Lecture notes on risk management, public policy, and the financial system

Leverage risk

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Defining and measuring leverage

Putting on leverage through carry trades

Incentive alignment and capital structure
Defining and measuring leverage

Definition of leverage
Leverage and risk

Putting on leverage through carry trades

Incentive alignment and capital structure
Economic balance sheet

- **Leverage**: funding of assets by issuing of debt in addition to equity (owner resources)
- Defined in terms of firm’s or investor’s *economic* balance sheet
- Accounting standards may not fully reflect economic reality; may, for example
  - Keep some exposures and obligations off-balance sheet
  - Display some assets and liabilities at inaccurate historical values
  - Permit use of possibly inaccurate valuation models
## Definition of leverage

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of the assets or exposure (A)</td>
<td>Equity ((E))</td>
</tr>
<tr>
<td></td>
<td>Debt ((D))</td>
</tr>
</tbody>
</table>

- Leverage \(L\) (in “turns”): ratio of assets to equity

\[
L = \frac{A}{E} = \frac{E + D}{E} = 1 + \frac{D}{E}
\]

- But several common alternative definitions, e.g. \(\frac{D}{A}\)
- Regulatory \(\rightarrow\) **leverage ratio** defined as \(\frac{\text{regulatory capital}}{\text{adjusted assets}}\)
  - With detailed definitions of numerator and denominator
- Distinct from **operating leverage** \(\frac{\%\Delta \text{operating income}}{\%\Delta \text{sales}}\)
  - High when income/net revenue increases rapidly with output and sales due to low variable and high fixed costs
Modigliani-Miller irrelevancy

- **Corporate financial policy**: decisions about capital structure
- Includes decisions about
  - Capital distributions—dividends, share repurchases
  - Forms of debt and equity issuance
- **Modigliani-Miller theorem** (Proposition I): equivalently,
  - Firm market value is independent of capital structure
  - Only firm asset choices matter for firm value
  - Weighted average cost of capital (WACC) = expected asset return
- Requires strong assumption of perfect capital markets, including
  - Complete arbitrage
  - No taxes
  - No bankruptcy costs apart from LGD
  - But debt may be risky
- Core argument: investors can borrow or lend at prevailing rates to undo corporate financial decisions
  - And achieve degree of leverage they desire
Why is leverage attractive?

- WACC can be expressed as
  \[ r^a = \frac{1}{L} r^e + \left(1 - \frac{1}{L}\right) r \]

  \[ r^e \] rate of return on equity
  \[ r^a \] WACC/rate of return on assets
  \[ r \] blended cost of debt financing
- \( \Rightarrow \) Leverage amplifies return on equity
  \[ r^e = L r^a - (L - 1) r = L(r^a - r) + r \]
  \[ \frac{\partial r^e}{\partial L} = r^a - r \]

- \( r^a > r \): no limit on \( r^e \) as leverage rises
- \( r^a < r \): but loss amplification as leverage rises
- “Degenerate cases”
  - \( r^a = r \Rightarrow r^e = r^a \): no effect of leverage
  - \( L = 1 \Rightarrow r^e = r^a \): no debt financing, earn exactly return on assets
Sources of the impact of leverage on returns

- Decompose $r^a$ into asset appreciation $\frac{\Delta S}{S}$ and cash flow $q$

$$r^e = L \left( \frac{\Delta S}{S} + q \right) - (L - 1)r = L \left( \frac{\Delta S}{S} + q - r \right) + r$$

- $q$ may represent bond coupon, foreign interest, dividend
- For commodities, real estate, $q < 0$: storage/maintenance
- $\Delta S, q, r$ defined as annual percent change or rate, for consistency with typical quoted rates
- The standalone $r$ arises from equity funding

- $q - r$ called the **carry**
- $r$ can also be seen as risk-free rate $\rightarrow$ leverage amplifies excess return
- $\frac{\Delta S}{S}$ can be decomposed into expected $E \left[ \frac{\Delta S}{S} \right]$ asset appreciation and surprise $\frac{\Delta S}{S} - E \left[ \frac{\Delta S}{S} \right]$
- Taxes may create additional complications
Leverage increases risk

- Both sides of balance sheet, not just assets, important to risk
  - VaR applied to assets only → does not reveal capital structure risk
  - VaR must be compared to equity to capture effect on firm viability
- Market risk: $\frac{\Delta S}{S} - \mathbb{E} \left[ \frac{\Delta S}{S} \right]$ sufficiently adverse to exceed carry
  - Greater vulnerability to price decline
    - If capital thinner or carry lower
    - If occurs over shorter time horizon
- (→) Funding liquidity risks
  - $r$ rises
  - Margin call
  - Rollover risk, forced unwind
Leverage and systematic risk

- Modigliani-Miller Proposition II: higher leverage increases required equity return
- Proposition I: independence of WACC and leverage
  - Assets a portfolio of equity and debt
    - With leveraged-based weights of the WACC definition
- Assume debt has no systematic risk (but possibly plenty of idiosyncratic risk)

\[
\beta^a = \frac{1}{L} \beta^e \iff \beta^e = L \beta^a
\]
Leverage risk

Putting on leverage through carry trades

Defining and measuring leverage

**Putting on leverage through carry trades**
- Defining carry trades
- Leveraged fixed-income trades
- Leveraged foreign exchange trades

Incentive alignment and capital structure
Carry trades and search for yield

- **Carry trades**: earn positive carry, avoid $\frac{\Delta S}{S}$ surprises
  - Leveraged trades generally expected to have positive carry
  - Many market participants will not/cannot bear negative carry

- **Hurdle** or **target rate of return**: often set by firms, pension funds, portfolio managers
  - $L$ set so as to achieve return target (→ reaching for yield)

\[
\text{required leverage} = \frac{\text{target } r^e - r}{r^a - r} = \frac{\text{target } r^e - r}{\frac{\Delta S}{S} + q - r}
\]

- **Example**: bank’s target return $r^e = 25$ percent, $r^a = 4$ percent, $r = 2$ percent, sets $L = 11.5$

- Financing: leverage often provided by broker-dealer
Scenario analysis

- Establish a set of baseline assumptions
- Scenarios: determine impact on equity return if outcome differs from baseline
- Decline in asset price sufficient to drive $r^e \rightarrow 0$: find $\frac{\Delta S}{S}$ such that
  \[
  r^e = 0 \iff \frac{\Delta S}{S} = - \left( q - r + \frac{r}{L} \right)
  \]
- Decline in asset price sufficient to wipe out equity: $r^e \rightarrow -1$ (-100 percent): $\frac{\Delta S}{S}$ such that
  \[
  r^e = -1 \iff \frac{\Delta S}{S} = - \left[ (q - r) + \frac{1}{L} (1 + r) \right]
  \]
- Further asset price decline equal to haircut $\frac{1}{L}$ compared to $r^e = 0$ scenario
Fixed-income carry trade

- **Trade thesis:**
  - Buy higher-yielding (longer-term and/or credit-risky) bond
  - Financed by short-term (→)repo financing
  - Earn difference between yield and financing cost

- **Key risks:** bond price decline \( \frac{\Delta S}{S} < 0 \), loss of funding

- **Audience:** hedge funds, prop desks, dealers

- **Roll-down:** maturity of bond shorter at unwinding of trade
  - Unless yield curve flat, value of shorter-term bond different—generally higher
  - Can be approximated using duration and term spread
  - Present to a small extent in floating-rate securities
  - Should also be taken into account in measuring carry trade gains, but can ignored for short-term trades
Example: fixed-income carry trade

- Parameters: long floating-rate AAA ABS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected change in bond price</td>
<td>$E \left[ \frac{\Delta S}{S} \right]$</td>
</tr>
<tr>
<td>Leverage</td>
<td>$L$</td>
</tr>
<tr>
<td>Repo rate</td>
<td>$r$</td>
</tr>
<tr>
<td>Coupon rate</td>
<td>$q$</td>
</tr>
</tbody>
</table>

- Risk scenarios (at annual rate, but monitored daily)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\frac{\Delta S}{S}$ (%)</th>
<th>$q - r$ (%)</th>
<th>$r^e$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline: zero bond price change</td>
<td>0.00</td>
<td>1.00</td>
<td>27.50</td>
</tr>
<tr>
<td>Break-even bond price change</td>
<td>$-1.10$</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Bond price change→100% loss</td>
<td>$-5.10$</td>
<td>1.00</td>
<td>$-100.00$</td>
</tr>
<tr>
<td>Funding rate ↑100 bps</td>
<td>0.00</td>
<td>0.00</td>
<td>3.50</td>
</tr>
</tbody>
</table>
Back-of-envelope risk calculations

- How large a spread widening → 100% loss?
  - Suppose spread01 = 400
  - +1 bp → $\frac{\Delta S}{S} = -0.04$ percent, +127.5 bp → $\frac{\Delta S}{S} = -5.1$ percent
  - $\Rightarrow$ Spread widening of $\frac{5.1}{0.04} = 127.5$ bps → wipe-out
Foreign exchange carry

- Trade thesis:
  - Invest in high-yield target currency (e.g. AUD, emerging markets)
  - Financed by borrowing in low-yield funding currency (e.g. CHF, ¥)
  - Earn difference between money market rates in target and funding currencies

- Key risk: funding currency appreciation $\frac{\Delta S}{S} < 0$

- Initiating and unwinding currency carry trade:
  - Buy $S_{t-1}$ units of target currency, with $S_t$ exchange rate in funding currency units, invest at rate $q$
  - Sell proceeds at rate $S_{t+1}$ to attain excess return

\[
(1 + q) \left(1 + \frac{\Delta S}{S}\right) - (1 + r) = (1 + q) \frac{\Delta S}{S} + q - r
\]

- If leverage is applied, the return is

\[
 r^e = L \left[(1 + q) \frac{\Delta S}{S} + q - r \right] + r
\]

- For foreign exchange trades, $S$ is the exchange rate in funding currency units
Example: foreign exchange carry trade

- Parameters: long AUD against ¥ (funding currency)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected exchange rate appreciation</td>
<td>$E \left[ \frac{\Delta S}{S} \right] = 0$</td>
</tr>
<tr>
<td>Leverage</td>
<td>$L = 25$</td>
</tr>
<tr>
<td>¥ money-market rate</td>
<td>$r = 0.005$</td>
</tr>
<tr>
<td>AUD money-market rate</td>
<td>$q = 0.025$</td>
</tr>
</tbody>
</table>

- Risk scenarios (at annual rate, but monitored daily)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\frac{\Delta S}{S}$ (%)</th>
<th>$r_e$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline: zero ¥ appreciation</td>
<td>0.000</td>
<td>50.500</td>
</tr>
<tr>
<td>Break-even ¥ appreciation</td>
<td>$-1.971$</td>
<td>0.000</td>
</tr>
<tr>
<td>¥ appreciation $\rightarrow$ 100% loss</td>
<td>$-5.873$</td>
<td>$-100.000$</td>
</tr>
</tbody>
</table>
Risks of foreign exchange carry trade

- Back-of-envelope risk calculations: informal assessments of risk based on rough approximations, unrealistic assumptions
- Probability of wipe-out over 1 year: suppose
  - Implied or historical vol of AUD-JPY exchange rate $\approx 6$ percent p.a.
  - Long AUD-JPY position held unchanged for 1 year
  - AUD-JPY exchange rate approximately normally distributed
- ¥ appreciation $\geq 6$ percent $\Leftrightarrow 1$ s.d. or greater decline in $S_t$
  - $\Rightarrow$ Probability 16 percent of 100 percent loss
Leverage risk

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**Incentive alignment and capital structure**

Leverage and moral hazard
Debt overhang
Risk-sharing impact of leverage

- For any firm, leverage a mechanism for distributing risk between equity owners and lenders to firm
- Merton model: debt and equity as options on firm assets
- Moral hazard in presence of limited liability generates inefficiencies
- Takes two forms that are inverse to one another
  - **Risk-shifting** or **asset substitution**: equity owners choose riskier projects/loans
  - **Debt overhang**: equity owners avoid riskier projects/loans
- Distribution as well as mean of outcomes of additional projects/loans important
- Capital regulation and presence of government guarantees can generate or exacerbate either one
Risk-shifting

- Limited liability generates incentive for equity owners to take on greater risk at expense of debtholders
  - Higher leverage shifts risk from equity owners to creditors
- Equity owners may have incentive to accept negative NPV to obtain positive upside debtholders
- Effect of guarantees (→deposit insurance, TBTF): equity owners incentivized to increase risk
  - Equivalent to “taxpayer put” provided to intermediaries
- Exacerbated by decline in firm asset value leading to gambling for redemption: incentive to make more/riskier loans
Definition of debt overhang

- High leverage and/or high default probability may have negative impact on firm’s willingness to invest
- Positive NPV has much larger impact on recovery and current market value of existing senior debt than on equity
  - Impact on equity may even be negative due to dilution or funding via new junior debt
  - Seniority of high existing debt would force new equity to share returns with debtholders
- Owners may avoid equity-financed positive NPV projects if it raises value of debt and reduces value of equity
  - New investment projects with positive net present value cannot be financed via equity
  - No incentive of current owners to finance projects via junior debt
Example of debt overhang: assumptions

- Existing debt: senior debt, par value 100
  - With covenants preventing increased issuance or subordination to new debt
  - Yield/interest rate $r = 0$
- Default event: default if asset value $= 80$, nondefault value $110$
  - $\Rightarrow$ Debt LGD 20, recovery 80
- Firm/asset value: default probability-weighted average of asset value
  \[
  \text{firm value} = \pi \times 80 + (1 - \pi) \times 110
  \]
- Debt value: default probability-weighted average of par (100) and recovery (80)
  \[
  \text{debt value} = \pi \times 80 + (1 - \pi) \times 100
  \]
- New investment opportunity: invest 5, certain future value of 15
  - Financing through issuance of new shares or junior debt
Example of debt overhang: results

<table>
<thead>
<tr>
<th>Default probability</th>
<th>1.000</th>
<th>0.500</th>
<th>0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Before new investment</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm value</td>
<td>80.000</td>
<td>95.000</td>
<td>109.850</td>
</tr>
<tr>
<td>Debt value</td>
<td>80.000</td>
<td>90.000</td>
<td>99.900</td>
</tr>
<tr>
<td>Equity value</td>
<td>0.000</td>
<td>5.000</td>
<td>9.950</td>
</tr>
<tr>
<td><strong>After new investment</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm value</td>
<td>95.000</td>
<td>110.000</td>
<td>124.850</td>
</tr>
<tr>
<td>Debt value</td>
<td>95.000</td>
<td>97.500</td>
<td>99.975</td>
</tr>
<tr>
<td>Equity and junior debt value</td>
<td>0.000</td>
<td>12.500</td>
<td>24.875</td>
</tr>
<tr>
<td>Equity gain net of funding cost</td>
<td>−5.000</td>
<td>2.500</td>
<td>9.925</td>
</tr>
</tbody>
</table>
Interpreting the example

- **Default certain or near-certain:**
  - All the future value of new investment goes to senior debt holders
  - Current or new equity and junior debt investors lose entire cost of investment

- **Default probability high:**
  - Much of the future value of new investment goes to senior debt holders
  - Little left over after cost of investment to incentivize firm owners

- **Default probability low:**
  - Senior debt recovery little changed
  - Owners have ample incentive to invest in a “sure thing”