Appendix A - the model

A1. The basic setup

In this section, we describe in more detail the main features of our framework, focusing on the principal elements of departure from previous treatments.

A1.1 Households

The representative household within a country is thought of as a continuum of members with names on the unit interval. Each household purchases consumption goods, holds money and supplies labor. Wages are fixed by bargaining, and, given the presence of involuntary unemployment, the labor supply is not binding. Household members can be employed or unemployed. To avoid distributional issues, we assume that households pool their income and consumption.

The representative household in country i ($i = H$ or F) seeks to maximize lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi_0 \frac{\left(N_t^H\right)^{1+\phi}}{1+\phi} \right\}, \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t^* - \chi_1 \frac{\left(N_t^F\right)^{1+\phi}}{1+\phi} \right\} \tag{1}$$

where variables with star refer to the foreign country. N_t^i denotes the number of employed individuals in the representative household of country i while C_t and C_t^* are the composite consumption indexes for the home and foreign country respectively, defined as:

$$C_t = \frac{\left(C_t^H\right)^{1-\alpha} \left(C_t^F\right)^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}}, \quad C_t^* = \frac{\left(C_t^{F,*}\right)^{1-\alpha} \left(C_t^{H,*}\right)^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}} \tag{2}$$

$C_t^{j,i}$ is the quantity of the good produced in country j and consumed by residents of country i. $\alpha \in [0,1]$ is the weight on the imported goods in the utility of private consumption; a value for α strictly less than $\frac{1}{2}$ reflects the presence of home bias in consumption.

The production sectors are characterized by monopolistic competition. The index of country i's consumption of the good produced in country j, $C_t^{j,i}$, is given by the usual CES aggregator:

$$C_t^{j,i} = \left(\int_0^1 \left(C_t^{j,i}(z) \right)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}, \ i = H \ or \ F; \ j = H \ or \ F$$

The parameter ϵ $(\epsilon^*) > 1$ is the elasticity of substitution between varieties produced within home (foreign) country.

Utility maximization for the home household is subject to a sequence of budget constraints of the form[1]:

$$\int_o^1 P_t^H(j) C_t^H(j) dj + \int_0^1 P_t^F(j) C_t^F(j) dj + E_t \left\{ Q_{t,t+1}^H V_{t+1}^H \right\} \leq V_t^H + W_t^H N_t^H + \Pi_t^H - T_t^H$$

for $t = 0, 1, 2, ...$, where $P_t^i(j)$ is the price of good j produced in country i (expressed in the units of the single currency). V_t^H is the nominal payoff in period t of the portfolio held at the end of period $t-1$; Π_t^H denotes the profits received by the Home households, which is the sum of dividends derived from retailers; W_t^H is the nominal wage and T_t^H denotes lump-sum taxes.

We assume complete securities markets; $Q_{t,t+1}^H$ is the stochastic discount factor for one-period ahead nominal payoffs, which is common across countries. Implicit in the budget constraint is the assumption that the law of one price holds across the union.

The demands for the generic goods produced at home and foreign are respectively:

$$C_t^{H,i}(z) = \left(\frac{P_t^H(z)}{P_t^H} \right)^{-\epsilon} C_t^{H,i}; \ C_t^{F,i}(z) = \left(\frac{P_t^F(z)}{P_t^F} \right)^{-\epsilon^*} C_t^{F,i} \tag{3}$$

for $i = H, F(^*)$, $z \in [0,1]$. The appropriate domestic (producer) price indexes of the home and foreign countries are:

$$P_t^H = \left(\int_0^1 \left(P_t^H(z) \right)^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}}; \ P_t^F = \left(\int_0^1 \left(P_t^F(z) \right)^{1-\epsilon^*} dz \right)^{\frac{1}{1-\epsilon^*}}$$

Since the law of one price holds, P_t^H represents both the price index for the bundle of goods imported from country H as well as H's domestic price index. From the

[1] The utility maximization problem for the foreign household is completely analogous.

demand functions (3), we get (for the home country): $\int_{o}^{1} P_t^H(j)C_t^H(j)dj = P_t^H C_t^H$ and $\int_{o}^{1} P_t^F(j)C_t^F(j)dj = P_t^F C_t^F$.

Furthermore, the optimal allocation of expenditures by country of origin implies, for the home country:

$$P_t^H C_t^H = (1-\alpha)P_t C_t, \quad P_t^F C_t^F = \alpha P_t C_t \tag{4}$$

while for the foreign country:

$$P_t^{H,*} C_t^{H,*} = \alpha P_t^* C_t^*, \quad P_t^{F,*} C_t^{F,*} = (1-\alpha) P_t^* C_t^* \tag{5}$$

where $P_t = \left(P_t^H\right)^{1-\alpha} \left(P_t^F\right)^{\alpha}$ and $P_t^* = \left(P_t^{H*}\right)^{\alpha} \left(P_t^{F*}\right)^{1-\alpha}$ are respectively the home and the foreign CPI indexes. As usual with Cobb-Douglas preferences, households allocate a fixed proportion of income to each consumption bundle.

Under the assumption of "home bias" in consumption (i.e. $\alpha < \frac{1}{2}$) different regions consume goods in different proportions; therefore, even if the Law of One Price holds for all goods, the Purchasing Power Parity (PPP) may not hold at the aggregate level ($P_t \neq P_t^*$).

Combining all previous results, we can write total consumption expenditures by home's households as $P_t^H C_t^H + P_t^F C_t^F = P_t C_t$. Thus, conditional on optimal allocation of expenditures, the period budget constraint is given by:

$$P_t C_t + E_t \left\{ Q_{t,t+1}^H V_{t+1}^H \right\} \leq V_t^H + W_t^H N_t^H + \Pi_t^H - T_t^H \tag{6}$$

The remaining optimality conditions for country i are given by:

$$\beta R_t E_t \left(\frac{C_t^i}{C_{t+1}^i} \frac{P_t^i}{P_{t+1}^i} \right) = 1 \tag{7}$$

$$\chi_i C_t^i \left(N_t^i\right)^{\phi} \leq \frac{W_t^i}{P_t^i} \tag{8}$$

where $R_t = \frac{1}{E_t Q_{t,t+1}^H}$ is the (gross) nominal interest rate.

A1.2 The Terms of Trade and the Real Exchange Rate

In this section we introduce some definitions and identities that are used extensively below. First, we define the bilateral terms of trade between the home and foreign countries as the ratio of the price of goods produced in country F to that produced in country H:

$$S_t = \frac{P_t^F}{P_t^H} \tag{9}$$

As the Law of One Price holds for all goods - which implies $P_t^F = P_t^{F,*}$ and $P_t^H = P_t^{H,*}$ - the CPI and the domestic price indexes in the two regions are related according to:

$$P_t = P_t^H (S_t)^{\alpha}, \quad P_t^* = P_t^F (S_t)^{-\alpha} \tag{10}$$

Let domestic (i.e. producer prices') inflation be defined as the rate of change of domestically produced goods, i.e. as $\pi_t^i \equiv \log \frac{P_t^i}{P_{t-1}^i} = p_t^i - p_{t-1}^i$, where $p_t^i = \log P_t^i$. Taking logs of the above identities, we obtain a relation between Domestic and CPI inflation:

$$\pi_t = \pi_t^H + \alpha \Delta s_t, \quad \pi_t^* = \pi_t^F - \alpha \Delta s_t \tag{11}$$

for the home and the foreign country respectively[2].

Finally, the real exchange rate RER_t is defined as the ratio between foreign and home CPIs and is related to the terms of trade according to:

$$RER_t = \frac{P_t^*}{P_t} = (S_t)^{1-2\alpha} \tag{12}$$

A1.3 International Risk Sharing

Capital markets are complete: each household has access to a complete set of contingent claims, traded internationally. Combining the first order conditions relative to state contingent securities in the two countries, we obtain the usual result:

$$RER_t = \psi \frac{u'(C_t^*)}{u'(C_t)} = \psi \frac{C_t}{C_t^*} \tag{13}$$

where $\psi = RER_0 \frac{u'(C_0^*)}{u'(C_0)} = \frac{P_0^*}{P_0} \frac{u'(C_0^*)}{u'(C_0)}$ is a constant, reflecting initial conditions regarding relative net asset positions. If PPP holds (and this will occur in this model for

[2] Notice that the distinction between CPI inflation and domestic inflation, while important at the country level, vanishes for the monetary union as a whole. In fact, summing up the equation for the logs in prices, yields the equality $p_{CPI,t}^U = p_t^U$.

$\alpha = 1/2$), the real exchange rate $RER_t = 1$ and the marginal utilities of consumption are equated up to a constant ψ. In general, movements in the real exchange rate will be reflected in different consumption rates:

$$C_t = \frac{1}{\psi} RER_t C_t^* \tag{14}$$

Therefore, even with complete financial markets, it is not efficient to equalize consumption across countries when there is a home Bias in consumption ($\alpha < \frac{1}{2}$).

Henceforth, to keep the analysis as simple as possible, we assume initial conditions are such that $\psi = 1$.

A1.4 Supply Side

The setup of the supply side of the two countries follows Blanchard and Galí (2008). There are two sectors of production in each economy. Firms in the wholesale sector are perfectly competitive and produce a homogeneous intermediate good using labor as the only input. This output is sold to retailers who are monopolistically competitive. Retailers transform the homogeneous goods one for one into differentiated goods at no cost.

A1.4.1 Wholesale Firms and the Labor Market

The wholesale sector in each country is composed of a continuum of firms, indexed by $j \in [0, 1]$. Each firm produces a homogeneous intermediate good with an identical CRS technology:

$$X_t^i(j) = A_t^i N_t^i(j), \; for \; i = H, F(^*)$$

where the variables A_t^i represent the state of technology in country i.

In each period a fraction δ^i of the employed loses their jobs and joins the unemployment pool. Employment in firm j evolves according to

$$N_t^i(j) = (1 - \delta^i) N_{t-1}^i(j) + h_t^i(j), \; for \; i = H, F(^*)$$

where $h_t^i(j)$ is the the number of new hires for firm j in country i. Notice that, following Blanchard and Galí (2008), Thomas and Zanetti (2008) and Ravenna and Walsh (2008) among others, we assume that workers hired in a period start producing before the end of the period. We believe the instantaneous-hiring assumption is more plausible given the quarterly frequency of our model[3].

At the aggregate level, employment $N_t^i \equiv \int_0^1 N_t^i(j)dj$ is given by:

$$N_t^i = (1-\delta^i)N_{t-1}^i + h_t^i$$

where $h_t^i \equiv \int_0^1 h_t^i(j)dj$ denotes the aggregate hiring level.

We assume all unemployed in the family look for a job. The analysis thus abstracts from any transition of people in and out the labor force, which we assume to be constant and equal to 1. The number of searching workers who are available for hire in country i, U_t^i, is defined as

$$U_t^i = 1-(1-\delta^i)N_{t-1}^i, \ for\ i = H, F(^*)$$

and we define unemployment in our model as the fraction of the population who are left without a job after hiring takes place, $u_t^i = 1 - N_t^i$.

Labor market frictions are introduced by assuming that hiring labor is costly. Total hiring costs for a firm in country i are given by $G_t^i h_t^i(j)$ where G_t^i, the cost per hire in country i, is taken as given by the individual firm[4].

Following Blanchard and Galí (2008), we define the labor market tightness index as the ratio of aggregate hires to the number of searching individuals, $x_t^i \equiv \frac{h_t^i}{U_t^i}$, and we assume that recruitment costs are an increasing function of of the labor market tightness index:

$$G_t^i = A_t^i B^i \left(x_t^i\right)^{\varphi}, \ for\ i = H, F(^*)$$

where $\varphi > 0$ and B^i is a positive constant. This specification of hiring costs leads to labor market frictions which are very similar to the ones obtained with the standard search and matching model[5].

The labor market tightness index will be a crucial variable in our analysis. Since by assumption firms can hire workers only from the pool of unemployed, $x_t^i \in [0,1]$. Note that, while from the viewpoint of firms the tightness index captures the conditions of the labor market, from the viewpoint of the unemployed it can be interpreted as the probability of finding a new job in period t, i.e. as the job-finding rate. In the

[3]See Thomas and Zanetti (2008) for a brief discussion of the advantages of the contemporaneous hiring assumption in a model with quarterly frequency.

[4]G_t^i is expressed in terms of the domestic CES bundle of goods.

[5]See Blanchard and Galí (2008), p.7. In the standard Diamond-Mortensen-Pissarides model the expected hiring cost is equal to the cost of posting a vacancy times the expected time to fill it. This expected time is an increasing function of the ratio of vacancy to unemployment, which can also be expressed as a function of labor market tightness.

following we use the terms labor market tightness and job-finding rate interchangeably.

The intermediate good produced at Home is sold to Home retailers at relative price $\mu_t^H = \frac{P_{I,t}}{P_t^H}$, with $P_{I,t}$ being the nominal price of the intermediate good.

Profit maximization gives the first order condition:

$$\mu_t^H A_t^H = W_t^{H,R} (S_t)^\alpha + G_t^H - (1-\delta) E_t \left\{ \beta_{t,t+1} G_{t+1}^H \right\} \tag{15}$$

where $\beta_{t,t+1} = \beta \frac{C_t (S_t)^\alpha}{C_{t+1}(S_{t+1})^\alpha}$ and where $W_t^{H,R} = \frac{W_t^H}{P_t}$ is the real wage expressed in terms of the consumption good. Equation (15) states that the real marginal revenue product of labor (the left-hand side) has to equal its real marginal cost, which now includes not only real wages but also a component associated with hiring costs. This new component is composed of two terms. The first, G_t^H, represents the additional cost the firm faces to hire a new worker; the second additional term reflects the savings in future hiring costs resulting from increasing the number of employees today. The cyclical behavior of marginal costs in a model with labor market frictions can depart substantially from that of real wages[6].

A1.4.2 Wage Determination

In this model, the presence of hiring costs creates a positive rent for existing employment relationships. Following much of the literature, we assume wages are bargained to split this rent between the firm and the employee, according to their respective bargaining power.

Consider the generic firm j in the home country. The value of a job for firm j is simply given by the hiring costs G_t^H. Notice however that hiring costs are expressed in terms of the domestic goods, while wages are set in terms of the consumption goods. The relevant firm's surplus - expressed in terms of consumption goods - is therefore

$$\frac{P_t^H G_t^H}{P_t} \tag{16}$$

Turning to the problem of the worker, let $W_t^{H,E}$ and $W_t^{H,U}$ denote the value of being employed or unemployed, expressed in consumption units.

[6] See Krause and Lubik (2007).

The marginal value of an employment relationship is given by:

$$\begin{aligned} W_t^{H,E} &= W_t^{H,Nash} - \chi_0 C_t \left(N_t^H\right)^\phi \\ &+\beta E_t \left\{ \frac{C_t}{C_{t+1}} \left[\left(1-\delta(1-x_{t+1}^H)\right) W_{t+1}^{H,E} + \delta(1-x_{t+1}^H) W_{t+1}^{H,U} \right] \right\} \end{aligned} \tag{17}$$

The first term, $W_t^{H,Nash}$, represents the worker's wage income (in real terms); the second the disutility of work and the last the discounted expected continuation value. $\delta(1-x_{t+1}^H)$ is the probability of being unemployed at time t conditional on being employed at time t.

The corresponding value for a member who remains unemployed after hiring take place is:

$$W_t^{H,U} = \beta E_t \left\{ \frac{C_t}{C_{t+1}} \left[x_{t+1}^H W_{t+1}^{H,E} + (1-x_{t+1}^H) W_{t+1}^{H,U} \right] \right\} \tag{18}$$

Combining both conditions we obtain the household's surplus from an established relationship:

$$W_t^{H,E} - W_t^{H,U} = W_t^{H,Nash} - \chi_0 C_t \left(N_t^H\right)^\phi + \beta(1-\delta) E_t \left\{ \frac{C_t}{C_{t+1}} \left[(1-x_{t+1}^H)(W_{t+1}^{H,E} - W_{t+1}^{H,U}) \right] \right\} \tag{19}$$

Let ζ denote the share of the surplus going to the worker. The bargaining solution is given by:

$$W_t^{H,E} - W_t^{H,U} = \frac{\zeta}{1-\zeta} \frac{P_t^H G_t^H}{P_t} = \eta G_t^H (S_t)^{-\alpha} \tag{20}$$

where we make use of the fact that $\frac{P_t^H}{P_t} = (S_t)^{-\alpha}$ and we define $\eta = \frac{\zeta}{1-\zeta}$ as the relative weight of workers in the Nash bargaining, which reflects workers' bargaining power.

Imposing this condition to (19) and rearranging, we get the *Nash wage schedule*:

$$\begin{aligned} \frac{W_t^{H,Nash}}{A_t^H} (S_t^H)^\alpha &= \frac{\chi_0 C_t^{H,W} \left(N_t^H\right)^\phi}{A_t^H} + \eta B \left(x_t^H\right)^\varphi \\ &- \beta(1-\delta) E_t \left\{ \frac{C_{Ht}^W}{C_{Ht+1}^W} \frac{A_{t+1}^H}{A_t^H} \left[(1-x_{t+1}^H)(\eta B \left(x_{t+1}^H\right)^\varphi) \right] \right\} \end{aligned} \tag{21}$$

where we use the fact that the total consumption of the Home final good $C_t^{H,W} = C_t^H + C_t^{H,*} = C_t (S_t)^\alpha$.

A1.4.3 Final Goods Sector

In each country there is a measure one of monopolistic retailers indexed by z on the unit interval, each of them producing one differentiated consumption good. Due to imperfect substitutability across goods, each retailer faces a Dixit Stiglitz demand function for its product:

$$Y_t(z) = \left(\frac{P_{Ht}(z)}{P_t^H}\right)^{-\epsilon} Y_t$$

Retailers share the same technology, which transforms one unit of wholesale goods into one unit of retail goods, so that $Y_t^F(z) = X_t(z)$. Firms on the retail sector purchase intermediate goods from wholesale producers at price μ_t^H and convert it into a differentiated final good sold to households and wholesale firms. Notice that the relative price of the intermediate goods μ_t^H represent the marginal cost for the final goods producers.

We introduce nominal price rigidity using a model à la Calvo (1983). Each period, a firm faces a fixed probability $(1-\theta)$ of adjusting its price, irrespective of the time elapsed since it last reset its price. The firm resets the price in order to maximize its present discounted value, while taking into consideration that the price it chooses will remain effective for a (random) number of periods. It can be shown that the optimal price setting rule for a home firm resetting prices in period t is given by:

$$E_t\left\{\sum_{s=0}^{\infty}\theta^s Q_{t,t+s}^H Y_{t+s/t}\left(\tilde{P}_t^H - \frac{\epsilon}{\epsilon-1}P_{t+s}^H\mu_{t+s}^H\right)\right\} = 0 \tag{22}$$

where $\tilde{P}_t^H$ denotes the price newly set at time t, $Y_{t+s/t}$ is the level of output in period t+s for a firm resetting its price in period t and $\frac{\epsilon}{\epsilon-1}$ is the gross desired markup.

Log-linearizing around a zero inflation steady state the optimal price setting rule and the price index equation $P_t^H = \left[(1-\theta)(\tilde{P}_t^H)^{1-\epsilon} + \theta(P_{t-1}^H)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$, we get the New Keynesian Phillips Curve:

$$\hat{\pi}_t^H = \beta E_t\hat{\pi}_{t+1}^H + \lambda\widehat{mc}_t^H \tag{23}$$

where $\hat{\pi}_t^H$ is domestic (i.e. producer prices') inflation, $\widehat{mc}_t^H = \hat{\mu}_t^H$ represent the log deviation of real marginal cost from its steady state value and $\lambda = (1-\beta\theta)(1-\theta)/\theta$. Note that while (23) looks like a standard New Keynesian Phillips Curve, the dynamics of the real marginal costs are now substantially different from the ones of a standard NK model, as they are deeply affected by the labor market institutions.

In fact, log-linearizing equation (32) we can rewrite marginal costs as:

$$\widehat{mc}_t^H = \frac{W(S)^\alpha}{\mu}\left(\hat{w}_t^{H,R} + \alpha\hat{s}_t - \hat{a}_t^H\right) + \frac{g\varphi}{\mu}\hat{x}_t^H - \beta(1-\delta)\frac{g}{\mu}E_t\left\{\hat{\beta}_{t,t+1} + \Delta a_{t+1}^H + \varphi\hat{x}_{t+1}^H\right\} \tag{24}$$

where variables with hat denote log-deviations from steady state, variables without subscript steady state values and g is the steady state value of unit hiring costs G_t^H. Marginal costs depend not only on the evolution of real wages, terms of trade and productivity, as in the standard New Keynesian model; they also depend on current labor market conditions (x_t) and on the future labor market conditions, as captured by the last term on the right-hand side.

A1.5 Market Clearing Conditions

Consider the home country. The clearing of the market for good j requires:

$$\begin{aligned} Y_t(j) - G_t^H h_t^H(j) &= C_t^H(j) + C_t^{H,*}(j) \\ &= \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\epsilon}\left[C_t^H + C_t^{H,*}\right] \\ &= \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\epsilon} C_t(S_t)^\alpha \end{aligned} \tag{25}$$

Aggregating across varieties, we obtain the aggregate goods market clearing condition for home:

$$Y_t - G_t^H h_t^H = C_t(S_t)^\alpha D_t^H$$

where $D_t^H \equiv \int_0^1 \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\epsilon} dj$.

Similar conditions hold for the foreign country:

$$Y_t^*(j) - G_t^F h_t^F = \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon} C_t^*(S_t)^{-\alpha} \tag{26}$$

$$Y_t^*(j) - G_t^F h_t^F = C_t^*(S_t)^{-\alpha} D_t^F \tag{27}$$

where $D_t^F \equiv \int_0^1 \left(\frac{P_t^F(z)}{P_t^F}\right)^{-\epsilon} dz$. Notice that the assumption of Cobb-Douglas preferences over the home and foreign goods allows us to derive a simple relation between the terms of trade and relative output:

$$S_t = \frac{P_t^F}{P_t^H} = \frac{Y_t - G_t^H h_t^H}{Y_t^* - G_t^F h_t^F} \frac{D_t^F}{D_t^H} \tag{28}$$

Equation (28) simply states the relative price of domestic (foreign) goods is inversely related to the quantity produced in the two regions (net of aggregate hiring costs)[7].

A2. The Social Planner 's Problem

In this section we derive the so-called "constrained efficient allocation". Following Blanchard and Galí (2008), we assume that the social planner maximizes the welfare of the Union, taking as given the technological constraints and the labor market frictions that are present in the decentralized economy. In other words, the social planner cannot eliminate or reduce hiring costs, which are simply taken as a fact of life; he can, however, internalize the effects of variations in employment on labor market tightness and, hence, on hiring costs.

Proposition 1 *Employment is invariant to productivity shocks under the constrained efficient allocation.*

Proof. Given simmetry in preferences and technology, the social planner chooses an equilibrium in which the goods, in each country, are produced and consumed in identical quantities $C_{jt}^i(z) = C_{jt}^i$.

Hence, the Union's optimal allocation can be described as the solution of the following social planner's problem:

$$\begin{aligned}
&Max \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi_0 \frac{\left(N_t^H\right)^{1+\phi}}{1+\phi} + \log C_t^* - \chi_1 \frac{\left(N_t^F\right)^{1+\phi}}{1+\phi} \right\} \\
&s.t. \\
&C_t^H + C_t^{H,*} = A_t^H N_t^H - G_t^H h_t^H \\
&C_t^F + C_t^{F,*} = A_t^F N_t^F - G_t^F h_t^F
\end{aligned}$$

[7]Notice that equation (28) allows us to pin down the steady state level of S_t:

$$S = \frac{A^H N^H (1 - g\delta)}{A^F N^F (1 - g^* \delta^*)}$$

where $g^i = B^i (x^i)^{\varphi}\ for\ i = H, F(^*)$.

where C_t^i, $C_t^{j,i}$, G_t^i and h_t^i are as defined before. Notice that the previous constraints already embed the optimal condition whereby the different good types in any given country should be produced and consumed in identical quantities.

Maximization with respect to consumption leads to the following optimality conditions:

$$\begin{aligned}\frac{1-\alpha}{\alpha} &= \frac{C_t^H}{C_t^{H,*}} = \frac{C_t^{F,*}}{C_t^F} \\ \frac{\nu_t}{\xi_t} &= \frac{\alpha}{1-\alpha}\frac{C_t^H}{C_t^F} = MRS^H = \frac{1-\alpha}{\alpha}\frac{C_t^{H,*}}{C_t^{F,*}} = MRS^F\end{aligned}$$

where ξ_t is the shadow value of an additional unit of the good produced at home and ν_t is the shadow value of an additional unit of the foreign good.

Solving the social planner's problem with respect to home employment, we obtain:

$$\begin{aligned}\chi_0 \frac{\left(N_t^H\right)^{\phi} C_t^{H,W}}{A_t^H} &\leq 1 - B(1+\varphi)\left(x_t^H\right)^{\varphi} \\ &+B(1-\delta)E_t\left\{\frac{C_t^{H,W}}{C_{t+1}^{H,W}}\frac{A_{t+1}^H}{A_t^H}B\left[\begin{array}{c}\left(x_{t+1}^H\right)^{\varphi} + \\ \varphi\left(x_{t+1}^H\right)^{\varphi}\left(1-x_{t+1}^H\right)\end{array}\right]\right\}\end{aligned} \tag{29}$$

which must hold with strict equality if $N_t^H < 1$.

The important point to note is that the above expression implies a *constant level of employment*. Note in fact that, world consumption of the home good is proportional to productivity

$$C_t^{H,W} \equiv C_t^H + C_t^{H,*} = A_t^H(N_t^H - B\left(x_t^H\right)^{\varphi} h_t^H) \tag{30}$$

It follows that the optimality condition does not depend on the productivity levels prevailing at home (or foreign). ■

This invariance is a consequence of the assumption of a log utility function, which implies offsetting income and substitution effects on the labor supply. The fact that employment is constant is a useful result, as it implies that any fluctuations in employment are inefficient[8].

To determine the efficient level of employment we can proceed in two steps. First, the efficient level for the labor market tightness indicator, x_E^H, is implicitely

[8]Blanchard and Galí (2008) get the same result in the context of a one-country model.

determined as the solution to

$$\chi_0 \left(\frac{x^H}{\delta + (1-\delta)x^H} \right)^{1+\phi} \left(1 - \delta B \left(x^H\right)^{\varphi}\right) \leq 1 - (1 - \beta(1-\delta))(1+\varphi) B \left(x^H\right)^{\varphi} - \beta(1-\delta)\varphi B \left(x^H\right)^{1+\varphi}$$

Second, the optimal level of employment at home is given by

$$N_E^H = \frac{x_E^H}{\delta + (1-\delta)x_E^H}$$

The optimal employment level depends therefore on the separation rate δ, on the hiring costs' scaling parameter B, on the sensitivity of hiring costs to labor market conditions φ and on parameters influencing the disutility of working (ϕ and χ_0).

A constant employment level implies that output is proportional to home productivity ($Y_t = A_t^H N_E^H$) while consumption depends on both home and foreign productivity ($C_t = \left(A_t^H\right)^{1-\alpha} \left(A_t^F\right)^{\alpha} N_E^H (1 - \delta B \left(x_E^H\right)^{\varphi}$).

Similar conditions and the same conclusions hold for the foreign country.

A3. Equilibrium under Flexible Prices

Proposition 2 *Under flexible prices and no real wage rigidities, employment is invariant to productivity shocks. The decentralized equilibrium corresponds to the constrained efficient equilibrium only if two conditions are satisfied: 1. The Hosios condition holds, i.e. $\varphi = \eta$; 2. Monopolistic distortions in the final goods market are eliminated through a production subsidy.*

Proof. Under flexible prices, the optimal price setting rule of final goods firms takes the form of a mark-up over the real marginal costs:

$$\frac{P_t^H(i)}{P_t^H} = \frac{\epsilon}{\epsilon - 1}\mu_t^H \tag{31}$$

where the firm's real marginal cost is (expressed in terms of domestic goods):

$$\mu_t^H = \frac{W_t^{H,R}}{A_t^H} (S_t)^{\alpha} + B \left(x_t^H\right)^{\varphi} - \beta(1-\delta) E_t \left\{ \frac{C_t (S_t)^{\alpha}}{C_{t+1} (S_{t+1})^{\alpha}} \frac{A_{t+1}^H}{A_t^H} B \left(x_{t+1}^H\right)^{\varphi} \right\} \tag{32}$$

and $W_t^{H,R} = \frac{W_t^H}{P_t}$ is the real wage expressed in terms of the consumption good.

The key difference between the supply side in our model and in a standard New Keynesian model with a neoclassical labor market is the behavior of the real marginal cost. In a model with a competitive labor market the real marginal cost is strictly related to the evolution of the real wage:

$$\mu_t^H = \frac{W_t^{H,R}}{A_t^H} (S_t)^\alpha \tag{33}$$

In our model, which embeds the NK model as a special case, the presence of hiring costs creates a wedge between the real wage and the marginal costs relevant for the firm, which in turn are essential to explain inflation dynamics.

In a symmetric equilibrium, $P_t^H(i) = P_t^H$ for all $i \in [0,1]$, and hence the optimal price setting implies:

$$\mu_t^H = \frac{\epsilon - 1}{\epsilon}$$

for all t. When shocks occur, each firm varies its prices and hiring decisions to keep the marginal cost constant. It follows that in equilibrium:

$$\frac{W_t^{H,R}}{A_t^H} (S_t)^\alpha = \mu - B\left(x_t^H\right)^\varphi + \beta(1-\delta)E_t\left\{\frac{C_t (S_t)^\alpha}{C_{t+1}(S_{t+1})^\alpha}\frac{A_{t+1}^H}{A_t^H} B\left(x_{t+1}^H\right)^\varphi\right\} \tag{34}$$

where μ is the inverse of the mark-up. Similar conditions hold for the foreign country.

Finally, substituting the Nash wage rule (21) in the (34), we obtain the *equilibrium under Nash bargaining*:

$$\begin{aligned} \frac{\chi_0 C_t^{H,W}\left(N_t^H\right)^\phi}{A_t^H} &= \mu - (1+\eta)B\left(x_t^H\right)^\varphi \\ &\quad + \beta(1-\delta)E_t\left\{\frac{C_t^{H,W}}{C_{t+1}^{H,W}}\frac{A_{t+1}^H}{A_t^H}\left[\left(1+\eta(1-x_{t+1}^H)\right)B\left(x_{t+1}^H\right)^\varphi\right]\right\} \end{aligned} \tag{35}$$

This condition determines the evolution of (un)employment under Nash bargaining. It is easy to verify that the decentralized equilibrium with Nash bargained wages involves a constant job-finding rate and, hence, a constant level of unemployment. Again, this crucial result derives from the assumption of a utility function that is log in consumption. Combining the equilibrium under Nash bargaining and the Nash wage rule, we can determine the actual behavior of real wages:

$$\frac{W_t^{H,R}}{A_t^H}(S_t)^\alpha = \mu - \left[1-\beta(1-\delta)\right]B\left(x_M^H\right)^\varphi \tag{36}$$

where x_M^H is the (constant) equilibrium job-finding rate, which is solution of (35).

Compare the equilibrium under the efficient allocation (29) and under the decentralized equilibrium (35). While the (un)employment level is constant in both cases, these levels generally differs. Mainly due to the monopolistic distortions, the unemployment level under the efficient allocation is higher than the one prevailing in the decentralized solution. It is easy to verify that the conditions under which the two equilibria correspond, are the following:

1. Perfect competition in the goods market, i.e. $\mu = 1$.

2. $\varphi = \eta$, i.e. the share of the surplus that goes to workers has to coincide with the elasticity of hiring costs with respect to the job-finding rate. ■

Similar conditions and exactly the same conclusions hold for the foreign country.

A3.1 Introducing Real Wage Rigidities

As first emphasized by Hall (2005) and Shimer (2005), the introduction of real wage rigidity considerably improves the performance of the matching models in terms of the dynamics of the labor market. The Nash bargained wage implies in fact a real wage volatility which is too high relatively to empirical evidence. As a consequence, the standard matching model finds it difficult to replicate the response of labor market variables - and in particular unemployment - to productivity shocks. This issue is especially important for the euro area, which is characterized by a considerable degree of wage rigidity[9].

To solve this problem, we follow much of the literature and introduce real wage rigidity by employing a version of Hall's (2005) notion of wage norm. A wage norm may arise as a result of social conventions that constrain wage adjustment for existing and newly hired workers. One way to model this is to assume that the real wage $W_t^{H,R}$ is a weighted average of the desired wage (the Nash bargained wage $W_t^{H,Nash}$) and a wage norm $\bar{W}^H$, which is simply assumed to be the wage prevailing in steady state. Specifically, we assume the real wage is determined as follows:

$$W_t^{H,R} = \left(W_t^{H,Nash}\right)^{1-\gamma} \left(\bar{W}^H\right)^{\gamma}, \quad W_t^{F,R} = \left(W_t^{F,Nash}\right)^{1-\gamma^*} \left(\bar{W}^F\right)^{\gamma^*} \tag{37}$$

where γ^i is an index of the real wage rigidities present in the economy, with $0 \leq \gamma^i \leq 1$. As shown by Hall (2005), this rule follows inside the range defined by the bargaining set and thus remains robust to the Barro (1977) critique.

[9]See e.g. Dickens et al. (2007) and Du Caju et al. (2008) for some evidence on nominal and real wage rigidity in the euro area.

The introduction of such a wage rule modifies the decentralized equilibrium solution. Consider for instance the home country. In equilibrium:

$$\frac{\left(W_t^{H,Nash}\right)^{1-\gamma}\left(\bar{W}^H\right)^{\gamma}}{A_t^H}\left(S_t\right)^{\alpha}=\mu-B\left(x_t^H\right)^{\varphi}+\beta(1-\delta)E_t\left\{\frac{C_t^{H,W}}{C_{t+1}^{H,W}}\frac{A_{t+1}^H}{A_t^H}B\left(x_{t+1}^H\right)^{\varphi}\right\} \tag{38}$$

As shown before, the Nash bargained wage varies proportionally to $A_t^H(S_t)^{-\alpha}$ and thus neutralizes the effect of productivity changes on employment. When real wage rigidities are present, instead, the wages do not move enough to absorb the impact of technology shocks. As a result, in a decentralized equilibrium with sticky wages, employment will *not* be constant. As in Blanchard and Galí (2007 and 2008), the presence of real wage rigidities introduces a substantial difference between the constrained efficient solution (where employment is constant) and the decentralized solution (where employment varies with productivity shocks). For this reason, to the extent that γ or γ^* are different from zero, it is not possible for the monetary authority to stabilize simultaneously inflation and unemployment. There is no "divine coincidence".

A4. Equilibrium Fluctuations under sticky prices

Let denote with $\hat{X}$ the deviation of a variable X from its steady state value, and let us define union-wide variables as $\hat{x}_t^U=\frac{\hat{x}_t^H+\hat{x}_t^F}{2}$. When prices are sticky, we can reduce the model to the following equations:

- The two Phillips curves:

$$\hat{\pi}_t^H = \beta E_t\hat{\pi}_{t+1}^H-h_0\hat{u}_t^H+h_L\hat{u}_{t-1}^H+h_FE_t\hat{u}_{t+1}^H-\gamma h_T\left(\hat{a}_t^H-\alpha\hat{s}_t\right) \tag{39}$$
$$\hat{\pi}_t^F = \beta E_t\hat{\pi}_{t+1}^F-h_0^*\hat{u}_t^F+h_L^*\hat{u}_{t-1}^F+h_F^*E_t\hat{u}_{t+1}^F-\gamma^* h_T^*\left(\hat{a}_t^F+\alpha\hat{s}_t\right) \tag{40}$$

where the parameters for Home are (analogous expressions hold for Foreign):

$$h_0 = \frac{1}{1-u}\frac{\lambda}{\mu}\left[\begin{array}{c}\varphi g\left(1+\eta(1-\gamma)\right)d_0+\beta(1-\delta_H)g\left[\varphi+\eta(1-\gamma)(1-x)\left(\varphi-\frac{x^H}{1-x^H}\right)\right]d_1\\ +(1-\gamma)\chi C_H^W N^{\phi}\left(\tau_0+\phi\right)-\beta(1-\delta_H)g\left(1+\eta(1-\gamma)(1-x)\right)\left(\tau_0-\tau_1\right)\end{array}\right]$$

$$h_L = -\frac{1}{1-u}\frac{\lambda}{\mu}\left[\begin{array}{c}-\varphi g\left(1+\eta(1-\gamma)\right)d_1\\ +\left[\left((1-\gamma)\chi C_H^W N^{\phi}-\beta(1-\delta_H)g\left(1+\eta(1-\gamma)(1-x)\right)\right)\right]\tau_1\end{array}\right]$$

$$
\begin{aligned}
h_F &= -\frac{1}{1-u}\frac{\lambda}{\mu}\left[\begin{array}{c} -\beta(1-\delta_H)g\left[\varphi+\eta(1-\gamma)(1-x)\left(\varphi-\frac{x^H}{1-x^H}\right)\right]d_0 \\ +\left[\beta(1-\delta_H)g\left(1+\eta(1-\gamma)(1-x)\right)\right]\tau_0 \end{array}\right] \\
h_T &= \frac{\lambda}{\mu}W^{H,R}\left(S\right)^{\alpha}
\end{aligned}
$$

- Union wide Euler:

$$
\hat{c}_t^U = E_t\hat{c}_{t+1}^U - (\hat{\imath}_t - E_t\pi_{t+1}^U) \tag{41}
$$

- Relative demand condition:

$$
(1-2\alpha)\,\hat{s}_t = \hat{c}_t - \hat{c}_t^* \tag{42}
$$

- Definition of the terms of trade:

$$
\Delta\hat{s}_t = \hat{\pi}_t^H - \hat{\pi}_t^F \tag{43}
$$

- Market clearing conditions:

$$
\hat{c}_t = \hat{a}_t^H - \frac{\tau_0}{1-u^H}\hat{u}_t^H - \frac{\tau_1}{1-u^H}\hat{u}_{t-1}^H - \alpha\hat{s}_t \tag{44}
$$

$$
\hat{c}_t^* = \hat{a}_t^F - \frac{\tau_0^*}{1-u^F}\hat{u}_t^F - \frac{\tau_1^*}{1-u^F}\hat{u}_{t-1}^F + \alpha\hat{s}_t \tag{45}
$$

where $\tau_0^i = \frac{1-g^i\left(1+\varphi^i\right)}{1-\delta^i g^i}$ and $\tau_1^i = \frac{g^i\left(1-\delta^i\right)\left(1+\varphi^i(1-x)\right)}{1-\delta^i g^i}$.

Given initial conditions, equations (39)-(45) and a monetary policy rule then solve for the equilibrium path of $\left\{\hat{\pi}_t^H, \hat{\pi}_t^F, \hat{u}_t^H, \hat{u}_t^F, \hat{c}_t, \hat{c}_t^*, \hat{s}_t, \hat{\imath}_t\right\}_{t=0}^{\infty}$.

Note that if the monetary policy rule does not contain the interest rate (such as, e.g., a targeting rule of the form $\pi_t^U = 0$), one can solve for the equilibrium path of $\left\{\hat{\pi}_t^H, \hat{\pi}_t^F, \hat{u}_t^H, \hat{u}_t^F, \hat{c}_t, \hat{c}_t^*, \hat{s}_t\right\}_{t=0}^{\infty}$ and then back out the equilibrium path of the interest rate by using the Euler equation (41). Moreover, for deriving optimal monetary policy, it is useful to characterize the equilibrium in as few variables as possible. For this reason, we would like to write the model in the 4 variables $\hat{\pi}_t^H, \hat{\pi}_t^F, \hat{u}_t^H$ and $\hat{u}_t^F$. Substituting equations (42), (44) and (45) into (39)-(41) and (43), we obtain equations (46)-(48):

1. Phillips curve at home:

$$\hat{\pi}_t^H = \beta E_t \left(\hat{\pi}_{t+1}^H\right) + \left[\begin{array}{c} -\kappa_0 \hat{u}_t^H + \kappa_L \hat{u}_{t-1}^H + \kappa_F E_t \hat{u}_{t+1}^H \\ +\kappa_1 \hat{u}_t^F + \kappa_2 \hat{u}_{t-1}^F - \kappa_a \left\{(1-\alpha)\hat{a}_t^H + \alpha \hat{a}_t^F\right\} \end{array} \right] \tag{46}$$

2. Phillips curve abroad:

$$\hat{\pi}_t^F = \beta E_t \left(\hat{\pi}_{t+1}^F\right) + \left[\begin{array}{c} -\kappa_0^* \hat{u}_t^F + \kappa_L^* \hat{u}_{t-1}^F + \kappa_F^* E_t \hat{u}_{t+1}^F \\ +\kappa_1^* \hat{u}_t^H + \kappa_2^* \hat{u}_{t-1}^H - \kappa_a^* \left\{(1-\alpha)\hat{a}_t^F + \alpha \hat{a}_t^H\right\} \end{array} \right] \tag{47}$$

3. Relative demand equation:

$$\begin{aligned} \hat{\pi}_t^F - \hat{\pi}_t^H &= \Delta \hat{a}_t^H - \Delta \hat{a}_t^F - \frac{\tau_0}{1-u^H} \Delta \hat{u}_t^H - \frac{\tau_1}{1-u^H} \Delta \hat{u}_{t-1}^H \\ &\quad + \frac{\tau_0^*}{1-u^F} \Delta \hat{u}_t^F + \frac{\tau_1^*}{1-u^F} \Delta \hat{u}_{t-1}^F \end{aligned} \tag{48}$$

where

$$\begin{aligned} \kappa_a &= \lambda \gamma \mu W^{H,R} (S)^{\alpha} ; \kappa_a^* = \lambda^* \gamma^* \mu^* W^{F,R} (S)^{-\alpha} \\ \kappa_0 &= \left[h_0 + \frac{\lambda \kappa_a \alpha \tau_0}{1-u^H} \right] ; \kappa_0^* = \left[h_0^* + \frac{\lambda^* \kappa_a^* \alpha \tau_0^*}{1-u^F} \right] \\ \kappa_L &= \left[h_L - \frac{\lambda \kappa_a \alpha \tau_1}{1-u^H} \right] ; \kappa_L^* = \left[h_L^* - \frac{\lambda^* \kappa_a^* \alpha \tau_1^*}{1-u^F} \right] \\ \kappa_F &= h_F ; \kappa_F^* = h_F^* \\ \kappa_1 &= \left[\frac{\lambda \kappa_a \alpha \tau_0^*}{1-u^F} \right] ; \kappa_1^* = \left[\frac{\lambda^* \kappa_a^* \alpha \tau_0}{1-u^H} \right] \\ \kappa_2 &= \left[\frac{\lambda \kappa_a \alpha \tau_1^*}{1-u^F} \right] ; \kappa_2^* = \left[\frac{\lambda^* \kappa_a^* \alpha \tau_1}{1-u^H} \right] \end{aligned}$$

Given initial conditions, equations (46)-(48) and a monetary policy targeting rule then solve for the equilibrium path of $\left\{\hat{\pi}_t^H, \hat{\pi}_t^F, \hat{u}_t^H, \hat{u}_t^F\right\}_{t=0}^{\infty}$.

Appendix B - Welfare

B1. Derivation of the Micro-Founded Loss Function

In this section, we derive the micro-founded loss function from the welfare criterion of the currency union. We define the welfare criterion of the currency union as the utilitarian welfare function:

$$U_0^{CU} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} U(C_t, N_t^H) + \frac{1}{2} U(C_t^*, N_t^F) \right] \tag{49}$$

Our utility specification for the home family is

$$u(C_t, N_t^H) = \log C_t - \chi \frac{\left(N_t^H\right)^{1+\phi}}{1+\phi}$$

which we can approximate up to a second order as

$$\begin{aligned} u(C_t, N_t^H) \;\simeq\;& \log C + \hat{c}_t \\ & - \chi \frac{\left(N^H\right)^{1+\phi}}{1+\phi} - \chi \left(N^H\right)^{1+\phi} \hat{n}_t - \frac{1}{2} \chi (1+\phi) \left(N^H\right)^{1+\phi} \hat{n}_t^2 \end{aligned} \tag{50}$$

We can express $\hat{c}_t$ in terms of $\hat{n}_t$ and other variables by using the market clearing condition for variety z, its demand equation and the law of one price:

$$\begin{aligned} A_t^H \left(N_t^H(z) - B \left(x_t^H\right)^{\varphi} h_t^H(z) \right) &= C_t^H(z) + C_t^{H,*}(z) \\ &= C_t^H \left(\frac{P_t^H(z)}{P_t^H} \right)^{-\epsilon} + C_t^{H,*} \left(\frac{P_t^{H,*}(z)}{P_t^{H,*}} \right)^{-\epsilon} \\ &= \left(C_t^H + C_t^{H,*} \right) \left(\frac{P_t^H(z)}{P_t^H} \right)^{-\epsilon} \\ &= C_t^{H,W} \left(\frac{P_t^H(z)}{P_t^H} \right)^{-\epsilon} \end{aligned}$$

and integrating over all varieties, we get

$$A_t^H \left(N_t^H - B \left(x_t^H\right)^{\varphi} h_t^H \right) = C_t^{H,W} D_t^H$$

where

$$D_t^H = \int_0^1 \left(\frac{P_t^H(z)}{P_t^H} \right)^{-\epsilon} dz$$

Using the risk sharing condition, we know that $C_t^{H,W} = S_t^\alpha C_t$ and we get

$$A_t^H \left(N_t^H - B \left(x_t^H \right)^\varphi h_t^H \right) = S_t^\alpha C_t D_t^H$$

and dividing by A_t^H, we get

$$N_t^H - B \left(x_t^H \right)^\varphi h_t^H = \frac{S_t^\alpha C_t D_t^H}{A_t^H}$$

and taking log differences, we get

$$\log(N_t^H - B \left(x_t^H \right)^\varphi h_t^H) - \log(N^H(1 - \delta g)) = \alpha \hat{s}_t + \hat{c}_t + d_t^H - \hat{a}_t^H \tag{51}$$

Note that in the zero inflation steady state, there is no price dispersion and hence $d^H = 0$.

In what follows, we find a second order approximation to the left-hand side of equation (51) (note that the right hand side is an exact log linear expression). Remember that $h_t^H = N_t^H - (1 - \delta_H) N_{t-1}^H$ and $U_t^H = 1 - N_{t-1}^H (1 - \delta_H)$ and, therefore, we get

$$\begin{aligned} N_t^H - B \left(x_t^H \right)^\varphi h_t^H) &= N_t^H - B \left(\frac{h_t^H}{U_t^H} \right)^\varphi h_t^H \\ &= N_t^H - B \left(N_t^H - (1 - \delta_H) N_{t-1}^H \right)^{\varphi+1} \left(1 - N_{t-1}^H (1 - \delta_H) \right)^{-\varphi} \end{aligned} \tag{52}$$

Let's define

$$f(N_t, N_{t-1}) = N_t - B \left(N_t - (1 - \delta) N_{t-1} \right)^{\varphi+1} \left(1 - N_{t-1}(1 - \delta) \right)^{-\varphi} \tag{53}$$

then one can show that in staedy state ($N_t = N_{t-1} = N$) the first and second derivatives of f are

$$
\begin{aligned}
f &= (1-\delta g)N &(54)\\
f_1 &= 1-(\varphi+1)g &(55)\\
f_{11} &= -\frac{(\varphi+1)\varphi g}{\delta N} &(56)\\
f_2 &= (1-\delta)g(1+\varphi(1-x)) &(57)\\
f_{22} &= -\frac{(\varphi+1)\varphi(1-\delta)^2 g\,(1-x)^2}{\delta N} &(58)\\
f_{12} &= \frac{(\varphi+1)\varphi(1-\delta)g(1-x)}{\delta N} &(59)
\end{aligned}
$$

Now, let's do a second order approximation to the left hand side of equation (51) around the steady state, using the fact that, up to a second order, $\frac{N_t-N}{N}=\hat{n}_t+\frac{1}{2}\hat{n}_t^2$:

$$
\begin{aligned}
\log(f(N_t,N_{t-1})) \simeq\ & \log(f)+\frac{f_1}{f}(N_t-N)+\frac{f_2}{f}(N_{t-1}-N)\\
&+\frac{1}{2}\frac{f_{11}f-(f_1)^2}{f^2}(N_t-N)^2+\frac{1}{2}\frac{f_{22}f-(f_2)^2}{f^2}(N_{t-1}-N)^2\\
&+\frac{f_{12}f-f_1f_2}{f^2}(N_t-N)(N_{t-1}-N)\\
\simeq\ & \log(f)+\left(\frac{f_1N}{f}\right)(\hat{n}_t)+\left(\frac{f_2N}{f}\right)(\hat{n}_{t-1})\\
&+\frac{1}{2}\left(\frac{f_1N}{f}+\frac{f_{11}f-(f_1)^2}{f^2}N^2\right)\hat{n}_t^2\\
&+\frac{1}{2}\left(\frac{f_2N}{f}+\frac{f_{22}f-(f_2)^2}{f^2}N^2\right)\hat{n}_{t-1}^2\\
&+\left(\frac{f_{12}f-f_1f_2}{f^2}\right)N^2\hat{n}_t\hat{n}_{t-1} \qquad (60)
\end{aligned}
$$

Lemma 3 *Up to a second order approximation,* $d_t^i \simeq \frac{\epsilon}{2}var_z\left(p_t^i(z)\right)$*. Proof: Blanchard and Galí (2008).*

Putting equations (50) - (60) and Lemma 3 together, we get a second order approximation to the utility function of the home family:

$$\begin{aligned} u(C_t, N_t^H) - u(C, N^H) \simeq\ & \hat{a}_t^H - \frac{\epsilon}{2} var_z \left(p_t^H(z)\right) - \alpha \hat{s}_t \\ & + \left(\frac{f_1 N^H}{f} - \chi \left(N^H\right)^{1+\phi}\right)\left(\hat{n}_t^H\right) + \left(\frac{f_2 N^H}{f}\right)\left(\hat{n}_{t-1}^H\right) \\ & + \frac{1}{2}\left(\frac{f_1 N^H}{f} + \frac{f_{11} f - (f_1)^2}{f^2}\left(N^H\right)^2 - \chi(1+\phi)\left(N^H\right)^{1+\phi}\right)\left(\hat{n}_t^H\right)^2 \\ & + \frac{1}{2}\left(\frac{f_2 N^H}{f} + \frac{f_{22} f - (f_2)^2}{f^2}\left(N^H\right)^2\right)\left(\hat{n}_{t-1}^H\right)^2 \\ & + \left(\frac{f_{12} f - f_1 f_2}{f^2}\right)\left(N^H\right)^2 \hat{n}_t^H \hat{n}_{t-1}^H \end{aligned} \tag{61}$$

and similarily for foreign, except that the sign on the terms of trade is inverted. Note that when the economy fluctuates around the efficient steady state, one can show that $\chi^i \left(N^i\right)^{1+\phi} - \frac{f_1^i + \beta f_2^i}{f^i} N^i = 0$ and, hence, the coefficients on the linear terms are equal to zero and we can simplify the coefficients on the quadratic terms. Also, note that up to a first order $\hat{n}_t^i = -\frac{1}{N^i}\hat{u}_t^i$.

Lemma 4 $\sum_{t=0}^{\infty} \beta^t var_z^i \left(p_t^i(z)\right) = \frac{1}{\lambda^i} \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t^i\right)^2$. *Proof: Woodford (2003).*

Finally, using the welfare criterion of the currency union (49), we get the following second order approximation to utility losses for the currency union

$$L_0 \simeq E_0 \sum_{t=0}^{\infty} \beta^t [\frac{\omega_\pi}{2}\left(\hat{\pi}_t^H\right)^2 + \frac{\omega_\pi^*}{2}\left(\hat{\pi}_t^F\right)^2 + \frac{\omega_u}{2}\left(\hat{u}_t^H\right)^2 + \frac{\omega_u^*}{2}\left(\hat{u}_t^F\right)^2 - \frac{\omega_{u_L}}{2}\hat{u}_t^H \hat{u}_{t-1}^H - \frac{\omega_{u_L}^*}{2}\hat{u}_t^F \hat{u}_{t-1}^F] + t.i.p. \tag{62}$$

where

$$\begin{aligned} \omega_\pi^i &= \frac{\varepsilon^i}{2\lambda^i} \\ \omega_u^i &= \left(\frac{\chi^i \phi \left(N^i\right)^{\phi-1}}{2} - \frac{f_{11}^i f^i - \left(f_1^i\right)^2}{2\left(f^i\right)^2} - \beta \frac{f_{22}^i f^i - \left(f_2^i\right)^2}{2\left(f^i\right)^2}\right) \\ \omega_{u_L}^i &= \frac{f_{12}^i f^i - f_1^i f_2^i}{\left(f^i\right)^2} \end{aligned}$$

for $i = H, F(^*)$, where f_{kl}^i is the derivative of (53) w.r.t. to elements $k, l \in (1, 2)$ evaluated in steady state and where t.i.p. refers to terms independent of policy. Note that because of log utility in consumption welfare losses in each country correspond to a percentage reduction in steady state consumption.

B2. Optimal monetary policy under commitment

Given the second order approximation to the welfare criterion (62), we find optimal monetary policy by minimizing L_0 subject to the equilibrium equations of our model. To simplify the optimization problem, we reduce the number of variables in our equilibrium equations to the four variables in the loss function (see Appendix above): $\left\{\hat{\pi}_t^H, \hat{\pi}_t^F, \hat{u}_t^H, \hat{u}_t^F\right\}$. Optimal monetary policy under commitment then solves the following problem:

$$\min_{\left\{\hat{\pi}_t^H, \hat{\pi}_t^F, \hat{u}_t^H, \hat{u}_t^F\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \frac{\omega_\pi}{2}\left(\hat{\pi}_t^H\right)^2 + \frac{\omega_\pi^*}{2}\left(\hat{\pi}_t^F\right)^2 + \frac{\omega_u}{2}\left(\hat{u}_t^H\right)^2 + \frac{\omega_u^*}{2}\left(\hat{u}_t^F\right)^2 \\ -\frac{\omega_{u_L}}{2}\hat{u}_t^H\hat{u}_{t-1}^H - \frac{\omega_{u_L}^*}{2}\hat{u}_t^F\hat{u}_{t-1}^F \end{array} \right]$$

subject to the two Phillips curves (46) and (47) and the relative demand equation (48) for all $t \geq 0$. The FOCs are (for all $t \geq 0$)

$$\left(\hat{\pi}_t^H\right) \ : \ \omega_\pi \hat{\pi}_t^H - \mu_t^H + \mu_{t-1}^H + \mu_t^D = 0 \qquad (63)$$

$$\left(\hat{\pi}_t^F\right) \ : \ \omega_\pi^* \hat{\pi}_t^F - \mu_t^F + \mu_{t-1}^F - \mu_t^D = 0 \qquad (64)$$

$$\begin{aligned} \left(u_t^H\right) \ : \ & 0 = \omega_u \hat{u}_t^H - \frac{\omega_{u_L}}{2}\hat{u}_{t-1}^H - \frac{\omega_{u_L}}{2}\beta E_t \hat{u}_{t+1}^H \\ & -\kappa_0 \mu_t^H + \kappa_L \beta E_t \mu_{t+1}^H + \kappa_F \beta^{-1}\mu_{t-1}^H + \kappa_1^* \mu_t^F + \kappa_2^* \beta E_t \mu_{t+1}^F \\ & -\frac{\tau_0}{1-u^H}\mu_t^D + \left(\frac{\tau_0 - \tau_1}{1-u^H}\right)\beta E_t \mu_{t+1}^D + \frac{\tau_1}{1-u^H}\beta^2 E_t \mu_{t+2}^D \end{aligned} \qquad (65)$$

$$\begin{aligned} \left(u_t^F\right) \ : \ & 0 = \omega_u^* \hat{u}_t^F - \frac{\omega_{u_L}^*}{2}\hat{u}_{t-1}^F - \frac{\omega_{u_L}^*}{2}\beta E_t \hat{u}_{t+1}^F \\ & -\kappa_0^* \mu_t^F + \kappa_L^* \beta E_t \mu_{t+1}^F + \kappa_F^* \beta^{-1}\mu_{t-1}^F + \kappa_1 \mu_t^H + \kappa_2 \beta E_t \mu_{t+1}^H \\ & +\frac{\tau_0^*}{1-u^F}\mu_t^D - \left(\frac{\tau_0^* - \tau_1^*}{1-u^F}\right)\beta E_t \mu_{t+1}^D - \frac{\tau_1^*}{1-u^F}\beta^2 E_t \mu_{t+2}^D \end{aligned} \qquad (66)$$

where μ_t^H, μ_t^F and μ_t^D are the Lagrange multipliers with the constraints (46)-(48) in period t.

Note that we adopt the "timeless perspective" (Woodford, 2003) and ignore the special constraints on the lagged lagrange multipliers in period t=0. Equations (46)-(48) and (63)-(66) at all $t \geq 0$ along with initial conditions then charaterize the equilibrium under optimal monetary policy with commitment (the unknowns are: $\left\{\hat{\pi}_t^H, \hat{\pi}_t^F, \hat{u}_t^H, \hat{u}_t^F, \mu_t^H, \mu_t^F, \mu_t^D\right\}_{t=0}^{\infty}$).

B3. The Policy Frontier

The policy frontier is defined as the set of feasible monetary policy choices. To compute the policy frontier, we use the following weights in the loss function:

1. To calculate the policy frontier between the volatility of union inflation and unemployment, we use

$$L1 \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[\omega \left(\hat{\pi}_t^U \right)^2 + (1-\omega) \left(\hat{u}_t^U \right)^2 \right]$$

2. To calculate the policy frontier between the volatility of home and foreign inflation, we use

$$L2 \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[\omega \left(\hat{\pi}_t^H \right)^2 + (1-\omega) \left(\hat{\pi}_t^F \right)^2 \right]$$

Similar to Appendix B2 above, we then compute the optimal monetary policy under commitment, but for all weights $\omega \in [0,1]$. Finally, we simulate the model and compute the variances for each $\omega \in [0,1]$.