Problem 1

For each of the following one-dimensional ODEs, do the following: 1) Find the fixed points, 2) Using the theorem from class about $\frac{df}{dx}|_{x^*}$, classify the stability of each of the fixed points if possible, and 3) for any fixed point you couldn’t classify using $\frac{df}{dx}|_{x^*}$, draw the phase line and try to classify stability that way.

(a) $x' = (x - 2)(x + 3)$

(b) $x' = \frac{x(x-1)}{1+x^2}$

(c) $x' = x^2(x - 2)$

(d) $x' = \sin x - x$

Problem 2

Assume $x' = Ax$. For each of the following matrices $A$, do the following: 1) classify the type of fixed point that $x = 0$ is, just using the trace and determinant to find the eigenvalues; and 2) determine whether $x = 0$ is stable. You don’t need to find eigenvectors or plot anything.

(a) $A = \begin{pmatrix} -2 & -3 \\ 5 & 1 \end{pmatrix}$

(b) $A = \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix}$
Problem 3

Consider the nonlinear system:

\[
x' = y + x(3/4 - y^2) \\
y' = 1 - x
\]

(a) Find the fixed points.

(b) Write the Jacobian matrix

\[
J(x, y) = \begin{pmatrix}
\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\
\frac{\partial g}{\partial x} & \frac{\partial g}{\partial y}
\end{pmatrix}
\]

(c) Using the eigenvalues of the Jacobian evaluated at each fixed point, determine whether each of the fixed points you find is stable, unstable, or undetermined given the eigenvalues.

Problem 4

Consider the nonlinear system:

\[
x' = (x - 1)y \\
y' = x + x^2 - 2y^2
\]

(a) Find the fixed points (hint: there are four).

(b) Write the Jacobian matrix

\[
J(x, y) = \begin{pmatrix}
\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\
\frac{\partial g}{\partial x} & \frac{\partial g}{\partial y}
\end{pmatrix}
\]

(c) Using the eigenvalues of the Jacobian evaluated at each fixed point, determine whether each of the fixed points you find is stable, unstable, or undetermined given the eigenvalues.