On the Boundary Between Mereology and Topology

Achille C. Varzi
Istituto per la Ricerca Scientifica e Tecnologica, I-38100 Trento, Italy
varzi@irst.it

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1. Introduction

Much recent work aimed at providing a formal ontology for the common-sense world has emphasized the need for a mereological account to be supplemented with topological concepts and principles. There are at least two reasons underlying this view. The first is truly metaphysical and relates to the task of characterizing individual integrity or organic unity: since the notion of connectedness runs afoul of plain mereology, a theory of parts and *wholes* really needs to incorporate a topological machinery of some sort. The second reason has been stressed mainly in connection with applications to certain areas of artificial intelligence, most notably naive physics and qualitative reasoning about space and time: here mereology proves useful to account for certain basic relationships among things or events; but one needs topology to account for the fact that, say, two events can be continuous with each other, or that something can be inside, outside, abutting, or surrounding something else.

These motivations (at times combined with others, e.g., semantic transparency or computational efficiency) have led to the development of theories in which both mereological and topological notions play a pivotal role. How exactly these notions are related, however, and how the underlying principles should interact with one another, is still a rather unexplored issue. One can see mereology and topology as two independent chapters; or one may grant priority to topology and characterize mereology derivatively, defining parthood in terms of connection; or, again, one may privilege mereology and explain connection in terms of parthood *and* other predicates or relations. It is also possible, on some assumptions, to develop a unified framework based on a single mereotopological primitive of connected parthood. The purpose of this paper is to offer a first assessment of these alternative routes, discussing their relative merits and examining to what extent their adequacy, and more generally the boundary between mereology and topology, depends on the ontological fauna that one is willing to countenance.

The Bounds of Mereology

Mereology is by definition concerned with parts—that is, with the relation holding between two things when one is part of the other. On a weak understanding this means that a mereological theory is first and foremost an attempt to explicate the meaning of the word 'part' and to set out the principles underlying our correct use of it (and of kindred notions). For instance, virtually every mereological theory agrees on treating parthood as a partial ordering, which in a way reflects some very basic meaning postulates for 'part'.[1] Here, however, I am interested in the stronger interpretation, according to which mereology may provide a fundamental framework for the task of ontological investigations. It is a view that influenced much Greek and scholastic philosophy, and that made its way into modern philosophy *via* Husserl's third *Logical Investigation*.[2] Mereology tells us how reality is *constituted*. In this sense not just any partial ordering will qualify as a part-relation, and the question of what additional principles are involved becomes a truly philosophical (as opposed to merely terminological) question. Modern formal systems of mereology also owe their birth to this view, regardless of whether the relation "*x* is (a) part of *y*" is taken as a primitive (as in Leśniewski's *Mereology*) or defined in terms of cognate relations such as, for instance, "*x* extends over *y*" (Whitehead's *Enquiry*), "*x* is disjoint from *y*" (Leonard and Goodman's *Calculus of Individuals*), or "*x* overlaps *y*" (Goodman's *Structure of Appearance*).[3] Sometimes this has been associated with a nominalistic stand and mereology has been presented as a parsimonious alternative to set theory, dispensing with abstract entities or, better, treating all entities as individuals. However there is no necessary internal link between mereology and nominalism. Mereology can be credited a fundamental ontological role whether or not we take the entire universe to be describable in terms of parthood relationships. I think the recent revival of mereology and its ascent in artificial intelligence and other cognitive sciences can be seen in this light. The question is, rather, how far we can go with it—how much of the universe can be grasped and described by means of purely mereological notions.

It is in this perspective that the limitations mentioned at the beginning become relevant, particularly if our concern is the ontology of the macroscopic, common-sense world. Our common-sense picture of reality requires some means of distinguishing between things that are all of a piece and things that are scattered in space or time, or between things that are continuous and things that are not. Yet it is not clear how this can be done mereologically, starting from the relational concept of part (or overlapping, disjointness, and the like). In spite of the natural tendency to present mereology as a theory of parts *and* wholes, wholeness cannot be explained in terms of parthood, hence of mereology, except in the trivial sense that everything qualifies as the complete whole of its parts. Moreover, according to classical mereology every class of parts determines a complete whole (its mereological sum, or fusion), which makes the latter an utterly ineffective notion.

This latter point is not in itself undisputed. Avoiding explicit reference to classes, the underlying principle is that every satisfied property or condition picks out a unique entity consisting of all things satisfying that property or condition. It is usually expressed as follows:[4]

(1) $\exists x\phi x \rightarrow \exists x\forall y(\mathrm{O}yx \leftrightarrow \exists z(\phi z \wedge \mathrm{O}yz))$

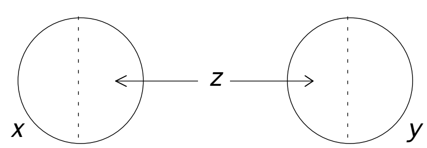
where 'O' stands for the relation of overlapping (i.e., sharing a common part). This principle is probably the most commonly criticised feature of classical mereology, the usual objection being that it has counter-intuitive instances, i.e., "unnatural" sums of widely scattered, disparate, unrelated, or otherwise ill assorted entities, such as the totality of red things, or my eyebrows and your favorite Chinese restaurant.[5] The classical mereologist's reply is simply that the criticism is off target, and I go along with that. If you already have some things, allowing for their sum is no further commitment: the sum *is* those things.[6] One may feel uncomfortable with treating unheard-of Goodmanian mixtures as individual wholes. But it is not a task of mereology to specify which wholes are more natural than others. In effect, telling which entities constitute natural wholes is presumably not even a metaphysical task, but the concern of empirical sciences [7] (just as ascertaining which sentences are true is not a semantic task but an empirical issue). The real source of difficulty, as I see it, is different. It is that the question of what constitutes a natural whole cannot even be *formu-*

lated in mereological terms. As soon as we allow for the possibility of scattered entities we lose the possibility of discriminating them from integral, connected wholes; but of course we cannot just keep the latter without some means of discriminating them from the former.

Whitehead's early attempts to characterize his ontology of events (the primary natural entities of the *Enquiry*) provide a good exemplification of this mereological dilemma.[8] Whitehead's system does not satisfy (1), for the intended domain is one which excludes scattered or disconnected entities (events). Of course it is not maintained that there are *no* mereological sums. Rather, the suggestion is that a necessary condition for two events to have a sum is that they be "joined" to each other. This relation of joining does not coincide with overlapping (for otherwise no event could be dissected into separated proper parts), and it is explicitly considered that two events may be *ad*joined, i.e., joined without sharing any common parts. Joining is thus a more general notion than overlapping: it is intuitively meant to hold whenever two events are continuous with each other, be they discrete or not. Whitehead eventually defines it along the following lines:

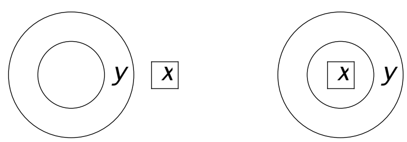
(2) $\mathrm{J}xy =_{\mathrm{df}} \exists z(\mathrm{O}zx \wedge \mathrm{O}zy \wedge \forall w(\mathrm{P}wz \rightarrow \mathrm{O}wx \vee \mathrm{O}wy))$

where 'P' indicates parthood. Now, this definition does indeed say that two events are joined just in case their mereological sum exists. But precisely for this reason, it is immediately verified that (2) falls well short of capturing the intended notion of topological connectedness or continuity. For there is nothing to guarantee that the piece overlaying two joined events be itself connected. The pattern reproduced below, with two disconnected discs *x* and *y* partially overlapping a disconnected piece *z*, illustrates a simple counter-example.[9]



These considerations apply *mutatis mutandis* to other attempts to subsume topological connectedness within a bare mereological framework.[10] Of course one can succeed if the assumption is made that only self-connected entities can inhabit the domain of discourse. This would indeed support a restricted conception of wholeness in which, withholding (1) in its generality, a plurality of entities can be said to make up an integral whole just in case they have a sum (or at least an upper bound relative to the part-relation [11]). But this is no satisfactory way out, for it just is not possible to make the assumption explicit. If the lack of any specific notion of whole is indicative of the neutrality of mereology, and hence of its strength and generality,[12] it is a fact that this lack is in turn a lack of expressiveness, hence a sign of weakness too.

Nor is this exclusively a metaphysical concern. This deficiency of mereology has shown up in various ways in recent work in linguistics, knowledge representation, and qualitative reasoning in artificial intelligence.[13] In all of these contexts mereology is now credited a central role in accounting for certain fundamental relationships among the entities in the domain of discourse—be they events, spatial regions, physical objects, or what have you. But as I already anticipated, these contexts also tend to confirm the limits of mereology when it comes to accounting for relations that entail a step into the territory of topology. Mereologically the two situations depicted below involve no difference, though of course we may want to keep track of the basic opposition in terms of spatial inclusion of the square, *x*, in the annulus, *y*.



The same difficulty arises when we consider relations among things that are just touching each other, or straddling one another, or neighbouring other things. All of these—and many others indeed—are relations that any theory concerned with spatio-temporal entities should supply and which cannot, however, be defined directly in terms of plain mereological primitives.

Three Ways to Topology

This need to overcome the bounds of mereology has been handled in the literature in various ways, but I think three main strategies can be distinguished. The first is, in a sense, the obvious one: if topology cannot be made to fit merelogy, and if its importance is to be fully recognized, then we may just *add* it to a mereological basis as an independent chapter. From this point of view, mereology can be seen as the ground theory on which theories of greater and greater complexity (including topology as well as, say, morphology or kinematics) can be built by supplying the necessary notions and principles. The second strategy is more radical: if topology eludes the bounds of mereology, we may try and turn things around: start from topology right away and define mereological notions in terms of topological primitives. From this point of view, just as mereology can be seen as a generalisation of the even more fundamental theory of identity (parthood, overlapping, and even fusion subsuming singular identity as a definable special case), likewise topology can be seen as a generalisation of mereology, where the relation of joining or connection takes over overlapping and parthood as special cases. Finally, the third strategy is a sort of vindication of mereology, building precisely on its formal generality: on this view topology is simply a domain-specific chapter of mereology, connection and kindred notions being accounted for in terms of part-relations among entities of a specified sort. I shall further scrutinize these three strategies in turn.

First way. The first strategy is, as I said, the most obvious and has in effect been followed by most authors (sometimes on quite independent grounds). It was pioneered by Tarski's work on the foundations of the geometry of solids (where a mereological basis is supplemented with a single primitive predicate "*x* is a sphere" to allow a definition of solid geometric correlates of all ordinary point-geometric notions) and was reproposed by Lejewski in his outline of a Leśniewskian theory of time, or *Chronology* (where a mereological basis is supplemented with a primitive relation "*x* is wholly earlier than *y*" to account for the main topological feature of time structures, viz. precedence).[14] The same approach underlies Tiles' analysis of events (where topology is introduced by means of the primitive "*x* lies in the interior of *y*") as well as much linguistics-oriented work on time, tense, and aspect, such as Kamp's analysis of temporal reference and Bach's or Link's algebraic semantics for event structures (where the relation of overlapping defined on temporal entities is typically paired with a strict ordering of total precedence).[15] Chisholm's recent work on spatial continuity can also be viewed in this light.[16] I suppose the basic idea has been exploited in other areas, though presumably the range of actual implementations and choice of primitives is not much wider.[17] It is however the mereo-topological framework recently proposed by Smith that can be regarded as the outstanding representative of this approach, also because it is effectively much more general—and with much more far-reaching foundational ambitions—than its special-purpose precursors.[18]

Smith uses the same primitives as Tiles, namely the relations of being a part and of being an interior part. The former has a standard interpretation, while the latter is intuitively meant to hold when an entity is part of another and does not overlap its boundary. I shall not go into the details of the axiomatization: it is rather straightforward and justifies the claim that topology can conveniently be grounded on mereology rather than set theory.[19] The aim, however, is "to go further and capture mathematically certain ontological intuitions pertaining to ordinary material objects […] to capture, if one will, the mathematical structures characteristic of the common-sense world".[20] Using 'P' and 'IP' to indicate parthood and interior parthood, respectively, here is for example how such basic notions as boundary ('B'), connection ('C'), or self-connectedness ('SC') are captured within the proposed theory:[21]

(3) $\mathrm{B}xy =_{\mathrm{df}} \forall z(\mathrm{P}zx \rightarrow \forall w(\mathrm{IP}zw \rightarrow \mathrm{O}wy \wedge \neg \mathrm{P}wy))$

(4) $\mathrm{C}xy =_{\mathrm{df}} \mathrm{O}xy \vee \exists z(\mathrm{O}zx \wedge \mathrm{B}zy \vee \mathrm{O}zy \wedge \mathrm{B}zx)$

(5) $\mathrm{SC}x =_{\mathrm{df}} \forall y\forall z(\forall w(\mathrm{O}wx \leftrightarrow \mathrm{O}wy \vee \mathrm{O}wz) \rightarrow \mathrm{C}yz)$.

We need not for the moment discuss the intuitive adequacy of these notions. They do the job, as far as topology goes. Moreover, the resulting framework does allow one to sketch a first formulation of some basic ontological intuitions that go well beyond the repertoire of standard topology. For instance, Smith suggests a first rendering of the Brentanian thesis that boundaries are dependent things, i.e., can only exist as boundaries *of* something[22] (contrary to the set-theoretic conception of boundaries as sets of ordinary, ontologically independent points):

(6) $\exists y\mathrm{B}xy \rightarrow \exists z\exists w(\mathrm{B}xz \wedge \mathrm{P}xz \wedge \mathrm{IP}wz)$

or, more strictly, that self-connected boundaries are boundaries of self-connected wholes:

(7) $\mathrm{SC}x \wedge \exists y\mathrm{B}xy \rightarrow \exists z\exists w(\mathrm{SC}z \wedge \mathrm{B}xz \wedge \mathrm{P}xz \wedge \mathrm{IP}wz)$.

On the other hand, one thing to be noticed is that much of this involves a conceptual detour that could effectively be avoided. We could just assume as primitive the very notion of a boundary (which we are actually to presuppose in the intuitive interpretation of interior parthood),[23] or the relation of connection, or even the property of being self-connected, and then define interior parts accordingly—as by the following general equivalences:

(8) $\mathrm{IP}xy \leftrightarrow \mathrm{P}xy \wedge \forall z(\mathrm{P}zx \rightarrow \neg \mathrm{B}zy)$

(9) $\mathrm{IP}xy \leftrightarrow \mathrm{P}xy \wedge \forall z(\mathrm{C}zx \rightarrow \mathrm{O}zy)$

(10) $\mathrm{IP}xy \leftrightarrow \mathrm{P}xy \wedge \forall z(\forall w(\mathrm{SC}w \wedge \mathrm{O}wy \wedge \neg \mathrm{P}wy \rightarrow \mathrm{O}wz) \rightarrow \neg \mathrm{O}xz)$.

I shall indeed come back to this point, for I think this is where a lesson is to be drawn. First, however, I shall move on to considering in greater detail the other two strategies mentioned above.

Second way. The second way to bridge the gap between merelogy and topology exploits the intuition that the latter is truly a more basic and more general framework subsuming the former in its entirety, at least relative to certain domains. This view can be traced back to De Laguna's work on solid geometry (based on the primitive relation "*x* connects *y* to *z*") and was taken over by Whitehead himself in the final version of his theory in *Process and Reality* (where all notions are explained in terms of the single topological primitive "*x* is extensionally connected with *y*").[24] The approach was fully worked out by Clarke in his resourceful reformulation of the calculus of individuals and has recently been employed by Randell, Cui and Cohn for work in spatio-temporal reasoning and naive physics, and by Aurnague and Vieu for the analysis of spatial prepositions in natural language.[25] (To my knowledge, not many other developments or applications have been put forward, if we exclude the interval logics for the representation of time—based on a primitive relation of temporal precedence—which have been a very active and yet independent research area in artificial intelligence, particularly under the impact of Allen's work[26]).

In all of these systems, parthood (and consequently the other principal mereological relations) is characterized derivatively in terms of topological connection ('C') in accordance with the following definition:

(11) $\mathrm{P}xy \quad =_{\mathrm{df}} \quad \forall z(\mathrm{C}zx \rightarrow \mathrm{C}zy).$

As is clear, much of the intuitive appeal of this reduction depends on the intended interpretation of the basic topological relation (which is axiomatized as a reflexive and symmetric relation—I shall again skip the details). If we take 'C' to mean the same as 'O', then (11) converts to a standard mereological equivalence; but things may change radically on different interpretations. Clarke follows Whitehead and explicitly suggests that one might "interpret the individual variables as ranging over spatio-temporal regions and the two-place primitive predicate, '*x* is connected with *y*', as a rendering of '*x* and *y* share a common point'".[27] This account has been subscribed to by other authors as well. Since points are not regions, sharing a point does not imply overlapping, which therefore does not coincide with (even though it is included in) connection. This means that things may be *externally* connected:

(12) $\mathrm{EC}xy =_{\mathrm{df}} \mathrm{C}xy \wedge \neg \mathrm{O}xy$

But whereas on Smith's more standard rendering of 'C' (4) this would be explained in terms of overlapping of a common boundary (though not of a common part, recalling that boundaries need not be parts of the things they bound), here the explanation is left open, for boundaries are just not included in the domain. Thus, on this account things can be topologically "open" or "closed" without there being any corresponding mereological difference—a feature that some have found philosophically unpalatable.[28]

Only recently, Randell, Cui and Cohn have proposed a modified version of their theory in which '*x* is connected with *y*' is taken as a rendering of 'the topological closures of regions *x* and *y* share a common point'.[29] The reason is precisely to do justice to the intuition that "from the naive point of view, the distinction between open, semi-open and closed regions is not drawn", as well as to avoid the consequence that "if we map bodies to closed regions (as the spaces they occupy), then their complements become open, which is a less agreeable result".[30] This shift of interpretation is reflected, formally, in the abandonment of the quasi-Boolean operation of complementation originally used by Clarke

(13) $x' =_{\mathrm{df}} \iota y \forall z(\mathrm{C}zy \leftrightarrow \neg \mathrm{P}zx)$

in favor of the following weaker variant:

(14) $x' =_{\mathrm{df}} \iota y \forall z((\mathrm{C}zy \leftrightarrow (\mathrm{P}xz \vee \neg \mathrm{IP}zx)) \wedge (\mathrm{O}zy \leftrightarrow \neg \mathrm{P}zx))$

where 'IP' is defined as follows (in contrast to (9) above):

(15) $\mathrm{IP}xy =_{\mathrm{df}} \mathrm{P}xy \wedge \neg \exists z(\mathrm{C}zx \wedge \mathrm{C}zy \wedge \neg \mathrm{O}zx \wedge \neg \mathrm{O}zy))$.

This guarantees that every (non-universal) region be connected with its own complement and, more generally, it avoids the above-mentioned feature of Clarke's original formulation: the "remainder principle" of classical mereology:

(16) $\mathrm{P}xy \wedge \neg \mathrm{P}yx \rightarrow \exists z(\mathrm{P}zy \wedge \neg \mathrm{O}zx)$

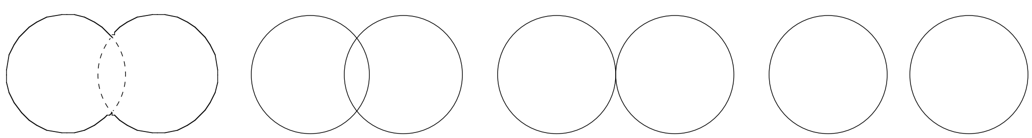
is in fact a theorem of the modified theory. However, there are some drawbacks as well. For instance, the resulting theory does not support models with atoms (regions with no proper parts). For an atom would have the property that every region connected with it would be connected with its complement, and by (11) that would imply the absurdity that every atom is part of its complement.[31]

So much for the intuitive modeling. It is apparent that the effective meaning of (11)—and consequently the mereological system that one effectively obtains—can drastically change depending on the particular interpretation that one considers. But from our present perspective the interesting question is even more fundamental; it concerns the very basic idea of relying on something like (11) when 'C' is not interpreted mereologically. And it is just here that I have reservations. *If* spatial regions are the only entities of our domain, then the proposed definition is really all there is to mereology, and the different interpretations reflect neither more nor less than a natural variety of possible implementations of the same idea (to which there corresponds a variety of more or less standard mereologized topologies). In fact both Clarke's original system and the developments that followed prove fit to account for a fair deal of mereo-topological reasoning. Simulation programs have also been built using which one can go as far as to model some rather complicated biological or mechanical processes, such as the cycle of operations in a force pump.[32] If, however, we are to take an open-faced attitude towards other entities than just regions, with or without boundaries, then we do not have much choice. Either we insist on the idea that things can be mapped to the regions that they occupy, or we maintain that the topology of regions is really all we need insofar as the same principles apply to the entities of a common-sense ontology as well. Both views seem to me rather difficult to defend, except perhaps for special purpose representations. A shadow does not overlap the wall onto which it is cast. And an object can be wholly located inside a hole, hence totally connected with it, without actually being part of it. The region that it occupies is part of the region occupied by the hole, but that's all. Or think of Lewis's angels dancing forever on the head of one pin: "At every moment, each occupies the same place as the other. Still they are two distinct proper parts of the total angelic content of their shared region".[33] For the purposes of naive mereo-topological reasoning, these are all cases of things that are connected but not overlapping. They are, following (12), externally connected. But they are not adjacent, which is what the notion of external connection is supposed to account for. The wall is in no sense abutting the shadow. And the hole does not squeeze to the side to leave room for its guest. Holes are immaterial and can be *interpenetrated*: if the object is inside the hole, then each part of the object is connected with some part of the hole and it makes no sense to characterize this as *external* connection. From here intuitions diverge rapidly: the notion of connection that we get by reasoning exclusively in terms of regions, no matter which specific interpretation we choose, is just too strong for the general case.

Indeed, from this point of view Smith's definition of connection in terms of boundaries and overlapping as given in (4) is likewise unacceptable. What is required is, rather, a weaker interpretation of connection as *co-localization at* (rather than *sharing of*) some point in space-time. We could try the following: A thing *x* is connected with a thing *y* iff either *x* and the closure of *y* or *y* and the closure of *x* are co-localized—but not necessarily overlapping—at some point (where the closure is the thing together with its boundaries). This would allow one to keep track of the distinction between being part-of and the more general relation of being spatially enclosed-in ('E'), hence between overlapping and the relation of spatially intersecting ('I'), hence, again, among things that are connected and things that are superimposed ('S') or merely adjacent ('A'):

(17) $\mathrm{E}xy \;=_{\mathrm{df}}\; \forall z\,(\mathrm{C}zx \rightarrow \mathrm{C}zy)$
(18) $\mathrm{I}xy \;=_{\mathrm{df}}\; \exists z\,(\mathrm{E}zx \wedge \mathrm{E}zy)$
(19) $\mathrm{S}xy \;=_{\mathrm{df}}\; \mathrm{I}xy \wedge \neg \mathrm{O}xy$
(20) $\mathrm{A}xy \;=_{\mathrm{df}}\; \mathrm{C}xy \wedge \neg \mathrm{I}xy$

These distinctions and the corresponding relations in terms of logical inclusion are schematically illustrated in the following diagrams.



C + I + O   C + I + ¬O (= S   C + ¬ I + ¬O (= A   ¬C + ¬I + ¬O

But of course this weaker interpretation—or others along these lines—would not support (11). If *x* is a part of *y* then everything connected with *x* would be connected with *y*, but the converse implication would fail. In this sense 'P' does elude 'C'. Hence the basic assumption of the "second way" falls short, and we are back to the first way (though with a new notion of connection).[34]

Third way. We thus come to the third possibility mentioned above, which to my knowledge has only been put forward in very recent work by Eschenbach and Heydrich.[35] Here the idea is that a topological framework like Clarke's can effectively be regained in a purely mereological setting (rather than vice versa), provided that we embed it in a less restrictive domain. The embedding is straightforward and exploits the non-mereological domain-specific concept of a region. This was the only ontological category admitted in Clarke's domains. In

Eschenbach and Heydrich, however, points and other boundaries are also admitted. Accordingly, connection is neither more nor less than overlapping of regions, and yet the topological idea of external connection is made safe by the fact that the common part of two overlapping regions need not itself be a region. Using 'R' to indicate the relevant (primitive) domain-specific region-predicate, the following definitions are all we need in order to reconstruct a mereologized topology of the sort discussed above:

(21) $\mathrm{C}xy =_{df} \mathrm{O}xy \wedge \mathrm{R}x \wedge \mathrm{R}y$

(22) $\mathrm{EC}xy =_{df} \mathrm{C}xy \wedge \forall z(\mathrm{P}zx \wedge \mathrm{P}zy \rightarrow \neg\mathrm{R}z)$.

As is clear, this approach allows one to retain standard mereology *holus bolus*. Some principles (like the "remainder principle" (16)) may not hold unrestrictedly in the restricted domain of regions; but this simply mirrors the fact that such a domain (the extension of the non-universal predicate 'R') is deprived of some topologically relevant elements, points and boundaries in the first place. In the comprehensive domain this principle is just as unproblematic as any other. In fact, the main point can be put in the form of a general translation theorem to the effect that the mereology resulting upon restricting the ontology to only include spatial regions is exactly the subtheory that can be obtained from the unrestricted mereology by uniformly restricting the range of quantifiers. Thus, for instance, principle (16) is valid but its restricted variant

(23) $\mathrm{P}xy \wedge \neg\mathrm{P}yx \rightarrow \exists z(\mathrm{R}z \wedge \mathrm{P}zy \wedge \neg\mathrm{O}zx)$

(with 'R' interpreted as an ordinary predicate constant) is not.[36]

I find this illuminating, for it shows that mereology needs very little help in order to cope with certain basic topological notions and principles. Formally it is only a matter of restricted quantification. Moreover, this way of looking at things is very general and one may consider exploiting different interpretations of 'R', or referring to other domain-restricting devices (I shall consider some possibilities in a moment). If this amounts to saying that topology is exclusively about regions of space and few other region-related entities such as points and boundaries (or about whatever selected entities we employ to fill in the extension of 'R'), then of course it contrasts the general desiderata expressed above. Whether we try to explain mereological relations among things in terms of topological relations among the corresponding regions, or topological relations among regions in terms of mereological relations among things of a kind, we miss out on something important for the ontology of the everyday world. However, the present approach draws no necessary reduction of parthood to spatial connection, and this gives new content to the idea that topological reasoning about ordinary things can be inferred from the topology of the regions they occupy: on this approach we may safely confine ourselves to reasoning about regions and yet keep track of the relevant difference between enclosure and parthood, or between intersection and overlapping. Thus, the limitation is not substantial: the third way is wide enough to support analogues of the general interpretation of connection suggested above.

The Fourth Way

There is also a fourth way. I only mention it now because I am not aware of any serious proposal in this direction, but the basic idea seems to me simple and attractive. If connection is too strong for the purpose of doing mereology, and parthood too weak for doing topology, why not just put the two notions together to get the right blend? Why not build a unified framework based on a single mereo-topological primitive covering both territories? An obvious possibility is to use as a primitive the ternary relation "*x* and *y* are connected parts of *z*". Indicating this with 'CP*xyz*', we can define parthood and connection as follows:

(24) $\mathrm{P}xy =_{\mathrm{df}} \mathrm{CP}xxy$
(25) $\mathrm{C}xy =_{\mathrm{df}} \exists z \mathrm{CP}xyz$.

From here we can then go on as we wish. For instance, we can define interior parthood using (9) and then proceed as in Smith's account. Or we can follow alternative routes, including an account to the effect that the equivalence corresponding to (11) holds.

This strategy has in fact one obvious advantage, namely it remains neutral with respect to the actual interpretation of the notions defined: 'part' and 'connection' can be characterized axiomatically (and interpreted intuitively) as if they were two independent primitives. The only mutual constraints are that for (24) to make good sense, connection must be a reflexive relation, whereas (25) presupposes that every pair of connected entities have a mereological sum. But these are perfectly uncontroversial presuppositions. In particular, the latter is not only an obvious consequence of the sum principle of classical extensional mereology (1), but also a principle held by the opponents of (1) (sometimes with the precise intent of stressing a distinction between admissible and inadmissible sums). From this point of view, this fourth "way" embodies the same formal generality as the first way, but since it only requires one primitive it also enjoys a certain conceptual economy that can be seen as an advantage of the second way.

The Ease of Mereo-Topology

We have, then, a rather comprehensive taxonomy of possible strategies and theories. Each of them reflects some way of overcoming the bounds of a plain mereology in dealing with topological notions and properties. And each does so without requiring a significant departure from the general outlook that led to the development of modern mereological systems (witness the fact that all theories considered above are compatible with a nominalistic stand).

Now *which* way is actually to be preferred is not a question that I here intend to address any further. The main strategies have been developed mostly on independent grounds and with disparate purposes, and putting them in the same sandbox and under the same light is only a first step towards a critical appraisal of their relative limits and potentialities. What I wish to emphasize, rather, is that the difference between the various alternatives is not only a matter of applicative purposes, or formal thoroughness, or computational efficiency. Although each of these concerns may have played an important role in the development of each single theory, I think the difference lies first and foremost in the ontological status that certain entities—from boundaries to ordinary things—are accorded. As we have seen, where and how the domains of merology and topology are bridged depends heavily on the ontological fauna that one is willing to countenance, on the variety of entities that one is ready to allow in the universe of discourse. And it is just here that I would like to add some remarks.

We have a picture that looks like a network connecting two extreme positions. At one extreme we find Whitehead's early stand as reflected in the definition of 'join' discussed at the beginning of this paper: If we only allow for self-connected entities, then mereology may even subsume topology, though we cannot expect it to be quite classical (the sum principle (1) must fail). At the other extreme we find Clarke's exploitation of Whitehead's late approach: If we only have regions, then topology alone suffices and mereology can be subsumed easily, though again we cannot expect the outcome to be very classical (the remainder principle (16) must be sacrificed). In between we have a variety of intermediate and perhaps not always directly comparable positions, each according greater weight to some entities over others. Now, one thing that is remarkable in this picture is that in spite of the apparent conflict the two extreme positions can be seen as implementing the very same idea. It is, indeed, a matter of restricted quantification in the spirit exemplified by Eschenbach and Heydrich. Just as Clarke's account can be viewed as the result of restricting classical mereology to a domain of regions ('R'), in the precise sense that every universal or existential statement amounts to a restricted quantification of the form '$\forall x(\mathrm{R}x \rightarrow \psi)$' and '$\exists x(\mathrm{R}x \wedge \psi)$' (respectively), so Whitehead's theory can be obtained by restricting quantified statements using a predicate for self-connectedness ('SC') i.e., by transforming them into restricted statements of the form '$\forall x(\mathrm{SC}x \rightarrow \psi)$' and '$\exists x(\mathrm{SC}x \wedge \psi)$'. Mereology has no "predicate" for self-connectedness, but nothing prevents us from borrowing it from somewhere else, just as we can borrow a region predicate 'R'. As a matter of fact, if we do so then the defective definition of 'join' (2) becomes perfectly adequate:

(26) $\mathrm{J}xy =_{\mathrm{df}} \exists z(\mathrm{SC}z \wedge \mathrm{O}zx \wedge \mathrm{O}zy \wedge \forall w(\mathrm{SC}w \wedge \mathrm{P}wz \rightarrow \mathrm{O}wx \vee \mathrm{O}wy))$.

Perhaps this comes as no surprise, as both systems are Whitehead's after all. It is, however, instructive that seemingly opposite positions support essentially the same interpretation (modulo specific differences in the axiomatization).

We can also see how this relates to the intermediate positions in the network. If, as seems to be the case, topology is to a great extent a matter of domain-specific predicates, then in the end the bounds of mereology—at least the ones considered above—do not seem to determine any dramatic conceptual limitation. As long as we are capable of specifying what we are talking about, the bounds are easily overcome. Now this brings us back to a remark that I left open when discussing Smith's choice of primitives. What I find interesting in the viability of many possible primitives for a system like Smith's (but the same applies to other systems of the sort) is that there is no *prima facie* ontological or methodological reason for preferring one choice to another—e.g., for preferring 'IP' to 'B' or 'C'. In fact we saw that we can even take as a topological primitive the predicate 'SC'. Thus, also in this case topology can be viewed as a business of simple predication and restricted quantification. The difference with the extreme positions mentioned above is that in this case the domain's composition is left open. No assumption is made concerning the extension of the distinguishing topological notions—and this supports a non-trivial (hence open-faced) use of 'SC'. From this perspective the bounds of mereology are the bounds of any fundamental theory. Mereology tells us how the world is constituted *in general*, but if we want to talk about certain things as opposed to others, if we want to pick out certain classes of entities instead of others, we need some means of referring to them. Whether we then make this into a theory in many chapters (first way), a compact monograph (fourth way), or something in between (second and third way) is of little importance as long as the choice is recognized to be a matter of ontological transparency.

There remains a question of "what's next", to use Lejewski's phrase.[37] I have been talking of topology as a necessary next step after mereology, and I have done this by concentrating on connection and related notions. However, there is of course much more than this to topology. I do not mean to say that we still need to do a lot of work to get close to the topology actually used by matematicians — I am not even sure that that is necessary. Rather, I want to underline that there are many important topological notions that cannot be captured by any of the systems outlined here and which nevertheless play a very basic role in our everyday reasoning about the world. For instance, how can we distinguish between things with holes and things without — between a sphere and a torus? We need, it seems, an additional predicate of "simple connectedness", or some means to distinguish surfaces of different genus. How can we account for such basic spatial relations as being inside or outside a given object (or region) when this is not a matter of pure topological closure? For instance, how can we say whether the fly is inside or outside the glass? We need, some authors have suggested, an additional topological operation capturing some notion of "convex hull", not definable in terms of "connection" and the like.[38] Whether or not such additions yield an adequate treatment of the examples mentioned is of course a complex matter. It seems, however, that from this point of view one can hardly feel satisfied with simply expanding a mereological framework with a notion of connectedness. One needs much more just to accomplish some very basic pieces of topological reasoning. And, more importantly, even when a satisfactory amount of topological reasoning could be regained, we would need to move into other provinces to account for equally basic commonsensical intuitions concerning, for instance, movement of parts or interactions among wholes. After all the bounds of topology are pretty narrow too. The world of topology is initially a world of spheres and toruses and little else, and we need to step into morphology—the theory of qualitative discontinuities—just to account for certain basic differences in shape; we need to step into kinematics just to account for certain basic differences of behavior.

My provisional conclusion is thus two-faced. On the one hand, the move from mereology to a mereo-topology is an important and yet rather easy matter of specialization (comparable to the move from, say, set theory to set-theoretical topology). On the other, if we go the way of saying that topology is required for the purposes of investigating the common-sense world, then we can hardly stop there. Many other boundaries have to be crossed—which in effect is a way of saying that many more things have to be taken at face value.

Notes

I am grateful to Gearge Bealer, Tony Cohn, Nicola Guarino and Barry Smith for helpful discussion and valuable comments on earlier drafts.

[1] There are exceptions. In particular, the transitivity of the part-relation has been disputed at least since Rescher 1955. Compare e.g. Cruse 1979, Winston *et al.* 1987, and the recent plea for naive mereology in Sanford 1993.

[2] Husserl 1901. On the role of mereology in ancient and scholastic philosophy, see e.g. Burkhardt and Dufour 1991, Henry 1991.

[3] See Leśniewski 1916, Whitehead 1919, Leonard and Goodman 1940, and Goodman 1951, respectively. For a thorough overview see Simons 1987.

[4] Some classical systems, such as Tarski 1929 or Leonard and Goodman 1940, give a formulation of the principle involving reference to classes of individuals rather than just using predicates or open formulas. Here I stick to a class-neutral formulation simply for expository convenience. The difference is nonetheless to be noted, since an ordinary first-order language has a denumerable supply of predicates or formulas, so that at most denumerably many classes (in any given universe of discourse) can be specified. For the nominalist this limitation is of course negligible insofar as classes do not exist except as *nomina*. Compare Eberle 1970, especially pp. 67f.

[5] See e.g. the early criticisms of Lowe 1953, Rescher 1955, or Chisholm 1976. Of course a similar complaint arises in set theory, as discussed in Smith 1991.

[6] The *locus classicus* is Goodman 1956, 1958. For a recent statement see Lewis 1991, who stresses that "it is in virtue of this thesis that mereology is ontologically innocent: it commits us only to things that are identical, so to speak, to what we were committed to before" (p. 82); the "so to speak" is explained as in Baxter 1988a, 1988b.

[7] This point is made in Simons 1982, p. 149.

[8] Whitehead 1919. The definition of joining given below is actually from Whitehead 1920, but the difference from the earlier account is inessential.

[9] The figure is adapted from Simons 1987, p. 337, where a similar point is made (compare the discussion at pp. 81-86). See also Simons 1991.

[10] Compare e.g. Needham 1981.

[11] Compare e.g. Bostock 1979.

[12] As recently emphasized by Eschenbach and Heydrich, 1993, p. 207.

[13] I shall give some detailed references in the next section.

[14] See Tarski 1929 and Lejewski 1982, respectively. Lejewski also hypothesises that combining *Chronology* with a cognate theory of space, or *Stereology*, would yield a favorable framework for developing a general *Kinematics*. Compare Lejewski 1986.

[15] See Tiles 1981, Kamp 1979, Bach 1986, Link 1987, as also van Benthem 1983.

[16] Chisholm 1992/93.

[17] I myself have recently followed this approach in joint work with R. Casati on the metaphysics of holes and holed things: see Varzi 1993, Casati and Varzi 1994.

[18] Smith 1992, 1993.

[19] Following Menger 1940.

[20] Smith 1993, p. 61.

[21] I deviate slightly—but inessentially—from the original notation and formulation to avoid unnecessary intermediate definitional detours.

[22] Compare Brentano 1976. Of course a full statement of the thesis requires further work, as is shown in White 1993. See also Chisholm 1984, 1989 (ch. 8).

[23] Smith himself considers the possibility of using a primitive closure operation.

[24] See De Laguna 1922 and Whitehead 1929, respectively.

[25] See Clarke 1981, 1985; Randell and Cohn 1989, 1992, Randell 1991, Cohn *et al.* 1993, Randell *et al.* 1992a, 1992b, 1992c; Vieu 1991, Aurnague and Vieu 1993a, 1993b.

[26] See Allen 1981, 1984, Allen and Hayes 1985 (though much can already be found in Hamblin 1969, 1971). See Galton 1993 for a unified theory of space, time and motion.

[27] Clarke 1981, p. 205. Cp. Gerla and Tortora 1992.

[28] Compare Simons 1987: "What we are being asked to believe is that there are two kinds of individuals, 'soft' (open) ones, which touch nothing, and partly or wholly 'hard' ones, which touch something. Yet we are not allowed to believe that there are any individuals which make up the difference. We can discriminate individuals which differ by as little as a point, but are unable to discriminate the point. It is hard to find satisfaction in this picture" (p. 98). One is remined here of Brentano's reprehension against the "monstruous doctrine that there would exist bodies with and without surfaces, the one class containing just so many as the other, because contact would be possible only between a body with a surface and another without" (Brentano 1976, pp. 146-47; the reference is to Bolzano 1851). A way of recovering the *notion* of a boundary within Clarke's framework (relative to finite domains) is indicated in Vieu 1991 and Aurnague and Vieu 1993.

[29] Randell *et al.* 1992a, 1992b, Cohn *et al.* 1993.

[30] Randell *et al.* 1992a, pp. 394-95.

[31] This is noted in Randell *et al.* 1992b, p. 173, correcting a mistake of Randell *et al.* 1992a. Three alternative ways of dealing with atoms are made available, but they all determine some departure from the basic framework.

[32] Compare Randell *et al.* 1992c.

[33] The story is from Lewis 1991, p. 75.

[34] This is the approach that I followed in Varzi 1993.

[35] Eschenbach and Heydrich 1993.

[36] To be more precise, I believe the point can be put as follows. First, let L be a mereological language with 'P' as primitive, let L_t be the language obtained from L by replacing 'P' with 'C', and let L_r be obtained from L by adding a new predicate symbol 'R'. Next, for any sentence ϕ of L, let ϕ_t be the sentence of L_t obtained from ϕ by replacing each atomic component of the form '$\mathrm{P}xy$' with '$\forall z(\mathrm{C}zx \rightarrow \mathrm{C}zy)$', and let ϕ_r be the sentence of L_r obtained from ϕ by recur-

sively replacing each quantified component of the form '$\forall x\psi$' or '$\exists x\psi$' (with 'x' free in ψ) with '$\forall x(Rx \rightarrow \psi)$' and '$\exists x(Rx \wedge \psi)$' respectively. Lastly, let M be a mereological system in L and let M_t and M_r be corresponding systems in L_t and L_r obtained by replacing each axiom ϕ of M with ϕ_t or ϕ_r, respectively. Then, for every thesis ϕ of M, the sentence ϕ_t is a thesis of M_t iff ϕ_r is a thesis of M_r.

[37] From Lejewski 1982.

[38] This is the strategy followed e.g. by Randell, Cui and Cohn in most of the works cited above. The fly example is from Vieu 1991 (adapted from Herskovits 1986).

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