

## **“The 4-quadrant diagram depicting the Heckscher-Ohlin model”**

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The 4-quadrant diagram is remarkable for its ability to represent succinctly and precisely the full equilibrium in the two-good, two-factor, two-country Heckscher-Ohlin (H-O) model. With four factor allocations, two outputs and two factor prices in each country and the common goods price, the model has 17 variables in all. The diagram manages to explicitly show the equilibrium value of the key variables explicitly, with the remaining ones determined indirectly.

I first introduce the basic diagram as applied to the autarky equilibrium. I then extend it to the two-country equilibrium and briefly mention further applications. In the penultimate section, I give a brief history of the diagram.

### **1 The Basic Diagram**

Consider an economy producing two goods, 1 and 2, using two factors, labor (L) and capital (K). Production functions exhibit the standard properties including constant returns to scale and diminishing returns to each factor. Cost minimization under these assumptions leads to a positive relationship between optimal capital-labor ratio and the wage-rental ratio in each good. Measuring the wage-rental ratio,  $w$ , along the horizontal axis and the capital-labor ratio in sector  $i$ ,  $k_i$ , along the vertical axis, I depict relationship for each good in the first quadrant of Figure 1.

The H-O model further assumes that one of the goods, say, good 2, is unambiguously capital intensive. This means that for each set of factor prices, cost-minimizing capital-to-labor ratio is higher in good 2 than good 1. This is the reason I have drawn  $Ok_2$  above  $Ok_1$  everywhere in Figure 1. In principle, it is possible for  $Ok_1$

and  $Ok_2$  to cross such that good 2 is capital intensive for lower values of  $w$  and labor intensive (meaning it would exhibit lower capital-labor ratio than good 1) for higher values of it. I will briefly return to this “factor-intensity reversal” later.

Ruling out the factor-intensity reversals, the Stolper-Samuelson (1942) property of the H-O model allows us to establish a one-to-one relationship between goods prices and factor returns. A higher relative price of good 1, the labor-intensive good, is associated with a higher wage-rental ratio. Intuitively, the increase in the relative price of good 1 leads to an expansion of good 1 and contraction of good 2. At the initial factor prices, good 1 demands more labor per-unit of capital than good 2 releases. This change leads to an excess demand for labor and excess supply of capital, raising the wage-rental ratio.  $FF'$  in the fourth quadrant of Figure 1 depicts this positive relationship between  $w$  and  $p$ , where  $p$  denotes the relative price of good 1.

Next, let  $k$  on the vertical axis in the first quadrant of Figure 1 represent the capital-labor-endowment ratio of the country. If the country specializes completely in good 2, we have  $k_2 = k$  as shown by point A. Cost minimization then implies  $w^0$  as the wage-rental ratio and  $p^0$  as the corresponding price. At prices equal to or less than  $p^0$ , it is unprofitable to produce good 1 and the economy specializes completely in good 2. The equilibrium is characterized by  $k_2 = k$ , requiring  $w = w^0$ .

We can now turn to tracing the full relative supply curve in the third quadrant. Measure the ratio of output of good 1 to that of good 2 by  $x$ . We already know that at prices equal to or less than  $p^0$ ,  $x = 0$ . Starting at  $p^0$ , suppose we increase  $p$ . Positive output of good 1 now appears with resources allocated from good 2 to good 1. As this happens,  $w$  and the capital-labor ratios in the two sectors rise. Both capital-labor ratios

can rise despite a constant endowment ratio because the shrinking output of the capital-intensive good. As expected, we have an upward-sloped relative supply curve RS. Eventually, as  $p$  reaches  $p^{00}$ , output of good 2 drops to zero with  $k_1 = k$  and  $w = w^{00}$ . The value of  $x$  approaches infinity as  $p$  approaches  $p^{00}$ .

The last step in completing the autarky equilibrium is to introduce demand. This is done most simply by assuming homothetic preferences, which ensures that the relative demand, depicted by RD in Figure 1, depends only on the relative price. The intersection of RS and RD gives the equilibrium price and relative output shown by  $p_e$  and  $x_e$ , respectively. Given  $p_e$ , we can determine  $w_e$  as the wage-rental ratio in the fourth quadrant and given  $w_e$ , we can read the equilibrium  $k_1$  and  $k_2$  along  $Ok_1$  and  $Ok_2$ , respectively, in the first quadrant. The values of  $k_1$  and  $k_2$  can be combined with full employment conditions to determine absolute levels of factor allocations. In turn, the factor allocations can be combined with the production functions to determine outputs and marginal products of labor and capital in each good. The latter equal the real factor returns in terms of the two goods.

## 2 The Two-Country Equilibrium

Let us think of the country just considered as Home and introduce a second country called Foreign. Identify the variables of Foreign by an asterisk. Although Ohlin (1933) did not originally assume this, to obtain precise predictions, assume Home and Foreign have identical tastes and technology and differ only in terms of relative factor endowments. Regarding the latter, assume Foreign has a lower capital-labor-endowment ratio than Home.

To avoid clutter, I depict the international equilibrium separately in Figure 2 where I suppress the autarky equilibriums and some of the labeling. All curves associated with Home and Foreign are the same except the relative supply curves. With the endowment ratio of Foreign at  $k^*$ , its supply curve is  $RS^*$ . Consistent with the Rybczynski (1955) theorem, at a given goods price, labor-abundant Foreign produces relatively larger quantity of the labor-intensive good, good 1.

It is immediate from Figure 2 that under autarky, the good using the abundant factor more intensively will be cheaper in each country: good 2 in Home and 1 in Foreign. Therefore, consistent with the Heckscher-Ohlin theorem, each country will export the good using its abundant factor more intensively. Under autarky, the lower price of the good using the abundant factor more intensively leads to a lower relative price of that factor in each country. Therefore, under autarky, the wage-rental ratio is lower in the labor-abundant Foreign than Home. As trade equalizes the good prices between the two countries ( $p^1$  in Figure 2), it also equalizes the relative factor prices ( $w^1$  in Figure 2). But equal wage-rental ratios lead to equal capital-labor ratios in each good across countries. In turn, this equalizes the marginal products of each factor in each good across countries. Therefore, consistent with the Factor-Price Equalization theorem, trade equalizes not just the goods prices but real factor prices internationally as well.

### **3 Further Applications**

Three further applications of the 4-quadrant diagram may be noted. First, the reader may verify that if the endowment ratios were sufficiently different, trade would result in complete specialization by at least one country. In this case, trade will lead to a

convergence of the factor prices but stop short of their complete equalization except in the borderline case in which the goods price is at the threshold of complete specialization.

Second, the diagram can also be employed to analyze factor-intensity reversals. Thus, for example, imagine  $Ok_1$  intersects  $Ok_2$  from below at point  $Z$  so that good 2 is capital intensive for the wage-rental ratios to the left of  $Z$  and labor-intensive to the right of it. This will result in  $FF'$  in the fourth quadrant taking U-shape with the bottom of  $U$  being vertically below  $Z$ .  $FF'$  will have a positive slope to the left of  $Z$  and negative slope to the right of it. Suppose further that the capital-labor-endowment ratio of Home is above  $Z$  and of Foreign below it. It is then clear that the wage-rental ratio of Home will be determined on the downward sloped section of  $FF'$  and of Foreign on the upward sloped section of it. Equalization of goods prices in a free-trade equilibrium will now not result in the equalization of factor prices. Besides, given good 1 is capital intensive in Foreign and labor intensive in Home, there is no meaningful way to relate exports to factor endowments and factor intensities.

Finally, technologies may differ between the two countries. For simplicity, suppose the technology for good 2 is the same but FC has a slightly superiority technology in good 1 in the Hicks-neutral sense. The relationships for HC in Figure 2 remain unchanged. Given the Hicks-neutral difference in technology, the relationships for FC in the first quadrant of Figure 2 are also unchanged. But since good 1 is labor intensive, at a given price ratio, FC has a higher wage-rental ratio. Therefore, the relationship in the fourth quadrant for FC now lies to the right of  $FF'$ . The relative supply curve of FC in the third quadrant lies to the left of where it is shown in Figure 2. All results of the H-O still hold except that while trade leads to the convergence of factor prices it falls short of

equalizing them. At a common goods-price ratio, factor prices in HC and FC are determined along *different* FF' curves in the fourth quadrant. Other forms of technological differences can be similarly analyzed.

#### **4 History of the Diagram**

Although the diagram is often associated with the names of Harrod (1958) and Johnson (1957), the constructions in the first and fourth quadrant in Figure 1 can be found in Samuelson (1949) who presents them in the first and second quadrants, respectively, with the wage-rental ratio shown on the vertical axis. Remarkably, in footnote 1 on p. 188, he even draws the two quadrants side by side depicting the factor-intensity reversal. Harrod only presents the first quadrant but elaborates on factor-intensity reversals. Johnson, who explicitly acknowledge building his analysis on the diagrams in Samuelson and Harrod, places the relationship between the goods prices and the wage-rental ratio in the fourth quadrant, explicitly introduces factor endowments of the countries and extensively applies it to analyze the patterns of trade and the implications of trade for factor-price convergence and equalization across countries. I am unable to track down when the full 4-quadrant diagram first appeared though I saw it for the first time in 1975 in my graduate course in international trade by Professor Carlos Rodriguez of Columbia University who drove down every Friday morning to teach the course at Princeton. I have subsequently used the four-quadrant diagram regularly in my own graduate courses.

## References

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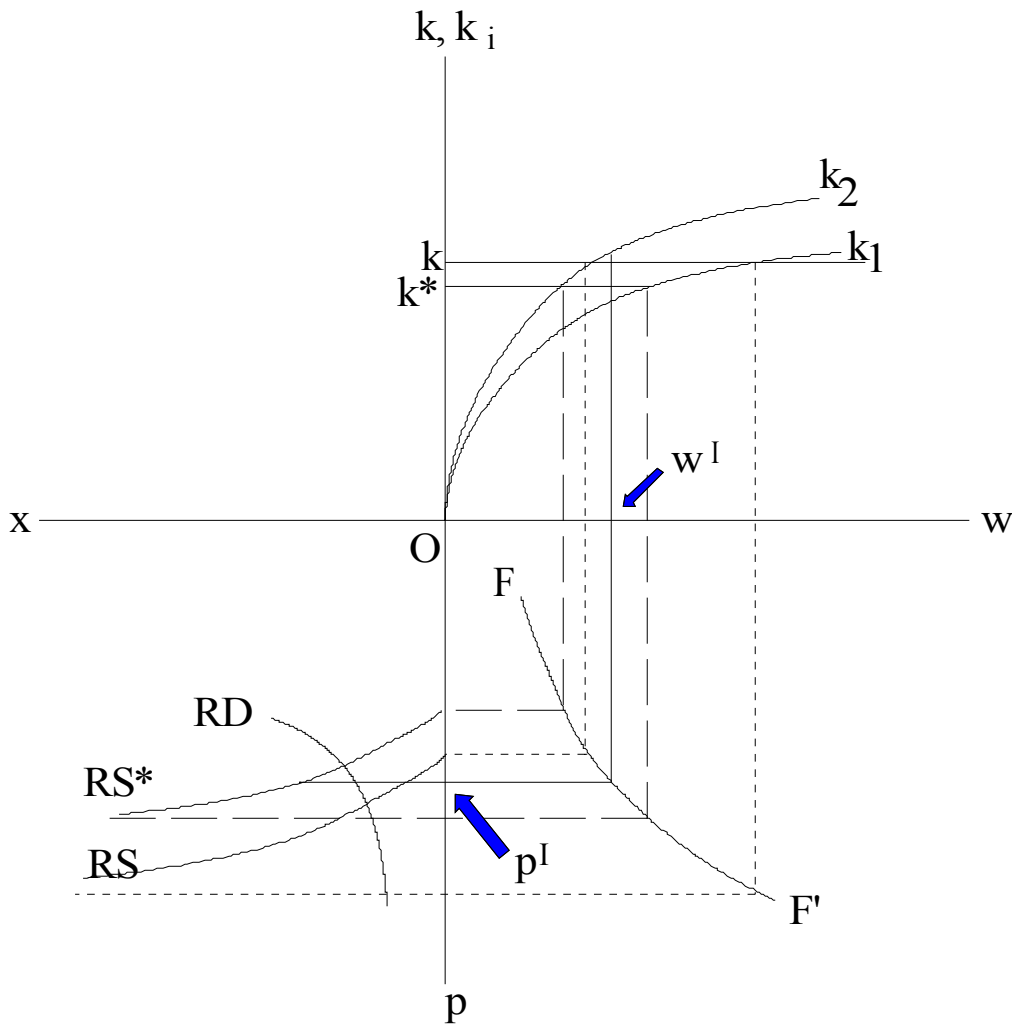


Figure 2: Two-country Equilibrium in the H-O Model