HERE ARE 20 SELECTED EXERCISES FROM THE TEXT.

THE SOLUTIONS WILL BE SENT LATE NEXT WEEK.

YOU MAY WANT TO DO NONE, SOME, OR ALL OF THESE.

I WILL BE IN MY OFFICE THROUGHOUT THE WEEK, BUT NOT EARLY, IF YOU HAVE ANY QUESTIONS.

	FROM EDITION 8	FROM EDITION 7
1	2.36,PAGE48	16, PAGE 39
2	5.17, PAGE 151	17, PAGE 125
3	7.4, PAGE 226	4, PAGE 191
4	7.20, PAGE 228	20, PAGE 193 → $\boxed{\text{Assume } \mu = 4}$.
5	8.59, PAGE 286	4, PAGE 228 (REVIEW)
6	9.34, PAGE 288	20, PAGE 246
7	9.36, PAGE 297	2, PAGE 255
8	9.66, PAGE 305	16, PAGE 263
9	9.83, PAGE 315	3, PAGE 280
10	9.87, PAGE 315	1, PAGE 281
11	10.16, PAGE 338	16, PAGE 299
12	10.36, PAGE 358	18, PAGE 320
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16	10.74, PAGE 370	8, PAGE 332
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12/11/06

Solutions to Sample Problems

2.36, P. 48 (1) (a) (Assuming the numbers cannot begin with 0)

$6 \times 6 \times 5 = 180$

(b) $5 \times 5 \times 3 = 75$ [3 in units position, 5 in hundreds position, 5 in tens position]

(c) 34x, 35x, 36x; 4xx; 5xx; 6xx

$15 + 30(3) = 105$

5.17, P. 151 (2) $p = 0.70$, $n = 5$. $X \sim b(x; 5, 0.70)$

$\mu = np = 5(0.70) = 3.50$

$\sigma^2 = np(1-p) = 5(0.70)(0.30) = 1.050$

Chebyshev: $\mu - k\sigma = 3.50 - 2\sqrt{1.050} = 1.451$

$\mu + k\sigma = 3.50 + 2\sqrt{1.050} = 5.549$

$P(1.451 < X < 5.549) \geq 1 - \frac{1}{4} = 3/4$

The probability that between 2 and 5 (inclusive) people in the sample think that tranquilizers do not cure but only cover up the real problem

7.4, P. 226 (3) $f(x_1, x_2) = \begin{cases} \frac{x_1 x_2}{18}, & x_1 = 1, 2;\ x_2 = 1, 2, 3 \\ 0, & \text{elsewhere} \end{cases}$

$Y = X_1 X_2$

x_1x_2:

X_1 \ X_2	1	2	3
1	1	2	3
2	2	4	6

Probability:

	1	2	3
1	1/18	2/18	3/18
2	2/18	4/18	6/18

$x_1 x_2$	Probability
1	1/18
2	2/9
3	1/6
4	2/9
6	1/3

Let $\mu = 4$.

(4)

7.20, P.228 $M_X(t) = e^{\mu(e^t-1)} = E\, e^{tX}$

$$M_X(t) = \sum_{x=0}^{\infty} \frac{e^{tx} e^{-\mu} (\mu)^x}{x!} = e^{-\mu} \sum_{x=0}^{\infty} \frac{(e^t \mu)^x}{x!}$$

$$= e^{-\mu} e^{(e^t \mu)} = \boxed{e^{\mu(e^t-1)}}$$. Because $\mu = \sigma^2 = 4$,

$$P(\mu - 2\sigma < X < \mu + 2\sigma)$$

$$= \boxed{P(\mu - 2\sqrt{\mu} < X < \mu + 2\sqrt{\mu})}$$

$$= P(0 < X < 8) = \sum_{x=1}^{7} \frac{e^{-4}(4)^x}{x!} =$$ (use Table A.3)

$$= .9489 - 0.0183 = 0.9306$$

(6)

9.34, P288 $N = 12$, $\bar{x} = 45.80$, $S = 1.5$.

$\gamma = 0.05$, $\alpha = 0.10$

From Table A.7, $K = 2.655$

$$\bar{x} \pm Ks = 48.50 \pm (2.655)(1.5) = 48.50 \pm 3.9825$$

$$= \boxed{(44.5175,\ 52.4825)}$$

(5)

8.59, P286 $N_1 = 8$, $N_2 = 12$

$$P(S_1^2/S_2^2 < 4.89) = P(F_{7,11} < 4.89) = f_{0.01}(7,11)$$

$$= 0.99$$

9.36, P297 (7) A: $\bar{X}_A = 78.3,\ S_A = 5.6,\ N_A = 50$

B: $\bar{X}_B = 87.2,\ S_B = 6.3,\ N_B = 50$

$$(\bar{X}_A - \bar{X}_B) \pm Z \sqrt{\frac{S_A^2}{N_A} + \frac{S_B^2}{N_B}}$$

N.B. $N_1, N_2 \geq 30$

$$(78.3 - 87.2) \pm 1.96 \sqrt{\frac{(5.6)^2 + (6.3)^2}{50}}$$

$$= -8.9 \pm 2.336 = \boxed{(-11.24, -6.56)}$$

9.66, P305 (8)

	women	men	total
electrical engineers:	80	170	250
chemical engineers:	40	135	175

$$\hat{P}_E = \frac{80}{250} = 0.32, \quad \hat{P}_C = \frac{40}{175} = \frac{8}{35}$$

$$(\hat{P}_E - \hat{P}_C) \pm 1.645 \sqrt{\frac{(\hat{P}_E)(1-\hat{P}_E)}{N_E} + \frac{\hat{P}_C(1-\hat{P}_C)}{N_C}}$$

$$= (0.091429) \pm 1.645 \sqrt{\frac{(.32)(.68)}{250} + \frac{(40/175)(135/175)}{175}}$$

$$= (0.0914) \pm 0.0713 = \boxed{(0.201, \ 0.1627)}$$

It appears there is a significantly higher proportion of women in electrical engineering than there is in chemical engineering

9.83, P315 (9.) $X_1, X_2, \ldots, X_N$

$$f(x) = \begin{cases} \frac{1}{(\sqrt{2\pi})\sigma x} e^{-[\ln(x) - \mu]^2 / 2\sigma^2}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

(a) $$L(x_1, x_2, \ldots, x_N; \mu, \sigma) = \begin{cases} \frac{1}{(2\pi\sigma^2)^{N/2}(x_1 x_2 \ldots x_N)} e^{-\sum_{i=1}^{N}[\ln(x_i) - \mu]^2 / 2\sigma^2} \\ \quad x_1, x_2, \ldots, x_N > 0 \\ 0, \text{ elsewhere} \end{cases}$$

$$\ln L = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma^2) - \sum \ln(X_i) - \frac{1}{2\sigma^2}\sum[\ln(X_i)-\mu]^2$$

$$\frac{\partial \ln L}{\partial \mu} = \frac{\sum \ln(X_i)}{\sigma^2} - \frac{N\mu}{\sigma^2} = 0, \quad \hat{\mu} = \frac{\sum \ln(X_i)}{N}$$

$$\frac{\partial \ln L}{\partial \sigma^2} = \frac{-N}{2\sigma^2} + \frac{\sum[\ln(X_i)-\mu]^2}{2\sigma^4} = 0$$

$$, \quad \sigma^2 = \boxed{\frac{\sum[\ln(X_i)-\hat{\mu}]^2}{N}}$$

10
9.87, P 315

$$Var(S^2) = Var\left[\frac{(N-1)S^2}{\sigma^2}\cdot\frac{\sigma^2}{(N-1)}\right] = Var\left[\chi^2_{N-1}\frac{\sigma^2}{(N-1)}\right]$$

$$= \frac{\sigma^4}{(N-1)^2}Var[\chi^2_{N-1}] = \frac{\sigma^4}{(N-1)^2}\, 2(N-1)$$

$$= \boxed{\frac{2\sigma^4}{(N-1)}}$$

Because $\frac{(N-1)S^2}{\sigma^2} \sim \chi^2_{N-1}$ And $Var(\chi^2_{N-1}) = 2(N-1)$

$$Var\ \hat{\sigma}^2 = Var\left[\frac{(N-1)}{N}S^2\right] = \frac{(N-1)^2}{N^2}Var(S^2)$$

$$= \frac{(N-1)^2}{N^2}\cdot\frac{2\sigma^4}{(N-1)} = \boxed{\frac{2(N-1)}{N^2}\sigma^4}$$

11
10.16, P 338

$N(x; 200, 15)$

$N = 25$

H_0: Machine operates successfully

Reject H_0 iff. $\bar{x} \le 191$ or $\bar{x} \ge 209$

(a) $1-\alpha = P(191 \le \bar{x} \le 209 \mid \mu = 200)$

$$= P\left(\frac{191-200}{3} \le Z \le \frac{209-200}{3}\right) = (-3 \le Z \le 3).$$

$$= 0.9987 - 0.0013 = \quad = 0.9974, \quad \alpha = 0.0026$$

(b) $\beta = P(191 \le \bar{X} \le 209 / \mu = 215)$

$$= P\left(\frac{191-215}{3} \le Z \le \frac{209-215}{3}\right)$$

$$= P(-8 \le Z \le -2) = 0.0228$$

12

10.36, P358

$\bar{X}_1 = 37{,}900$ $\bar{X}_2 = 39{,}800$

$S_1 = 5100$ $S_2 = 5900$ $N_1 = N_2 = 12$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \ne \mu_2$

$$t_{22} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(5100)^2 + (5900)^2}{12}}} = \frac{-1900}{2251.295923} = -0.84$$

Reject H_0 if $|t| \ge t_{.025, 22\,df} = 2.074$

$\boxed{\text{Do Not Reject } H_0}$

13

10.52, P361

$\alpha = 0.05$, $\beta = 0.10$, 2-sided, $\sigma = 1.25$, $\delta = 0.5$

$\Delta = \frac{|\delta|}{\sigma} = \frac{0.5}{1.25} = 0.40$,

Use TABLE A.8, $N \ge 68$

14

10.64, P366

Population 1: $N_1 = 300$, $X_1 = 240$

Population 2: $N_2 = 400$, $X_2 = 288$

$H_0 : P_1 = P_2$

$H_1 : P_1 > P_2$

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}}$$

$P_1 = 0.80$, $\hat{P}_2 = 0.72$

$$\hat{P} = \frac{X_1 + X_2}{N_1 + N_2} = \frac{528}{700}$$

$$Z = \frac{0.08}{\sqrt{\frac{528}{700}\frac{(172)}{700}\left(\frac{1}{300} + \frac{1}{400}\right)}} = 2.433$$

$$P\text{-value} = P(Z > 2.433) = \boxed{0.0075}$$

Reject Ho.

15

10.72, P370

$N \geq 30 \qquad Z = \dfrac{S - \sigma_0}{\sigma_0/\sqrt{2N}}$

(a) $N = 12$, $\bar{X} = 42$, $S = 11.9$

$H_0: \sigma^2 = 100$
$H_1: \sigma \neq 100$

$$\chi^2_{N-1} = \frac{(N-1)S^2}{\sigma_0^2} = \frac{(11)(11.9)^2}{100} = \boxed{15.5771}$$

Reject Ho if $\chi^2_{11} \geq \chi^2_{.025,11} = 21.925$

or $\chi^2 \leq \chi_{.975,11} = 3.816$

$\boxed{\text{Fail to Reject Ho.}}$

(b) $N = 72$, $S^2 = 4.41$, $\sigma^2 = 6.25$

$$Z = \frac{S - \sigma_0}{\sigma_0/\sqrt{2N}} = \frac{2.1 - 2.5}{2.5/\sqrt{144}} = \frac{-0.4}{(2.5)/12} = \boxed{-1.92}$$

Reject Ho if $Z < -1.645$ Reject Ho

$$P\text{-value} = P[Z < -1.92] = \boxed{0.0274}$$

16

10.74, P370

$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$

$N_1 = 16$, $\Sigma X_1 = 158{,}360$, $\Sigma X_1^2 = 2{,}497{,}444{,}000$

$N_2 = 12$, $\Sigma X_2 = 49{,}450$, $\Sigma X_2^2 = 271{,}403{,}500$

$S_1^2 = 62{,}005{,}060$

$S_2^2 = 6{,}147{,}935.606$

$$F = \frac{62{,}005{,}060}{6{,}147{,}935.606} = 10.086$$

$$P\text{-value} = 2P[F_{15,11} > 10.086] = 2 \times 0.0002 = \boxed{0.0004}$$

Variances are significantly different.

$$e_{11} = \frac{(83)(45)}{200} = 18.675$$

(17)

10.93, P384

	0-1	2-3	over 3	
Elem,	14	37	32	83
Secondary	19	42	17	78
College	12	17	10	39
Total	45	96	59	200

N.B. $(3-1)\times(3-1) = 4$ d.f

expected

	0-1	2-3	over 3	
Elem,	(18.675)	(39.84)	(24.485)	83
Secondary	(17.55)	(37.44)	(23.01)	78
College	(8.775)	(18.72)	(11.505)	39
Total	45	96	59	200

$$\chi^2 = \frac{(14-18.675)^2}{18.675} + \ldots + \frac{(10-11.505)^2}{11.505}$$

$$= 1.1703 + 0.2024 + 2.3065 + 0.1198 + 0.5554 + 1.5698 + 1.1853 + 0.1580 + 0.1969 = 7.464$$

Reject if $\chi^2 > \chi^2_{4\,d.f.,\,0.05} = 9.488$

Fail to Reject H_0

(18)

10.99, P385

Sentiment	City: Richmond	Norfolk	Total
Favor A	204 (214.5)	225 (214.5)	429
Favor B	211 (204.5)	198 (204.5)	409
Undecided	85 (81)	77 (81)	162
Total	500	500	1000

H_0: Proportions the same for each city

H_1: Not the same

$$\chi^2 = \frac{(204-214.5)^2}{214.5} + \frac{(225-214.5)^2}{214.5} + \frac{(211-204.5)^2}{204.5} + \frac{(198-204.5)^2}{204.5} + \frac{(85-81)^2}{81} + \frac{(77-81)^2}{81} =$$

$$= 0.513986 + 0.513986 + 0.206601 + 0.206601 + 0.197531 + 0.197531 \cong 1.836$$

Reject H_0 if $X^2 > X^2_{2df, .05} = 5.991$

Fail to Reject H_0.

(19)

10.128, P386

$$Z = \frac{\hat{P}_1 - \hat{P}_2 - d_0}{\sqrt{\frac{\hat{P}_1 \hat{q}_1}{N_1} + \frac{\hat{P}_2 \hat{q}_2}{N_2}}}$$

$\hat{P}_1 = \frac{120}{200} = 0.60$, $\hat{q}_1 = 0.40$, $N_1 = 200$

$\hat{P}_2 = \frac{240}{500} = 0.48$, $\hat{q}_2 = 0.52$, $N_2 = 500$

H_0: $P_1 - P_2 = 0.03$

H_1: $P_1 - P_2 > 0.03$

$$Z = \frac{0.12 - 0.03}{\sqrt{\frac{(.60)(.40)}{200} + \frac{(.48)(.52)}{500}}} = \frac{0.09}{0.04122} = 2.183$$

P-value $= P(Z > 2.183) = 0.01451$

Reject H_0 The difference in voters favoring the proposal exceeds 3%

(20)

Plant A: $N_1 = 10$, $\Sigma X_1 = 215$, $\Sigma X_1^2 = 4877$

10.113, P387

Plant B: $N_2 = 10$, $\Sigma X_2 = 283$, $\Sigma X_2^2 = 8319$

$\bar{X}_1 = 21.5$, $\bar{X}_2 = 28.3$, $S_1^2 = 28.277\overline{7}$, $S_2^2 = 34.45\overline{55}$

H_0: $\mu_A - \mu_B = 0$ Note

H_1: $\mu_A - \mu_B < 0$

test for equal variances:

$F = \frac{34.45\overline{55}}{28.27\overline{77}} = 1.2185$. P-value $= 2P[F_{9,9} > 1.2185]$

$= 2(0.39) = 0.78$

The variances may be considered equal.

$S_P^2 = \frac{9(28.27\overline{77}) + 9(34.45\overline{55})}{18} = 31.3\overline{66}$

$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_P^2\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}} = \frac{-6.8}{2.5047} = -2.71$

P-value $= P(t_{18} < -2.71)$

$= 0.007$ Reject H_0