Introduction to Set Theory (W4431, Fall 1996, MW 4:10–5:25, HAM 309)

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Office Hours: T 3-5, or by appointment

General Outline

Set theory is the foundation of mathematics: all mathematical concepts can be characterized in terms of the primitive notions of set and membership. (Some would go as far as saying that *all* rigorous concepts—whether belonging to mathematics or to other disciplines—should be so characterizable.) But set theory is also a branch of mathematics, like algebra or geometry, with its own subject matter, basic results, open problems. The aim of this course is to give a general introduction to both aspects, with an eye for the unifying philosophical issues that lie behind them.

The first part of the course focuses on the question of providing an axiomatic formulation of set theory. The specific axiom system we shall examine is a version of ZAC, Zermelo set theory with the Axiom of Choice, eventually supplemented with Fraenkel's Axiom of Replacement (ZFAC). In the second part, the strength of theory will be tested and applied: topics covered include the natural numbers, well-ordered sets, transfinite induction and recursion, the continuum hypothesis, up to the basic concepts and results about infinite cardinal and ordinal arithmetic. The final part of the course will be devoted to questions of consistency and relative independence. We shall focus on the study of natural models of various set-theoretic principles and, if time permits, we shall compare them with some non-standard set universes, including Aczel's "antifounded universe".

Prerequisites

One term of formal logic (V3411/G4415, *Introduction to Symbolic/Formal Logic*, or G4801, *Mathematical Logic I*) and a willingness to master technicalities and to work at a certain level of abstraction.

Requirements

There will be four take-home examinations. The first three assignments will count for 20% of the final grade; the last assignment, at the end of the course, will account for the remaining 40%. I shall also give optional assignments every week. These will not count for the final grade, but everybody is encouraged to do them on a regular basis and to hand them in for correction.

Text

The textbook for the course is Yiannis N. Moschovakis's *Notes on Set Theory* (Springer-Verlag, 1994). Copies are available at Posman Books (Brodway & 116th). A copy is also available on reserve in the Math Library.

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Schedule (tentative)

1.	W	Sept 4	Presentation	/
2.	M	Sept 9	Naive set theory: introduction and basic notions	Ch. 1
3.	W	Sept 11	Naive set theory: equinumerosity; Cantor set	Ch. 2(a)
4.	M	Sept 16	Naive set theory: Schröder-Bernstein Theorem	Ch. 2(b)
5.	W	Sept 18	Axiomatic set theory: Foundations	Ch. 3
6.	M	Sept 23	Are sets all there is?	Ch. 4(a)
7.	W	Sept 25	Cardinal numbers and structured sets	Ch. 4(b)
8.	M	Sept 30	The natural numbers: existence, uniqueness, recursion	Ch. 5(a)
9.	W	Oct 2	The natural numbers: basic properties	Ch. 5(b)
10.	M	Oct 7	Partially ordered sets: basic notions	Ch. 6(a)
11.	W	Oct 9	Partially ordered sets: fixed point theorems	Ch. 6(b)
12.	M	Oct 14	Well ordered sets: basic notions	Ch. 7(a)
13.	W	Oct 16	Well ordered sets: transfinite induction and recursion	Ch. 7(b)
14.	M	Oct 21	Well ordered sets: comparability results	Ch. 7(c)
15.	W	Oct 23	The Axiom of Choice: formulations	Ch. 8(a)
16.	M	Oct 28	The Axiom of Choice: weakenings	Ch. 8(b)
17.	W	Oct 30	The Axiom of Choice: consequences	Ch. 9
18.	M	Nov 4	*Academic holiday*	/
19.	W	Nov 6	The Axiom of Replacement	Ch. 11(a)
20.	M	Nov 11	The hereditarily finite sets	Ch. 11(b)
21.	W	Nov 13	The Axiom of Foundation	Ch. 11(c)
22.	M	Nov 18	Ordinal numbers: basic properties	Ch. 12(a)
23.	W	Nov 20	Ordinal numbers: characterization	Ch. 12(b)
24.	M	Nov 25	Ordinal numbers: the cumulative hierarchy	Ch. 12(c)
25.	W	Nov 27	Interpreting the theory: Zermelo universes	App. B(a)
26.	M	Dec 2	Interpreting the theory: Rieger universes	App. B(b)
27.	W	Dec 4	Interpreting the theory: Aczel's antifounded universe	App. B(c)
28.	M	Dec 9	Retrospectus and conclusion	/