o\}$), where $S2^o = S2^*$ and $S1^o$ is defined as follows.

$$S1^o = \{C1^o(\alpha^i_{-j}, q\,; \widetilde{D}^i), I^o(\widehat{D}^i)\},$$

where $I^0(\widehat{D}^i)$ is an N-dimensional vector with all the d-th elements (d $\in \widehat{D}^i$) being equal to 0 and with all the others being equal to 1, which is to disclose to all but the revealed insurer(s) who sell an individual *i* at a price q lower than $\frac{\bar{P}}{1-\bar{P}}$; and where the contract strategy $C1^o(\alpha^i_{-j}, q\,; \widetilde{D}^i)$ is defined as

$$C1^o(\alpha^i_{-j}, q; \widetilde{D}^i) = \mu(\alpha^i_{-j}, q)(1, \frac{\bar{P}}{1-\bar{P}}), \qquad \text{for} \quad \widetilde{D}^i = \emptyset \qquad (13)$$
$$= 0 \qquad \text{for} \quad \widetilde{D}^i \neq \emptyset$$

where

$$\mu(\alpha^i_{-j}, q\,; \widetilde{D}^i) = \ Max\{\ \bar{\alpha} - \alpha^i_{-j}, 0\} \qquad \text{for} \ \ q \geq \frac{\bar{P}}{1-\bar{P}}$$

$$= [0, \alpha'] \qquad \text{for} \quad q < \frac{\bar{P}}{1-\bar{P}}\ .$$

where $\alpha' \ \geq \bar{\alpha}$. In other words, $C1^o(\alpha^i_{-j}, q; \widetilde{D}^i\,)$ is the same as $C1^*(\alpha^i_{-j}; \widetilde{D}^i)$ as defined by (11) when the price of insurance set by a deviant contract (or other contracts more generally) is not lower than $\frac{\bar{P}}{1-\bar{P}}$, while offering up to α'

otherwise. Thus, S^o contains a latent strategy, which entails the contract strategy $C1^o(\alpha_{-j}^i, q)$ for $q < \frac{\bar{P}}{1-\bar{P}}$ and the information strategy $I^o(\widehat{D}^i)$ of not disclosing to the deviant firms charging a price lower than $\frac{\bar{P}}{1-\bar{P}}$. We can then see that S^o can determine the equilibrium allocation E^* as Proposition 5 can apply given S^o. As for the robustness of an equilibrium in response to a deviant firm k with {AB, G}, we can first note that a high-risk individual i choosing G would not reveal to k his purchases of pooling insurance from other firms, who also do not reveal to the deviant firm k their sales to i. We can then show that the deviant firm k cannot make non-negative profits as follows. The maximum profit that a deviant firm k can get with {AB,G} will be made when $G \approx A \approx B$, and the amount of maximum profit will not be greater than $\bar{\alpha} \cdot \left(\frac{\bar{P}}{1-\bar{P}} - \frac{P_L}{1-P_L}\right) \cdot (1-\theta)$, where θ is the portion of high-risk individuals. If a high-risk individual would like to purchase supplemental insurance (from $S2^o$) after purchasing G from k and $\bar{\alpha}$ pooling insurance from a non-deviant firm, the contract that induces high-risk types not to choose G with the minimum loss will be B' in Figure 10. The loss that the deviant firm makes by offering B' for high-risk individuals will be greater than $\bar{\alpha} \cdot \left(\frac{P_H}{1-P_H} - \frac{\bar{P}}{1-\bar{P}}\right) \cdot \theta$, implying that the total profit for the deviant firm should not be positive.

We have thus established

Theorem 4

If deviant firms are allowed to offer multiple insurance contracts, there always exists a Nash equilibrium that sustains the unique equilibrium outcome $E^* \equiv \{(\alpha_H^*, \beta_H^*), (\alpha_L^*, \beta_L^*)\}$. The Nash equilibrium entails the use of latent policies.

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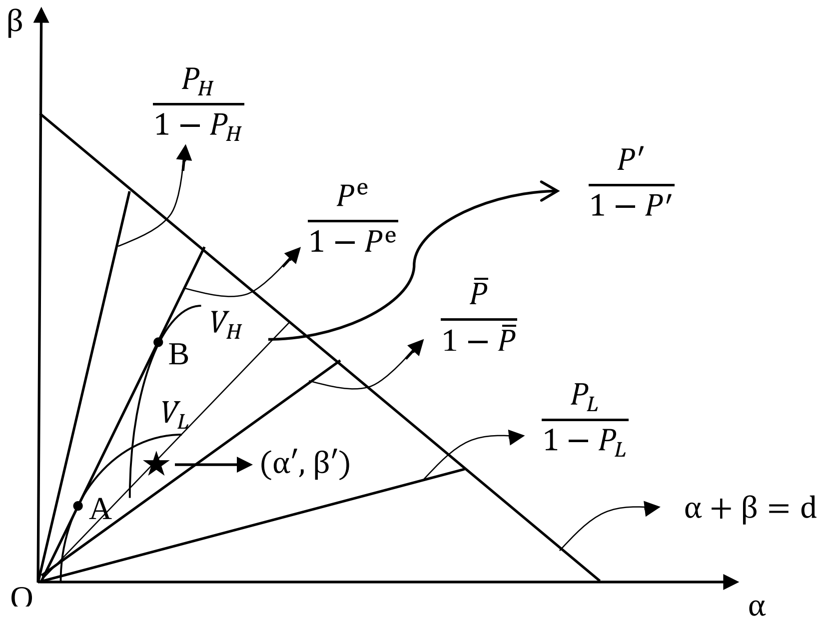
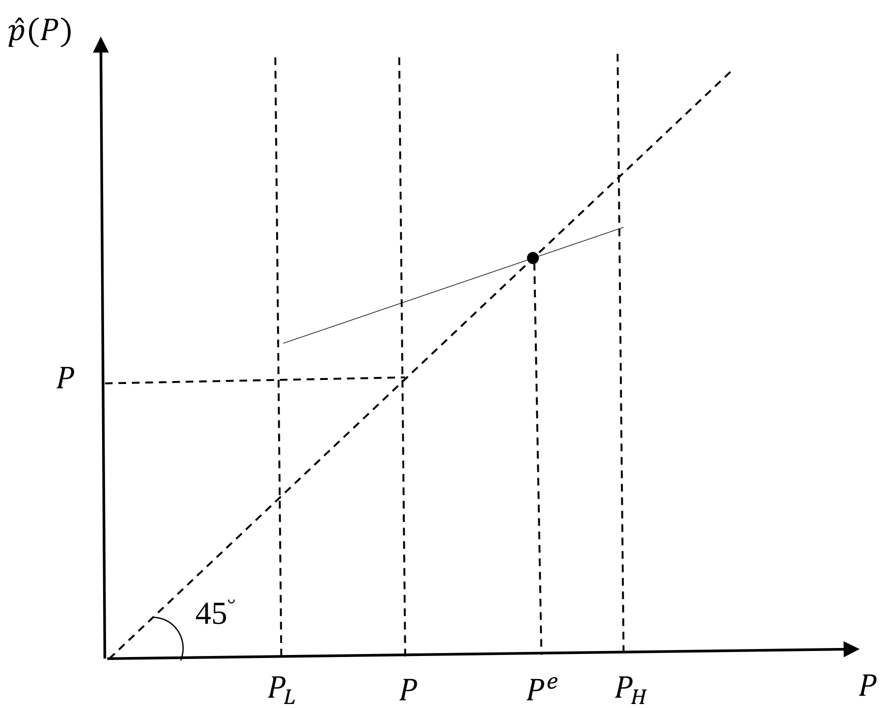
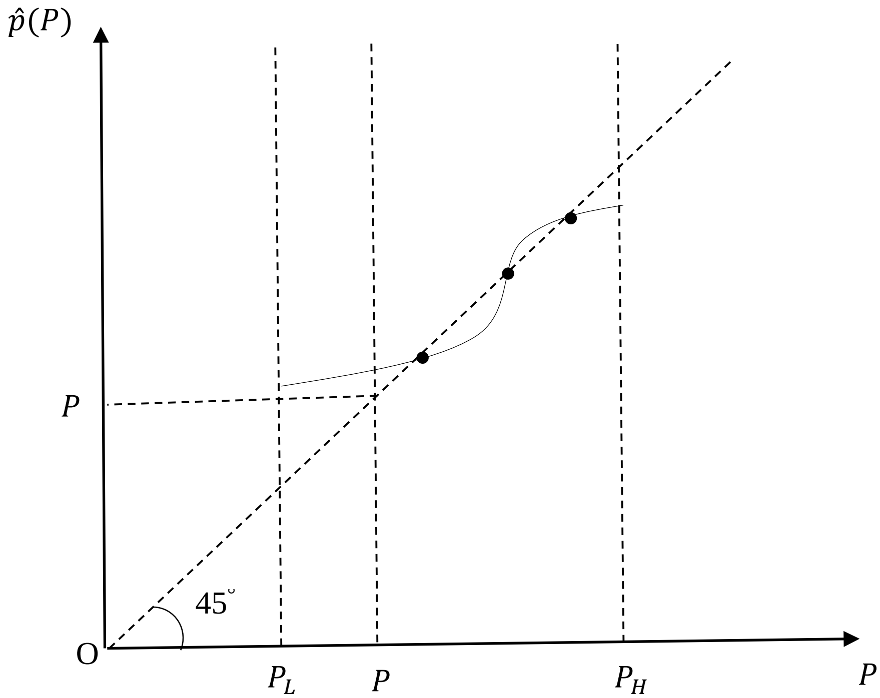
Figures



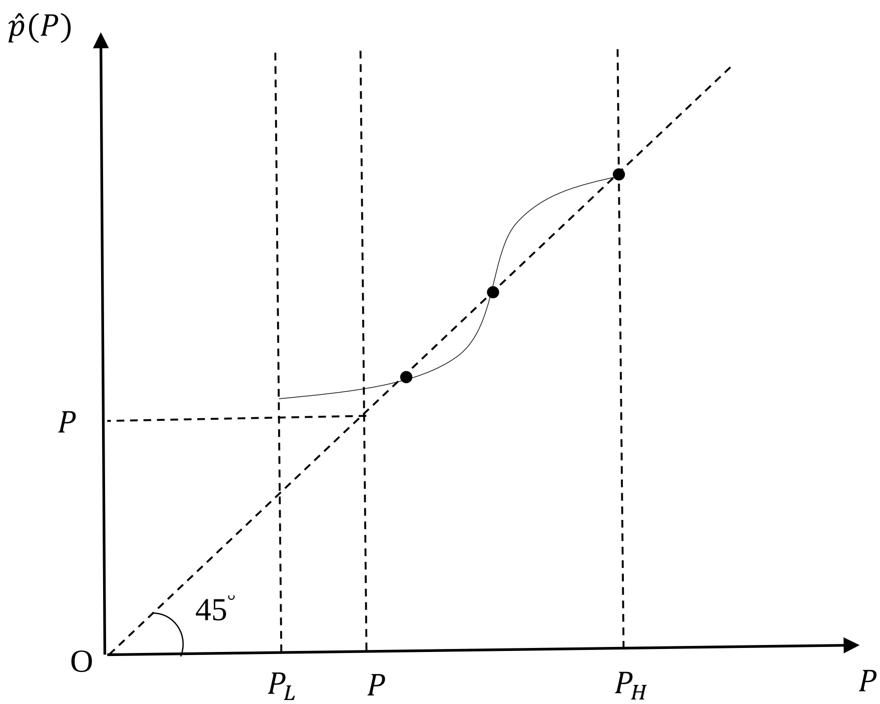
< Figure 1a > Basic Adverse Selection Model



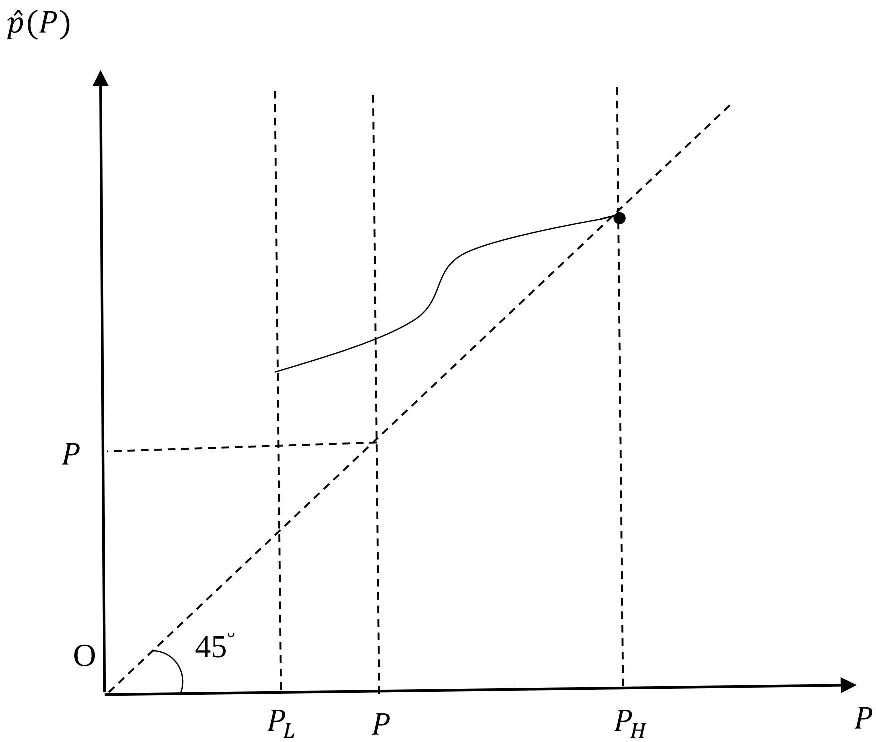
< Figure 1b > One interior equilibrium



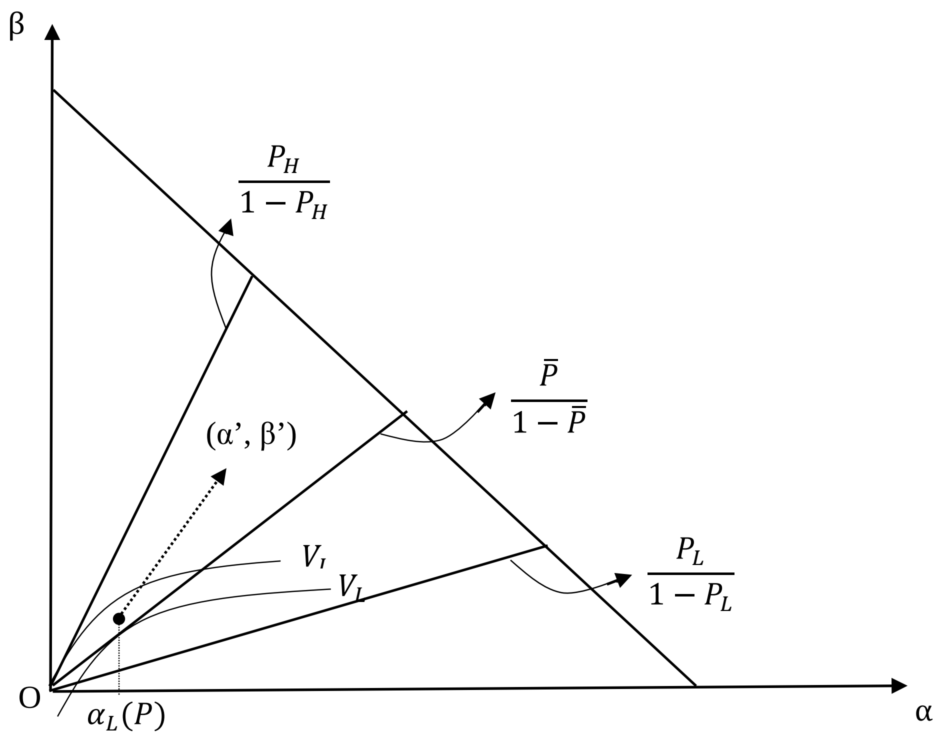
< Figure 1c > Several (here, three) interior equilibria



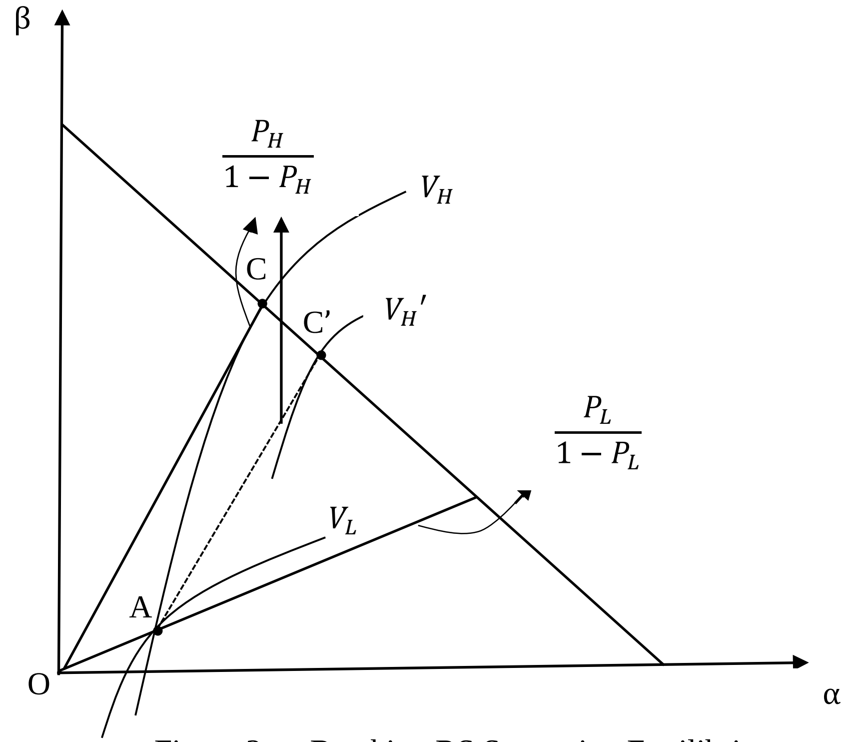
< Figure 1d > Several equilibria; both corner and interior solutions



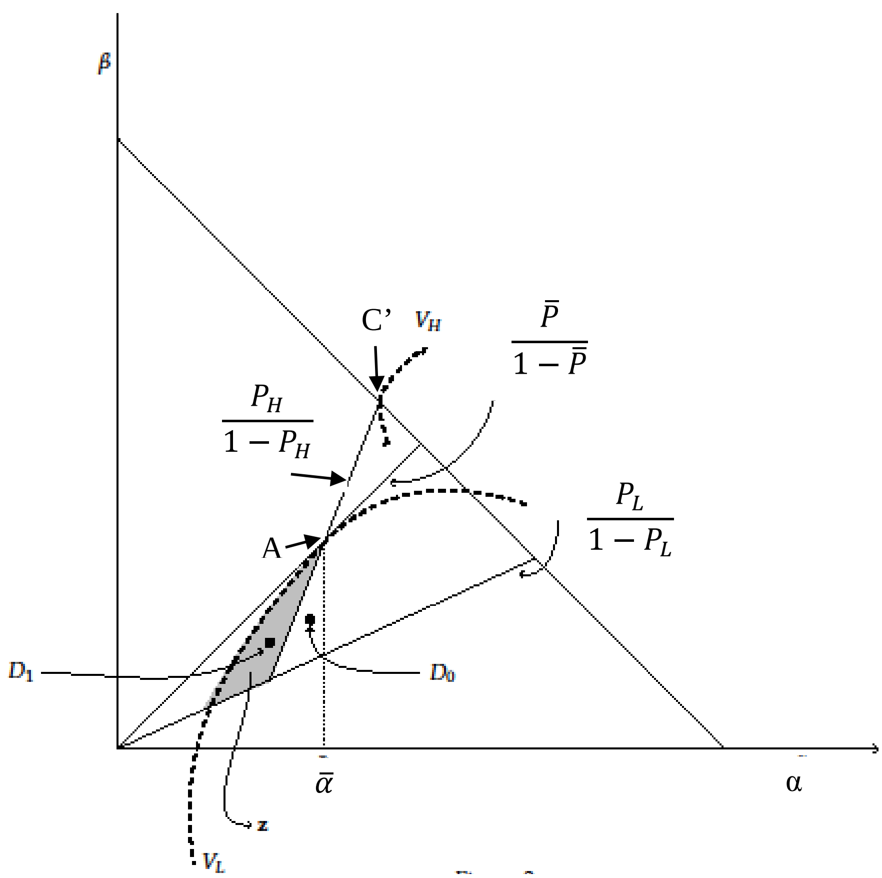
< Figure 1e > One equilibrium; corner solution



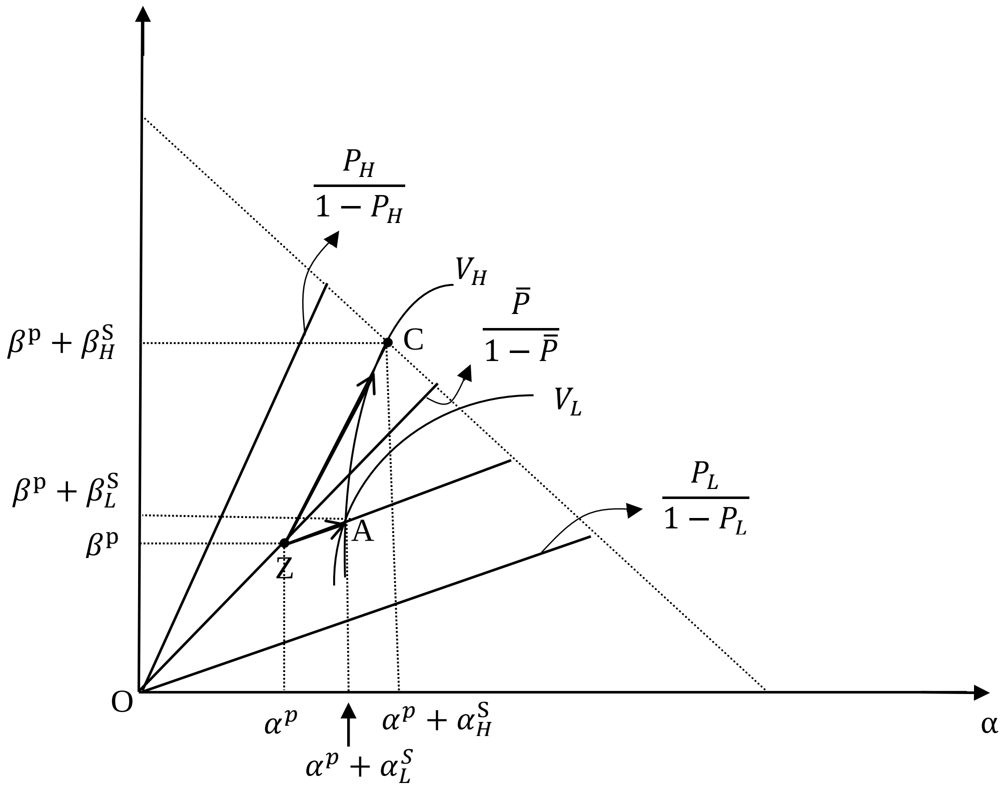
< Figure 1f > Breaking Akerlof Equilibrium



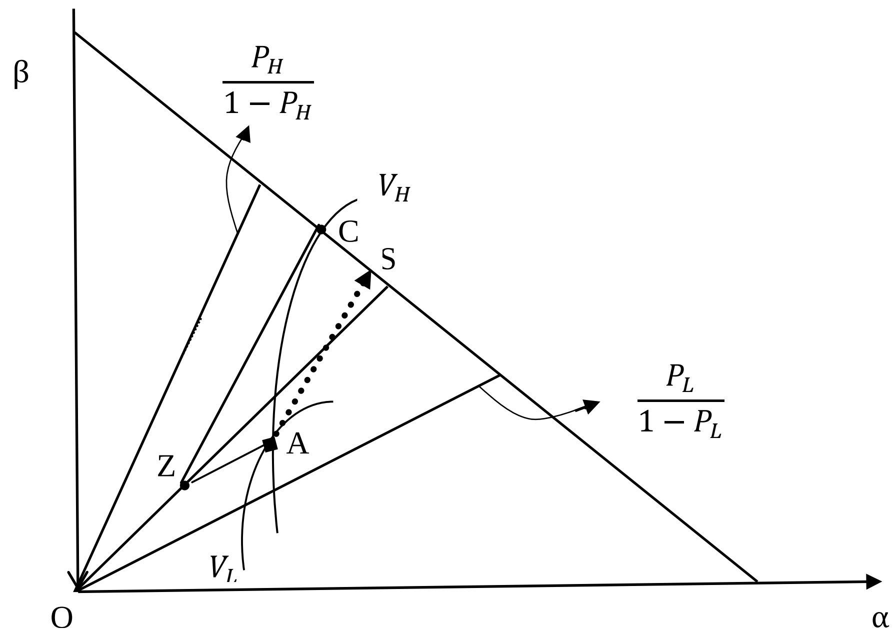
< Figure 2a > Breaking RS Separating Equilibrium



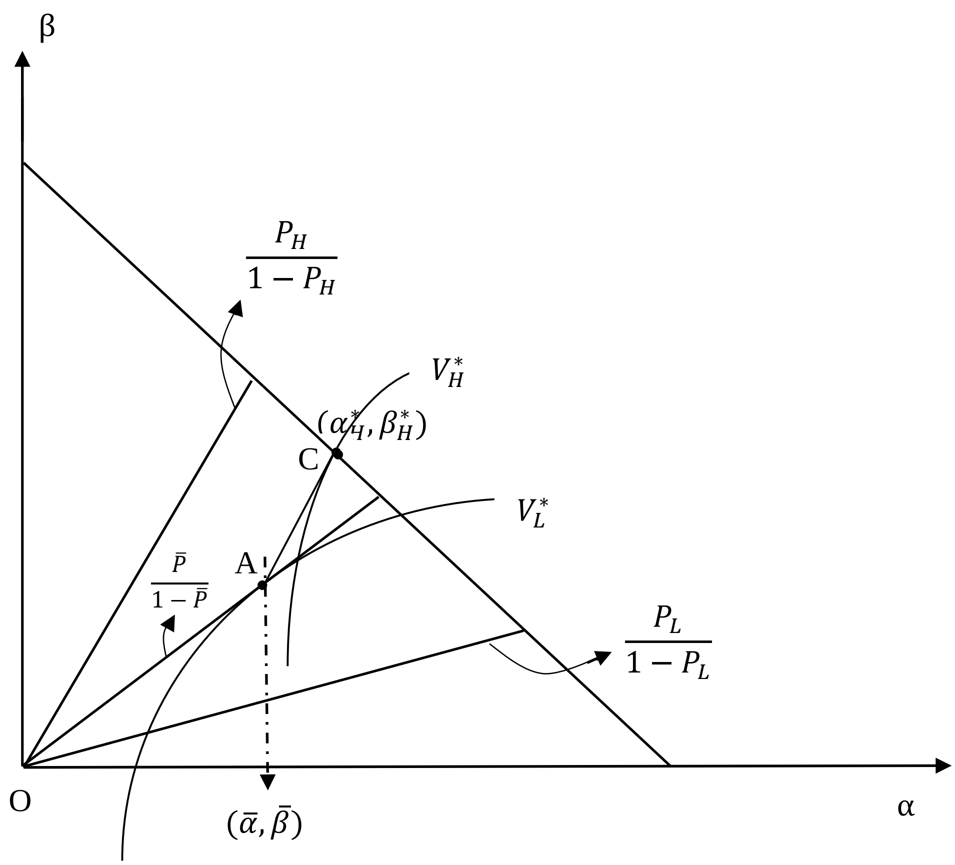
<Figure 2b> Breaking RS pooling equilibrium



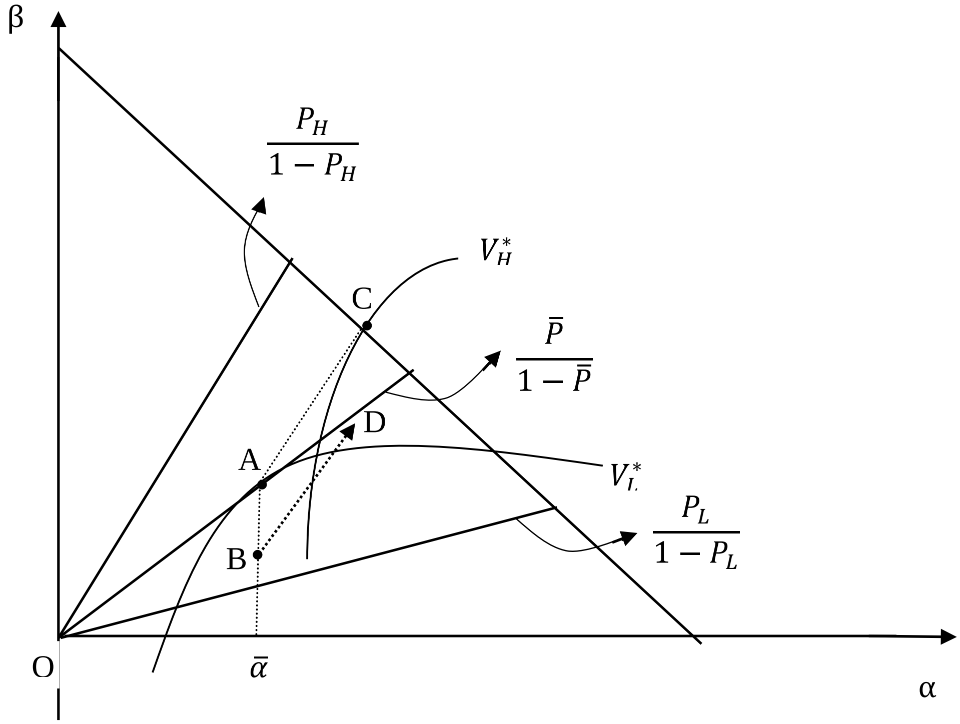
< Figure 3 > Decomposing any pair of contracts into a pooling contract and supplemental contracts



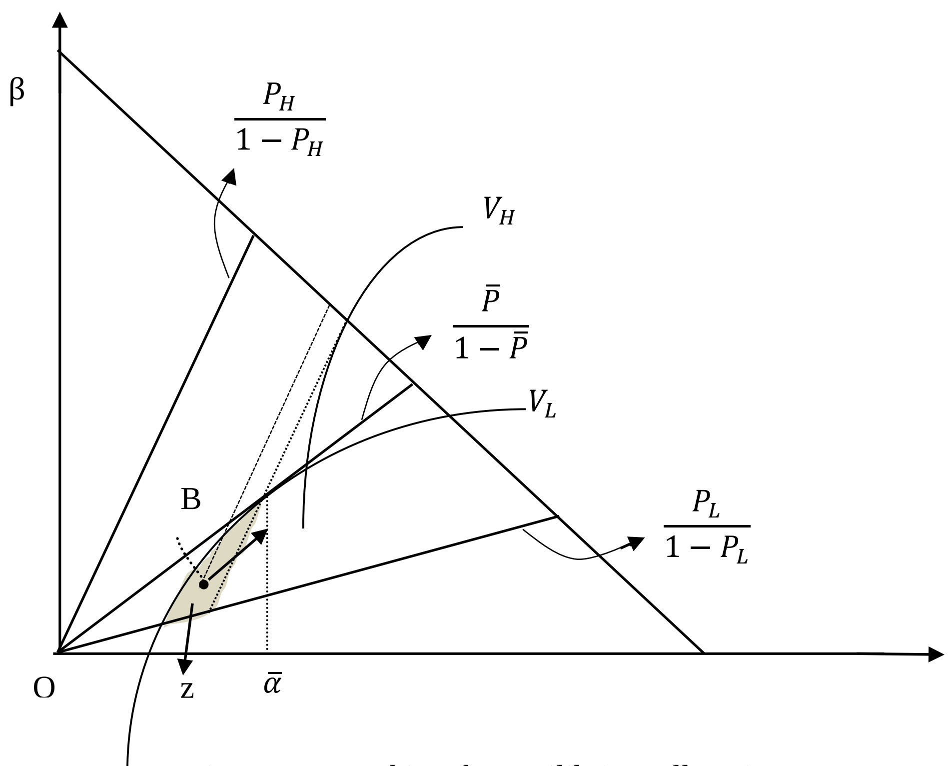
< Figure 4 > Equilibrium cannot entail supplemental insurance for the low risk type and must involve full insurance for the high risk type



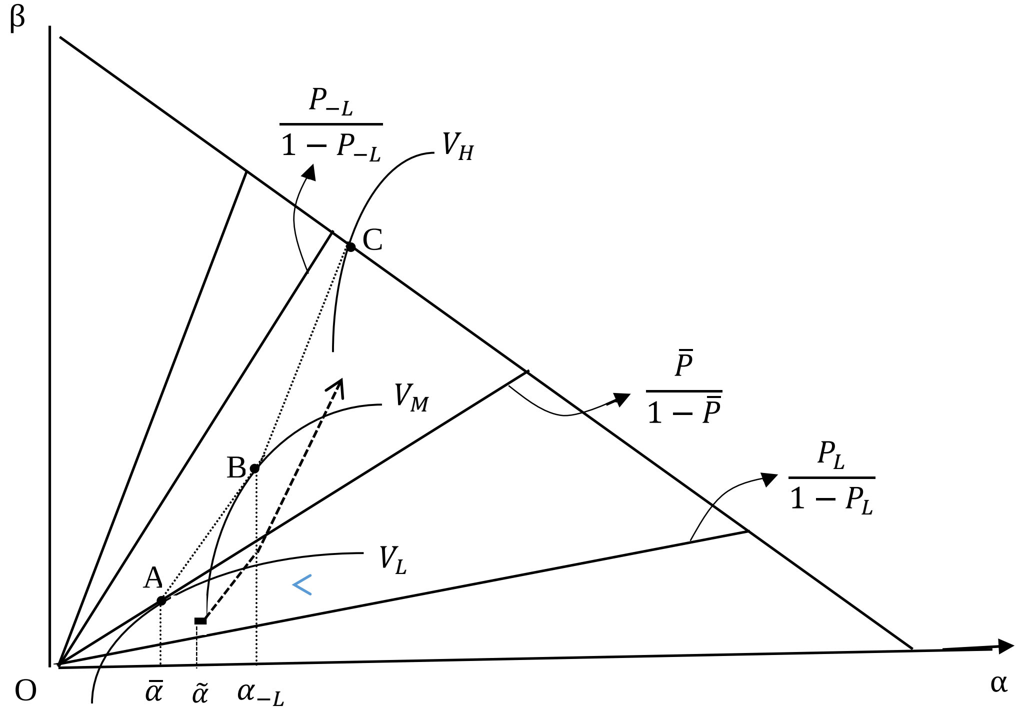
< Figure 5 > Equilibrium allocation



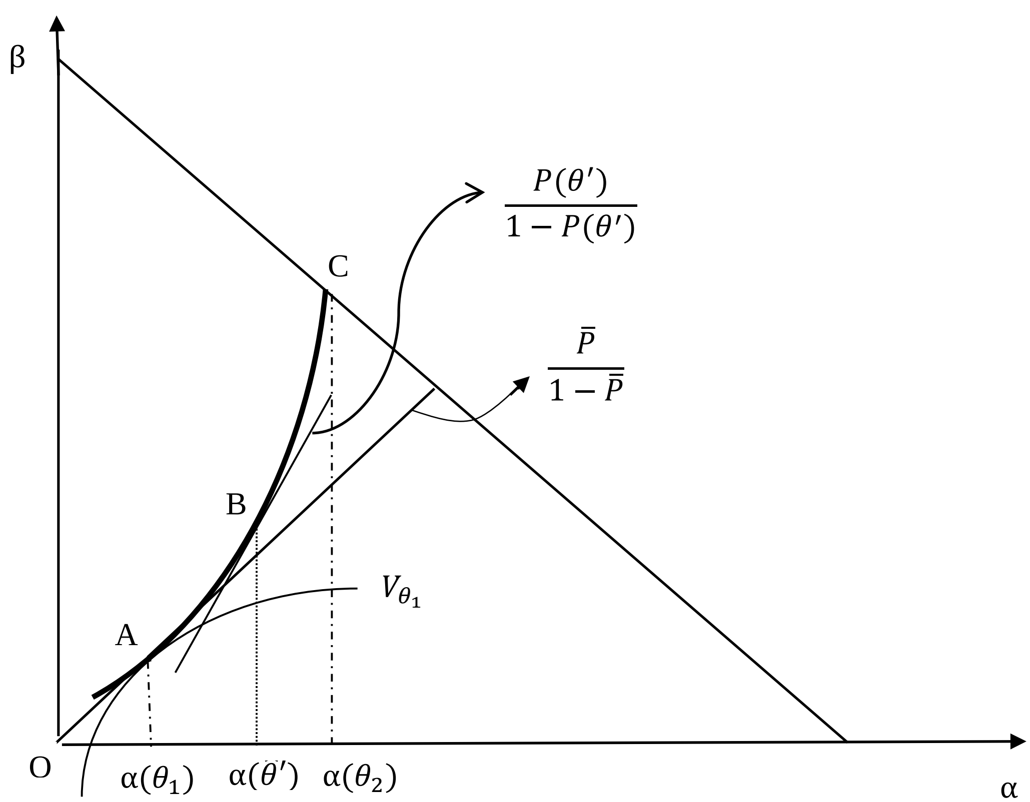
< Figure 6 > Breaking the equilibrium allocation



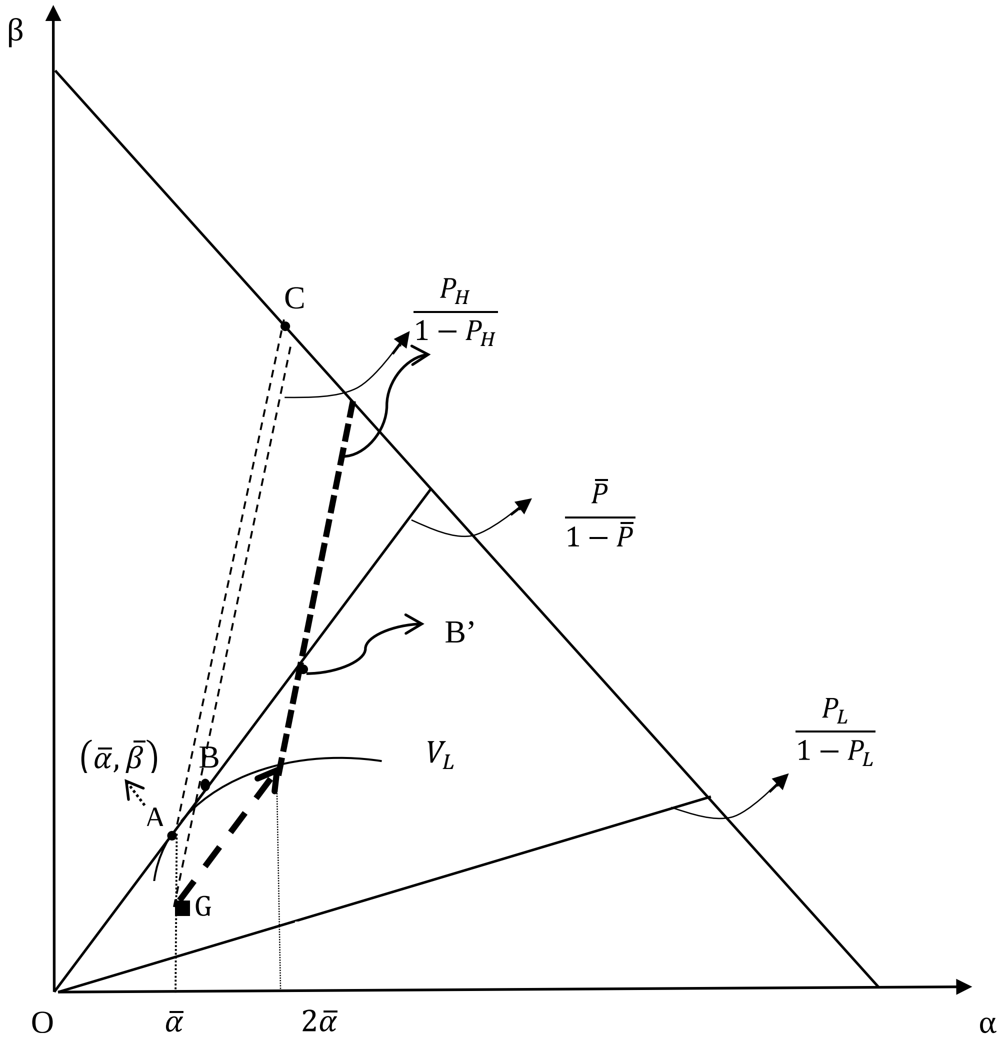
< Figure 7 > Breaking the equilibrium allocation



< Figure 8 > Equilibrium with three types



<Figure 9> Equilibrium with continuum types



<Figure 10 > Nash Equilibrium with both multiple and latent contracts