Beth Too, But Only If

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Instructions. Today's test will be on the conditional connective. As always, I will just give you a sentence in English and you will have to symbolize it in the language of sentential logic. Remember that symbolization is a procedure whereby you extract the logical *form* of a sentence. This is not just a translation procedure and there is no straightforward algorithm for it; it sometimes involves a difficult process of interpretation. But it is crucially important for logic, for the logical techniques that we are going to develop will apply to well-formed formulas of the language of sentential logic and only indirectly to the sentences of English. It will apply, that is, to the logical forms of the sentences of English.

The sentence you have to symbolize is this:

(*) If Alf went to the movie then Beth went too, but only if she found a taxi cab.

Please supply your answer with a brief description of the reasons why you think your symbolization captures the meaning (i.e., the truth-conditions) of (*). For uniformity, please use the following key:

A = Alf went to the movie.
B = Beth went to the movie.
C = Beth found a taxi cab.

Student 1. This looks like a conditional, but it isn't. The word 'but' is, from a logical point of view, equivalent to 'and', and there is no question that in this case the 'and' has a distributive reading. So 'but' expresses the conjunction connective and (*) can be rewritten as

(**) If Alf went to the movie then Beth went to the movie, and Beth went to the movie only if she found a taxi cab.

Accordingly, the symbolization is:

(1) $(A \rightarrow B) \wedge (B \rightarrow C)$.

Student 2. The word 'but' is not doing any work here. We might be tempted to symbolize (*) as

(1) $(A \to B) \wedge (B \to C)$,

but this sentential expression would have the wrong truth-conditions. For example, if Alf went to the movie and Beth did not succeed in finding a cab (i.e., if *A* is true and *C* false), then (1) would be false regardless of whether or not Beth went to the movie, whereas the intuitive truth-conditions for (*) would seem to depend on what Beth did (at least in case Alf went to the movie). Thus, I am inclined to think that 'but only if' here means just 'only if' and expresses a necessary condition for the truth of the sentence that comes before, which is marked by the comma and is itself in conditional form. Thus, a better rendering of (*) is

(**) If Alf went to the movie then Beth went to the movie, only if she found a taxi cab.

Accordingly, the symbolization is:

(2) $(A \to B) \to C$.

Student 3. The word 'but' is not doing any work here. One might be tempted to symbolize (*) as

(1) $(A \to B) \wedge (B \to C)$,

but this sentential expression would have the wrong truth-conditions. It would be F whenever *A* is T and *C* is F, contrary to intuition. So I take it that 'but only if' here just means 'only if'. What is the antecedent of this 'only if'? *Prima facie*, the comma suggests that the antecedent is the whole sentence that comes before the connective itself. Thus, since that sentence is in conditional form, one might be tempted to symbolize (*) as

(2) $(A \to B) \to C$.

However, this sentential expression would also have the wrong truth-conditions. It would be T whenever *C* is T, contrary to intuition. Perhaps this is just another example of the paradoxes of material implication, but I don't think so. I don't think so because—as Michael Dummett observed—we hardly have any use, in natural language, of conditionals in which the antecedent is itself a conditional—and I don't think (*) is an exception. (A conditional might occur as a premise in an argument, but that is another matter.) So I do not think that 'only if' is the main connective of (*). It is, rather, the connective of what I take to be the consequent of (*). The whole sentence can be re-written more perspicuously as

> (**) If Alf went to the movie, then Beth went to the movie only if she found a taxi cab.

And this can be symbolized as follows:

> (3) $A \rightarrow (B \rightarrow C)$.

Student 4. This is tricky. We might be tempted to symbolize (*) as

> (1) $(A \rightarrow B) \wedge (B \rightarrow C)$

or as

> (2) $(A \rightarrow B) \rightarrow C$

or as

> (3) $A \rightarrow (B \rightarrow C)$,

but all of these sentential expressions have the wrong truth-conditions. In particular, (3) is just as bad as (2), since both would get the value T as long as C gets the value T, whereas intuitively Beth's finding a cab is not a sufficient condition for the truth of (*). This is not just a sign of the inadequacy of the material conditional. Rather, I think that the source of the difficulty lies in the 'but only if C' clause, which requires a different analysis. As I see it, the purpose of this clause is not only to specify a necessary condition for B. Is also adds a restriction on the sufficiency of A for the obtaining of B. If A holds, and if the additional clause C is satisfied, then the speaker is committed to the truth of B. Thus, given A, 'B but only if C' really means 'B only if C, and if C then B' – that is, 'but only if' amounts to 'if and only if', and we can rewrite (*) as

> (**) If Alf went to the movie, then Beth went to the movie if and only if she found a taxi cab,

whose symbolization is (somewhat surprisingly):

> (4) $A \rightarrow (B \leftrightarrow C)$.

In other words, 'but only if' has a double effect: it introduces a necessary and sufficient condition, and it imposes a certain parsing of the sentence which ties the additional clause to the consequent of the conditional introduced by 'if'.

Student 5. Some *prima facie* options are:

(1) $(A \to B) \wedge (B \to C)$
(2) $(A \to B) \to C$
(3) $A \to (B \to C)$
(4) $A \to (B \leftrightarrow C)$.

However, none of these expressions captures the intuitive truth-conditions of the English sentence (*). This is obvious for (1), (2), and (3), all of which fail to do justice to the relevance of 'Beth went too' in the evaluation of (*). But the same applies to (4). For this last expression would be true in a scenario where Alf did not go to the movie while Beth found a cab and went alone, which is not what the sentence says. This is not just a byproduct of the material reading of the conditional; it's the word 'too' that is not accounted for. To rule this case out, we must take C to be a necessary (and, as it turns out, sufficient) condition, not of B, but of the conjunction of A with B. Accordingly, my proposed reading of (*) is

(**) If Alf went to the movie, then Alf *and* Beth went to the movie if and only if Beth found a taxi cab,

which is symbolized as

(5) $A \to ((A \wedge B) \leftrightarrow C)$.

Student 6. I am not sure how to do this. I considered a number of options that correspond to plausible parsings of the given English sentence:

(1) $(A \to B) \wedge (B \to C)$
(2) $(A \to B) \to C$
(3) $A \to (B \to C)$
(4) $A \to (B \leftrightarrow C)$
(5) $A \to ((A \wedge B) \leftrightarrow C)$.

However, none of these seems to capture the intuitive truth-conditions of (*), either because the sentential expression can be false regardless of the truth-value of B (as in (1)), or because it can be true regardless of the truth-value of B (as in

(2) and (3)), or because it fails to capture the contribution made of the word 'too' (as in (4) and (5)). Indeed, although (5) might seem better, it is tautologically equivalent to (4). So, instead of trying to figure out the answer by directly trying to parse (*) in some way or other, I think it's better to turn things around and begin by considering the situations with which the truth of (*) is compatible. On my reckoning, there are only four such situations:

A = T	B = T	C = T
A = T	B = F	C = F
A = F	B = F	C = T
A = F	B = F	C = F

The other four situations appear to be incompatible with (*):

A = T	B = T	C = F	(the 'only if' clause is violated)
A = T	B = F	C = T	(Beth must go if A and C hold)
A = F	B = T	C = T	(Beth cannot go *too* if Alf does not go)
A = F	B = T	C = F	(ditto)

So, now, taking these situations to correspond to the rows in a truth table, the (shortest) corresponding sentential expression is

(6) $B \leftrightarrow (A \wedge C)$

This looks very different in structure form the surface grammar of (*) but, on the face of it, (6) captures exactly the right truth-conditions.

Discussion. Dear students, I have decided not to assign any grade for yesterday's test. I did not realize that the sentence I assigned was so recalcitrant to regimentation. As it turns out, were it not for the occurrence of the word 'too', Student 4 gave the best answer. But even so, I agree that (4) is rather surprising as a rendering of the logical form of an English sentence in which there is no clear indication of a biconditional. On the other hand, with the word 'too' properly assessed, it is Student 6 who gave the answer that I find most appropriate *vis-à-vis* the intuitive meaning of (*). However, in this case the answer is even more surprising. We know that it would be a mistake to suppose that every ordinary-language sentence wears all of its connectives on its sleeves. But it is very rare that the underlying logical form is so different from the grammatical form. And it is peculiar that to figure it out one has to start from scratch and go through a complete chart of the possible truth-conditions, ignoring parsing altogether. It is peculiar because, after all, in understanding an English sentence such as (*) we do not normally do so. Or do we?

Student 5 (Addendum). Professor, I realize that my symbolization was mistaken, and that Student 6's solution is the only one that fully captures the intuitive truth-conditions of (*). Still, I think the analysis I offered was basically correct, except that I shouldn't have said that to account for the meaning of 'too' we must modify (4) by taking *C* to be necessary and sufficient for *A and B*. (That yields an expression that is indeed equivalent to (4).) Instead, I should have simply said that to account for the meaning of 'too' we must *supplement* (4) with the missing bit, namely, with a conjunct that makes it explicit that *B* implies *A* (Beth went *too* only if Alf actually went). In other words, this is how I should have phrased my reading of (*):

(**) If Alf went to the movie, then Beth went to the movie if and only if she found a taxi cab, and she went only if Alf went.

This is symbolized, not as (5), but as

(7) $(A \rightarrow (B \leftrightarrow C)) \wedge (B \rightarrow A)$.

Now, (7) is tautologically equivalent to (6), so it must be equally correct. I am not sure that it is closer to the surface grammar of (*) than (6) is, especially if you think that (4) is already "surprising". On the other hand, I don't think I came up with this solution by merely surveying the chart of all possible truth-conditions, so I don't think I ignored parsing altogether. Or did I?

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