# 4150 Solutions to Homework Assignment 9

(1) $X(1), X(2), \ldots, X(N)$, $Y = X(1)X(2)\cdots X(N)$

$$Y = \operatorname{Log}(X_1 X_2 \cdots X_N)$$
$$= \log(X_1) + \log(X_2) + \cdots + \log(X_N)$$

N.B. $\log(X_1), \log(X_2), \ldots, \log(X_N)$ are independent and identically distributed

Let $Y_i = \log(X_i)$, $\underline{\mu = E(Y_i)}$, $\sigma = \underline{\text{standard deviation}(Y_i)}$

Then $\left[\text{or } \dfrac{\bar{Y} - \mu}{\sigma/\sqrt{N}} \to n(y;0,1)\right]$

$$\frac{\log Y - N\mu}{\sigma\sqrt{N}} = \frac{Y_1 + \cdots + Y_N - N\mu}{\sigma\sqrt{N}} \longrightarrow n(y;0,1)$$

by the central limit theorem. Thus $Y$ is (suitably normalized) asymptotically $n(y;0,1)$

(2) $N = 50$, $\bar{X} = 78.3$, $S = 5.6$

95% prediction interval:

$$\bar{X} \pm t_{.025;49\,d.f.} \frac{S}{\sqrt{1+1/N}} \approx \bar{X} \pm z \frac{S}{\sqrt{1+1/50}}$$

$$= 78.3 \pm 1.96 \frac{5.6}{\sqrt{51/50}} \approx 78.3 \pm 10.868 \approx \boxed{(67.43, 89.17)}$$

lower 95% tolerance limits: use $N = 50$, $\bar{X} = 78.3$, $S = 5.6$, $\gamma = .05$, $\alpha = .01$ $\boxed{\cancel{\text{ONE}}\ \text{TWO-SIDED}}$

From Table A.7, $K = 3.126$.

Thus we have $\bar{X} \pm Ks = 78.3 \pm 3.126(5.6)$

$$= 78.3 \pm 17.51 = \boxed{(60.79, 95.81)}$$

$\boxed{\text{ONE-SIDED}}$: From Table A.7, $K = 2.863$

$\bar{X} - Ks \qquad 78.3 - 2.863(5.6)$

$$= \boxed{62.2672}$$

(3) $X_1$: 103 94 110 87 98

$X_2$: 97 82 123 92 175 88 118

SAMPLE 1: $N=5$, $\Sigma X_i = 492$, $\boxed{\bar{X} = \frac{492}{5} = 98.4}$

$\Sigma X_i^2 = 48718$, $S^2 = \frac{N(\Sigma X_i^2) - (\Sigma X_i)^2}{N(N-1)}$

$= \frac{5(48718) - (492)^2}{5(4)} = \frac{1526}{20} = \boxed{76.3}$

SAMPLE 2: $N=7$, $\Sigma X_i = 775$, $\bar{X} = \frac{775}{7} = \boxed{110.7142857}$

$\Sigma X_i^2 = 92{,}019$, $S^2 = \frac{7(92019) - (775)^2}{7(6)} = \frac{43{,}508}{42}$

$= \boxed{1035.904762}$

$$\nu = \frac{(S_1^2/N_1 + S_2^2/N_2)^2}{[(S_1^2/N_1)^2/(N_1-1)] + [(S_2^2/N_2)^2/(N_2-1)]}$$

$$\frac{26{,}649.38534}{[58.2169] + [3649.995496]} = \frac{26{,}649.38534}{3708.212396} = 7.186585486$$

use $\nu = 7$

$$(\bar{X}_1 - \bar{X}_2) \pm t_{7,.05}\sqrt{\frac{S_1^2}{N_1} + \frac{S_2^2}{N_2}} = -12.3142857 \pm (1.895)(12.77619125)$$

$= -12.3142857 \pm 24.21201941$

$\boxed{(-36.526, -11.898)}$

4. (a) $(20)(.337) = 6.74$ 1990

$(20)(.362) = 7.24$ 1994

(b) 1990: $(\hat{p}_1 - \hat{p}_2) \pm 1.96\sqrt{\frac{(\hat{p}_1)(1-\hat{p}_1)}{20} + \frac{(\hat{p}_2)(1-\hat{p}_2)}{20}}$

$= (.337 - .362) \pm 1.96\sqrt{.01117155 + .01154478}$

$= -.025 \pm .29542961$

$= \boxed{(-.320, +.270)}$

not significantly higher because 0 is in the 95% confidence interval

5. $f(x) = \begin{cases} \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$ $\alpha, \beta > 0$.

$L(x_1, \ldots, x_N; \alpha, \beta) = \prod_{i=1}^{N} f(x_i) = \begin{cases} (\alpha\beta)^N (x_1 x_2 \ldots x_N)^{\beta-1} e^{-\alpha \sum x_i^\beta} & (\text{all } x_i > 0) \\ 0, & \text{elsewhere} \end{cases}$

(b) Take logarithms; (for all $x_i > 0$)

$N[\ln(\alpha) + \ln(\beta)] + (\beta - 1)[\ln x_1 + \ldots + \ln x_N] - \alpha \sum x_i^\beta$

$x_i^\beta = e^{\beta \ln x_i}$

differentiate w.r.t. $\alpha$:

$\boxed{\frac{N}{\alpha} - \sum x_i^\beta = 0}$

differentiate w.r.t. $\beta$:

$\boxed{\frac{N}{\beta} + \sum_{i=1}^{N} \ln x_i - \alpha \sum_{i=1}^{N} (\ln x_i) x_i^\beta = 0}$

# 4150 Solution to Homework Assignment 10

1. $N=25$

$H_0: p=0.10$

$H_1: p>0.10$

$Y>5$ is the rejection region

$Y=$ # of defective meters in the sample

| | Probability | | |
|---|---|---|---|
| | 0.10 | 0.20 | 0.40 |
| $P(Y\le 0)$ | 0.0718 | 0.0038 | 0.0000 |
| $P(Y\le 1)$ | 0.2712 | 0.0274 | 0.0001 |
| $P(Y\le 2)$ | 0.5371 | 0.0982 | 0.0004 |
| $P(Y\le 3)$ | 0.7636 | 0.2340 | 0.0024 |
| $P(Y\le 4)$ | 0.9020 | 0.4207 | 0.0095 |
| $P(Y\le 5)$ | 0.9667 | 0.6167 | 0.0294 |

(a) $\alpha = 1-.9667 = 0.0333$

(b) $\beta = 0.6167$ Power $= 1-0.6167 = 0.3833$

(c) $\beta = 0.0294$ Power $= 1-.0294 = 0.9706$.

2.

(a) $H_0: \mu = 2500$ $\quad \alpha = 0.10$

$H_1: \mu > 2500$

$N = 6,\ \Sigma X_i = 15{,}490,\ \Sigma X_i^2 = 40{,}065{,}100.$

$$Z = \frac{\bar{X} - 2500}{S/\sqrt{N}} =$$

$$\bar{X} = \frac{15{,}490}{6} = 2581.6\overline{6}$$

$$S^2 = \frac{6(40{,}065{,}100) - (15490)^2}{6(5)} = \frac{450{,}500}{30} = 1501\overline{6.66}$$

$$t = \frac{81.66}{\sqrt{\frac{15016.\overline{66}}{6}}} = \boxed{1.632} \qquad \frac{S}{\sqrt{N}} = \sqrt{\frac{15016.66}{6}} = 50.02777007$$

Reject $H_0$ if $t > t_{5\,df,\,.10} = \boxed{1.476}$

$\boxed{\text{Reject } H_0}$. $\quad$ P-value $= \boxed{.0818}$

(b) $\text{Beta}(\mu = 2525)$

$$= P\left(\frac{\bar{X} - 2500}{S/\sqrt{N}} < 1.476 \,\middle|\, \mu = 2525\right)$$

$$= P\left(\bar{X} < 2500 + 1.476(S/\sqrt{N}) \,\middle|\, \mu = 2525\right)$$

$$= P\left(\frac{\bar{X} - 2525}{S/\sqrt{N}} < \frac{2500 - 2525 + 1.476(S/\sqrt{N})}{S/\sqrt{N}} \,\middle|\, \mu = 2525\right)$$

$$= P\left(t_5 < \frac{48.8498862}{50.02777007}\right) = P(t_5 < 0.976277546) = \boxed{.8131}$$

$\boxed{\text{Power} = .1869}$

(c) $\text{Beta}(\mu = 2600)$

$$= P\left(t_5 < \frac{-26.15901138}{50.02777007}\right) = P(t_5 < -.5228849813) = \boxed{0.3117}$$

$\boxed{\text{Power} = 0.6883}$

(d) $\text{Beta}(\mu = 3000) = P\left(t_5 < \frac{-426.159014}{50.02777007}\right) = P(t_5 < -8.51844069) \approx \boxed{0.000}$

$\boxed{\text{Power} \approx 1.0000}$

8th Edition

$\bar{x}_1 = 70,750 \quad \bar{X}_2 = 65,200$

$s_1 = 6000 \quad S_2 = 5000$

(3)

$$Z = \frac{70,750 - 65,200}{\sqrt{\frac{(6000)^2}{200} + \frac{(5000)^2}{200}}} = \frac{5500}{\sqrt{305,000}} = 9.96 \quad \boxed{\text{Reject } H_0}$$

7th edition

3. $N_1 = 200$, $\bar{X}_1 = 51,750$, $S_1 = 5000$

$N_2 = 200$, $\bar{X}_1 = 47,500$, $S_2 = 5000$ $\quad \alpha = 0.01$

$H_0: \mu_1 - \mu_2 \le 0$

$H_1: \mu_1 - \mu_2 > 0$

It is obvious that we may use the pooled-t test (why?)

Thus the test statistic is

$$t \text{ (or } Z) = \frac{\bar{X}_1 - \bar{X}_2 - 2000}{\sqrt{S_p^2\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}} = \frac{2250}{\sqrt{\frac{(5000)^2}{100}}} = \boxed{4.5}$$

Reject if $t > t_{398, 0.01} \approx Z_{0.01} = \boxed{2.326}$

$\boxed{\text{Reject } H_0}$ Associate Professors at research institutions make > 2000 more than Associate professors at other institutions.

4.

| Subject | with Co | without Co | $d_i$ | $d_i^2$ |
|---|---|---|---|---|
| 1 | 30 | 30 | 0 | 0 |
| 2 | 45 | 40 | -5 | 25 |
| 3 | 26 | 25 | -1 | 1 |
| 4 | 25 | 23 | -2 | 4 |
| 5 | 34 | 30 | -4 | 16 |
| 6 | 51 | 49 | -2 | 4 |
| 7 | 46 | 41 | -5 | 25 |
| 8 | 32 | 35 | 3 | 9 |
| 9 | 30 | 28 | -2 | 4 |
| Totals | 319 | 301 | -18 | 88 |

$\bar{d} = \frac{\Sigma d_i}{N} = \frac{-18}{9} = -2 \qquad S_d^2 = \frac{9(88) - (-18)^2}{9(8)} = \frac{468}{72} = 6.5$

$$t = \frac{\bar{d} - 0}{\sqrt{S_d^2/N}} = \frac{-2}{\sqrt{\frac{(6.5)}{9}}} = -2.35$$

Reject $H_0$ if $t < -t_{8\,d.f., 0.05} = -1.860$ $\quad$ p-value $= \boxed{0.0233}$

Reject $H_0$

The mean breathing frequency is greater with CO.

5) We estimate $\sigma$ to be 1.25.

$\alpha = 0.05$, $\sigma = 1.25$, $\beta = 0.10$, $\delta = 0.5$

Two-Sided

$$\Delta = \frac{|\delta|}{\sigma} = \frac{0.5}{1.25} = 0.4$$

Use Table A.8: $\boxed{N = 68}$