INBOX: Fwd: SIEO W4150.001 (566 of 603) Move | Copy This message to

Date: Thu, 07 Sep 2006 18:56:10 -0400
From: alw2113@columbia.edu
To: alw2113@columbia.edu
Subject: Fwd: SIEO W4150.001

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----- Forwarded message from alw2113@columbia.edu -----
    Date: Wed, 06 Sep 2006 18:00:17 -0400
    From: alw2113@columbia.edu
Reply-To: alw2113@columbia.edu
 Subject: SIEO W4150.001
      To: alw2113@columbia.edu
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SIEO 4150 SYLLABUS
FALL 2006

INSTRUCTOR: DR. A. LARRY WRIGHT

OFFICE/OFFICE HOURS: MUDD 318, TUESDAYS 10:30-12:30 AND BY APPOINTMENT

E-MAIL: alw2113@columbia.edu

TEACHING ASSISTANT: KA CHUN MA, MUDD 313A

TEXT: PROBABILITY AND STATISTICS FOR ENGINEERS AND SCIENTISTS, 8TH EDITION, BY
WALPOLE,MYERS,MYERS, AND YE.

WE WILL COVER THE FIRST 11 CHAPTERS, PLUS SOME EXTRA MATERIAL OUTSIDE THE BOOK.

HOMEWORK WILL BE ASSIGNED WEEKLY, AND COLLECTED ON TUESDAYS AT THE BEGINNING OF CLASS

EXAMINATION SCHEDULE:

EXAMINATION 1: THURSDAY, OCTOBER 5
EXAMINATION 2: THURSDAY, NOVEMBER 9
FINAL(TENTATIVE): THURSDAY, DECEMBER 21, FROM 9:00-12:00.

GRADING:
HOMEWORK: 15%

EXAMINATION 1: 25%
EXAMINATION 2: 25%
FINAL: 35%

STUDENTS WILL NEED A CALCULATOR WITH AT LEAST THE BASIC MATHEMATICAL FUNCTIONS. EXAMS WILL BE CLOSED BOOK. YOU ARE ALLOWERD TWO 8.5 BY 11 INCH SHEETS OF PAPER FOR FORMULAS, ETC. THERE IS NO EXTRA CREDIT.

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----- End forwarded message -----

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Date: Sat, 09 Sep 2006 17:08:49 -0400
From: alw2113@columbia.edu
To: alw2113@columbia.edu
Subject: SIEO W4150.001

HOMEWORK EXERCISES DUE TUESDAY, SEPTEMBER 12
HINTS AND REWORDING

1. For each of the following functions f(x), calculate the integral from 0 to infinity, (0,infinity) and from -infinity to infinity (-infinity,infinity). If the integral does not exist, explain.

(a) exp(-x)

(b) xexp(-x)

(c) exp(-(x**2))

(d) xexp(-(x**2))

Here exp(x) is the exponential function.

2. Sum the following values from x=0 to infinity (x=0,1,2,...), where lambda is a positive constant

(a) [lambda**x]/(x!)

(lambda to the xth power, divided by x factorial)

(b) x[lambda**x]/(x!)

The function in (a) is very famous! Just name it.

3. Given data points x(1),x(2),...,x(n), show that the following equalities hold, where xbar is the sample mean:

(a) (x(1)-xbar)+(x(2)-xbar)+...+(x(n)-xbar)
=0.

(b) (x(1)**2)+(x(2)**2)+...+(x(n)**2)-
{[x(1)+x(2)+...+x(n)]**2}/n=
[(x(1)-xbar)**2]+[(x(2)-xbar)**2]+...

```
+[(x(n)-xbar)**2]
```

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4150 Homework Solutions
1st Set

(1)

(a) $\int_0^\infty e^{-x}dx = -e^{-x}\Big|_0^\infty = 0-(-1) = 1$

(2) $\int_{-\infty}^\infty e^{-x}dx = \int_0^\infty e^{-x}dx + \int_{-\infty}^0 e^{-x}dx = 1+\infty = \infty$

(b) $\int_0^\infty xe^{-x}dx =$ $u=x$, $dv = e^{-x}dx$

$du = dx$, $v = -e^{-x}$

$= -xe^{-x}\Big|_0^\infty + \int_0^\infty e^{-x}dx = 0+1 = 1$

$\int_{-\infty}^\infty xe^{-x}dx = \int_0^\infty xe^{-x}dx + \int_{-\infty}^0 xe^{-x}dx = 1+\infty = -\infty$

(c) $\int_0^\infty e^{-x^2}dx = \frac{1}{2}\int_{-\infty}^\infty e^{-x^2}dx$

(2)

Let $I = \int_{-\infty}^\infty e^{-x^2}dx$

Then $I^2 = \left(\int_{-\infty}^\infty e^{-x^2}dx\right)\left(\int_{-\infty}^\infty e^{-y^2}dy\right)$

use Polar co-ordinates: $x = r\cos\theta$, $y = r\sin\theta$

$x^2+y^2 = r^2$

$dxdy = r\,dr\,d\theta$

$I^2 = \int_0^{2\pi}\int_0^\infty re^{-r^2}d\theta\,dr$

$= 2\pi\int_0^\infty re^{-r^2}dr = 2\pi\frac{e^{-r^2}}{2}\Big|_0^\infty$

$= \pi$. Thus $I = \sqrt{\pi}$

And $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$

$\int_{-\infty}^\infty e^{-x}dx = \sqrt{\pi}$

(d) $\displaystyle\int_0^{\infty} x e^{-x^2} dx = \quad -\tfrac{1}{2} e^{-x^2}\Big|_0^{\infty} = \tfrac{1}{2}$

(1)

$\displaystyle\int_{-\infty}^{\infty} x e^{-x^2} dx = \underline{\underline{0}}.$

2 (a)

(1) $\displaystyle\sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{\lambda}$ by definition,

(b) $\displaystyle\sum_{x=0}^{\infty} x \frac{\lambda^x}{x!} = \sum_{x=1}^{\infty} \frac{x\lambda^x}{x!}$

(1)

$\displaystyle= \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$ Let $y = x-1$

$\displaystyle= \sum_{y=0}^{\infty} \frac{\lambda^{y+1}}{y!} = \lambda \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} = \lambda e^{\lambda}$

(a)

3 $\displaystyle\sum_{i=1}^{N} (X_i - \bar{X}) = \sum_{i=1}^{N} \left(X_i - \frac{\left(\sum_{j=1}^{N} X_j\right)}{N}\right)$

(1)

$\displaystyle= \sum_{i=1}^{N} X_i - \sum_{i=1}^{N} \frac{\left(\sum_{j=1}^{N} X_j\right)}{N}$

$\displaystyle= \sum_{i=1}^{N} X_i - \sum_{j=1}^{N} X_j = 0.$

(b)

4 $\displaystyle\sum_{i=1}^{N} (X_i - \bar{X})^2 = \sum_{i=1}^{N} \left(X_i^2 - 2\bar{X}(X_i) + (\bar{X})^2\right)$

(1).

$\displaystyle= \sum_{i=1}^{N} X_i^2 - 2\bar{X}\left(\sum_{i=1}^{N} X_i\right) + N(\bar{X})^2$

$\displaystyle= \sum_{i=1}^{N} X_i^2 - \frac{2}{N}\left(\sum_{i=1}^{N} X_i\right)^2 + \frac{\left(\sum_{i=1}^{N} X_i\right)^2}{N}$

$\displaystyle\sum_{i=1}^{N} X_i^2 - \frac{\left(\sum_{i=1}^{N} X_i\right)^2}{N}$

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Date: Thu, 14 Sep 2006 09:35:28 -0400

From: alw2113@columbia.edu

To: alw2113@columbia.edu

Subject: SIEO W4150.001

SECOND HOMEWORK ASSIGNMENT. DUE TUESDAY, SEPTEMBER 19

1. Exercise 2.34

2. EXERCISE 2.70

3. How many distinct permutations can be made from the letters of the word engineering? How many end in the letter e?

4. Exercise 2.100

5. Exercise 2.128

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4150 Homework Solutions
$2^{\underline{nd}}$ Set

2.34 (a) $6! = 720$

(b) The three persons can line up in 4 ways: in positions $(1,2,3)$, $(2,3,4)$, $(3,4,5)$, $(4,5,6)$.

They may line up in $3! = 6$ ways

The remaining 3 can line up in $3! = 6$ ways.

Answer $= 4(3!)(3!) = 4\times6\times6 = 144$

(c) The 2 persons can sit in 10 different orders: $(1,3)$, $(1,4)$, $(1,5)$, $(1,6)$

$(2,4)$, $(2,5)$, $(2,6)$, $(3,5)$, $(3,6)$, $(4,6)$

They may be ordered in $2! = 2$ ways.

The remaining 4 can be ordered in $4! = 24$ ways

Answer: $10\times2\times24 = 480$

2.70 (a) 0.32

(b) 0.87

(c) 0.13

(d) 0.63

<u>Engineering Problem</u>:

e	g	i	n	r
3	2	2	3	1

(a) $\dfrac{11!}{3!\,2!\,2!\,3!\,1!} = 277{,}200$

(b)

e	g	i	n	r
1	2	2	3	1

$\dfrac{10!}{1!\,2!\,2!\,3!\,2!} = 75{,}600$

2.100

$P(\text{system works})$

$= (0.7)^2 + (0.8)^3 - (0.7)^2(0.8)^3 =$

$0.490 + 0.512 - (0.49)(0.512) =$

$1.002 - .25088 = \boxed{0.75112}$

$P(\text{component A does not work} / \text{system does not work})$

$$= \frac{P(\text{component A does not work and system does not work})}{P(\text{system does not work})}$$

$$= \frac{P(\text{component A does not work})P(\text{C, D and E do not \underline{all} work})}{P(\text{system does not work})}$$

$$= \frac{(0.3)(1-(.8)^3)}{1-.75112} = \frac{(0.3)(0.488)}{(0.24888)}$$

$= 0.58824$

2.128 $+$: tests positive

D : has the disease

$P(D) = 0.002$, $P(+/D) = 0.95$, $P(+/D') = 0.01$

$$P(D/+) = \frac{P(D \cap +)}{P(+)} = \frac{P(D)P(+/D)}{P(D)P(+/D) + P(D')P(+/D')}$$

$$= \frac{(0.002)(0.95)}{(0.002)(0.95) + (0.998)(0.01)}$$

$$= \frac{0.0019}{0.0019 + 0.00998} = \frac{0.0019}{0.01188}$$

$= 0.15993$

Date: Thu, 21 Sep 2006 15:01:04 -0400
From: alw2113@columbia.edu
To: alw2113@columbia.edu
Subject: SIEO W4150.001

SIEOR 4150 HOMEWORK

DUE TUESDAY, SEPTEMBER 26

1. The lifetime in hours of an electronic tube is a random variable having a probability density function given by

    f(x)=xe(-x), x>=0 (and equal to 0 for x<0)

Compute the expected lifetime of such a tube.

2. Suppose that 3 balls are chosen without replacement from an urn consisting of 5 white balls and 8 red balls. Let X(i) equal 1 if the ith ball selected is white, and let it equal 0 otherwise. Find the joint probability distribution of X(1),X(2), and X(3).

3. Exercise 3.30 (a),(b) on page 90 of the text.

4. Show that f(x,y)=1/x, 0<y<x<1 (and equal to 0 otherwise)is a joint density function. If f is the joint density function of X and Y, find

(a) the marginal distribution of X

(b) E(X).

5. Exercise 3.82 on page 106 of the text.

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SIEO Homework Solutions
3rd Set

(1)

$$f(x) = \begin{cases} xe^{-x}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} xe^{-x}\,dx. \quad \text{Let } u = x,\ dv = e^{-x}dx$$
$$du = dx,\ v = -e^{-x}$$

$$\int_0^{\infty} xe^{-x}\,dx = -xe^{-x}\Big|_0^{\infty} + \int_0^{\infty} e^{-x}\,dx = 0 + 1 = 1$$

$$EX = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^{\infty} x^2 e^{-x}\,dx. \quad \text{Let } u = x^2,\ dv = e^{-x}dx$$
$$du = 2x\,dx,\ v = -e^{-x}$$

$$EX = -x^2e^{-x}\Big|_0^{\infty} + 2\int_0^{\infty} xe^{-x}\,dx = 0 + 2 = 2$$

(2)

$X(1)$	$X(2)$	$X(3)$	Probability
0	0	0	$8/13 \times 7/12 \times 6/11 = \frac{336}{1716} = 84/429$
0	0	1	$8/13 \times 7/12 \times 5/11 = \frac{280}{1716} = 70/49$
0	1	0	$70/429$
1	0	0	$70/429$
0	1	1	$8/13 \times 5/12 \times 4/11 = \frac{160}{1716} = 40/429$
1	0	1	$40/429$
1	1	0	$40/49$
1	1	1	$5/13 \times 4/12 \times 3/11 = \frac{60}{1716} = 15/429$

(3)

$$f(x) = \begin{cases} k(3-x^2), & -1 \leq x \leq 1, \\ 0, & \text{elsewhere} \end{cases}$$

(a) $$\int_{-\infty}^{\infty} f(x)\,dx = k\int_{-1}^{1}(3-x^2)\,dx = 2k\int_0^1 (3-x^2)\,dx$$
$$= 2k\left[3x - x^3/3\right]\Big|_0^1 = 2k\left[3 - 1/3\right] = 2k\left[8/3\right] = \frac{16k}{3}$$
$$\Rightarrow k = 3/16$$

(b) $$P(X < 1/2) = 1/2 + \frac{3}{16}\int_0^{1/2}(3-x^2)\,dx$$
$$= 1/2 + 3/16\left[3x - x^3/3\right]\Big|_0^{1/2} = 1/2 + 3/16\left[3/2 - 1/24\right]$$
$$= 1/2 + 3/16\left[\frac{35}{24}\right] = 1/2 + \frac{35}{128} = 99/128$$

(4) $f(x,y) = \begin{cases} 1/x, & 0<y<x<1, \\ 0 & \text{otherwise} \end{cases}$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = \int_0^1 \frac{1}{x}\left[\int_0^x dy\right]dx = \int_0^1 \frac{1}{x}\,x\,dx = \int_0^1 1\,dx = 1$$

(a) $g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^x \frac{1}{x}\,dy = 1, \quad g(x) = \begin{cases} 1, & 0<x<1, \\ 0, & \text{elsewhere} \end{cases}$

$$EX = \int_{-\infty}^{\infty} x\,g(x)\,dx = \int_0^1 x\,dx = \frac{x^2}{2} = \frac{1}{2}$$

(5) $p = 0.92$, $1-p = 0.08$, $N=5$

P(system is operational) = P(≥ 3 components are operational)

Probability K components are operational

$= \binom{5}{k}(0.92)^k(0.08)^{5-k}$, $k = 0, 1, \ldots, 5$

K	f(K)
0	0.0000032768
1	0.000188416
2	0.004333568
3	0.049836032
4	0.286557184
5	0.659081523

P(system is operational) $= f(3) + f(4) + f(5)$

$= 0.9955$

ASSIGNMENTS

ASSIGNMENTS CREATE ASSIGNMENT

Assignments
Listing of assignments associated with this course.

ASSIGNMENT 3. DUE 9/26 Hide| Edit| Delete

Chebyshev's Inequality Hide| Edit| Delete

SIEO 4150 HOMEWORK ASSIGNMENT 4, DUE TUESDAY, OCTOBER 3 Hide| Edit| Delete

Notes:

1. A satellite system contains n components and functions on a given day if at least k of the n components function on that day. On a rainy day each of the components independently functions with probability p1, whereas on a dry day they each function independently with probability p2. If the probability of rain tomorrow is q, what is the probability that the system will function tomorrow?

2. Exercise 3.79 on page 105 of the text.

NOTE: This problem is: Consider the random variables X and Y that represent the number of vehicles that arrive at 2 separate street corners during a certain 2-minute period. These street corners are fairly close together so that it is important that traffic engineers deal with them jointly if necessary. The joint distribution of X and Y is known to be

f(x,y)=(9/16)[(1/4)**(x+y)], for x=0,1,2..., and y=0,1,2...

(a) Are the two random variables X and Y independent? Explain why or why not.

(b) What is the probability that during the time period in question less than 4 vehicles arrive at the two street corners?

3. Suppose 110 students are driven in 3 buses to a football game. There are 35 students in one of the buses, 42 in another, and 33 in the third bus. One of the 110 students is picked at random. Let X denote the number of students that were on the bus carrying the randomly selected student. One of the 3 bus drivers is also randomly selected. Let Y denote the number of students on her bus.

(a) Which of E(X) or E(Y) do you think is larger? Explain.

(b) Compute E(X) and E(Y).

4. Let X be the number of 1's and Y be the number of 2's in 3 rolls of a fair die. Compute the covariance of X and Y.

5. A random variable X has a mean E(X)=50 and a variance Var(X)=16. Using Chebyshev's theorem, estimate

(a) P(42<X<58).

(b) $P(X>=54)$.

(c) The value of the constant c such that $P(-c<X-50<c)=0.99$.

SIEO 4150 Solutions
4th Assignment

(1) Let $S = \{$ the system will function tomorrow $\}$

$R = \{$ it will be a rainy day $\}$

Then $P(S) = P(S \cap R) + P(S \cap R')$

$= P(R)P(S/R) + P(R')P(S/R')$

$P(R) = g \quad P(R') = 1-g.$

$P(S/R) = \sum_{i=K}^{N} \binom{N}{i} P_1^i (1-P_1)^{N-i}$

$P(S/R') = \sum_{i=K}^{N} \binom{N}{i} P_2^i (1-P_2)^{N-i}$

$$P(S) = g\left[\sum_{i=K}^{N} \binom{N}{i} P_1^i (1-P_1)^{N-i}\right] + (1-g)\left[\sum_{i=K}^{N} \binom{N}{i} P_2^i (1-P_2)^{N-i}\right]$$

(2) Note that $\sum_{x=0}^{\infty} \frac{1}{4^x} = \frac{1}{4^0} + \frac{1}{4^1} + \frac{1}{4^2} + \cdots = \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = 4/3$

and $\sum_{y=0}^{\infty} \frac{1}{4^y} = 4/3.$

Then $(9/16) \sum_{x=0}^{\infty} \sum_{y=0}^{\infty} \frac{1}{4^{(x+y)}}$

$= \left[\frac{3}{4} \sum_{x=0}^{\infty} \frac{1}{4^x}\right]\left[\frac{3}{4} \sum_{y=0}^{\infty} \frac{1}{4^y}\right] = 1 \cdot 1 = 1,$

so $f(x,y)$ is a joint probability density function.

(2) (a) $g(x) = \sum_{y=0}^{\infty} f(x,y) = \frac{9}{16} \cdot \frac{1}{4^x} \sum_{y=0}^{\infty} \frac{1}{4^y}$

$= \frac{9}{16} \cdot \frac{1}{4^x} \cdot \frac{4}{3} = \frac{3}{4} \cdot \frac{1}{4^x}, \quad x = 0, 1, 2, \ldots$

And $h(y) = \frac{3}{4} \cdot \frac{1}{4^y}, \quad y = 0, 1, 2, \ldots$

So that $f(x,y) = g(x)h(y)$, and X and Y are independent.

(b) $P[X+Y<4] = P[X+Y=0] + P[X+Y=1] + P[X+Y=2] + P[X+Y=3]$

x	y	$f(x,y)$
0	0	$9/16\ (1/4^0)$
0	1	$9/16\ (1/4^1)$
0	2	$9/16\ (1/4^2)$
0	3	$9/16\ (1/4^3)$
1	0	$9/16\ (1/4^1)$
1	1	$9/16\ (1/4^2)$
1	2	$9/16\ (1/4^3)$
2	0	$9/16\ (1/4^2)$
2	1	$9/16\ (1/4^3)$
3	0	$9/16\ (1/4^3)$

$P[X+Y<4] = 9/16\,[(1/4^0) + \cdots + 1/4^3]$

$= 9/16 \left[\frac{64+16+4+1+16+4+1+4+1+1}{64}\right]$

$= (9/16)(112/64) = \frac{9(7)}{64} = 63/64$

(3)

Bus 1	Bus 2	Bus 3
35	42	33

(a) $E(X)$ will be greater than $E(Y)$.

The distribution of Y is uniform:

$\frac{1}{3} = h(33) = h(35) = h(42)$, whereas

The distribution of X is weighted toward the larger values

$$g(42) = 42/110 > g(35) = 35/110 > g(33) = 33/110$$

(b) $E(X) = 33\left(\frac{33}{110}\right) + 35\left(\frac{35}{110}\right) + 42\left(\frac{42}{110}\right) = \frac{4078}{110} = 37.0727$

(b) $E(Y) = \frac{35+42+33}{3} = 36.66$

(4)

X	Y	XY	Probability	
0	0	0	$(2/3)^3$	$= 64/216$
0	1	0	$3(1/6)(2/3)^2$	$= 48/216$
0	2	0	$3(1/6)^2(2/3)$	$= 12/216$
0	3	0	$(1/6)^3 =$	$1/216$
1	0	0	$3(1/6)(2/3)^2$	$= 48/216$
1	1	1	$\binom{3}{1}\binom{2}{1}(1/6)^2(2/3)$	$= 24/216$
1	2	2	$\binom{3}{2}(1/6)^2(1/6)$	$= 3/216$
2	0	0	$\binom{3}{2}(1/6)^2(2/3)$	$= 12/216$
2	1	2	$\binom{3}{2}(1/6)^3$	$= 3/216$
3	0	0	$(1/6)^3$	$= 1/216$

X	g(X)
0	125/216
1	75/216
2	15/216
3	1/216

Y	h(Y)
0	125/216
1	75/216
2	15/216
3	1/216

XY	f(X,Y)
0	186/216
1	24/216
2	6/216

$EX = EY = 0(^{125}/_{216}) + 1(^{75}/_{216}) + 2(^{15}/_{216}) + 3(^{1}/_{216}) = \frac{108}{216} = \frac{1}{2}$

$EXY = 0(^{186}/_{216}) + 1(^{24}/_{216}) + 2(^{6}/_{216}) = ^{36}/_{216} = \frac{1}{6}$

$\sigma_{xy} = EXY - (EX)(EY) = \frac{1}{6} - \frac{1}{4} = \frac{2}{12} - \frac{3}{12} = -\frac{1}{12}$

(5) $\boxed{P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - 1/k^2 \quad \text{or} \quad P(|X-\mu| \geq k\sigma) < 1/k^2}$

$\underline{\mu} = E(X) = \underline{50}$, $\sigma_x^2 = \sigma^2 = 16$, $\underline{\sigma = 4}$.

(a) $P(42 < X < 58) = P(50 - 2(4) < X < 50 + 2(4))$

$= P(50 - 2\sigma < X < 50 + 2\sigma) \geq 1 - \frac{1}{2^2} = \frac{3}{4}$.

$$\boxed{\begin{array}{l} P(X - \mu \geq a) \leq \frac{\sigma^2}{\sigma^2 + a^2} \quad \text{or} \\ P(X - \mu \geq k\sigma) \leq \frac{1}{1+k^2} \quad \text{or} \\ P(X \geq \mu + k\sigma) \leq \frac{1}{1+k^2} \end{array}}$$

$P(X \geq 54) = P(X \geq 50 + 1\cdot 4)$

$= P(X \geq \mu + 1\cdot\sigma) \leq \frac{1}{2}$.

(c) $P(-c < X - 50 < c) = 0.99 = 1 - 1/k^2 \quad k = 10$.

$k\sigma = c$, $10(4) = 40 = c$.

Date: Tue, 10 Oct 2006 12:30:41 -0400
From: alw2113@columbia.edu
To: alw2113@columbia.edu
Subject: SIEO W4150.001

FIFTH HOMEWORK ASSIGNMENT DUE TUESDAY, OCTOBER 17

1. A communication system has n components, each functioning (independently) with probability p. The total system will operate successfully if at least 1/2 of its components function.

(a) Find the probability that a 3-component system will operate successfully.

(b) Find the probability that a 5-component system will operate successfully.

(c) For which values of p is a 5-component system more likely to operate successfully than a 3-component system?

2. The potential buyer of a particular engine requires (among other things) that the engine start successfully 10 consecutive times. Suppose the probability of a successful start is 0.990. Let us assume that the outcomes of successful starts are independent.

(a) What is the probability that the engine is accepted after only 10 starts?

(b) What is the probability that exactly 12 attempted starts are made during the acceptance process?

(c) What is the probability that exactly 14 attempted starts are made, given that more than 10 attempted starts are made?

3. A local drugstore owner knows that, on average, 100 people per hour stop by his store.

(a) Find the probability that in a given 3-minute period nobody enters the store.

(b) Find the probability that in a given 3-minute period more than 5 people enter the store.

4. For the normal curve with mean mu and standard deviation sigma:

(a) Show that the curve has points of inflection at x=mu +/- sigma.

(b) Show that the curve is concave downward if mu-sigma< X <mu +sigma, and is concave upward otherwise.

(This message is associated with INTRO-PROBABILITY & STATISTIC)

SIEO 4150

5th Homework Assignment Solutions

① (a) $\binom{3}{2}p^2(1-p) + \binom{3}{3}p^3 = 3p^2 - 3p^3 + p^3 = 3p^2 - 2p^3$

(b) $\binom{5}{5}p^5 + \binom{5}{4}p^4(1-p) + \binom{5}{3}p^3(1-p)^2 =$

$p^5 + 5p^4 - 5p^5 + 10p^3 - 20p^4 + 10p^5 = 6p^5 - 15p^4 + 10p^3$

(c) The 5-component system is better if

$6p^5 - 15p^4 + 10p^3 > 3p^2 - 2p^3$, or (dividing by p^2)

$6p^3 - 15p^2 + 12p - 3 > 0$

$3(2p^3 - 5p^2 + 4p - 1) > 0$

$3(p-1)^2(2p-1) > 0.$

Thus it is better if $\boxed{p > 1/2.}$

② (a) $(.99)^{10} = \boxed{.904382075}$

(b) P(12 Attempts Needed) =

P(2nd Attempt fails, next 10 succeed)

$= (.01)(.99)^{10} = \boxed{.00904382075}$

(c) P(14 Attempts Needed/More than 10 Needed)

$= \dfrac{P(\text{4th Attempt fails, next 10 succeed})}{1-(.99)^{10}}$

$= \dfrac{(.01)(.99)^{10}}{1-(.99)^{10}} = \dfrac{0.00904382075}{.095617924}$

$= \boxed{.094582893}$

③ $\lambda = 100 \times \frac{3}{60} = 5$. Use Table A.2 on page 748

(a) $e^{-5} = \boxed{.0067379}$

(b) $1 - P[X \le 5] = 1 - 0.6160 = \boxed{0.3840}$

④ Let
$$f(x) = n(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty$$

$$\frac{\partial f}{\partial x} = \frac{1}{(2\pi)^{1/2}\sigma}\, e^{-(x-\mu)^2/2\sigma^2}\,\frac{[-(x-\mu)]}{\sigma^2}$$

$$= \frac{-1}{(2\pi)^{1/2}\sigma^3}\, e^{-(x-\mu)^2/2\sigma^2}(x-\mu)$$

$$\frac{\partial^2 f}{\partial x^2} = -\frac{1}{(2\pi)^{1/2}\sigma^3}\, e^{-(x-\mu)^2/2\sigma^2}\left[1 - \frac{(x-\mu)^2}{\sigma^2}\right]$$

$$= \frac{-1}{(2\pi)^{1/2}\sigma^5}\, e^{-(x-\mu)^2/2\sigma^2}\left[\sigma^2 - (x-\mu)^2\right]$$

$$\frac{\partial^2 f}{\partial x^2} \text{ is: } \begin{cases} < 0, \text{ if } \sigma^2 > (x-\mu)^2, \text{ i.e. } \sigma > |x-\mu| \text{ or} \\ \quad \mu-\sigma < x < \mu+\sigma \\ = 0 \text{ if } x = \mu \pm \sigma \\ > 0 \text{ if } (x-\mu)^2 > \sigma^2, \text{ i.e. } x > \mu+\sigma \text{ or } x < \mu-\sigma \end{cases}$$

Thus

(a) The points of inflection are $x = \mu \pm \sigma$

(b) The curve is concave downward if $\mu - \sigma < x < \mu + \sigma$ and is concave upward otherwise.

INBOX: SIEO W4150.001 (660 of 674)

Date: Fri, 20 Oct 2006 10:26:48 -0400
From: alw2113@columbia.edu
To: alw2113@columbia.edu
Subject: SIEO W4150.001

SIEO 4150 6TH HOMEWORK ASSIGNMENT
DUE THURSDAY, OCTOBER 26

1. Suppose a bank is staffed by two tellers. Suppose that when Mr. Gallego arrives at the bank, he discovers that Mr. Kou is being served by one of the tellers and Ms. Mack by the other. Suppose also that Mr. Gallego is told that his service will begin as soon as either Mr. Kou or Ms. Mack leaves. If the amount of time a teller spends with a customer is exponentially distributed with parameter lambda, what is the probability that, of the three customers, Mr. Gallego is the last to leave the bank?

2. Derive the variance of the geometric distribution with parameter p.

3. A fair die is tossed 1000 times. Estimate the probability that the number 6 will appear between 150 and 200 times inclusively. If the number 6 appears exactly 200 times, estimate the probability that the number 3 will appear less than 150 times.

4. The length of time, in seconds, that a computer user reads his (or her)e-mail is distributed as a lognormal random variable with mu=1.8 and sigma=2.

(a) What is the probability that the user reads the mail for more than 20 seconds?

(b) What is the probability that the user reads the mail for a length of time that is less than or equal to the mean of the underlying lognormal distribution?

5. Suppose the service life, in years, of a hearing aid battery is a random variable having a Weibull distribution with alpha=1/3 and beta=2.

(a) How long can such a battery be expected to last?

(b) What is the probability that such a battery will be operating after 3 years?

(This message is associated with INTRO-PROBABILITY & STATISTIC)

6th Homework Assignment Solutions

(1) Consider the time at which Mr. Gallego first finds a free teller. At this point either Mr. Kou or Ms. Mack would have just left and the other one would still be in service. However, by the lack of memory of the exponential, it follows that the <u>additional</u> amount of time that this other person would still have to spend in the bank is exponentially distributed with parameter λ. That is, it is the same as if service for this person were just starting at this point. Hence, by symmetry, the probability that the remaining person finishes before Mr. Gallego is $1/2$.

(2) We have seen that the mean of the geometric distribution is $1/p$. We use the formula $\sigma^2 = E(X^2) - \mu^2 = EX^2 - (1/p^2)$.

Now $EX^2 = \sum_{x=1}^{\infty} x^2 f(x) = p \sum_{x=1}^{\infty} x^2 q^{x-1}$.

But $\frac{d}{dq}(xq^x) = x^2 q^{x-1}$ for $x = 1, 2, \ldots$

Thus $EX^2 = p \frac{d}{dq}\left(\sum_{x=1}^{\infty} x q^x\right) = p \frac{d}{dq}\left[\frac{q}{(1-q)} \sum_{x=1}^{\infty} x q^{x-1}\right]$

$= p \frac{d}{dq}\left[\frac{q}{(1-q)} E(x)\right] = p \frac{d}{dq}\left[\frac{q}{(1-q)^2}\right]$

$= p\left[\frac{(1-q)^2 + 2q(1-q)}{(1-q)^4}\right] = p\left[\frac{1}{p^2} + \frac{2(1-p)}{p^3}\right]$

$= \frac{1}{p} + \frac{2(1-p)}{p^2} = \boxed{\frac{2}{p^2} - \frac{1}{p}}$

Thus $\sigma^2 = E(X^2) - 1/p^2 = 1/p^2 - 1/p = \frac{(1-p)}{p^2}$

(3)

$N = 1000 \qquad p = P(6) = 1/6$

$X = $ # of sixes in 1000 tosses.

X is $b(x; 1000, 1/6) \qquad \mu = E(X) = \frac{1000}{6} = 166.\overline{66}, \quad \sigma^2 = Np(1-p) = \frac{1000(5)}{36} = 138.\overline{88}$

(a) $P[150 \le X \le 200] = P[149.5 \le X \le 200.5]$

$\approx P\left[\frac{149.5-\mu}{\sigma} \le Z \le \frac{200.5-\mu}{\sigma}\right]$

$= P\left[\frac{149.5-166.\overline{66}}{\sqrt{138.\overline{88}}} \le Z \le \frac{200.5-166.\overline{66}}{\sqrt{138.\overline{88}}}\right]$

$= P\left[\frac{149.5-166.\overline{66}}{11.785112} \le Z \le \frac{200.5-166.\overline{66}}{11.785112}\right]$

$\approx P[-1.46 \le Z \le 2.87] = .9979 - 0.0722 = 0.9257$

(b) Here we have 800 non-sixes. $\quad \mu = 800(1/5) = 160$

$Y = $ # of 3s $\quad Y$ is $b(x; 800, 1/5) \quad \sigma^2 = 800(1/5)(4/5) = 128$

$P[Y < 150] = P[Y \le 149] = P[Y \le 149.5]$

$P\left[\frac{Y-160}{\sqrt{128}} \le \frac{149.5-160}{\sqrt{128}}\right] \approx P\left[Z \le \frac{149.5-160}{11.3137085}\right]$

$= P[Z \le -.93] = 0.1762$

(4) $\mu = 1.8, \ \sigma = 2.0$

(a) $P[X > 20] = P[\log X > \log(20)]$

$= P\left[\frac{\log X - 1.8}{2} > \frac{\log(20) - 1.8}{2}\right]$

$= P\left[Z > \frac{\log(20) - 1.8}{2}\right] = P\left[Z > \frac{2.9957322735539 - 1.8}{2}\right]$

$\approx P[Z > 0.60] = 1 - P[Z \le 0.60]$

$= 1 - 0.7257 = 0.2743$

$\boxed{P(X < e^{\mu+\sigma^2}) \quad P(\log X < \mu + \sigma^2/2) \quad P(\log X - \mu < \sigma^2/2) \quad P(Z < \sigma/2)}$

The mean $= e^{\mu + \sigma^2/2} = e^{1.8+2} = e^{3.8} = 44.701184493300$

$P[X > 44.7011\ldots] = P[\log(X) > \log(44.7011\ldots)]$

$= P[\log(X) > 3.8]$

$= P\left[\frac{\log X - 1.8}{2} > \frac{3.8-1.8}{2}\right] = P[Z > 1] = 1 - P[Z \le 1]$

$= 1 - .8413 = 0.1587$

(4) Let $f(x) = n(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}, \; -\infty < x < \infty$

$$\frac{\partial f}{\partial x} = \frac{1}{(2\pi)^{1/2}\sigma} e^{-(x-\mu)^2/2\sigma^2} \frac{[-(x-\mu)]}{\sigma^2}$$

$$= \frac{-1}{(2\pi)^{1/2}\sigma^3} e^{-(x-\mu)^2/2\sigma^2} (x-\mu)$$

$$\frac{\partial^2 f}{\partial x^2} = -\frac{1}{(2\pi)^{1/2}\sigma^3} e^{-(x-\mu)^2/2\sigma^2}\left[1 - \frac{(x-\mu)^2}{\sigma^2}\right]$$

$$= \frac{-1}{(2\pi)^{1/2}\sigma^5} e^{-(x-\mu)^2/2\sigma^2}\left[\sigma^2 - (x-\mu)^2\right]$$

$$\frac{\partial^2 f}{\partial x^2} \text{ is: } \begin{cases} < 0, \text{ if } \sigma^2 > (x-\mu)^2, \text{ i.e. } \sigma > |x-\mu| \text{ or } \\ \quad \mu - \sigma < X < \mu + \sigma \\ = 0 \text{ if } X = \mu \pm \sigma \\ > 0 \text{ if } (x-\mu)^2 > \sigma^2, \text{ i.e. } x > \mu + \sigma \text{ or } x < \mu - \sigma \end{cases}$$

Thus

(a) The points of inflection are $X = \mu \pm \sigma$

(b) The curve is concave downward if $\mu - \sigma < X < \mu + \sigma$ and is concave upward otherwise.

(5)

Weibull: $\alpha = 1/3, \beta = 2.$

MEAN $= \mu = \alpha^{-1/\beta}\Gamma(1+1/\beta) = (1/3)^{-1/2}\Gamma(1+1/2) = (\sqrt{3})\Gamma(1.5) =$

$$\mu = (\sqrt{3})\underbrace{(.8862269254 53)}_{\Gamma(1.5)} = (1.7320508)(0.886\ldots) = 1.535$$

(a) $\boxed{1.54 \text{ years}}$ (the MEAN)

(b) $P(X>3) = \int_3^\infty \frac{1}{3}(2)\,x\,e^{-\frac{1}{3}x^2}dx$

$= 2/3\int_3^\infty x\,e^{-x^2/3}dx$, Let $u = x^2/3$
$du = \frac{2x}{3}dx$

$= \int_3^\infty e^{-u}du = e^{-3} = 0.04979$

Date: Thu, 26 Oct 2006 13:58:58 -0400
From: alw2113@columbia.edu
To: alw2113@columbia.edu
Subject: SIEO W4150.001

SIEO 4150 7TH HOMEWORK ASSIGNMENT

DUE THURSDAY, NOVEMBER 2

1. Let X have a continuous uniform distribution on (0,1), that is,

$$f(x) = 1, \quad 0<x<1,$$
$$\quad = 0, \quad \text{otherwise.}$$

(a) Find the probability distribution of X**3.

(b) Show that Y=-2lnX has a chi-squared distribution with 2 degrees of freedom.

2. If X1 and X2 are independent Poisson random variables with parameters lambda1=1
and lambda2=2 respectively, find the probability that X1=2 given that X1+X2=5.

3. Let X1 and X2 have independent exponential distributions with parameter lambda=1.

(a) Show that U=X1+X2 has a Gamma(2,1) distribution.

(b) Show that V=X1/(X1+X2) has the uniform(0,1) distribution.

4. If X has a geometric distribution with parameter p, $0<p<1$:

(a) Find the moment-generating function of X.

Using the result in (a)

(b) Find the mean of X.

(c) Find the variance of X.

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4150 7th Homework Assignment Solutions

(1) $f(x) = \begin{cases} 1, & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$

(a) $Y = X^3$

$u(x) = x^3$

$w(y) = y^{1/3}$ $\quad w'(y) = \begin{cases} \frac{1}{3} y^{-2/3}, & 0<y<1 \\ 0, & \text{elsewhere} \end{cases}$

$g(y) = f(w(y))\,|J| = \begin{cases} \frac{1}{3} y^{-2/3}, & 0<y<1 \\ 0, & \text{otherwise} \end{cases}$

(b) $Y = -2\ln X$

$u(x) = -2\ln x$

$\ln x = -\frac{y}{2}, \quad x = e^{-y/2}$

$w(y) = e^{-y/2}, \quad w'(y) = -\frac{1}{2} e^{-y/2}$

$g(y) = f[w(y)]\,|J| \qquad |J| = \frac{1}{2} e^{-y/2}$

$= \begin{cases} e^{-y/2}\,\frac{1}{2} e^{-y/2}, & 0<y<1 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \frac{1}{2} e^{-y}, & 0<y<1 \\ 0, & \text{otherwise} \end{cases}$

which (see page 200) is the density function of a χ^2 random variable with 2 degrees of freedom. ($\nu = 2$)

(4) $g(x;p) = p q^{x-1}$ for $x = 1, 2, 3, \ldots$

(a) $M_X(t) = E(e^{tX}) = \sum_{x=1}^{\infty} p e^{tx} q^{x-1} = \frac{p}{q} \sum_{x=1}^{\infty} (e^t q)^x$

Now $0 < e^t q < 1 \iff e^t < \frac{1}{q} \iff \boxed{t < -\ln q}$

In this case, $\sum_{x=0}^{\infty} (e^t q)^x = 1 + e^t q + e^{2t} q^2 + \cdots$

$= \frac{1}{1 - qe^t}$, so that

$M_X(t) = \frac{p}{q}\left[\frac{1}{1-qe^t} - 1\right] = \frac{p}{q}\left[\frac{qe^t}{1-qe^t}\right] = \frac{pe^t}{1-qe^t}$

Thus $M_X(t) = \begin{cases} \frac{pe^t}{1-qe^t}, & \boxed{t < -\ln q} \\ 0, & \text{elsewhere} \end{cases}$

(b) $\mu = \frac{d M_X(t)}{dt}\Big|_{t=0} = \frac{pe^t(1-qe^t) - pe^t(-qe^t)}{(1-qe^t)^2}\Big|_{t=0}$

$= \frac{pe^t}{(1-qe^t)^2}\Big|_{t=0} = \frac{p}{p^2} = \frac{1}{p}$

(c) $EX^2 = \frac{d^2 M_X(t)}{d^2 t}\Big|_{t=0} = \frac{d}{dt}\left[\frac{pe^t}{(1-qe^t)^2}\right]_{t=0} = \frac{(1-qe^t)^2 pe^t + pe^t(2)(1-qe^t)qe^t}{(1-qe^t)^4}\Big|_{t=0}$

$= \frac{p^3+2p^2q}{p^4} \quad \frac{p+2q}{p^2}$

$\sigma^2 = EX^2 - \mu^2 = \frac{p+2q-1}{p^2} = \frac{q}{p^2} = \frac{1-p}{p^2}$.

(3) $f(x) = \begin{cases} e^{-x}, & 0<x<\infty \\ 0, & \text{elsewhere} \end{cases}$ $\quad g(y) = \begin{cases} e^{-y}, & 0<y<\infty \\ 0, & \text{elsewhere} \end{cases}$ $\quad f(x,y) = \begin{cases} e^{-(x+y)} & 0<x<1,\ 0<y<1 \\ 0, & \text{elsewhere} \end{cases}$

(a) $U = X+Y$

Let $Y_1 = u_1(X_1, X_2) = X_1 + X_2$

$Y_2 = u_2(X_1, X_2) = X_2$

This defines a 1-1 transformation

$x_1 = w_1(y_1, y_2) = \begin{cases} y_1 - y_2, & 0<y_1<1,\ 0<y_2<y_1 \\ 0 & \text{elsewhere} \end{cases}$

$x_2 = w_1(y_1, y_2) = \begin{cases} y_2, & 0<y_2<y_1 \\ 0 & \text{elsewhere} \end{cases}$

$x_1 = w_1(y_1, y_2) = y_1 - y_2$

$x_2 = w_1(y_1, y_2) = y_2$

$J = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1, \quad |J| = 1.$

$g(y_1, y_2) = f[w_1(y_1, y_2), w_2(y_1, y_2)]\,|J|$

$= e^{-(y_1-y_2)-y_2} = \begin{cases} e^{-y_1}, & 0<y_2<y_1,\ 0<y_1<\infty \\ 0, & \text{otherwise} \end{cases}$

$h(y_1) = \int_{-\infty}^{\infty} g(y_1, y_2)\, dy_2 = \int_0^{y_1} e^{-y_1} dy_2 = \begin{cases} y_1 e^{-y_1}, & 0<y_1<\infty \\ 0, & \text{elsewhere} \end{cases}$

GAMMA (2, 1)

(see page 195)

(b) $V = \frac{X}{(X+Y)}$. For $0 < v < 1$,

$P[V<v] = \int_0^\infty \left[\int_0^{\frac{vy}{(1-v)}} e^{-x} dx\right] e^{-y} dy$

$-\left[\frac{vy}{1-v} + y\right] = -\left[\frac{vy+(1-v)y}{1-v}\right] = -\frac{y}{(1-v)}$

$\frac{x}{x+y} \le v \quad x \le vx + vy$

$(1-v)x \le vy$

$x \le \frac{vy}{1-v}$

$P[V<v] = \int_0^\infty \left[1 - e^{-\frac{vy}{(1-v)}}\right] e^{-y} dy = \int_0^\infty e^{-y} dy - \int_0^\infty e^{\frac{-y}{(1-v)}} dy$

$= 1 - \left[-(1-v)e^{-\frac{y}{(1-v)}}\Big|_0^\infty\right]$

$= 1 - (1-v) = v,$

Thus $f_V(v) = \begin{cases} 1, & 0<v<1 \\ 0 & \text{elsewhere} \end{cases}$. V is uniformly distributed on (0, 1).

(2) $X_1 \sim P(x_1; 1)$, $X_2 \sim P(x_2; 2)$

$$P[X_1=2 / X_1+X_2=5] = \frac{P[X_1=2, X_2=3]}{P[X_1+X_2=5]}$$

$$= \frac{\frac{e^{-1}(1)^2}{2!} \frac{e^{-2}(2)^3}{3!}}{\frac{e^{-3}(3)^5}{5!}} = \left[\frac{8}{2!3!}\right] \frac{5!}{3^5} = \left(\frac{8}{12}\right)\left(\frac{120}{243}\right) =$$

$$\left(\frac{2}{3}\right)\left(\frac{40}{81}\right) = \frac{80}{243}$$

Date: Fri, 17 Nov 2006 14:08:46 -0500

From: alw2113@columbia.edu

To: alw2113@columbia.edu

Subject: SIEO W4150.001

4150 HOMEWORK ASSIGNMENT DUE TUESDAY, NOVEMBER 21.

1. A maker of a certain brand of low-fat cereal bars claims that their average saturated fat content is 0.5 grams. In a random sample of 8 cereal bars of this brand the saturated fat content was 0.6,0,7,0.7,0.3,0.4,0.5,0.4, and 0.2. Would you agree with this claim? Assume a normal distribition.

2. Exercise 8.57 on page 266 of the text (8th edition). This is Exercise 2 on page 228 of the 7th edition.

3. Suppose X is a random variable with unknown mean mu and variance [sigma**2]=1.0. A random sample of size n is to be used as an estimate of mu. When the data are taken and the sample mean is measured, we wish it to be within 0.03 units of the true mean with probability 0.98. That is, we want the probability that the sample mean xbar and the population mean mu differ by more than 0.03 to be 0.02. Assuming a normal distribution, what sample size is required?

4. In the magazine Chance (Fall 2002) a patent infringement case is described. The case rested on determining whether a patent witness's signature was written on top of a key text in a notebook or under a key text. Zinc measurements for 3 notebook locations were taken. The results follow:

```
Text line:      .335   .374   .440
Witness line:          .210 .262 .188 .329 .439 .397
Intersection:   .393 .353 .285 .295 .319
```

(a) Compute a 95% confidence interval for each set of data.

(b) From the results in (a), what can you infer about the mean zinc measurements at the three locations?

(c) What assumptions are required for the inferences in (b) to be valid?

5. a random sample of 16 graduates of a secretarial school typed an average of 75.5 words per minute with a standard deviation of 7.8 words per minute. Assuming a normal distribution:

(a) Find a 98% confidence interval for the average number of words typed by all graduates of the school.

(b) Find a 95% prediction interval for the next observed number of words per minute typed by a member of the secretarial school.

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4150 8th Homework Assignment Solutions

① $N=8$, $\Sigma X_i = 3.8$, $\Sigma X_i^2 = 2.04$, $\bar{X} = \frac{3.8}{8} = .475$

$$S^2 = \frac{N\Sigma X_i^2 - (\Sigma X_i)^2}{N(N-1)} = \frac{8(2.04)-(3.8)^2}{56} = \frac{1.88}{56} = \frac{.235}{7}$$
$$= .033571428$$

A 95% confidence interval for μ is
$(\bar{X} - t_{.025,7} \cdot \frac{S}{\sqrt{N}}, \bar{X} + t_{.025,7} \frac{S}{\sqrt{N}})$

$t_{.025, 7\,d.f.} = 2.365$

$= (0.475 - 2.365\sqrt{\frac{.235}{56}}, 0.475 + 2.365\sqrt{\frac{.235}{56}})$

$= (0.475 - .153204338, 0.475 + .153204338)$

$(.3218, .6282)$

We agree with the claim that the (population) mean $= 0.5$, because 0.5 is in the 95% confidence interval.

2.) For the gamma with parameters α and β, and $\beta t < 1$,

$$M_X(t) = E(e^{tX}) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^\infty e^{tx} x^{\alpha-1} e^{-x/\beta} dx$$

$$= \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^\infty x^{\alpha-1} e^{-x(\frac{1}{\beta}-t)} dx$$

$x = \frac{\beta y}{(1-\beta t)}$

let $y = x(\frac{1}{\beta} - t)$, $dy = (\frac{1}{\beta} - t)dx$, $dx = \frac{\beta dy}{(1-\beta t)}$

$$M_X(t) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^\infty \left[\frac{\beta y}{(1-\beta t)}\right]^{\alpha-1} e^{-y} \frac{\beta dy}{(1-\beta t)}$$

$$= \frac{1}{(1-\beta t)^\alpha \Gamma(\alpha)} \underbrace{\int_0^\infty y^{\alpha-1} e^{-y} dy}_{=\Gamma(\alpha)} = \boxed{(1-\beta t)^{-\alpha}}$$

the moment-generating function of the exponential distribution is $M_X(t) = E(e^{tX}) =$

$$\int_0^\infty e^{tx} x^{\alpha-1} e^{-x} dx.$$

This is the moment-generating function of the gamma distribution with $\alpha=1$. Thus

$$M_X(t) = (1-\beta t)^{-1}$$

For a random sample of size N, $M_Y(t) = M_{X_1+\cdots+X_N}(t)$

$$= \boxed{(1-\beta t)^{-N}}$$

For $\alpha = N$, the moment-generating function of the gamma with parameters N and β is $\boxed{(1-\beta t)^{-N}}$ (Theorem 7.10, page 223). By the Uniqueness Theorem 7.7 the density functions agree.

3. $\sigma^2 = 1.0$. We want $P[|\bar{X}-\mu| \le 0.03] = .98$,

or $P\left[\frac{|\bar{x}-\mu|}{1/\sqrt{N}} < \frac{0.03}{1/\sqrt{N}}\right] = .98$, $P\left[|Z| \le (0.03)\sqrt{N}\right] = .98$.

From Table A.3, $P[|Z| < 2.326] = .98$ (why?)

We solve $(0.03)\sqrt{N} = 2.326$,

$$\sqrt{N} \geq \frac{2.326}{0.03}, \quad N \geq \left(\frac{2.326}{0.03}\right)^2 = 6011.4177$$

$$N \geq 6012.$$

(a)
(1)

4.) <u>text line</u>: .335 .374 .440

$N_1 = 3 \quad \Sigma X_i = 1.149, \ \Sigma X_i^2 = 0.445701, \ \bar{X} = 0.383$

$$S^2 = \frac{\Sigma X_i^2 - (\Sigma X_i)^2/N}{N-1} = \frac{0.445701 - (1.149)^2/3}{2}$$

$$= \frac{0.005634}{2} = 0.002817, \qquad t_{.025,2} = 4.303$$

A 95% confidence interval for μ is

$$\bar{X} \pm t_{.025,2}\sqrt{\frac{S^2}{N}} = 0.383 \pm 4.303\sqrt{\frac{0.002817}{3}}$$

$$= 0.383 \pm 0.13185728 = \boxed{(.251, .515)}$$

(2) <u>witness line</u>: .210 .262 .188 .329 .439 .397

$N = 6, \ \Sigma X_i = 1.825, \ \Sigma X_i^2 = .606659$

$\bar{X} = \frac{1.825}{6} = .30416\overline{6}, \ S^2 = .010310966$

$t_{.025,5} = 2.571$

A 95% confidence interval for μ is

$$\bar{X} \pm t_{.025,5}\sqrt{\frac{S^2}{N}} = .30416\overline{6} \pm 2.571\sqrt{\frac{.010310966}{6}}$$

$$= .30416\overline{6} \pm .10658010l = \boxed{(.196, .411)}$$

(3) <u>Intersection</u>: .393 .353 .285 .295 .319

$N = 5, \ \Sigma X_i = 1.645, \ \Sigma X_i^2 = .549069$

$t_{.025,4} = 2.776 \qquad \bar{X} = \frac{1.645}{5} = 0.329$

$S^2 = 0.001966$

A 95% confidence interval for μ is

$$\bar{X} \pm t_{.025,4}\sqrt{\frac{S^2}{N}} = 0.329 \pm 2.776\sqrt{\frac{0.001966}{5}}$$

$$= 0.329 \pm .055046 \qquad \boxed{(.274, .384)}$$

(4) (b) The 3 confidence intervals intersect in the interval $\boxed{(.274, .384)}$ (the 3rd interval).
Note that the three means (0.383, 0.304 and 0.329) all lie in this interval. Thus we do not have enough evidence to conclude that the population means differ.

(c) We assume that we are sampling from independent normally distributed populations

(5.) $N=16$, $\bar{x}=75.5$, $s=7.8$

(a) $\bar{x} \pm t_{.01,15}\sqrt{s^2/N} = 75.5 \pm 2.602\sqrt{\frac{(7.8)^2}{16}} =$

$75.5 \pm 2.602\left(\frac{7.8}{4}\right) = 75.5 \pm 5.0739 = \boxed{(70.43, 80.57)}$

(b) $\bar{x} - t_{\alpha/2}\, s\sqrt{1+\frac{1}{N}} < x_0 < \bar{x} + t_{\alpha/2}\, s\sqrt{1+\frac{1}{N}}$

$75.5 - 2.131\,(7.8)\sqrt{\frac{17}{16}} < x_0 < 75.5 + 2.131\,(7.8)\sqrt{\frac{17}{16}}$

$75.5 - 17.13359 < x_0 < 75.5 + 17.13359$

$\boxed{(58.366, 92.633)}$

Date: Mon, 27 Nov 2006 14:33:50 -0500
From: alw2113@columbia.edu
To: alw2113@columbia.edu
Subject: SIEO W4150.001

4150 HOMEWORK ASSIGNMENT DUE TUESDAY, DECEMBER 5

1. Let X(1),X(2),,,,X(n) be a random sample that can take on only positive values. Use the central limit theorem to produce an argument that if n is sufficiently large, then Y=X(1)X(2)...X(n) has approximately a lognormal distribution.

2. A type of thread is being studied for its tensile strength properties. Fifty pieces were tested under similar conditions and the results showed an average tensile strength of 78.3 kilograms and a standard deviation of 5.6 kilograms. Assuming a normal distribution of tensile strength, give a lower 95% prediction interval on a single observed tensile strength value. In addition, give a lower 95% tolerance limit that is exceeded by 99% of the tensile strength values.

3. Exercise 9.46 on page 298 of the text (8th Edition). (This is exercise 10 on page
256 of the 7th edition.)

4. Exercise 9.70 on page 306 of the text (8th Edition). (This is exercise 20 on page 264 of the 7th edition.)

5. Exercise 9.82 On page 315 of the text (8th Edition). (This is exercise 2 on page 280 of the 7th edition.)

(This message is associated with INTRO-PROBABILITY & STATISTIC)

Date: Fri, 01 Dec 2006 11:20:23 -0500
From: alw2113@columbia.edu
To: alw2113@columbia.edu
Subject: SIEO W4150.001

IEOR 4150 HOMEWORK ASSIGNMENT
DUE TUESDAY, DECEMBER 5

1. A manufacturer of power meters claims that when its production process is operating correctly, only 10% of the power meters will be defective. A vendor has just received a shipment of 25 power meters from the manufacturer. Suppose that the vendor wants to test H(0): p=0.10 against H(1):p>0.10, where p is the true proportion of power meters that are defective. Use Y>5 as the rejection region, where Y is the number of defective power meters in the sample.

(a) Determine the value of alpha.

(b) Find beta if in fact p=0.20. What is the power of the test for this value of p?

(c) Find beta if in fact p=0.40. What is the power of the test for this value of p?

2. The building specifications in a certain city require that the sewer pipe used in residential areas have a mean breaking strength of more than 2500 pounds per lineal foot. An independent contractor selected six sections of a manufacturer's pipe and tested each for breaking strength. The results (pounds per lineal foot) follow:
2610 2750 2420 2540 2490 2680. Let H(0): mu=2500 be tested against H (1):mu>2500.

(a) Is there sufficient evidence to conclude that the manufacturer's sewer pipe meets the required specifications? Use alpha=0.10.

(b) Find the value of beta for mu=2525. What is the power of the test?

(c) Find the value of beta for mu=2600.

(d) Find the value of beta for mu=3000.

3. Exercise 10.32 on page 358 of the 8th edition (exercise 14 on page 320 of the 7th edition).

4. Exercise 10.54 on page 361 of the 8th edition (exercise 36 on page 323 of the 7th edition).

5. Exercise 10.52 on page 361 of the 8th edition (exercise 34 on page 323 of the 7th edition).

*****EXPLAIN WHAT ASSUMPTIONS (NORMALITY, INDEPENDENCE,ETC.) YOU ARE MAKING IN EACH PROBLEM*****

(This message is associated with INTRO-PROBABILITY & STATISTIC)