Campaign Spending with Office-Seeking Politicians, Rational Voters, and Multiple Lobbies

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I introduce a microfounded model of campaign finance with office-seeking politicians, a continuum of voters, and a large number of heterogeneous lobbies. Lobbies make contributions to politicians according to a common agency framework. Politicians use contributions to finance their electoral expenditures. Voters are not fooled by electoral expenditures: they are influenced in a way that is consistent with the equilibrium behavior of lobbies and politicians. The model is used to: (i) determine the relation between campaign spending and the deviation from the median voter’s preferred policy; (ii) show the informational value of lobbies’ contributions; (iii) evaluate the welfare implications of restricting campaign spending; and (iv) interpret the empirical finding that campaign expenditures have a very low effect on election outcome. Although in equilibrium advertising provides voters with useful information, under reasonable parameter values, a ban on campaign contributions makes the median voter better off. Journal of Economic Literature Classification Numbers: D72, D82, M37. © 2001 Elsevier Science (USA)
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1. INTRODUCTION

Campaign finance is a hotly debated topic. People are generally uncomfortable with the mixing of money and politics. They fear that wealthy lobbies can obtain favors from politicians through implicit or explicit promises of campaign contributions. This fear is reinforced by the fact that some of the biggest contributors are unpopular industries, such as tobacco manufacturing.

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To respond to voters’ concerns, most Western countries have introduced some form of campaign finance regulation.²

It would be important to have a model to evaluate the opportunity of alternative forms of regulation. As Austen-Smith [4], Morton and Cameron [27], Laffont and Tirole [21, p. 634], and Baron [7] have stressed, we need a microfounded theory of political competition which gives a role to campaign expenditures. Most of the existing models assume that electoral spending influences voters in an ad-hoc way, while in order to make welfare comparisons one needs to know the voters’ primitives.

Two types of model have been proposed. First, Austen-Smith [3] has developed a model of directly informative advertising. Voters observe candidates’ positions with a certain error, and campaign expenditures reduce the variance of that error. Thus, spending is assumed to be directly informative. An equilibrium is derived in which candidates’ positions are affected by the desire of receiving contributions. Contributions help candidates to get elected because risk-averse voters prefer candidates with a clearer policy position. Second, Potters, Sloof, and van Winden [30], Prat [31], and Dharmapala [14] use nondirectly informative advertising. Each candidate is characterized by a nonpolicy dimension (valence), which lobbies can observe more precisely than voters. The amount of campaign money that a candidate collects signals his valence to voters. Hence, the role of campaign advertising is not to convey a direct message but to credibly “burn” campaign money.³

The present paper follows the second strand in that it uses nondirectly informative advertising. While it cannot be denied that campaign advertising often provides voters with some kind of direct, verifiable information, many ads contain mostly vague claims and nonbinding promises. Experimental evidence indicates that television ads influence voters’ choice, even when, by design, they are devoid of any direct information (Ansolabehere and Iyengar [2]).

One limitation of the existing papers in the second strand is that they assume that there is only one lobby. Besides the lack of realism, this

² See Levitt [25] for a discussion of existing regulation in the US. In principle one may restrict candidates’ entries (campaign contributions) or candidates’ expenditures (campaign spending), or both. The US Supreme Court has ruled that limits on spending are unconstitutional because they restrict the right to free speech. In contrast, limits on spending are in place in most European countries.

³ An entirely different way of microfounding campaign expenditures is proposed in Austen-Smith [5, 6]. Lobbies give contributions in exchange for access to politicians. Politicians are not interested in campaign money per se. They only care about the information that lobbies can provide them with. However, in the spirit of Crawford and Sobel’s [12], the extent of truthful information transmission is increasing in the preference congruence between a lobby and the politician. Campaign contributions signal preference congruence and therefore induce the candidate to grant access to the lobbies that make them.
assumption skews results against campaign advertising. A unique lobby has a strong bargaining power, which it is likely to use to force candidates into choosing policies that are very detrimental to voters. The main original contribution of this paper consists in developing a microfounded model of campaign advertising with multiple lobbies by combining a signaling model of noninformative advertising with common agency.

The model can be sketched as follows. There are three types of agents: politicians, lobbies, and voters. I employ a retrospective model of voting in which politicians maximize their chances of being re-elected. Voters are fully rational. They are heterogeneous and care about policy and valence of the politician in office. Valence denotes politicians’ personal attributes, such as competence or charisma, which affect all voters in a similar way.

There are a large number of organized interest groups. Policy is multi-dimensional but each lobby is interested only in a small subset of policy dimensions. Given the large number of lobbies, each lobby is small enough that the influence of its contributions on the electoral outcome is negligible. Therefore lobbies’ contributions are service-induced. They offer money to politicians in exchange for favors which take the form of policy positions beneficial to lobbies.

Asymmetric information plays a central role. Professional lobbyists are in a better position than common voters to observe the valence of politicians. Lobbies take into account information on valence when they make their contribution choices. They do not do it because they care about valence directly, but because voters may end up finding out about the politicians’ valence. From the viewpoint of lobbies, the return of a dollar of contributions to a certain politician depends on the probability that the politician is re-elected. A high-valence politician is a better bet than a low-valence politician. Hence, lobbies are more willing to give to politicians they have positive information about.

Voters observe campaign expenditures and form beliefs about the valence of politicians. I require voters’ beliefs to be consistent with the equilibrium behavior of lobbies and politicians. Campaign spending can influence voters only inasmuch as it brings them information.

One can say that, within the class of models of non-directly informative advertising, the present paper stacks the odds in favor of campaign contributions. Several key assumptions go in the direction of making campaign expenditures useful: voters are not fooled; lobbies have negligible market power; and politicians do not pocket the contributions but only use them.

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5 The service-induced model has been shown to have a good predictive power by Snyder [33].
for informative purposes. One could have assumed that, confronted with campaign advertising, voters act on impulse and then regret it. Or that some large lobbies have bargaining power against single candidates. Or, still, that contributions are actually just a form of corruption, with the connected wasteful transaction costs.

It is therefore surprising that in this model, under reasonable parametric assumptions, a ban on campaign contributions improves voter welfare (in a Utilitarian sense). The intuition for this result does not have to do with the fact that campaign spending is a wasteful activity. Rather, it is due to the policy bias that the incumbent generates in order to collect money to signal her type. In equilibrium, a candidate who makes low campaign expenditures is perceived as a candidate who could not get money from lobbies because he or she was seen as inept or uncharismatic. As this perception is correct in equilibrium, all good candidates are forced to sell out to lobbies. Under certain conditions (high informational asymmetry between lobbies and voters), the cost to voters of the candidate's selling out is higher than the expected informational benefit. The problem would be solved if voters could commit before the election not to look at advertising, but, as this commitment is not credible, there is scope for regulation.

The main simplifying assumption of this paper—which is worth stating upfront—is that only incumbents can receive campaign contributions. It is unclear whether this shortcut skews results in favor of or against campaign contributions. On the one hand, because strong incumbents deter prospective challengers, the valence of challengers tends to be negatively correlated with the valence of incumbents. Then, when two candidates compete for contributions and lobbies have insider information about both, advertising may become more informative because a dollar spent by one candidate is a positive signal for that candidate and a negative signal for the other. On the other hand, competition between candidates also shifts some bargaining power back to lobbies. In a one-lobby model, Prat [31] has analyzed the case in which two candidates can receive money. Competition between candidates allows the lobby to extract political favors, even when the lobby has no insider information. This happens because if one candidate rejects the lobby's offer, he is perceived as low quality while the other candidate is perceived as high quality.

This paper draws from two distinct strands of literature. The first is common agency theory. Introduced by Bernheim and Whinston [9], common agency offers a general framework to study the strategic interaction between interest groups and politicians. Grossman and Helpman [18] use common agency to analyze a general model of spatial voting with two candidates and two interest groups. The model allows politicians to derive utility both from policy and from office tenure and generates an endogenous distinction between electoral motive and influence motive. However,
the existing applications of common agency do not model voters' decision making or (as in the case of Grossman and Helpman [18]) assume that voters are influenced by campaign spending in an ad-hoc way.

The second strand is the industrial organization literature on commercial advertising (Kihlstrom and Riordan [19] and Milgrom and Roberts [26]). Advertising is assumed to be not directly informative. Consumers are affected not by its message, but by the amount of money spent on it (advertising on mass media is the most visible and credible way of burning money). For instance, a firm that is launching a new product may want to burn money in order to show its intention to stay in the market on the long-term. This in turn signals to consumers that the firm believes to have a high-quality product. I adopt the spirit of the literature on commercial advertising in assuming that voters (instead of consumers) interpret campaign expenditures (instead of advertising) in a way that is consistent with the structure of the model.

The plan of this paper is as follows. Section 2 presents the model and discusses its main assumptions. Section 3 deals with the supply-side of campaign finance. The goal of the politician is to collect a given amount of money (war chest) with the lowest political cost possible (policy bias). The game between the politician and the lobbies is modeled as a common agency problem. As a result, policy bias is determined as a function of desired war chest, concentration of lobbies' fundraising abilities, and the lobbies' belief on the probability that the politician is re-elected. Section 4 looks at the interaction between politicians and voters given the supply function of contributions. I construct a revealing equilibrium in which the amount of campaign spending fully reveals the politician's valence to voters. The game has other equilibria as well but they do not survive the Intuitive Criterion refinement. Section 5 analyzes the welfare properties of the revealing equilibrium in comparison with the equilibrium that would arise if campaign spending (or contributions) were forbidden by law. I identify a sufficient condition for campaign spending to decrease median voter welfare. If the precision of voter information is low enough, then the median voter is better off if spending is forbidden. Section 6 uses the present model to interpret the empirical finding that campaign spending has very little effect on electoral outcome. A low observed effectiveness of campaign spending is shown to imply that campaign spending decreases median voter welfare. Section 7 concludes.

2. MODEL

The model introduced in this section includes features of spatial voting, retrospective voting, and signaling games. In the spirit of retrospective
voting, there are two terms. In the first term an incumbent (denoted with \( I \)) is exogenously put in power. During the first term, lobbies offer \( I \) monetary contributions in exchange for favorable policy choices. Lobbies’ offers will be modeled in a common agency framework a la Bernheim and Whinston [9]. After observing the lobbies’ offers, \( I \) chooses a position, which he or she will not be able to modify in the future. As a result of his or her policy choice, \( I \) collects campaign contributions which will form his or her war chest. In the end of the first term, a challenger appears. The incumbent spends his or her war chest on campaign expenditures. Voters cast their ballot. In the second term, the election winner rules. The model includes asymmetric information. \( I \) is characterized by a valence variable (charisma, integrity, ability, etc.) which benefits all voters independently of their political opinions. The valence variable is perfectly observed by lobbies and by \( I \) but only imperfectly observed by voters.

**Incumbent.** In the first term, a politician is exogenously put in power. Her valence is given by \( g + \theta \), \( g \in \mathbb{R} \) is observed by all players including the voters and represents personal characteristics that can be verified (such as political record or performance in the first term). Instead, \( \theta \in [0, h] \) is perfectly observed by lobbies and by the candidate and imperfectly observed by voters. \( \theta \) captures those personal characteristics that cannot be verified (such as personal ability). The probability that \( \theta = h \) is denoted with \( q \in (0, 1) \).

The policy space has \( n \) dimensions, where \( n \) is a large positive integer. \( I \) chooses \( p \in \mathbb{R}^n \). Examples of policy dimensions are tariff rate on textiles, amount invested in education, strictness of abortion rule, etc. The choice of \( p \) is made in the first term and is valid for two terms. This assumption is meant as a reduced form of a reputational model in which politicians face a cost if they alter their policy positions significantly.

**Voters.** There exist a continuum of voters uniformly distributed on \([ -t, t] \) with \( t \geq 0 \). Given two vectors \( x = (x_1, ..., x_n) \) and \( y = (y_1, ..., y_n) \), let the Euclidean distance between them be denoted as \( \| x - y \| = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2} \). If \( I \) has valence \( \theta \), chooses \( p \), and is re-elected, Voter \( j \in [ -t, t] \) receives a second-term utility

\[
u_j(p, g, \theta) = g + \theta - \| p - j \| .\]

Thus, all voters equally benefit from valence but they have different opinions on policy. In this (nongeneric) multi-dimensional policy space, a voting equilibrium exists and the median voter theorem holds (Davis, DeGroot, and Hinich [13]).

One may doubt that utility is separable in policy and valence. A left-wing voter may prefer an inept right-wing politician to an effective right-wing
politician because the latter is more likely to live up to his or her promises and pass right-wing legislation. Still, an inept politician creates pure inefficiencies which are costly to all citizens. I assume that this generalized inefficiency effect dominates the partisan effect.

**Lobbies.** Both in the US and in the EU, there exist a very large number of organized lobbies. Typically each lobby cares only about a limited set of issues. For instance, a group representing textile producers will try to influence tariff rates on textile products but will have no interest in the strictness of abortion rules. Vice versa, an anti-abortionist lobby will have no concern for textile tariffs. I capture this specialization phenomenon by assuming that each lobby cares only about one policy dimension.\(^6\)

However, on each policy dimension there is usually more than one active lobby. To keep things simple, I assume that on each dimension exactly two lobbies are active and they have opposite interests. Each lobby minimizes the Euclidean distance from a bliss point, which is \(-1\) for one lobby and \(+1\) for the other. Hence, on dimension \(i\), there is a right lobby \(R_i\) with payoff \((\text{net of contributions}) - k_R^i |1 - p_i|\) and a left lobby \(L_i\) with payoff \(-k_L^i |1 + p_i|\).\(^7\) The parameters \(k_R^i\) and \(k_L^i\) are both positive and will be discussed shortly.

Let \(\pi\) denote the probability of \(I\) winning the race, estimated by lobbies. Clearly, \(\pi\) is determined in equilibrium. However, each lobby is small enough to take this probability as given. Without this feature, a multilobby model combined with candidate signaling would be intractable. The right lobby on \(i\) offers contribution schedule \(c_R^i(p)\), while the left lobby on \(i\) offers contribution schedule \(c_L^i(p)\). Contributions must be nonnegative. The expected payoffs of the two lobbies on policy line \(i\) are

\[
\Pi^R_i = -k_R^i (1 + \varepsilon) |1 - p_i| - c_R^i(p_i) \quad (1)
\]

\[
\Pi^L_i = -k_L^i (1 + \varepsilon) |1 + p_i| - c_L^i(p_i), \quad (2)
\]

where the \(1 + \varepsilon\) factor comes from the fact that policy \(p_i\) is enacted with probability \(1\) in the first term and with probability \(\varepsilon\) in the second term.

The \(k\)-parameters play a crucial role in this analysis. They capture the importance of each lobby. A crucial aspect of interest group politics (See Schlozman and Tierney \([32, \text{Chap. 4}]\)) is that lobby membership is not uniformly distributed across voters. Some segments of voters are over-represented in lobbying. This may be due to institutional settings (people

\(^6\) Lobbies represent voters, but only along one dimension. For instance, one voter may be represented on a dimension by his or her trade union, on another dimension by his or her religious affiliation, and yet on a third dimension by a car drivers’ association.

\(^7\) The labels “left” and “right” have no relation with ideological positions. A left lobby on policy \(i\) has nothing in common with a left lobby on policy \(j \neq i\).
who are in contact for other reasons are more likely to form a lobby) or to informational constraints (wealthy, better educated people are more likely to form a lobby). Thus the \( k \)-parameters can be viewed as the fundraising abilities of single lobbies.

Given the contribution schedules of all lobbies, \( I \) selects \( p \in \mathcal{R} \) and receives \( a = \sum_{i=1}^{n} \left[ c^{R}_{i}(p^*) + c^{L}_{i}(p^*) \right] \). The amount \( a \) represents the incumbent’s war chest.\(^8\)

**Voter information.** Voter information is as follows. With probability \( \rho \) voters have full information, that is, they observe \( a, p, \theta \). With probability \( 1 - \rho \) they have no direct information, that is, they only observe \( a \). In that case, voters form belief \( \beta(a) \) on the characteristics of \( I \). \( \beta(a) \) is determined in equilibrium.

To understand these (strong) assumptions, assume that information can be either verifiable (signals supported by hard evidence) or nonverifiable. Suppose there are neutral mass media that are as informed as lobbies but are constrained by libel law to publish only verifiable information.\(^9\) In the first term, the media have published all verifiable information, which is described by \( g \). The nonverifiable information left is \( \theta \) and \( p \) (assuming that the policy stance of the incumbent does not become immediately public is realistic because of the complex working of the legislative system: while roll calls are clearly verifiable, strategic behavior in committee work is hard to detect, and yet highly effective).

The media cannot publish \( \theta \) or \( p \) as long as they are nonverifiable. However, during the term, some new evidence may appear. This can take many forms: from discoveries about the incumbent’s past to the way she handles a foreign policy crisis or to his or her voting behavior on current issues. The new evidence creates verifiable information and the media will compete to make it public. Thus, voters will receive additional signals on \( \theta \) and \( p \).

These signals should be modeled in a general way as random variables generated by probability distributions conditional on the state of the world (e.g. Prat [31]). However, such a complex formulation would be intractable in the presence of multiple lobbies. Hence, I assume that the evidence that can appear during the term either provides full information or provides no information. A feature of this restriction is that information on valence is perfectly correlated with information on policy. Without it, there

\(^8\) In reality, lobbies have other instruments to influence policy, besides contributions: information provision to policy makers, ballot initiatives, issue ads, etc. As Bennedsen and Feldman [8] have shown, the use of these instruments is not independent. Contributions may crowd out information provision.

\(^9\) But see Strömberg [34] for an analysis of the policy distortions arising in a world with independent but profit-maximizing mass media.
would be cases in which evidence appears on some policy dimensions but not on valence, and this possibility would make the equilibrium of the signaling game hard to pin down. Yet, there do not seem to be obvious reasons why the arguments presented in this simple model would not extend to richer informational environments.

**Challenger.** At the end of the first term a challenger appears. As the challenger has not been in office, he or she has not had the opportunity to collect a war chest. Therefore, the challenger makes zero campaign expenditures. I also assume that lobbies do not receive any insider information about the challenger. Thus, the expected valence of the challenger is common knowledge among all players and is given by expected valence $x = E[g_c + \theta_c]$. The assumption that only the incumbent can receive money is made for analytical simplicity.\(^{10}\)

The challenger selects a policy vector $p_c$. Voter $j$ casts his vote for the incumbent if and only if $g + E[\theta - \|p - j\|] \geq x - E[\|p_c - j\|]$. As the median voter theorem holds in this environment, the challenger is elected if and only if the median voter votes for him. The challenger maximizes his probability of being elected by selecting the median voter’s preferred policy, independent of the valence and the policy of the incumbent. Hence, $p_c = 0$ is a dominant strategy for the challenger, and, for the rest of the paper, I assume that the challenger caters to the median voter. Then, the incumbent is elected if and only if $g + E[\theta - \|p\|] \geq x$.

During the first term, the identity of the challenger is not known. $I$ and the lobbies have common prior on $x$ given by the cumulative distribution $F(x)$ continuous and with full support on $[0, \infty)$. $F(x)$ is assumed concave in $x$, which means that the frequency of challengers is decreasing with quality. The average quality of challengers may be lower, equal, or higher than the average quality of incumbents.

**Probability of election.** From the preceding discussion, it follows that the ex ante probability of election for $I$ is

$$e(g, \theta, p, a) = \rho F(g + \theta - \|p\|) + (1 - \rho) F(g + \beta(a)),$$

that is, with probability $\rho$, $I$ faces fully informed voters, while, with probability $1 - \rho$, she faces voters who only observe $a$. Clearly, $I$ will face a tradeoff between increasing her war chest $a$ and keeping $p$ not too

\(^{10}\)An additional complexity is pointed out by Morton and Myerson [28], who show that, when both candidates can receive money and there are multiple lobbies, a coordination game among lobbies creates a generalized indeterminacy in the identity of the election winner.
far away from the median voter. \( I \) has no policy preferences and chooses the policy vector \( p \) in order to maximize his or her probability of election.

**Policy Bias.** The term \( \|p\| \) represents the Euclidean distance of the policy selected by \( I \) from the median voter’s ideal policy. It can be seen as the policy bias of campaign contributions and will often be denoted with \( D \) (as in deviation). If campaign contributions are forbidden, the incumbent caters to the median voter by setting \( D = 0 \). Hence, a deviation from 0 is only motivated by the desire of attracting higher contributions.

**Timing of the race.** To summarize:

1. **First term:** The incumbent \( I \) is in office. Everybody observes \( g \). Lobbies and \( I \) observe \( \theta \). Lobbies choose contribution schedules. \( I \) sets \( p^* \) and receives a war chest \( a = \sum_{i=1}^{\infty} [c^R_i(p^*) + c^L_i(p^*)] \).

2. **Electoral campaign:** A challenger with expected valence \( x \) appears. \( I \) uses \( a \) for campaign expenditures. With probability \( \rho \) voters are perfectly informed. With probability \( 1 - \rho \) they only observe \( a \). Voters cast their ballot.

3. **Second term:** If \( I \) is elected, \( p^* \) is implemented. If the challenger is elected, 0 is implemented. Payoffs are made.

### 3. THE SUPPLY OF CAMPAIGN FUNDS

This section deals with the supply of campaign contributions. It is only a partial equilibrium analysis in which the incumbent’s probability of election is taken as exogenous. Voters are not in the picture yet. The next section will consider the equilibrium of the whole model. Each lobby offers a contribution schedule to \( I \). A lobby interested in dimension \( i \) offers a payment contingent on \( p_i \). This payment will also depend on the estimated probability that \( I \) is re-elected. As there is a large number of lobbies,

**Assumption 1.** Each single lobby takes \( I \)’s probability of re-election as given.

As all lobbies have the same information, they estimate the same probability of re-election, which we denote with \( \varepsilon \). Clearly, \( \varepsilon \) will depend on all the variables lobbies can observe, that is, \( g \) and \( \theta \).

Consider an incumbent with an estimated re-election probability \( \varepsilon \) who wants to raise a campaign war chest \( a \). From (3), her goal is to
minimize the Euclidean distance of \( p \) from the median voter’s preferred policy 0. Such distance is \( \| p \| \). Then, \( I \)'s problem can be rewritten as: \(^{11}\)

\[
\begin{align*}
\min_{p} & \sum_{i=1}^{n} p_i^2 \\
\text{subject to} & \sum_{i=1}^{n} [c^R_i(p_i) + c^L_i(p_i)] \geq a.
\end{align*}
\]

We suppose that contribution schedules are continuous and differentiable (in the proof of Proposition 1, it will be shown that this is the case) and we form the Lagrangian for problem (4):

\[
L(p, \lambda) = -\sum_{i=1}^{n} p_i^2 + \lambda \left( \sum_{i=1}^{n} [c^R_i(p_i) + c^L_i(p_i)] - a \right).
\]

Each single lobby takes the contribution schedules of other lobbies and the Lagrange multiplier \( \lambda \) as given. Then, from the point of view of a lobby on dimension \( i \), \( I \)'s problem becomes:

\[
L_i(p_i, \lambda) = -p_i^2 + \lambda [c^R_i(p_i) + c^L_i(p_i)].
\]

Lobbies on \( i \) know that \( I \) gets a marginal benefit \( \lambda \) for each dollar of contributions from lobbies on \( i \). This marginal benefit is independent of the contribution schedules of lobbies on \( i \). If \( I \) reduces the amount of money she gets from dimension \( i \) she can increase the amount she gets from all other lobbies by an infinitesimal amount. As there are a large number of lobbies and contribution schedules are assumed to be differentiable, this has an infinitesimal effect on \( \lambda \). I assume that there exist some dimensions on which \( k^R_i \neq k^L_i \). This excludes the extreme case in which \( \frac{d}{dp_i} c^R_i(p_i) + c^L_i(p_i) = 0 \) for all dimensions and all policies. In that case \( \lambda \to \infty \) and the following analysis is not applicable.

Thus, lobbies \( R \) and \( L \) solve respectively: \(^{12}\)

\[
\begin{align*}
\max_{p_i} & k^R_i(1 + \varepsilon) p_i^* - c^R_i(p_i^*) \\
\text{subject to} & p_i^* \in \text{argmax}_{p_i} L_i(p_i, \lambda),
\end{align*}
\]

\(^{11}\) Clearly, minimizing \( \sum_{i=1}^{n} p_i^2 \) is equivalent to minimizing \( \| p \| \). I use the former because it simplifies algebra.

\(^{12}\) For simplicity, I assume that the bliss points of the two lobbies \((-1 \text{ and } 1)\) are never reached, so that the problem is equivalent to a problem in which the right lobby maximizes \( p_i \) and the left lobby minimizes \( p_i \).
and
\[
\max_{\hat{c}_i(\cdot)} -k_i^r(1+\varepsilon)p_i^* - c_i^r(p_i^*)
\]
subject to: \( p_i^* \in \arg\max_{p_i} L_i(p_i, \lambda) \).

The game played between lobby \( R_i \) and lobby \( L_i \) according to (7) and (8) is a common agency problem (Bernheim and Whinston [9], Dixit, Grossman, and Helpman [15], Konishi, Le Breton, and Weber [20] and Laussel and Le Breton [22, 23]). Several principals (the lobbies on dimension \( i \)) offer money to the agent (the incumbent) conditional on the agent’s action \( (p_i) \). Common agency games tend to have multiple equilibria. However, one class of equilibria, called truthful equilibria, has been shown to be payoff-equivalent to the set of coalition-proof equilibria (Bernheim and Whinston [9]). A truthful equilibrium is an equilibrium in which the contribution schedule of each lobby follows the shape of the payoff function of that lobby less a constant and save for nonnegativity constraint. This property avoids equilibria in which lobbies fail to coordinate. In this paper I will only consider truthful equilibria.\(^{13}\)

If on each policy dimension the two lobbies and the incumbent play truthful strategies, then it is possible to provide a simple characterization of the policy bias:

**PROPOSITION 1.** Suppose a truthful equilibrium arises on each policy dimension \( i \). Then, given \( a \), the total policy bias needed to raise \( a \) is

\[
D \equiv \|p^*\| = \frac{a\delta}{1+\varepsilon},
\]

where

\[
\delta = 2 \frac{\sqrt{\sum_{i=1}^{n} (k_i^R - k_i^L)^2}}{\sum_{i=1}^{n} \left[ (k_i^R)^2 + (k_i^L)^2 \right]}.
\]

An important implication of Proposition 1 is that \( D \) is decreasing in the estimated probability of election \( \varepsilon \). With a higher \( \varepsilon \), lobbies believe that it is more likely that the policy vector \( p \) will be implemented for a second term. Therefore, they are willing to make higher contributions in order to

\(^{13}\) An important property of truthful equilibria is that they maximize the sum of payoffs of the principals and the agent. Here, this efficiency property means that the equilibrium maximizes the sum of payoffs of the lobbies and the incumbent (given \( \theta \)). However, it says nothing about voters’ payoffs, because the latter are not direct players in the common agency game.
steer ρ to their advantage. As we will see, ε depends positively on θ. Hence, given a, a candidate with θ = h incurs a lower policy bias than a candidate with θ = 0 in order to raise a. This will be the mechanism through which the amount of campaign contributions that one candidate has collected signal the candidate’s θ to voters.

Also, the policy bias is increasing in δ. Recall that the k’s represent the fundraising abilities of each single lobby. Then, δ can be seen as a concentration index of fundraising abilities. There are two extreme cases. If, on every policy dimension, both lobbies have the same fundraising ability, then δ → 0 (recall that the case in which δ is exactly zero cannot be treated here). This is the best-case scenario for the incumbent. She collects the highest amount of contributions by choosing the median voter’s ideal policy. The other extreme case occurs if on each dimension one lobby has no fundraising ability, which is as if only one lobby were active on each dimension. Let us assume without loss of generality that only right lobbies are active. Then, δ = 2/√∑_{i=1}^{n} (R_i^2). This is the worst-case scenario for the incumbent. Each lobby is a monopolist on its own dimension. Hence, for a given a, the policy bias increases as fundraising ability is more unequally distributed on each dimension.

4. EQUILIBRIUM ON THE CONTRIBUTION MARKET

After examining the supply side of campaign finance, we turn to the demand side and we find the equilibrium of the whole model. The incumbent wants money to be able to influence voters. Voters are influenced by campaign expenditures because they take them as signals that the incumbent has a high valence. This section makes this argument formally by determining the existence and the properties of revealing equilibria.

Let

\[ D_\theta(a) = \frac{a\delta}{1 + \epsilon_\theta} \]  

(9)

\[ e_\theta(a) = \rho F(g + \theta - D_\theta(a)) + (1 - \rho) F(g + \beta(a)). \]  

(10)

The function \( D_\theta(a) \) comes from Proposition 1 and represents the policy bias I must choose to collect a from lobbies if she is of type \( \theta \). The function \( e_\theta(a) \) comes from (3) and gives the election chance of I if she collects a war chest a and if her type is \( \theta \). \( \beta(\cdot) \) and \( \epsilon \) are beliefs and will be determined in equilibrium.

Let us define a revealing equilibrium as \( \langle a_0, a_h, \epsilon_0, \beta(\cdot) \rangle \) such that \( a_0 \neq a_h \) and, for \( \theta \in \{0, h\}, \)
where (11) gives the incumbent’s best reply given the beliefs of voters and lobbies and (12) and (13) require beliefs to be consistent. By combining (11) and (13), we immediately see that

**Lemma 1.** In a revealing equilibrium: (i) \( e_0(a_h) \leq e_0(0) \); (ii) \( e_h(a_h) \geq e_h(0) \); and (iii) \( a_h > a_0 \).

Conditions (i) and (ii) are familiar incentive-compatibility constraints. Condition (iii) guarantees that the high type takes a more costly action (in terms of policy bias) than the low type. If this were not the case, then the low type would gain by pretending to be a high type, which would violate (i).

To find a revealing equilibrium we guess the form of voters’ beliefs and we construct the equilibrium from there. The resulting equilibrium has the property of achieving revelation with the smallest policy bias. Suppose that voters have beliefs

\[
\beta(a) = \begin{cases} 
-D_0(a) & \text{if } a < a^* \\
h - D_h(a) & \text{if } a \geq a^*
\end{cases}
\]

for some \( a^* > 0 \).

With this type of beliefs, any \( a \in (0, a^*) \) is dominated by \( a = 0 \) and any \( a \in (a^*, \infty) \) is dominated by \( a = a^* \). Therefore, we can restrict our attention to \( a \in \{0, a^*\} \). The three conditions of Lemma 1 become two: \( e_0(a^*) \leq e_0(0) \) and \( e_h(a^*) \geq e_h(0) \). However, we will assume that the former condition holds as an equality (this assumption is motivated in footnote 14).

Thus, the conditions for a revealing equilibrium are:

\[
e_0(a^*) = e_0(0) \quad (15)
\]

\[
e_h(a^*) \geq e_h(0). \quad (16)
\]

By combining (10) and (14) and noticing that \( D_0(0) = D_h(0) = 0 \), we have

---

14 This type of beliefs is common in the industrial organization literature on advertising with rational voters. See, for instance, Milgrom and Roberts [26].
\( e_0(0) = F(g); \quad (17) \)
\( e_0(a^\ast) = \rho F(g - D_0(a^\ast)) + (1 - \rho) F(g + h - D_0(a^\ast)); \quad (18) \)
\( e_0(0) = \rho F(g + h) + (1 - \rho) F(g); \quad (19) \)
\( e_0(a^\ast) = F(g + h - D_0(a^\ast)). \quad (20) \)

\( e_0(0) \) and \( e_0(a^\ast) \) are probabilities of election if \( I \) behaves according to his or her type. \( e_0(a^\ast) \) is the probability of election of a low type who collects a war chest \( a^\ast \) in order to be perceived as a high type. With probability \( \rho \) voters will call her bluff, otherwise they will fall for it. \( e_0(0) \) is the probability of election for a high type who does not bother to collect campaign funds. With probability \( \rho \) voters will find out her valence anyway, otherwise they will think she is a low type.

Now, we are ready to prove that the incentive-compatibility constraint for the low type implies the incentive-compatibility constraint for the high type:

**Lemma 2.** (15) implies (16).

From Proposition 1, for any \( a^\ast > a' \) the following inequality holds

\[ D_\theta(a^\ast) - D_\theta(a') < D_0(a^\ast) - D_0(a'). \]

This is a single-crossing condition. The policy cost of additional campaign finance is higher for the lower type, which explains why Lemma 2 holds.

Again, by Proposition 1, there is a simple relation between \( D_\theta(a^\ast) \) and \( D_\theta(a^\ast) \):

\[ D_0(a^\ast) = \frac{1 + e_h}{1 + e_0} D_\theta(a^\ast). \]

However, by (12), lobbies’ beliefs must be consistent. Therefore,

\[ D_\theta(a^\ast) = \frac{1 + F(g + h - D_h(a^\ast))}{1 + F(g)} D_\theta(a^\ast). \quad (21) \]

By putting together (15) and (21), we have

\[ F(g) = \rho F \left( g - \frac{1 + F(g + h - D_h(a^\ast))}{1 + F(g)} D_\theta(a^\ast) \right) + (1 - \rho) F(g + h - D_\theta(a^\ast)) \quad (22) \]

in which the only unknown is \( D_h(a^\ast) \). Let the minimum positive solution of (22) be denoted with \( D^\ast \). A fixed point argument on \( D_\theta(0) = 0 \) and \( \lim_{a \to \infty} D_\theta(a) = \infty \) shows that \( D^\ast \) exists.
By using Proposition 1, we have that 

\[ a^* = \left(1 + \frac{F(g + h - D^*)}{\delta} \right) D^* \]  

which, by (12), implies that

\[ a^* = \left(1 + \frac{F(g + h - D^*)}{\delta} \right) D^* \]  

(23)

The preceding discussion is summarized in:

**Proposition 2.** There exists a revealing equilibrium in which \( a_0 = 0 \) and \( a_0 = a^* \), where

\[ a^* = \left(1 + \frac{F(g + h - D^*)}{\delta} \right) D^* \]  

(24)

and \( D^* \) is the minimum positive solution of

\[ F(g) = \rho F \left( g - \frac{1 + F(g + h - D^*)}{1 + F(g)} D^* \right) + (1 - \rho) F(g + h - D^*) \]  

(25)

Collecting a war chest is more “costly” for a low-valence incumbent because, for the same sum, she has to deviate further from the median voter’s ideal policy. Proposition 2 finds the level of advertising \( a^* \) at which a low-valence incumbent is indifferent between selling out and not selling out. Instead, at \( a^* \), a high-valence incumbent is strictly better off advertising because the loss she incurs when voters observe valence and policy directly is more than compensated by the benefit she gets when they do not receive a signal and they rely on advertising. This generates a fully revealing equilibrium in which a low-valence incumbent receives no money and chooses the median voter’s ideal policy while a high-valence candidate receives enough money to credibly signal that she is not a low-valence candidate. With probability \( 1 - \rho \), voters do not observe the valence and policy of the incumbent directly, but they infer it from the amount of advertising \( a \). Thus, in equilibrium, voters are fully informed whether or not they observe valence and policy directly.

The equilibrium of Proposition 2 is not the only perfect Bayesian equilibrium of the game considered here. In particular, there is a pooling equilibrium in which voters refuse to draw inferences from campaign expenditures and therefore the incumbent has no reason to make any (and therefore there are no campaign contributions either). In a pooling equilibrium, when voters are uninformed, they do not learn the type of the candidate. The pooling equilibrium is supported by voters’ beliefs \( \hat{\beta}(a) = qh \) for any \( a \) (recall that \( q = \Pr(\theta = h) \)).
To verify the robustness of the pooling equilibrium, one can apply the well-known Intuitive Criterion by Cho and Kreps [11] which checks the plausibility of out-of-equilibrium beliefs. Suppose the incumbent makes a positive level of campaign expenditures \( a' \). Voters should ask themselves what he or she is trying to signal. Suppose that, if voters believed that \( a' \) could only come from a high-type, then in fact a high type would be better off with \( a = a' \) rather than with \( a = 0 \), while a low type would be worse off with \( a = a' \) than with \( a = 0 \). Then, voters should conclude that only a high type may want to undertake such a deviation. In this case, one would say that the pooling equilibrium does not survive the Intuitive Criterion. Indeed, I prove that this is the case.

**Proposition 3.** The pooling equilibrium does not survive the Intuitive Criterion.

Hence, the fully revealing equilibrium of Proposition 2 appears to be more robust than the pooling equilibrium. From this point on, I will focus exclusively on the revealing equilibrium.\(^\text{15}\)

Let us conclude the section with some comparative statics:

**Proposition 4.** An increase in \( \rho \) causes a decrease in \( D^* \) and in \( a^* \).

Both policy bias and campaign expenditures go down as voters receive more direct information. A higher \( \rho \) reduces the incentive for a low-quality candidate to mimic a high-quality candidate and the latter can signal his or her type with a lower war chest. As a result, the high-quality candidate needs to make fewer concessions to lobbies. The negative relation between voter direct information and amount of campaign advertising predicted by Proposition 4 may be empirically testable. Ceteris paribus, a constituency with better informed voters (as measured before the effect of advertising) should experience lower levels of campaign spending.

5. MEDIAN VOTER WELFARE

Is the possibility of contributions beneficial or detrimental for voters? With campaign spending, I have argued that a revealing equilibrium is likely to arise. Voters learn the type of the candidate even when they are\(^\text{178}\)
uninformed, but high-type candidates deviate from the median voter’s ideal policy in order to secure campaign contributions from lobbies. Without campaign spending, the incumbent cannot signal her type. With probability \(1 - \rho\) voters ignore \(\theta\). On the other hand, the incumbent can always choose the median voter’s ideal policy.

In this section I focus on the expected utility of the median voter. Here, this measure has both a normative and a positive interpretation. The normative interpretation is that, as voters are symmetrically distributed, the median voter welfare is a reduced form of the integral of utility over the continuum of voters, that is, the Utilitarian social welfare. Note that this definition of welfare does include the utility of lobby members. They are counted in the same way as other voters. Moreover, the definition does not include the direct inefficiency arising from campaign spending per se. Electoral expenditures are unproductive and can use scarce resources such as labor or paper. This additional cost should be counted in the welfare measure, but is not. Thus, if we find that welfare is higher when campaign spending is prohibited, this result holds a fortiori if we include the direct inefficiency arising from campaign spending.

The median voter’s expected utility also answers the positive question: If, before the game is played a referendum is held to decide whether campaign contributions should be allowed, what would the outcome of the referendum be?

Let \(w_0\) be the median voter’s expected utility if it is known that the incumbent has no policy bias \((p_i = 0 \text{ for all } i)\) and type \(\theta\). Then, \(w_0 = E_x[\max(g, x)]\) and \(w_h = E_x[\max(g + h, x)]\). Similarly, let \(\bar{w}\) be the median voter’s expected utility if voters do not know what type the incumbent is (neither directly because they are informed nor indirectly because they infer it from campaign expenditure): \(\bar{w} = E_x[\max(g + qh, x)]\). Finally, let \(\bar{w}_h\) be the median voter’s expected utility if it is known that \(I\) is a high type who has chosen a policy bias \(D^*\): \(\bar{w}_h = E_x[\max(g + h - D^*, x)]\).

Let \(W\) be the median voter’s ex-ante utility, that is, his expected utility before receiving any information about the incumbent. Let \(W_n\) denote the case in which campaign spending (or campaign giving) is forbidden and let \(W_s\) denote the case in which it is allowed:

\[
W_n = \rho(1 - q) q_0 q + q w_h + (1 - \rho) \bar{w}, \tag{26}
\]

\[
W_s = (1 - q) w_0 q + q \bar{w}_h. \tag{27}
\]

By putting together (26) and (27) we have:

**Proposition 5.** \(W_n > W_s\) if and only if

\[
q (w_h - \bar{w}_h) > (1 - \rho)(1 - q) w_0 q + q w_h - \bar{w}. \tag{28}
\]
The left-hand side of (28) represents the expected cost for the median voter of allowing campaign spending, while the right-hand side represents the expected benefit. The expected cost is the difference between having a high-type incumbent with no policy bias and a high-type incumbent with \( D^* \). The \( q \) indicates that this cost is not sustained when the incumbent is low-type. The expected benefit comes from the ability of discriminating between high-types and low-types. It is immediate to see that it is always positive. The benefit is pre-multiplied by \((1 - \rho)\) because if voters receive direct information, they can discriminate between high-types and low-types anyway.

Both sides of (28) are decreasing in the probability that voters are informed: \( \rho \). The expected cost is decreasing because \( D^* \) is decreasing in \( \rho \). The expected benefit is decreasing because campaign spending is useful only when voters do not receive information directly. Therefore, Proposition 5 does not tell us whether the inequality \( W_s > W_n \) is more likely to hold for low \( \rho \)'s or for high \( \rho \)'s. However, we can prove the following:

**Proposition 6.** For any \( g \) and \( h \), if \( \rho \) is small enough, then \( W_n > W_s \).

**Proof.** From Proposition 5, \( \lim_{\rho \to 0} (W_n - W_s) > 0 \) if

\[
\lim_{\rho \to 0} \left[ q\left(w_h - \bar{w}\right) - (1 - \rho)((1 - q) w_0 + qw_h) - \bar{w}\right] > 0
\]

which—as \( w_0, w_h, \) and \( \bar{w} \) do not depend on \( \rho \)—is equivalent to

\[
\bar{w} - (1 - q) w_0 - q \lim_{\rho \to 0} \bar{w}_h > 0. \tag{29}
\]

Consider (25) and take the limit as \( \rho \to 0 \). The left-hand side is unchanged. The right-hand side tends to \( F(g + h - D^*) \). We have that

\[
\lim_{\rho \to 0} F(g + h - D^*) = F(g)
\]

\[
\lim_{\rho \to 0} (g + h - D^*) = g
\]

\[
\lim_{\rho \to 0} E_s[\max(g + h - D^*, x)] = E_s[\max(g, x)]
\]

\[
\lim_{\rho \to 0} \bar{w}_h = w_0.
\]

Thus, (29) reduces to \( \bar{w} > w_0 \), which is true because \( h > 0 \).

If voters are unlikely to receive direct information, then welfare is higher if campaign spending is forbidden. This may seem surprising because campaign spending brings the highest informational benefit when voters are the
least informed. However, also the policy bias associated with campaign spending is at its highest when voters are least informed. If voters are unlikely to get a direct signal about the incumbent’s type, then a low-type incumbent has a high incentive to mimic a high type. In the limit, as $\rho$ tends to zero, the benefit from campaign spending is equal for a low type and for a high type. To separate herself from the low type, the high type must “burn” through policy bias the whole benefit of being revealed as a high-type, that is, $h$.

One may wonder about the rationality of voters: if campaign spending reduces their welfare, why don’t they just refuse to listen to it? Unfortunately, they cannot commit before the election to disregard electoral advertising. Once the incumbent has sustained the expenditure, it is in the interest of voters to extract as much information as they can. This negative welfare effect due to the impossibility of committing not to use a costly signal has a parallel in other signaling games such as Akerlof’s [1] rat race.

6. THE OBSERVED EFFECTIVENESS OF CAMPAIGN SPENDING

Several authors have estimated the effect of campaign spending on voting behavior (for a survey, see Levitt [25]). Various datasets and econometric methodologies have been used. The vast majority of these studies suggest that campaign expenditures by an incumbent have very little influence on the incumbent’s probability of re-election. For instance, Levitt [24] cannot reject the null hypothesis that campaign spending has no effect at all on election outcome. Although most of these studies use US data, Palda and Palda [29] find analogous results for French legislative elections.

This section examines this somewhat surprising empirical finding in the light of the present model. As we will see, the fact that the observed effectiveness of campaign spending is low has stark welfare implications.

Consider a political environment that behaves according to the game theoretical model described so far. An external observer ignores the underlying parameters as well as the function $F(\cdot)$. The only data the observer has are election outcomes and campaign spending. Because the model is binary, only two levels of campaign spending will be observed: 0 and $a^*$. By regressing election outcomes on campaign spending, the observer obtains the gross effectiveness of campaign spending. The effectiveness is called “gross” because it overlooks the fact that spending and policy are determined together. In our binary model the gross effectiveness of campaign spending consists of two election probabilities: one when $a = 0$, which we denote with $\Psi_0$, and one when $a = a^*$, which we denote with $\Psi_h$. 

CAMPAIGN SPENDING

181
Based on these two election probabilities, what conclusions can the observer draw about median voter welfare?\textsuperscript{16}

**Proposition 7.** Suppose that $h \gg 0$, there exists a $k > 0$ such that if $\Psi_h - \Psi_0 > k$ then $W_n > W_s$.

**Proof.** By Proposition 1, low-type incumbents choose $a = 0$ and high types choose $a = a^*$. Thus, by (17) and (20),

$$\Psi_0 = e_d(0) = F(g),$$

$$\Psi_h = e_d(a^*) = F(g + h + D^*).$$

Therefore, $\Psi_h - \Psi_0$ implies $F(g + h + D^*) \rightarrow F(g)$. The result that $W_n > W_s$ can now be derived as in the proof of Proposition 6.

The intuition for Proposition 7 is straightforward. If one observes a low gross effectiveness of campaign spending, one should conclude that, in the eyes of the median voter, most of the informational benefit of spending is offset by the policy bias needed to raise contributions. Then, it is as if voters were faced with two types of candidates: low types with no policy bias and high types with a bias which almost makes up the difference between a high type and a low type. This situation is certainly worse than the situation without contributions. There, voters are faced with “average” candidates with no policy bias.

In this model a low observed effectiveness of campaign spending does not mean that spending is useless. Electoral expenditures still provide voters with valuable information. Unfortunately, the policy bias needed to raise contributions more than offsets the informational benefit. If we observe a low effectiveness, we should infer that the political system is in the rat race dilemma discussed in the end of the previous section. In such circumstances, forbidding campaign spending improves median voter welfare.

7. CONCLUSIONS

In this paper, I have attempted to build a coherent story of campaign advertising as money burning. However, campaign giving is only one of the activities of lobbies (Schlozman and Tierney \[32\]). Lobbies also provide information to policy makers and keep their members informed on the political process. This activity includes the possibility of suggesting to their

\textsuperscript{16} I assume however that the parameter $g$ is constant across observations. This can be achieved in panel data such as those used by Levitt.
members who to vote for, that is, to endorse candidates, which is studied by Grossman and Helpman [17].

Both the present model and Grossman and Helpman’s model are signaling games. In their model, some credible direct information communication between lobbies and voters is possible. That is because the optimal policy for the lobby and the optimal policy for the median voter both depend on the state of the world, which is a random variable. This partial congruence of interests may allow for some meaningful cheap talk à la Crawford and Sobel [12]. Instead, in my model the optimal policy for the voter is independent of the private information of lobbies because it is fixed at zero. Moreover, the lobby does not care about valence. This excludes the possibility of cheap talk. Thus, the present paper and Grossman–Helpman’s look at two polar cases: only money burning and only cheap talk.

Yet, there is a continuum between these two extremes. Suppose that in my model lobbies also benefit directly from the valence of the candidate. If they care about valence much more than about policy, then cheap talk is enough and lobbies would just be able to indicate to voters which candidate they should elect. If they have only a limited interest in valence, then money burning still plays a role. Lobbies still use political contributions to steer policy in their preferred direction. However, they are willing to give more to a high-valence candidate than to a low-valence candidate because of their concern for valence.

Another limitation of the present model is that it does not include political parties. Especially in Europe, campaign finance is often managed by parties. Party leaders receive contributions (whether public financing, legal private funds, or illegal private funds) and make political expenditures on behalf of candidates of that party. The signaling role of expenditures is mediated by the signaling role of parties (Caillaud and Tirole [10]). It would be important to examine the interaction between the two types of signals.

APPENDIX: PROOFS

Proof of Proposition 1. Lobbies on $i$ offer truthful contribution schedules

\[
c^L_i(p_i) = \max(0, k^L_i(1 + \epsilon) p_i - \pi^L_i))
\]

\[
c^R_i(p_i) = \max(0, k^R_i(1 - \epsilon) p_i - \pi^R_i)),
\]

where $\pi^L_i$ and $\pi^R_i$ are constants.

We now combine Lemma 3 (see statement and proof below) and minimization problem (4) to determine the policy vector $p^*$ that is optimal for
I given $a$ and given the contribution schedules of all lobbies. Let $c_i(p) = c_i^R(p) + c_i^L(p)$. Then, from Lemma 3,

$$c_i(p^*_i) = \frac{\lambda(1 + \epsilon)^2}{4} [(k_i^R)^2 - (k_i^L)^2].$$  \hspace{1cm} (32)

By summing (32) over $i$ and by recalling that $\sum_{i=1}^{n} c_i(p^*_i) = a$,

$$\lambda = \frac{4a}{(1 + \epsilon)^2 \sum_{i=1}^{n} [(k_i^R)^2 - (k_i^L)^2]},$$  \hspace{1cm} (33)

which shows that $\lambda > 0$. From (30) and (31), we have that $I$'s first-order condition for dimension $i$ is

$$p_i^* = \frac{\lambda(1 + \epsilon)(k_i^R - k_i^L)}{2}.$$  \hspace{1cm} (34)

If we substitute (33) into (34), we have

$$p_i^* = \frac{2a(k_i^R - k_i^L)}{(1 + \epsilon) \sum_{i=1}^{n} [(k_i^R)^2 + (k_i^L)^2]}. $$

By summing over policy dimensions, the proposition is proven.

**Statement and Proof of Lemma 3**

**LEMMA 3.** Given $\lambda > 0$ and $\epsilon$, in the common agency game on policy dimension $i$ defined by (7) and (8), there exists a unique truthful equilibrium. In the truthful equilibrium, contribution schedules are as in (30) and (31) with

$$\pi_i^L = \frac{\lambda(1 + \epsilon)^2}{4} [(k_i^R - k_i^L)^2 - (k_i^R)^2],$$

$$\pi_i^R = \frac{\lambda(1 + \epsilon)^2}{4} [(k_i^R - k_i^L)^2 - (k_i^L)^2];$$

and $I$ chooses policy

$$p_i^* = \frac{\lambda(1 + \epsilon)}{2} (k_i^R - k_i^L).$$
Proof. By (30) and (31),
\[ \hat{\rho} \equiv \arg\max_{\rho_i} \lambda \left[ c_i^R(\rho_i) + c_i^L(\rho_i) \right] - \rho_i^2 = \frac{\lambda (1 + \epsilon)}{2} (k_i^R - k_i^L) \]
\[ p_i^R \equiv \arg\max_{\rho_i} \lambda c_i^R(\rho_i) - \rho_i^2 = \frac{\lambda (1 + \epsilon)}{2} k_i^R, \]
\[ p_i^L \equiv \arg\max_{\rho_i} \lambda c_i^L(\rho_i) - \rho_i^2 = -\frac{\lambda (1 + \epsilon)}{2} k_i^L. \]

Suppose that \( c_i^R(\hat{\rho}_i), c_i^L(\hat{\rho}_i), c_i^R(p_i^R), \) and \( c_i^L(p_i^L) \) are all nonnegative (we will check later that this is indeed the case). Then we can disregard the nonnegativity constraints in (30) and (31) and we have
\[ c_i^R(\hat{\rho}_i) = \frac{\lambda (1 + \epsilon)^2}{2} k_i^R (k_i^R - k_i^L) - \pi_i^R, \quad (35) \]
\[ c_i^L(\hat{\rho}_i) = -\frac{\lambda (1 + \epsilon)^2}{2} k_i^L (k_i^R - k_i^L) - \pi_i^L, \quad (36) \]
\[ c_i^R(p_i^R) = \frac{\lambda (1 + \epsilon)^2}{2} (k_i^R)^2 - \pi_i^R, \quad (37) \]
\[ c_i^L(p_i^L) = \frac{\lambda (1 + \epsilon)^2}{2} (k_i^L)^2 - \pi_i^L. \quad (38) \]

To find \( \pi_i^R \) and \( \pi_i^L \), we apply Dixit, Grossman, and Helpman [15, Proposition 4]. Truthful equilibria satisfy
\[ \max_{\rho_i} \lambda \left[ c_i^R(\rho_i) + c_i^L(\rho_i) \right] - \rho_i^2 = \max_{\rho_i} \lambda c_i^R(\rho_i) - \rho_i^2 = \max_{\rho_i} \lambda c_i^L(\rho_i) - \rho_i^2, \]
which corresponds to
\[ \lambda \left[ c_i^R(\hat{\rho}_i) + c_i^L(\hat{\rho}_i) \right] - (\hat{\rho}_i)^2 = \lambda c_i^R(p_i^R) - (p_i^R)^2 = \lambda c_i^L(p_i^L) - (p_i^L)^2. \quad (39) \]

By substituting (35), (36), (37), and (38) into (39), we have a system of two equation in two unknowns, \( \pi_i^R \) and \( \pi_i^L \):
\[ \frac{\lambda^2 (1 + \epsilon)^2}{4} (k_i^R - k_i^L)^2 - \pi_i^R - \pi_i^L = \frac{\lambda^2 (1 + \epsilon)^2}{4} (k_i^R)^2 - \pi_i^R \]
\[ \frac{\lambda^2 (1 + \epsilon)^2}{4} (k_i^L)^2 - \pi_i^L. \]

Provided that \( \lambda > 0 \), the statement of the lemma is proven.
It remains to check that $c^R_i(\hat{\beta}_i)$, $c^L_i(\hat{\beta}_i)$, $c^R_i(p^R_i)$, and $c^L_i(p^L_i)$ are indeed nonnegative. By substituting $\pi^R_i$ and $\pi^L_i$ in (35), (36), (37), and (38), we have

\[
c^R_i(\hat{\beta}_i) = \frac{\hat{\lambda}(1+\epsilon)^2}{4} (k^R_i)^2 > 0,
\]

\[
c^L_i(\hat{\beta}_i) = \frac{\hat{\lambda}(1+\epsilon)^2}{4} (k^L_i)^2 > 0,
\]

\[
c^R_i(p^R_i) = \frac{\hat{\lambda}(1+\epsilon)^2}{4} [(k^R_i)^2 + k^R_i k^L_i] > 0,
\]

\[
c^L_i(p^L_i) = \frac{\hat{\lambda}(1+\epsilon)^2}{4} [(k^L_i)^2 + k^R_i k^L_i] > 0.
\]

By (30) and (31), this also proves that $c^R_i(\cdot)$ is differentiable on an open interval containing $\hat{\beta}_i$ and $p^R_i$ and that $c^L_i(\cdot)$ is differentiable on an open interval containing $\hat{\beta}_i$ and $p^L_i$.

**Proof of Lemma 2.** Equations (15) and (16) rewrite as

\[
p F(g - D_\delta(a^*)) + (1 - p) F(g + h - D_\delta(a^*)) = F(g) \tag{40}
\]

\[
F(g + h - D_\delta(a^*)) \geq p F(g + h) + (1 - p) F(g); \tag{41}
\]

Concavity of $F(\cdot)$ and nonnegativity of $D_\delta(a^*)$ imply

\[
F(g + h - D_\delta(a^*)) - F(g - D_\delta(a^*)) \geq F(g + h) - F(g)
\]

which (because, from Proposition 1, $D_\delta(a^*) \leq D_\delta(a^*)$) implies

\[
F(g + h - D_\delta(a^*)) - F(g - D_\delta(a^*)) \geq F(g + h) - F(g).
\]

Multiply both sides of the latter inequality by $\rho$ and add the resulting inequality to (40). The result is (41).

**Proof of Proposition 3.** In a pooling equilibrium, from (10), if $a = 0$, $I$’s probability of election is given by

\[
e_\delta(0) = p F(g) + (1 - p) F(g + qh)
\]

\[
e_\delta(0) = p F(g + h) + (1 - p) F(g + qh).
\]
Consider the deviation \( a = a' \) and suppose voters (if uninformed) believe that an incumbent with \( a' \) must be a high type. Then,

\[
e_d(a') = p F(g - D_d(a')) + (1 - p) F(g + h - D_d(a'))
\]

\[
e_d(a') = p F(g + h - D_d(a')) + (1 - p) F(g + h - D_d(a')).
\]

The deviation is profitable for a high type but not for a low type if

\[
e_d(0) \geq e_d(a') \quad \text{and} \quad e_d(0) < e_d(a'),
\]

which is true if

\[
p(F(g) - F(g - D_d(a'))) = (1 - p)(F(g + h - D_d(a')) - F(g + qh))
\]

\[
(1 - p)(F(g + h - D_d(a')) - F(g + qh)) \geq p(F(g + h) - F(g + h - D_d(a'))).
\]

From this point on, the proof proceeds as in Lemma 2. It is proven that such \( a' \) exists and, therefore, that the pooling equilibrium does not survive the Intuitive Criterion.

**Proof of Proposition 4.** Let

\[
\psi(D^*, \rho) \equiv \rho F\left( g - \frac{1 + F(g + h - D^*)}{1 + F(g)} D^* \right)
\]

\[
+ (1 - \rho) F(g + h - D^*) - F(g).
\]

From the fact that \( F(g - D_0) < F(g + h - D^*) \), we see that \( \frac{\partial \psi}{\partial \rho} < 0 \). Also, as both \( F(g - D_0) \) and \( F(g + h - D^*) \) are decreasing in \( D^* \) (it is easy to see that concavity of \( F \) implies \( \frac{d}{dD^*}((1 + F(g + h - D^*)) D^*) > 0 \), \( \frac{\partial \psi}{\partial D^*} < 0 \). By the Implicit Function Theorem,

\[
\frac{dD^*}{d\rho} = -\frac{\partial \psi}{\partial \rho} < 0.
\]

From (24), we see that \( \frac{d\psi}{dD^*} > 0 \), which implies that

\[
\frac{da^*}{d\rho} = \frac{da^*}{dD^*} \frac{dD^*}{d\rho} < 0.
\]
REFERENCES