The Magic of Holes

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Introduction

There is no doughnut without a hole, the saying goes. And that's true. If you think you can come up with an exception, it simply wouldn't be a doughnut. Holeless doughnuts are like extensionless color, or durationless sound—nonsense. Does it follow, then, that when we buy a doughnut we really purchase two sorts of thing—the edible stuff *plus* the little chunk of void in the middle? Surely we cannot just take the doughnut and leave the hole at the grocery store, as we cannot just eat the doughnut and save the hole for later. But then, again, surely when we eat a doughnut we do not also eat the hole. Or do we?

There is a lot of metaphysical mystery in a doughnut, as there is mystery in every object that comes with holes—a flute, a piggy bank, a genuine piece of Emmentaler. Back in the 1920s, Kurt Tucholsky mused upon the question, "Where do the holes in cheese come from?", and it is safe to say that many of us wouldn't know where to look for an answer.[1] Yet that is only the beginning. The real mystery is not where the holes *come from*; it is whether they *are there* in the first place. After all, holes are a paradigm example of absences, non--entities, nothingnesses, things that aren't there. As Tucholsky himself put it a few years later, "a hole is there where something isn't".[2] Perhaps only a dry-minded philosopher would hazard questioning the reality of the stuff we eat, along with that of other ordinary material objects. But ought we seriously contenance the reality of the holes, too?

Horror Vacui and Object Topology

Traditional wisdom says we oughtn't. We speak with the vulgar, but we should think with the learned. And the learned says that holes are just *façons de* *parler*, mere *entia representationis*, linguistic noise. Yes, we do say that there is a hole in this doughnut, or that there are several holes in this piece of Emmentaler, but that is not to imply that there are holes. "When *I* say there are holes in something—says David and Stephanie Lewis's materialist philosopher, Argle—I mean nothing more nor less than that it is perforated."[3] The expression 'there are holes in …' should not be taken literally as expressing an idiom of existential quantification, as when we say 'there are mountains in Switzerland'. Rather, on this view 'there are holes in …' is merely a variant of '… is perforated', which in turn is just an ordinary shape predicate, like '… is flat' or '… is a dodecahedron'. It is a perfectly innocuous shape predicate that may truly be predicated of a doughnut or of a piece of Swiss cheese without any implication that its shape depends on the presence of occult, immaterial entities called 'holes'.

That is a common strategy, when it comes to ontological disputes. We may say, "There is an age difference between John and Mary", but deep down our words should be interpreted without any commitment to the existence of such things as age differences; we are just talking about John and Mary, we are claiming that either he is older than her, or she is older than him.[4] We may say, "There are good chances that the Yankees will win again", but deep down our words should be interpreted without any commitment to the existence of such things as chances; we are simply saying that it is very likely that the Yankees will win.[5] Very well, says traditional wisdom: when we say that there are holes in a doughnut, or in a piece of Emmentaler, the situation is not different; we are just talking about the edible stuff, we are describing *it*.

One good thing about this way of thinking is that, unlike so many forms of *horror vacui*, it appears to admit of a thorough implementation. For, in a way, the whole mathematical discipline of topology may be regarded as providing a way to account for perforations along these lines. Topology is a sort of rubber geometry. It is concerned with the way the shape of an object can be transformed into another by pure elastic deformation. Topologically, you can stretch your object and distort it, but you are not allowed to connect what was disconnected (e.g., by pasting two surfaces or two parts of the same surface) or to disconnect what was connected (by making a cut). For instance, a cube can be transformed into a sphere in this way: just imagine it is made of plasticine and gradually smooth out its edges and corners. By contrast, suppose your doughnut is also made of plasticine. Then again you can deform it into various shapes. You can even transform it into a coffee mug (Figure 1). However, you cannot transform your doughnut into a sphere by mere elastic deformation, for you cannot get rid of the hole without cutting or pasting somewhere. That is precisely what its being perforated amounts to, topologically speaking. And this is a claim about the object itself, i.e., the doughnut; it does not require that we also treat the hole as an entity in its own right.

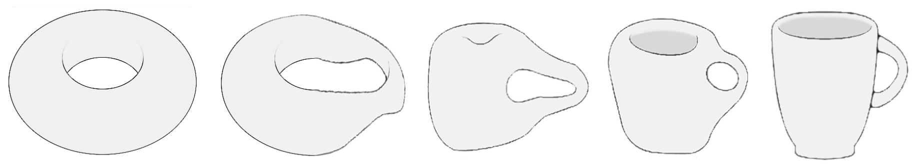

Figure 1: Deforming a doughnut into a mug.

There are, in fact, various ways of making this more precise, depending on how we define a spherical object. A customary definition is this: an object is spherical (or can be deformed into a sphere) if you cannot draw a circle or a closed curve on its surface without dividing the surface itself into two disjoint regions, namely, the part inside the circle and the part outside. In some cases the opposition between 'inside' and 'outside' may be inappropriate (think of the equator separating the globe into two hemispheres), but the concept of *division* still applies: there are points on the surface of a sphere that cannot be connected by a continuous path without intersecting the circle (you cannot drive from Paris to Cape Town without crossing the equator). If we use this definition, it is clear that all sorts of objects qualify as spherical: a cube, a champagne glass, even a baroque chandelier may pass the test. However, a doughnut does not. On a doughnut, there are several ways one can draw a closed curve without dividing the surface into two disjoint regions (Figure 2). So a doughnut is not spherical.

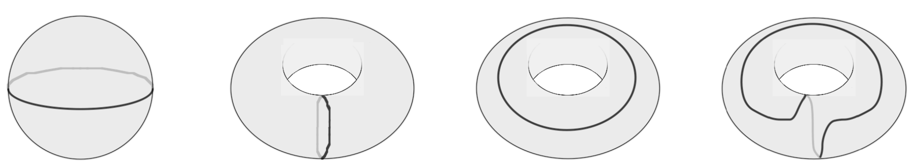

Figure 2. On a sphere, every closed curve separates the surface into two distinct regions; not so on a doughnut.

Equivalently, we can say that the property that topologically distinguishes a sphere from a doughnut is this: any circle or closed curve on a sphere can be shrunk to a single point by elastic deformation; on a torus (the topologist's word for the surface of a doughnut) this is not the case: the three curves in Figure 2 cannot be reduced to a point without "cutting" through the surface. Or again, the property in question can be characterized as follows: if two circles on a sphere intersect, they intersect in *two* points (this means "intersect" in the sense of going right through, not just touching); by contrast, on a torus two circles may intersect in just one point: consider for instance the circles in the two middle diagrams of Figure 2 and imagine drawing them on the surface of the same doughnut. This is why even a bug can figure out if it lives on a torus or on a sphere. The bug has no notion of a hole, for that would require stepping out of the surface and looking at the object from the outside, which it cannot do. But the bug knows that if it goes on a straight trip, it eventually gets back to the same point from the opposite side. So it only needs to go on two different trips and then check: do the trails that it leaves behind intersect only once, at the starting point, or twice? (At least, the bug can *hope* to figure things out this way. Strictly speaking, there is no guarantee that if it lives on a doughnut, the two paths will not intersect; but it is a possibility. See Figure 3.[6])

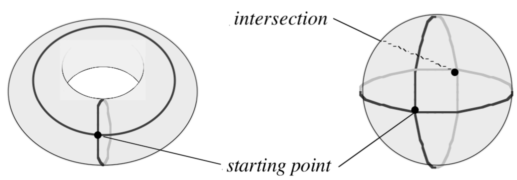

Figure 3. Even a bug can figure out whether it lives on a doughnut or on a sphere.

It does not matter which of these characterizations we work with. They are all equivalent. What matters, here, is that they lend direct support to what I have called "traditional wisdom". For they all serve the same purpose: they provide a means for distinguishing an object with a hole and an object without holes *exclusively in terms of the properties of the objects*, in fact, of their surfaces. No reference to the hole is necessary. In other word, topology provides a clear and precise framework for maintaining that the locution 'there is a hole in ...' really is just a shape predicate, albeit of a very special sort. Of course, this would be a modest achievement if one could not do the same with other hole-based locutions, such as 'there are seven holes in ...' or 'there are more holes in ... than in ...'. But topology can do that easily. The topologist can count *any* number of holes without ever mentioning them. For instance, one can count the maximum number of disjoint circles or closed curves that can be drawn on the surface of the object without separating it into disjoint regions. On a regular doughnut that number is 1; on a doughnut with two holes, it is 2; and on an object with n holes, the total number is n. That number is called the *genus* of the object's surface. And the fundamental theorem of topology asserts that all surfaces of three-dimensional objects can be classified exclusively (and completely) by their genus.

When the Void Matters

It is precisely this notion of genus that shows the full strength of the topological method for dealing with holes, hence of the "eliminativist" strategy that informs the traditional wisdom. The number of holes in an object is reflected in the object's genus. The question we must ask is: is it reflected in the right way? Is reference to holes made fully redundant by an analysis of the object's genus? Unfortunately, the answer is not quite in the affirmative. Some, perhaps most hole-statements can be handled in this way. But there are cases where the topology of the object delivers the wrong answer, or an answer that is badly incomplete. And in those cases direct reference to the holes seems necessary. Let us then turn to this side of the story.

A simple case in point is that the method is incapable of discriminating between straight and knotted holes. In other words, the two objects in figure 4 have the same topological genus. The reason is that the genus is only informative with regard to the *intrinsic* topology of an object, the topology as it can be figured out by a bug that lives on the surface. The object's *extrinsic* topology, i.e., the way the object is embedded in three-dimensional space, lies beyond that. The bug has no way of figuring it out, just as we generally cannot tell if we are walking through a straight tunnel or through a knotted one. Nevertheless there is a significant difference between the two cases, and one would like to be able to account for it adequately.

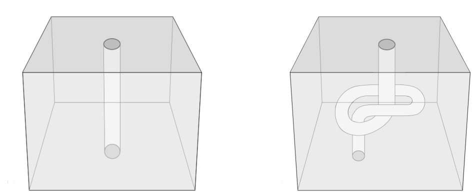

Figure 4. A straight hole, a knotted hole: where is the difference?

It may be replied that this is not a limit of topology as such. One could easily capture the difference between the two cases by considering the object's complement. Clearly, the complementary topology of an ordinary doughnut and that of a doughnut with a knotted hole are distinct. Yet this shift—from the object to its complement—is crucial. *If* we were only talking about regions of space (as topologists often do), then all is fine: there is no ontological difference between a region and its complement, and no reason to restrict oneself to one or the other. But if we are talking about *objects*—things such as doughnuts and chunks of Swiss cheese—then the shift to complementary topology is quite significant. An object's complement is, after all, just as immaterial as a hole. In fact, mereologically the hole in a doughnut is just a proper part of the doughnut's complement. So the complementary topology of the object is, to some extent, the topology of the hole. The expressive power of the topologist's language is safe. But it doesn't save us from explicit reference to the immaterial. From the perspective of traditional wisdom, that is bad news. We may want to focus on the doughnut; but we must also keep an eye on the hole.

Here is another, more interesting example. Consider the four objects depicted in Figure 5. As it turns out, they all have the same topological genus, namely 2, and indeed they can all be transformed into one another by mere elastic deformation, without cutting or pasting. (Check that!) However, we do want to make distinctions here. For instance, we do want to say that the object on the left (*a*) has *two* holes, whereas the other objects have only *one* hole (curiously shaped). The topology of the object simply delivers the wrong answer here: the objects may well be equivalent; the holes are not.

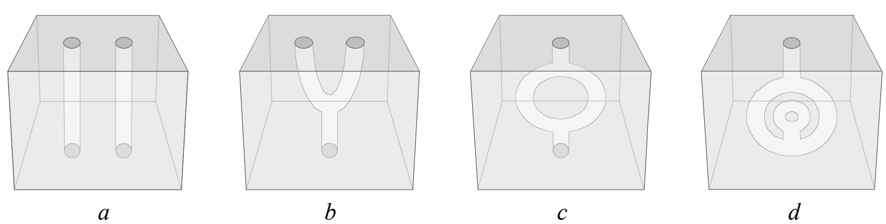

Figure 5. Same genus, different holes.

It is tempting to say that this outcome is simply a sign of the counterintuitiveness of certain topological equivalences (the same counterintuitiveness that underlies the equivalence of a cube and a baroque chandelier, or that of a doughnut and a coffee mug). Once again, however, the problem does not lie in the conceptual apparatus of topology *per se*. It lies in the application of the apparatus. It lies in the idea that the only topology that matters is that of the object's surface. If we look at the topology of the hole's surface instead, we get a completely different picture—indeed one that makes all the correct distinctions. By the surface of a hole I really mean its "skin", i.e., that part of the object's surface that envelops the hole, and that can only be individuated by reference to the hole. In a straight perforation, that superficial part is a cylinder:

its normalized topological figure is a sphere with two punctures. In the case of a Y-shaped hole, it is a sphere with three punctures. And in the cases corresponding to Figures 5*c* and 5*d*, the skin of the hole is not a punctured sphere but a torus with two punctures and a bitorus with one puncture, respectively. (Figure 6)

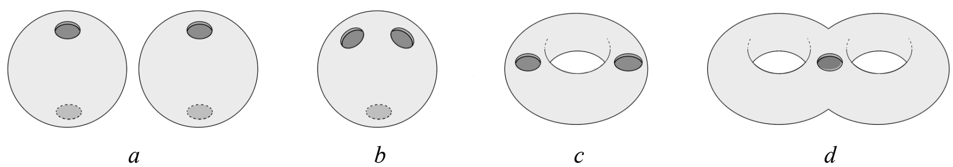

Figure 6. The holes in Figure 5 have topologically different skins.

Note that a puncture is not a genuine hole, but a hole of lower dimension—what topologists also call an 'edge' or 'boundary'. The fundamental theorem mentioned above can be formulated more fully using this vocabulary: orientable surfaces are completely characterized up to equivalence by their genus and by the number of their boundaries. Now, the surfaces of ordinary material objects don't have any boundaries in this sense. But the surfaces of holes—their skins—do. And that makes a great deal of difference.

Looking directly at the topology of the hole is also helpful when it comes to seeing the family resemblance between the type of holes considered so far—perforations—and other kinds of hole. For there are also holes that are purely superficial, like a hole in a golfing green, or the nostrils of a baby-doll; and there are holes that are entirely hidden in the interior of their material hosts, like a cavity inside a wheel of Swiss cheese. (See Figure 7.) These are all part of the big family of holes and indeed they all give rise to the same sort of puzzle we started with: you cannot buy the doll without the nostrils, and you cannot buy the cheese and leave its inner cavities at the grocery store. Holes are parasitic entities, no matter where they live. Now, if we only look at the objects hosting such holes, we need to come up with something else than their genus to describe the relevant geometric features. For instance, the presence of an inner cavity is reflected in the fact that the object (cheese) has two separate surfaces: the one that binds it on the outside (the crust) and the one that binds it on the inside (where the cavity is). And the presence of an external hollow is reflected in the fact that the surface of the object presents an abrupt change in its curvature pattern, from positive (convex) to negative (concave). These may all be effective ways of describing what is going on in such situations. Yet they would introduce a disturbing asymmetry among the various cases, as they go beyond the resources of topology. By contrast, a hole-based perspective handles all cases in a uniform way. Indeed, if we look at the skins of such holes we get exactly the patterns that were missing in the case of perforating holes (Figure 8): inner cavities have unbounded skins (spheres, toruses, bitoruses, and so on); superficial hollows have skins with one boundary; and then there are the mixed cases. *Holes come in various species, but they are all species of the same genus.*

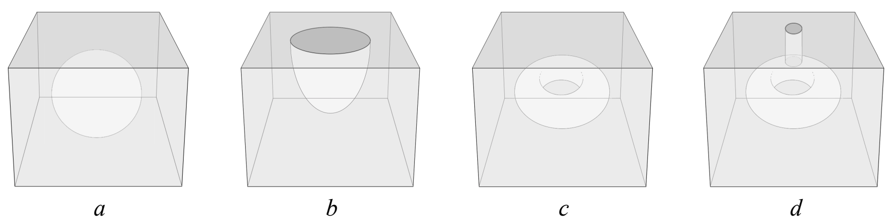

Figure 7. Holes come in various species, besides perforations.

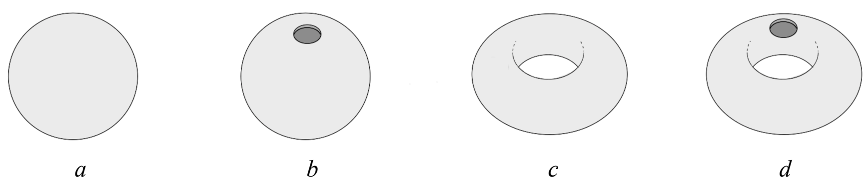

Figure 8. Different species, different skins.

A Tricky Interplay

This is not, of course, to say that one should desert the topology of the objects and switch to the topology of the holes instead. One has to be very careful at this point, for in some cases the surface of the hole may itself be deceptive. Just consider again the patterns in 7*a* and 7*c*. In such cases we have the same problem we had before, except that the difficulty now concerns the inner surface—the skin of the hole. What if the cavity in 7*a* had a dent? What if the torus-shaped cavity in 7*c* involved a knot? More generally, what if our wheel of Swiss cheese had inner cavities whose skins are exactly like the problematic surfaces of the objects in Figure 5? What if it had extravagant interlocking cavities with such skins as those depicted in Figure 9 below? In all these cases, and many moire indeed, the same arguments apply: one point of view (whether doughnut-based or hole-based) is not enough. The interplay between void and matter can be awfully complex, and the only way to address it properly and in a systematic way is to grant equal dignity to both characters: the void *and* the matter. Still, this is bad news enough for traditional wisdom, and for the eliminativist stance it advocates.

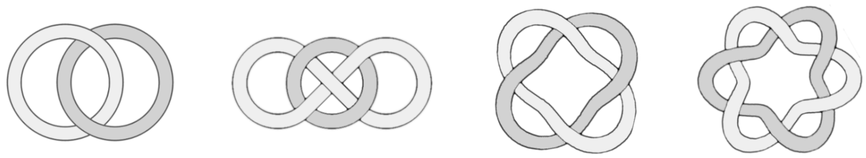

Figure 9. One point of view is not enough.

One might protest that this conclusion is still unwarranted. All we have seen is that topology provides an account of the locution 'there is a hole in …' that does not fully support the view that holes are merely a *façon de parler*. But this falls short of establishing that holes are not, in fact, a *façon de parler*. One could still try to do away with such nothings by relying on richer representation systems. For instance, one can resort to a description of the doughnut that combines topology with geometry broadly understood. Eventually, one could paraphrase every sentence of the form "There is such-and-such a hole in that object" by means of a point-by-point description of the object in question, including a thorough account of the properties that are exemplified *at each point*. That should do. That should allow one to stick to the solid, edible stuff and avoid any commitment to its immaterial intrusions. But there is an obvious drawback to such a strategy: a point-by-point paraphrase is simply too powerful a tool. You can use it to get rid of the hole; but you can also use it to get rid of the doughnut. You could just as well paraphrase every sentence about a doughnut by means of a thorough point-by-point description of the region of space that it occupies, combined with a complete account of the properties (of material constitution, color, texture, electric charge, etc.) that are exemplified at each point of that region. Surely this would hardly be compatible with the idea that doughnuts are not *façons de parler*. But that is the unavoidable boomerang effect of such an eliminative strategy. For this is the dilemma of every radical strategy of this sort: if successful, it ends up eliminating *everything* just in order to eliminate *nothings*.

Welcome to the Magic

We thus come to the moral of our little exercise. We cannot get rid of the holes? Too bad for traditional wisdom and the *horror vacui* that sustains it, but so be it. Let us welcome such things. Let us take our holes seriously, as we do with the edible stuff that hosts them. And let us take them for what they are—

voids. After all, that is the beauty of every doughnut and every genuine piece of Emmentaler: they defy regimentation. We don't need to leave the kitchen to get deep into genuine metaphysical mystery.

Now, what is the mystery, exactly? In a way, we have seen that holes appear to have all the features of ordinary spatiotemporal particulars. They can be counted. They have shapes, sizes, and locations. They have birthplaces and histories, and many things can happen to them. In short, they are not abstract entities. On the other hand, surely holes are *not* ordinary particulars. For the fact remains that they are not made of matter; they are *made of nothing*, if anything is. And this gives rise to *a lot* of mystery, over and above the puzzle we started with. Consider:[7]

— We all see holes. We see where they are and we see how they are—round, square, straight, knotted. Yet it is difficult to explain how holes can in fact be *seen*. If perception is grounded on causation, as Locke urged (*Essay* II, viii, 6), and if causality has to do with materiality, then immaterial bodies cannot be the source of any causal flow. So a causal theory of perception would not apply to holes. Does it follow that our impression of perceiving holes is a sort of systematic illusion, on pain of rejecting causal accounts of perception?[8]

— It is difficult to specify identity criteria for holes—more difficult than for ordinary material objects. Being immaterial, we cannot account for the identity of a hole *via* the identity of any constituting stuff. But neither can we rely on the identity conditions of its material "host" (the stuff around the hole), for we can imagine changing the host, partly or wholly, without affecting the hole. And we cannot rely on the identity conditions of its "guest" (the stuff inside it, be it air, water, of something else), for it would seem that we can empty a hole of whatever might partially or fully occupy it and leave the hole intact.

— Indeed, how do holes go out of existence? Never mind Tucholsky's worry concerning where they come from; if filling a hole is not a way of killing it, how do holes die, if ever? Perhaps they go out of existence when the matter that surrounds them contracts to the point of closing up on itself? Or perhaps filling a hole *may* be a way of killing it after all, e.g., if the matter of the filler is exactly the same as (and merges with) the matter surrounding the hole. Or consider: A stone falls on the ground, creating a small hole. Then a bigger stone falls down onto the same spot, producing a bigger hole. Shall we say that the first hole is destroyed upon creation of the second? Shall we say that the first hole has been enlarged? That it has become part of the second hole?

— Surely holes move: as you move your doughnut or your piece of Swiss cheese, you are also thereby moving the holes in them. Do they *always* move along with their material hosts? Take this doughnut and spin it clockwise. Is the hole spinning, too? Take a wedding ring, put it inside the hole in the doughnut, and spin it the other way. Is the little hole spinning counter-clockwise? But the little hole is part of the big hole, isn't it? So would the little hole be spinning in both directions at once?[9]

— Perhaps, when you place a hole inside another one, it does not become part of it after all. Holes are immaterial, which means that they can be interpenetrated by other things. That's why they can be filled, and that's why you can put a wedding ring inside the hole of a doughnut. But then, perhaps holes can also be interpenetrated by other holes? Perhaps, when you put the ring inside the doughnut, the small hole does not become part of the bigger hole; it merely ends up being partly co-located with it, i.e., exactly co-located with part of it. If so, however, does it follow that holes are a counterexample to general principle according to which two entities of the same kind cannot be (wholly or partly) co-located?

— How do we count holes? Take a card and punch a hole in it. You have made one hole. Now punch again next to it. Have you made another hole? In a way, yes: now the card is doubly perforated. But if holes are not to be understood in terms of the shapes of the objects hosting them, what prevents us from saying that we still have one hole, though a hole that comes in two disconnected parts? After all, material objects can be disconnected: a bikini, my copy of the *Recherche*, a token of the lowercase letter 'i'. Perhaps holes may be disconnected, too? If so, perhaps we have just punched a single, disconnected hole. How can we tell?

— What exactly is the relationship between a hole and its material host? Surely you cannot have a doughnut without a hole, as the saying goes. But that is a form of conceptual dependence, nothing more. If you cut your doughnut into pieces, the hole goes, the doughnut stays—though its shape is now different. You can still eat *the whole* doughnut. The dependence of a hole on its doughnut, by contrast, is genuinely ontological. When you eat your doughnut, the hole is gone, too. Yet this form of *de re* dependence does not seem to be as strong as other forms, such as the dependence of a smile on a face. *That* smile can only exist as an expression on *that* face (with the possible exception of the grin on the Cheshire cat[10]). But we have said that one can in principle change the entire host without affecting the hole in it. In what sense, then, is the hole ontologically parasitic upon the doughnut?

— You can cut a doughnut in half and save both halves for later. You can also eat half of it and keep the other half. But can you cut a hole in half? What would you get then—two half holes? Or two whole holes, though half the size?

As we sit in our kitchen and look at our plate, this is enough mystery to get started. Philosophy begins in wonder, said Aristotle, and sure enough our dougnhuts and Swiss cheese have a lot of wonder to offer. Let's not say No. Let's be dismissive. And never mind if we do not get all the answers. Holes are truly puzzling creatures. When the characters of Tucholsky's story decide to check the encyclopedia to see where the holes in cheese come from, the find the page is missing. Disappointed? Not really. For there may *be* no answer. Or rather, the answer may be exactly what is missing: a gap in the truth of things, a void surrounded by wisdom, a tastless, ineliminable, slippery, mind-blowing void.[11]

Notes

[1] See Tucholsky (1928), published under the pseudonym Peter Panter.
[2] See Tucholsky (1931), published under the pseudonym Kaspar Hauser.
[3] See Lewis & Lewis (1970, pp. 206–207).
[4] The example is from White (1956, pp. 68–69).
[5] This example is from Burgess & Rosen (1997, pp. 222–233).
[6] For more discussion on the bug's limits, see Varzi (2011).
[7] The philosophical territory defined by such questions is examined more fully in Casati & Varzi (1994).
[8] This question is also addressed at length in Sorensen (2008).
[9] This puzzle is from Lewis & Lewis (1970, p. 208).
[10] See Carroll (1865, pp. 93–94).
[11] Parts of this paper draw from of two earlier, shorter pieces. One appeared as 'Doughnuts', in M. Chadha & A.K. Raina (Eds.), *Basic Objects. Case Studies in Theoretical Primitives* (pp. 41–51), Shimla: Indian Institute of Advanced Study, 2001; the other is forthcoming as 'Qu'est-ce qu'un trou dans l'Emmental?', in A. Meylan & O. Massin (Eds.), *Aristote chez les Helvètes. Onze essais de métaphysique helvétique*, Paris: Ithaque.

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