

Vagueness in Geography

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Abstract. Some have argued that the vagueness exhibited by geographic names and descriptions such as ‘Albuquerque’, ‘the Outback’, or ‘Mount Everest’ is ultimately ontological: these terms are vague because they refer to *vague objects*, objects with fuzzy boundaries. I take the opposite stand and hold the view that geographic vagueness is exclusively semantic, or conceptual at large. There is no such thing as a vague mountain. Rather, there are many things where we conceive a mountain to be, each with its precise boundary, and when we say ‘Everest’ we are just being vague as to *which* thing we are referring to. This paper defends this view against some plausible objections.

Introduction

If you ask

- (1) What is the shortest mountain on Earth?

you will raise a laugh. It is a ludicrous question. It is ludicrous because it presupposes—falsely—that there is a precise lower bound to the height of a mountain, a precise boundary demarcating short mountains from high hills. In other words, (1) presupposes that the predicate ‘mountain’ is precise when in fact it isn’t. It is a vague predicate. And its vagueness is well reflected in the fact that it gives rise to paradoxical arguments of the sorites variety:

- (2) A 30,000-foot-high parcel of land is a mountain.
For all k : if a $(k + 1)$ -foot-high parcel of land is a mountain, so is a k -foot-high parcel of land.
Ergo, a 1-foot high parcel of land is a mountain.

(The premises are plausible and the argument is logically valid, as one can verify by 29,999 applications of the rules of universal instantiation and *modus ponens*; yet the conclusion is plainly false.¹) This is no surprise. Vagueness is a pervasive phenomenon of human thought and language and the world of geography is not exempted from its grasp. Not only ‘mountain’ but virtually every geographic word and concept suffers from it: How small can a *town* be? How long must a *river* be? How many islands does it take to have an *archipelago*?

Suppose you now ask:

- (3) What is the highest mountain on Earth?

This is a perfectly legitimate question. Although the predicate ‘mountain’ has borderline cases, it has clear instances as well: Kilimanjaro, Mont Blanc, Mount Everest, and lots more. All clear instances of the predicate ‘mountain’ are higher than any borderline case (that much is clear), and the highest of all is the highest mountain—Mount Everest. Perhaps we may still hesitate on the grounds that there are different ways of measuring the height of a mountain. From base to peak, for example, the Hawaiian volcano Mauna Kea rises 33,480 feet in total against Everest’s 29,035 feet. But this is an ambiguity in the relevant concept of height that can easily be resolved. Let us agree to measure heights from sea level and let us answer (3) accordingly: then the highest mountain on Earth is Everest and Mauna Kea is only 13,800 feet or so, like Gannett Peak.

We still have a problem, however. For just what is this mountain which is higher than any other? What exactly is Everest? Where does it begin and where does it end? On closer inspection, the name ‘Everest’ is just as vague as the predicate ‘mountain’ and gives rise to its own kind of soritical paradox. Consider:

- (4) Hunk h_0 is part of Everest.

For all k : if hunk h_k is part of Everest, then so is hunk h_{k+1} .

Ergo, any hunk h_n is part of Everest.

Here we may think of the h_i ’s as forming a long sequence of adjacent, very tiny, precise hunks of matter stretching along a straight path from near the top of Mount Everest, at 27°59' N 86°56' E, to your favorite spot in the center of Katmandu. And just as it seems ludicrous to suppose that there exists a number k such that a $(k+1)$ -foot-high parcel of land is a mountain and a k -foot-high parcel of land is not, so it seems ludicrous to suppose that there exists a number k such that hunk h_k is part of Everest and hunk h_{k+1} is not. Some hunks of matter are clearly part of Everest and others clearly aren’t, but there are lots of borderline hunks for which it is indeterminate whether or not they are part of Everest. One could object that the vagueness here lies in the relational predicate ‘part of’, but it is easy to reconstruct the paradox using precise predicates instead—e.g., the predicate ‘has a smaller volume than’, with a soritical sequence consisting of hunks of increasing size. The point is simply that ‘Everest’ does not refer to a precise part of reality. It does not refer to a volume of earthy matter sharply demarcated from its surrounding. ‘Everest’ is vague.

In a similar fashion we can see that many geographical names and descriptions are vague. Perhaps we all know exactly what we mean when we speak of Utah or of the Northern Hemisphere. But what about the Outback or the Sahara desert? Where

exactly do they begin and where do they end? Where exactly does the Missouri enter the Mississippi? What exactly are the boundaries of Albuquerque? Of Rio de Janeiro? Of Greenwich Village?

There is a sense, of course, in which these are ludicrous questions too. Normally we refer to a region without any need to *think* of its limit or boundary, never mind considering the boundary as being at some definite place. We know how to use geographic terms without being able to provide a precise explanation of the grounds for this competence. To paraphrase Wittgenstein, we know how to use these words just as we know what sound a clarinet makes, though we are unable to say it.² This much is clear. The philosophically interesting and by no means ludicrous question is: How does this work? What is a good model of this sort of competence—of our ability to live with vagueness in cases such as these? Why, for example, do we all agree that Everest is definitely in Asia and definitely not in Europe, though we may disagree or suspend judgment as to how much of it is in Tibet? And how do we avoid the slippery slope represented by such soritical reasonings as (4)? Presumably the model of family resemblance shows how, in ordinary circumstances, a word can be used successfully regardless of whether or not it meets the Fregean (or Tractarian) ideal of precision. We say that something is a mountain because it resembles several things that have hitherto been called mountain, even if the exact nature of this resemblance may give rise to borderline cases. Likewise we say that something is part of Everest because it resembles (in some relevant respect) several things that have hitherto been said to be part of Everest. But this is no way out of the sorites paradox. The model of family resemblance does not show how to avoid the slippery slope. To the contrary, it provides an excellent explanation of why we fall into the paradox in the first place.³

Partly, these are questions that arise in every domain—not just geography. Historians, for example, are deeply soaked in vagueness. The Renaissance, the Industrial Revolution, World War II—all of these are perfectly objective sub-totalities within the totality of all events making up universal history, but their spatial reach as well as their initial and terminal temporal boundaries may be indeterminate. Or take the philosopher concerned with the mind-body problem. The relevant question is whether John is identical with his body. But what exactly is that body? Surely it comprises John's heart and surely it does not comprise mine. But what about the olive that John is presently chewing: Is it part of John? Will it be part of John only after he has swallowed it? After he has started digesting it? After he has digested it completely? And exactly when did John come into existence? Exactly when will it be correct to say that John no longer exists?

I think there is indeed a deep analogy between these cases and the sort of vagueness familiar from the geographic world. For example, in all cases the indeter-

minacy does not seem to be purely epistemic, as if it were just a matter of our ignorance.⁴ It's not that there is this body, John's, whose exact spatiotemporal boundaries cannot yet be determined because of lack of relevant information. It's not that there is this event, the Industrial Revolution, which has perfectly precise and yet unknown spatiotemporal boundaries, boundaries that historians have not been able (and will never be able) to locate. Likewise, it's not that the boundaries of Everest are unknown to us. Rather, there is no fact of the matter about whether certain chunks of reality are part of Everest because the referent of the term 'Everest' is not sharply demarcated, just as in the case of the terms 'John' and 'the Industrial Revolution'.

In spite of these analogies, however, I think it is worth focusing on the case of geographic terms in its own right. As Barry Smith has argued, in the domain of geography we have a good understanding of the twofold nature of our boundary concept.⁵ On the one hand we have Australia, the Bikini atoll, Lake Ontario, the planet Earth—entities whose boundaries correspond to some *bona fide* qualitative differentiations or spatial discontinuities in the underlying territory. On the other hand we have Iraq, the state of Wyoming, Dogger Bank, the North Sea, whose borders are at least in part the result of human *fiat* and lie skew to any physical discontinuities. (Such *fiats* may correspond to laws, political decrees, or acts of human decision—as when Iraq was called into being in 1922 by the British High Commissioner, Sir Percy Cox, through the drawing of a few lines in the sand, or when the borders of Europe were redefined upon signing the final act of the Vienna congress on June 9, 1815; but they may also correspond to weaker forms of cognitive projection—as in the case of the Bermuda Triangle, Northern Italy, or a picnic spot in Central Park on Sunday afternoon). The articulation of these two notions of a boundary is—I think—crucial to our understanding of the vagueness of geographic names and descriptions, and my purpose in this paper is to clarify some aspects of this view. Briefly put: in geography *bona fide* boundaries tend to be crisp, but *fiat* boundaries are often vaguely drawn and the vagueness of this *fiat* process is reflected in the vagueness of much geographic terminology. Perhaps every other case of vagueness is to be explained along the same lines. I am indeed inclined to think so. However that is a general, difficult question that I wish to leave to another occasion.

Vagueness, *de re* and *de dicto*

Let us focus on Mount Everest. To say that the vagueness of 'Everest' is not purely epistemic is to say that its referent is genuinely indeterminate: it lacks sharp boundaries. There is, however, some dispute concerning the nature of this indeterminacy. The statement that the referent of a term *t* is indeterminate can be given a *de re* reading, as in (5a), or a *de dicto* reading, as in (5b):

- (5a) The referent of *t* is such that it is indeterminate whether certain chunks of reality lie within its boundaries.
- (5b) It is indeterminate whether certain chunks of reality lie within the boundaries of the referent of *t*.

On the *de re* reading the indeterminacy is ontological. A vague term is one that refers to a vague object, an object the spatial or temporal boundaries of which are genuinely “fuzzy”.⁶ Accordingly, on this reading ‘Everest’ is vague insofar as Everest is vague: there is no objective, determinate fact of the matter about whether the borderline hunks are inside or outside the mountain called ‘Everest’. The same applies to deserts, lakes, islands, rivers, forests, bays, streets, neighborhoods, and many other sorts of geographic entities: On the *de re* reading these entities have vague names because they are genuinely vague denizens of reality. Like the figures depicted in an impressionist painting, they do not fit into the standard topological picture according to which every object has an interior surrounded by an exterior. Like old soldiers, “they just fade away”.⁷

I reject this view. All geographic vagueness—I hold—is purely semantic. It lies in the representation system (our language, our conceptual apparatus), not in the represented entity, and to say that Everest must be vague insofar as ‘Everest’ is vague is to commit what Russell called the “fallacy of verbalism” and Roy Sorensen calls “the genetic fallacy”: one mistakenly infers that the *product* of a representation is vague because the representation *process* is vague.⁸ More: to me the claim that the vagueness of ‘Everest’ lies entirely in the vagueness of Everest is unintelligible. For just *what* would this vague object be? How could we be so precise as to designate *it*? At what distance from the summit would *its* parts begin to fade away? To me these are questions that cannot be properly answered and the claim that mountains, deserts, and the like are vague objects is only meaningful as a *façon de parler*.⁹

Accordingly, it is the *de dicto* reading that I favor. On this reading, corresponding to (5b), to say that the referent of a geographic term is not sharply demarcated is to say that the term vaguely designates an object, not that it designates a vague object. When we say ‘Everest’ we are speaking vaguely because there are several different ways of tracing the geographic limits of Mount Everest, all perfectly compatible with the way the name is used in ordinary circumstances. Likewise, when the members of the Geodetic Office of India baptized the mountain after the name of their British founder, they simply did not specify exactly which parcel of land they were referring to. The referent of their term was vaguely fixed. If we wish, we can add that it is ultimately the vagueness of the relevant sortal concept (the concept *mountain*, in this case) that was responsible for the way in which the referent of ‘Everest’ was vaguely fixed. But it is not the stuff out there that is vague. Each one of a large variety of slightly distinct aggregates of molecules has an equal claim to being

the referent of the vaguely introduced name 'Everest'. And each such thing is precisely determinate.

It is here that the categorial distinction between fiat and bona fide boundaries introduced above becomes relevant. Fiat boundaries play a central role in our articulation of the geographic world. They play a role especially when it comes to geopolitical entities such as nations, states, counties, or postal districts. Fiat boundaries of this sort can be very precise, as with Wyoming: the question of whether something is inside or outside the state of Wyoming is at least in principle (and leaving aside geological changes on the surface of the earth) determined. In some cases, however, and perhaps in most cases, the process leading to the definition of a fiat boundary is not so precise. Where are the borders of the Carribean? Of Downtown Manhattan? Of the Roman Empire? Where exactly do the borders of Germany, Switzerland, and Austria meet in the vicinity of Lake Constance? In cases such as these the question of whether something is on one side of the border or on the other may be indeterminate. Does this mean that the bounded region is vague? No: it simply means that many regions, each perfectly precise in its own right, have an equal claim to being the intended product of that fiat process. Many borders, each perfectly precise in its own right, could be traced out in a way that conforms to the intended stipulation. If the need arises we can make our stipulations more precise, the ultimate goal being to reduce the number of admissible candidates to a minimum. This need not be a purely academic operation, though. There may be genuine political disputes as concerns fishing and sailing rights in different parts of Lake Constance.¹⁰ Indeed, as Helen Couclelis has pointed out, people kill each other over boundaries of all sorts.¹¹ People fight wars because they disagree concerning the relevant concept of an admissible candidate, or because they think that a certain political boundary should reflect some sort of discontinuity (relating to culture, religion, race, language) that others do not recognize. This is not to say that people kill each other over vague borders. Wars are fought because there is disagreement as to the location of allegedly sharp borders (just as they are fought over the arbitrariness of certain borders¹²).

Finally, fiat boundaries also play a role when it comes to geo-physical entities such as deserts and mountains. When the name 'Everest' was introduced, a referent was carved out in such a way that part of the relevant boundary (that part which separates the mountain from the atmosphere) was of the bona fide sort: it existed independently of the geographers' conceptualizing activity. But part of the boundary (that part which demarcates the foothills) was of the fiat sort. It did not correspond to any natural physical discontinuity and the members of the Geodetic Office did not—apparently—bother to make themselves precise.

Perhaps even the bona fide boundary of Everest, that which separates it from the atmosphere, involves some degree of arbitrariness. Perhaps fiat boundaries are *al-*

ways at work in articulating the objects with which we have to deal in our everyday commerce with the world.¹³ If we think of a mountain—or of any other physical object—as an intricate system of minute particles, then even its physical boundary is not quite of the bona fide sort. It too becomes a fiat boundary, and its exact shape turns out to involve the same degree of arbitrariness as that of a mathematical graph smoothed out of sparse data. The boundary demarcating the foothills of Everest would be the result of a somewhat imprecise *social* fiat, involving some cooperation on the part of geographers and other social agents; the boundary delimiting the slopes would be the result of a somewhat imprecise *individual* fiat, of a demarcation process stemming exclusively from our personal cognitive activity. Indeed, as Smith has emphasized, an important motor for the drawing of fiat boundaries is perception, which has the function of articulating reality in terms of sharp boundaries even when these are not genuinely present in the autonomous (which is to say mind-independent) physical world. (Think again of the figures depicted in an impressionist painting.) The same may well be true for all sorts of seemingly bona fide boundaries, including the boundaries of islands, lakes, and planets. To the extent that such entities can be said to have vague boundaries their boundaries must be of the fiat sort, and to say the opposite is to commit the genetic fallacy. All vagueness is *de dicto* precisely in that it pertains exclusively to the realm of fiat articulations.

Supervaluationism

The *de dicto* conception of vagueness is not new, of course. It has amply been defended by David Lewis, among others, and it is rather popular among vagueness theorists.¹⁴ But how is it to be handled? What sort of treatment can be offered for the semantics of vaguely referring expressions?

I think the natural answer is to be found in a supervaluational semantics of the sort initially advocated by Kit Fine with regard to the vagueness of ordinary language predicates.¹⁵ A vague predicate such as ‘mountain’, on this view, is one that admits of various alternative “precisifications”. For instance, it can be made precise by deciding that a parcel of land counts as a mountain if and only if it is at least 5,000 feet high. Or it could be made precise by deciding that a parcel of land counts as a mountain if and only if it is at least 4,000 feet high. And so on. The predicate is vague exactly because there is indeterminacy between these various ways of picking out a precise cut-off value. With individual names the same intuition applies. A vague name such as ‘Everest’ can be made precise by drawing a precise boundary around its referent, but there are many ways of doing this and all of them are compatible with the way we use the name. As we have seen, there are many admissible candidates, and ‘Everest’ is vague because of this indeterminacy.

Given this understanding of vagueness, the basic idea of a supervenient semantics is that the truth-value of a statement involving vague terms (or vague expressions more generally) is a function of its truth-values under the various admissible precisifications of those terms. If the statement is true under all such precisifications, then we may take it to be true *simpliciter*; the unmade semantic stipulations do not matter. In other words, it makes no difference what those terms could designate had their semantic values been defined more precisely: what the statement says is true regardless (or *super-true*, as Fine has it). Likewise, if the statement comes out false under every precisification then we may regard it as false (or *super-false*), in spite of its vagueness. It is only when the statement is true under some precisifications and false under others that there is trouble. In such cases, nothing will settle the question for us and the statement will fall into a truth-value gap.

The notion of a precisification is a complex one and some authors find its unrestricted application problematic.¹⁶ However, in the case of geographic names and descriptions this notion appears to be clear enough, for every geographic term of that sort is introduced through some boundary-drawing process. (Whether other names and descriptions are introduced through such a process is, by contrast, a deep and controversial philosophical question.) This is why the method appears to apply naturally to our topic. A precisification of a geographic term t is just a way of assigning a precise referent to t by (re-)drawing a fiat boundary of the appropriate sort, subject to the relevant sortal restrictions. A precisification of ‘Everest’, in particular, will correspond to any acceptable way of drawing a precise boundary around a regular, self-connected, mountain-like region beginning from the peak of the soritical series, h_0 , and extending downwards to include some of the borderline hunks. This is because the term ‘Everest’ is meant to designate an entity of the sort mountain, and every mountain includes a peak by definition. Thus, in spite of the vagueness of ‘Everest’, the statement corresponding to the first premise of the relevant soritical argument is sure to be true under every precisification, and hence super-true:

- (6) Hunk h_0 is part of Everest.

Likewise, the conclusion of the argument,

- (7) Hunk h_n is part of Everest,

is sure to be false if h_n is located in Katmandu. On the other hand, there is no way in which we can settle the issue when it comes to the borderline hunks of matter, since the latter may turn out to be inside Everest or outside it depending on how we draw up a precise boundary for the mountain. In those cases nothing will settle the issue for us, and the statement that those borderline hunks are part of Everest will fail to receive a definite truth-value.

The same explanation applies to the case mentioned earlier: no matter how we precisify ‘Everest’, statements such as

- (8) Everest is entirely in Asia
- (9) Everest is entirely in Europe

are sure to come out true and false, respectively, and that explains our confidence in attributing such truth-values in spite of the actual vagueness of the term. For the same reason we can be confident that Everest supplies the right answer to the question in (3):

- (10) Everest is the highest mountain on Earth.

This is because no matter how we draw a fiat boundary to demarcate the referent of ‘Everest’, we are sure to include the peak h_0 , which is where the height is measured. By contrast, not all precisifications of ‘Everest’ will induce the same evaluation of a statement such as

- (11) Everest is located mostly in Tibet.

Some precisifications will make (11) true and some will make it false—whence our reluctance in attributing a definite truth-value to this statement.

This account is natural and is clearly generalizable to all sorts of cases, at least within the domain of geography. Whether we are speaking of Everest, Albuquerque, or the Sahara desert, the vagueness of our talk can be explained along the same lines: the truth-value of what we say is a function of the truth-value of what we would be saying if we were speaking precisely, according to some way or other of re-drawing all the relevant (fiat) boundaries. In Lewis’s words:

Whatever it is that we do to determine the “intended” interpretation of our language determines not one interpretation but a range of interpretations. . . . What we try for, in imparting information, is truth of what we say under all the intended interpretations.¹⁷

Moreover, the account works well. Among other things, it preserves a standard metaphysics of parts and wholes as well as classical logic, since precisifications yield models that are thoroughly classical. For example, in spite of the indeterminacy of (11), the corresponding instance of the excluded middle,

- (12) Either Everest is located mostly in Tibet, or it isn’t,

will come out true under every precisification, hence super-true.

There are, however, a number of objections that have or can be raised against this account, including some that would seriously affect its intuitive appeal. In the

remainder of the paper I want to consider four of them in particular, and show where they go wrong.¹⁸ This will provide direct evidence in favor of the supervenience account and thus also, indirectly, further elements in support of a fully *de dicto* conception of geographic vagueness: genuine *de re* vagueness, if intelligible at all, is unnecessary.

Are There Any Precifications?

The first objection concerns the basic notion of a precisification.¹⁹ We have seen that supervenience treats vagueness as ambiguity on a grand scale: we use ‘Everest’ in order to refer to a certain parcel of land, but we refrain from taking a precise decision about *which* piece we are talking about. This exploits the idea that many candidates have equal claim to being the referents of a vague name. On a closer look, however, one could object that there are no candidates at all. For—one could argue—let x be any precisely demarcated parcel of land. By assumption, it is indeterminate whether a borderline hunk of matter h_k is part of Everest. However, it cannot be indeterminate whether h_k is part of x . Hence, Everest and x would seem to have different properties, and by Leibniz’s law they would have to be distinct. Hence it would seem that when we say ‘Everest’ we cannot be talking about x . And by generalization it would follow that we cannot be talking about *any* (precisely demarcated) portion of reality.²⁰

The reply to this objection is that it rests on a fallacious use of Leibniz’s law. Generally speaking, the fallacy is analogous to a familiar one involving modal contexts. We know, for example, that the following statements have opposite truth-values:

- (13a) It is contingent that 20,000 feet is less than the height of Everest.
- (13b) It is contingent that 20,000 feet is less than 29,035 feet.

Yet this is not enough to conclude that the height of Everest and 29,035 feet have different properties (viz. different modal properties), hence that they are distinct heights, unless we also assume the equivalence between statements of the forms (14a) and (14b):

- (14a) It is contingent that 20,000 feet is less than t .
- (14b) t is an x such that it is contingent that 20,000 feet is less than x .

And this equivalence holds when ‘ t ’ is replaced by ‘29,035 feet’ (a rigid designator) but not when it is replaced by ‘the height of Everest’.²¹ Likewise, if h_k is a borderline hunk of matter and x a precisely demarcated parcel of land, then the following two statements have opposite truth-values:

- (15a) It is indeterminate whether h_k is part of Everest.
 (15b) It is indeterminate whether h_k is part of α .

But this is not enough to conclude that Everest and α have different properties (different mereological properties, in this case) unless we also assume the equivalence between statements of the forms (16a) and (16b):

- (16a) It is indeterminate whether h_k is part of t ,
 (16b) t is an x such that it is indeterminate whether h_k is part of x ,

where ' t ' can be replaced by 'Everest' or by ' α '. And do we have reasons to assume such an equivalence? Clearly, to ask this question is to ask the question whether a *de dicto* statement implies its *de re* counterpart. In the modal case the implication holds when t is a rigid designator. And sure enough, in the case of (16a) and (16b) the implication holds when ' t ' is a precise designator, such as ' α ': in that case, both statements are false, since h_k must either be determinately inside α or determinately outside. (Recall that ' h_k ' itself designates a precise hunk of matter.) However, why should the implication hold when ' t ' is replaced by a vague designator such as 'Everest'? To warrant such an implication would be to endorse the view that the vagueness of 'Everest' lies in the vagueness of Everest, and this is not entailed by the supervenient account. To the contrary, the supervenient account allows us to deny that view: the vagueness of 'Everest' is purely linguistic and does not extend to the world. To assume the implication from *de dicto* to *de re* vagueness is to assume that there are vague objects, and this just begs the question against the supervenient account.

So the opposition between (15a) and (15b) does not allow us to infer that Everest must be distinct from α . All that follows is that it is *indeterminate* whether Everest is α , and this intuitive consequence is perfectly coherent with supervenientism. The statement

- (17) Everest = α

is true under one precisification of the name 'Everest' (the one which picks out α as an admissible referent) and false under all the others.²² Indeed, from this perspective the initial line of objection can be turned into a serious objection against ontological vagueness. This can be done by adapting a much debated argument due to Gareth Evans.²³ For, on the one hand, to say that (17) is indeterminate is to say that the following is true:

- (18) It is indeterminate whether Everest = α .

On the other hand, there can be no question that Everest is self-identical, even if it turns out to be a genuinely vague object; hence the following must be false:

(19) It is indeterminate whether Everest = Everest.

But then, if indeed the vagueness of 'Everest' were ontological, Leibniz's law would allow us to conclude that Everest and α are distinct. This would be a fallacious move if the vagueness of 'Everest' were understood *de dicto*, as we have just seen. But it is not fallacious if this vagueness is understood *de re*. For in that case (18) and (19) imply (20) and (21), respectively:

(20) α is an x such that it is indeterminate whether $x = \text{Everest}$.

(21) Everest is an x such that it is indeterminate whether $x = \text{Everest}$.

And in the case of statements such as these Leibniz's law is perfectly applicable. We must therefore conclude that if 'Everest' is genuinely *de re*, then the identity statement in (17) is not indeterminate but false, contrary to our initial supposition. The same would be true if α were not a precise parcel of land but a vague one, for nothing in the argument rests on the specific nature of α . In fact, by reasoning in a similar fashion we must also conclude that on the ontological conception of vagueness *no identity statement* will be indeterminate. The world would be full of vague entities but each such entity would have identity conditions that are absolutely precise. And this is very strange indeed.

What Are the Precifications?

Consider now a second objection. If there were candidates, just what would they be? How can we set apart the admissible referents for a given vague term t ? Suppose that α is an admissible referent of 'Everest' and suppose that α' is a slightly larger parcel of land which differs from α only in that it includes an extra tiny bit of matter along the border. Surely α' must also count as an admissible referent of 'Everest' if α does, for there is no tangible difference between the two. But then, by the same pattern, any parcel of land would end up counting as an admissible referent of 'Everest'. And that would be absurd.

The argument, it may be noted, may also be put in the form of a sorites paradox for the relational predicate 'admissible referent':

(22) Parcel of land α_0 is an admissible referent of 'Everest'.

For all k : if parcel of land α_k is an admissible referent of 'Everest', then so is parcel of land α_{k+1} .

Ergo, any parcel of land α_n is an admissible referent of 'Everest'.

(where we may suppose that the α_i 's form a sequence of marginally different, gradually increasing parcels of land going from a small region around the peak of Mount Everest to an arbitrarily larger portion of the Asian continent). Thus, the ob-

jection effectively amounts to the observation that ‘admissible referent’ is a vague predicate, exactly like ‘mountain’ or ‘desert’. This is apparent if we consider that the result of putting quotation marks around the vague name ‘Everest’ is a precise name (a name of the word ‘Everest’, that is), so that the source of the trouble in (22) can only lie in the phrase ‘admissible referent’.

Now, is this a problem for the supervaluational account? In a way it is. Surely, if it is not determinate what chunks of reality count as admissible referents of a vague name, then the supervaluational account is incomplete. Supervaluationism says that the truth-value of a vague statement is a function of the truth-values of its precisifications, and this is no account unless the set of admissible referents for its terms is clearly determined. However, this difficulty simply bears witness to the fact that supervaluationism suffers from the phenomenon of higher-order vagueness. This is not an objection to supervaluationism but rather a proof that the language in which the theory is formulated—the semantic metalanguage—is itself vague. And bad news as this may be, it comes as no surprise. As J. L. Austin put it, ‘vague’ is vague,²⁴ and one can verify this by constructing a suitable sorites:²⁵

(23) The predicate ‘either small, or smaller than s_1 ’ is vague.

For all k : if ‘either small, or smaller than s_k ’ is vague, so is ‘either small, or smaller than s_{k+1} ’.

Ergo, the predicate ‘either small, or smaller than s_n ’ is vague for any n .

(where the s_i ’s again designate marginally different parcels of land of increasing size). Here the first premise is true, since the predicate in question is just as vague as the predicate ‘small’: both are true of the initial parcel of land in the sequence, s_0 , and for the other s_i ’s they behave exactly in the same way. Moreover, there seems to be no point in the infinite series of all these predicates where we encounter a vague predicate followed by a sharp one (what difference can a tiny bit make?)—so the second premise of (23) appears to be true also. The conclusion, however, is plainly false, since if n is large enough the predicate in question is just as precise as the predicate ‘smaller than s_n ’: both predicates are true of every parcel of land smaller than s_n , and false of all others. A similar argument, of course, can be constructed to reveal the vagueness of other semantic predicates, including ‘true’ and its derivatives.

One could point out that the vagueness of ‘vague’ in (23) is piggybacking on the vagueness of ‘small’.²⁶ But this is no objection. The vagueness of ‘admissible referent’ in (22) is likewise dependent on that of ‘Everest’. If ‘Everest’ were a precise name, then the second premise in (22) would be plainly false (for s_k =Everest). The point is simply that the semantic metalanguage inherits some vagueness from the object language and that makes the overall picture look messy. But is this a

problem? It is, for it implies that there is no end to the propagation of vagueness in the metalinguistic hierarchy. Yet (23) shows that this is a general problem for semantics and not a special problem for supervaluationism. For each way of choosing a metalanguage supervaluationism delivers a semantics for the object language. The choice may not be well defined and the range of options may itself be somewhat indeterminate. But that's the best we can do if the metalanguage is vague.

How Many Mountains?

So there is nothing in the way of assuming the existence of a (possibly indeterminate) number of precisifications for our vague geographic terms. Let us, then, consider what sort of things these precisifications can be. For the notion of a precisification is not an abstract one. In each individual case, the result of drawing a boundary would not count as an admissible precisification unless it also complies with certain constraints that are part of the overall background geographical vocabulary, vague as that may be. For example, we already mentioned that each parcel of land corresponding to a precisification of 'Everest' would have to be a *mountain*. If it didn't so qualify—one could argue—it wouldn't be an *admissible* referent, since 'Everest' is meant to name a mountain. (This is a fact about the correct application of the term 'Everest' that counts as "penumbral connection", in Fine's terminology.²⁷) Now, doesn't this presuppose the existence of several mountains in the area where the admissible referents of 'Everest' are to be found? All of these mountains would have the same peak but they would not coincide completely and would therefore be distinct. This would entail that when Edmund Hillary and Tenzing Norgay reached the top of Everest on May 29, 1953, they actually reached the top of several mountains, not one. And wouldn't this be absurd?

The answer to this worry is that the natural constraint on the admissible precisifications of 'Everest'—that each one of them pick out an object that qualifies as a mountain—does not in fact presuppose the existence of a cluster of overlapping mountains with a common peak. To say that each precisification of 'Everest' must pick out a mountain is to say that each precisification must verify

(24) Everest is a mountain.

But this is not to say that there must be many mountains to choose from. There must be many mountain-shaped parcels of land, and each precisification will select one of them and treat it as a mountain. Each precisification will select one *and only one* mountain-shaped parcel of land because mountains are meant to be mutually exclusive. This exclusiveness condition is part of our concept of a mountain (vague as it may be) and reflects another important sense in which the relevant notion of an ad-

missible precisification is restrictive. A precisification of the predicate ‘mountain’ is admissible only if it verifies the statement

(25) No two mountains overlap,

which means that (25) must be super-true. This is why we find it absurd to say that Hillary and Norgay reached the summit of several mountains at the end of their historical climb in 1953. They reached the common summit of several overlapping mountain-shaped chunks of reality, only one of which is a mountain because in the vicinity of every mountain-shaped region there is only one mountain. Surely there is no saying *which one* of them is a mountain, for that has not been settled. But this indeterminacy is just a restatement of the fact that ‘mountain’ is a vague predicate, whence ‘Everest’ is a vague name.

In this connection one could also point out that the relevant constraints on the admissible precisifications of ‘Everest’ are, to some degree, context relative. Our examples so far have presupposed that the admissible precisifications of ‘Everest’ are parcels of land, but that is a simplification. Is the ice on the summit part of Everest? Are the trees at the foothills part of it? What about the grass, the flowers, the rabbits, the worms that fill up every accessible nook and cranny of the relevant territory—are they part of Everest? Arguably, these are questions that do not admit of a unique, precise answer. They do not admit of a definite answer because our concept of mountain is indefinite in this respect. In some contexts, our supervalutary practices parcel out the admissible referents of ‘Everest’ without paying attention to the intuition that, say, rabbits are not part of mountains. In other contexts we do pay attention to such an intuition—we take it to be a conceptual restriction on the predicate ‘mountain’ that its precisifications satisfy

(26) No rabbits are parts of mountains.

And since we take it that Everest is a mountain, as per (24), it follows that the class of admissible referents of ‘Everest’ must, in such contexts, verify the statement

(27) No rabbits are parts of Everest.

This sort of context relativity introduces a further complicating element of metalinguistic indeterminacy. But again this is not only a complication for supervalutism but for every theory that attempts to do justice to the fact that the referent of ‘Everest’ (and the extension of ‘mountain’) is not fully determined.²⁸

One last complication is this. The Tibetans don’t say ‘Everest’ but ‘Chomolungma’, and the Nepalese say ‘Sagarmatha’, and presumably the range of admissible precisifications for these two names do not coincide with each other or with that of ‘Everest’. (At least, it will be an indeterminate matter whether they coincide, in

view of the vagueness of what counts as an admissible precisification.) Now ‘Chomolungma’ and ‘Sagarmatha’ are meant to name a mountain, too. But there is no guarantee that they are meant to name the same mountain, or the same mountain as ‘Everest’, since these three names were introduced independently of each other. So how can that be, if there is only one mountain in the neighborhood?

The answer is that there is only one mountain for the English speaker, one mountain for the Tibetans, and one mountain for the Nepalese (as long as the Tibetan and Nepalese concepts of mountain are also such as to satisfy (25))—but these need not be the same portion of reality. Different linguistic practices can determine different fiats, and these can bring different families of admissible precisifications. Mountains are not ready-made entities—they have no hidden essence. A mountain is nothing over and above a mountain-shaped parcel of land, and every mountain-shaped parcel of land could be a mountain. The claim that only one of them is a mountain because mountains are mutually exclusive is entirely internal to a linguistic community and there is no obvious way of establishing links between one community and another. For an English speaker, the referents of ‘Chomolungma’ and ‘Sagarmatha’ are mountain-shaped objects whose peaks coincide with that of Everest, no matter how these names are precisified. And for Tibetan or Nepalese speakers the referent of ‘Everest’ is a mountain-shaped object whose peak coincides with that of Chomolungma, or Sagarmatha, no matter how these names are precisified. But the statement

(28) Everest = Chomolungma = Sagarmatha

is likely to be supervalueationally indeterminate for the English speaker as well as for the Tibetan and the Nepalese. It is likely to be supervalueationally indeterminate, that is, unless these speakers know or otherwise stipulate that their fiats coincide.

Back to the Sorites Paradox

As a fourth and final objection, let us go back to the paradoxical argument in (4)—the sorites paradox for ‘Everest’. Supervalueationally, the rules of universal instantiation and *modus ponens* turn out to be valid, so the argument cannot be rejected on logical grounds: it is a perfectly valid piece of reasoning.²⁹ However, the argument can be rejected as unsound: the first premise is true but the second premise,

(29) For all k : if hunk h_k is part of Everest, then so is hunk h_{k+1} ,

turns out to be false under every precisification of ‘Everest’, hence super-false. Now, this blocks the paradox but—one could object—the price is unacceptable. For the second premise of a soritical argument is the one that captures the intuition that

the relevant vague terms are, in Crispin Wright's felicitous phrase, tolerant to marginal change.³⁰ So how can that premise be false? How could its negation

(30) There is some k such that hunk h_k is part of Everest but hunk h_{k+1} is not

be true if 'Everest' is vague? (This objection runs deep, for it would apply to the supervaluational account of vague predicates as well: the second premise of (2) turns out to be false for similar reasons, and how can that be right? How can it be right to say—prior to any extra stipulation—that there exists a number k such that a $(k + 1)$ -foot-high parcel of land is a mountain and a k -foot-high parcel of land is not?)

This line of objection is regarded by many as a knock down refutation of supervaluationism. As such, however, I think that the objection has already been answered by a number of authors.³¹ We have a tendency to assent to the second premise of a sorites argument because we have an impulse to demand a counterexample whenever a generalized statement is denied. We have a tendency to withhold (30) because we have an impulse to demand an instance whenever an existential statement is asserted. Such impulses are justified in the ordinary semantics for precise languages. However, the demand for instances and counterexamples is illegitimate if the object language includes vague expressions. Just as the law of excluded middle (31a) should not be confused with the semantic principle of bivalence (31b),

(31a) 'Either p or not p ' is true

(31b) Either ' p ' or 'not p ' is true

(compare (12)), so for example an existential law of the form (32a) should not be confused with the corresponding principle of specification (32b):

(32a) 'There is some k such that ... k ...' is true.

(32b) There is some k such that '... k ...' is true.

These are distinctions without a difference in the ordinary semantics for classical logic. But in the presence of vagueness (and perhaps other forms of semantic indeterminacy) they become significant and their confusion gives rise to paradox. Vague expressions are tolerant, not because they satisfy the second, inductive premise of a sorites argument, but because they falsify it without it being possible for anybody to exhibit a specific counterexample. They are tolerant, not because they falsify existential statements of the form (32a), but because they satisfy them without satisfying the corresponding statement (32b).

I do not mean to suggest that this sort of reply is straightforwardly acceptable. We are, after all, dealing with a paradox and it is hard to come up with a happy-face solution.³² However, I do say that in the geographic domain this explanation is the right one—and the only possible one—if we agree on the *de dicto* understanding

of vagueness. For the logical relationship between statements of the form (32a) and (32b) is precisely what distinguishes the two conceptions of vagueness, as illustrated with reference to (5a) and (5b) and again with reference to (16a) and (16b). Consider Mount Everest. In this case, the relevant instances of (32a) and (32b) are:

(33a) ‘There is some k such that hunk h_k is part of Everest but hunk h_{k+1} is not’ is true.

(33b) There is some k such that ‘hunk h_k is part of Everest but hunk h_{k+1} is not’ is true.

If the vagueness of ‘Everest’ were *de re*, then (33a) would have to be false because it would imply (33b), which is false. One would then have to find a different way of explaining away the paradox—normally one that involves a departure from classical logic. By contrast, if the vagueness of ‘Everest’ is *de dicto*—if it is merely a sign of our poor fiat boundary drawing, as I have been arguing—then the falsehood of (33b) has no effect on (33a), for the implication does not hold. Hence the solution to the sorites.

In fact, on the *de dicto* conception it is perfectly reasonable to say that (33a) and (33b) have opposite values. For, on the one hand, if Everest is not a vague object then it does have a sharp boundary. (Every sharp object has a sharp boundary somewhere, whether of the fiat or bona fide sort.) Hence the statement that there must exist a cut-off number k , i.e., the statement ‘There is some k such that hunk h_k is part of Everest but hunk h_{k+1} is not’ must be true. Hence (33a) is true. On the other hand, ‘Everest’ is vague, which means that it is impossible for anybody to *specify* a k that does the job. Hence there is no k such that the statement ‘Hunk h_k is part of Everest but hunk h_{k+1} is not’ is true, for the truth-value of that statement depends on how ‘Everest’ is precisified. Hence (33b) is false. In short, it is true that there exists a k that marks the boundary, but there is no k such that it is true of *it* that it marks the boundary. This is the only reasonable thing to say if Everest is sharp but ‘Everest’ is vague. And it is precisely the answer delivered by the supervaluational account. To reject it is to fall for a *de re* account of vagueness, hence for an ontology of vague objects. And there is no need for such entities in geography.³³

Notes

¹ See Sainsbury and Williamson (1997) and Keefe and Smith (1997) for a history of the paradox and its numerous variants.

² Compare Wittgenstein (1953), §78.

³ The sorites paradox is briefly mentioned by Wittgenstein in (1964), § 211, but no explicit account is given.

⁴ Such a view has been defended by Sorensen (1988) and Williamson (1994), among others.

⁵ See Smith (1995). The distinction between fiat and bona fide boundaries is further elaborated in Smith and Varzi (2000).

⁶ See e.g. Tye (1990) and Van Inwagen (1990). For an explicit geographic perspective see the papers collected in Burrough and Frank (1996).

⁷ The phrase is from Sylvan and Hyde (1993), p. 19.

⁸ See Russell (1923), p. 87, and Sorensen (2001), p. 10 of manuscript.

⁹ Many other reasons to be doubtful about vague objects are scattered in the philosophical literature. See e.g. Sainsbury (1995) and Heller (1996). One more, important reason will be examined below with reference to Evans's (1979) argument.

¹⁰ I owe this example to Smith and Brogaard (2001).

¹¹ Coclelius (1996), p. 55. See also Smith (1997) for an articulated account of the relevance of "cognitive geometry" to the aetiology of war.

¹² Compare Geyer and Green (1992) on the Gulf War, or Sahlins (1989) on the making of France and Spain in the Cerdanya.

¹³ Some (e.g., Lakoff 1987) would go as far as saying that the very idea of a world of bona fide objects is the expression of a realist metaphysics that is inadequate to our post-modern age. I think this is based on a confusion between what there is and what we single out through our conceptual articulations, though I will not elaborate on this issue here.

¹⁴ See e.g. Lewis (1986). A recent proponent is McGee (1997).

¹⁵ See e.g. Fine (1975) and McGee and McLaughlin (1995). The notion of a supervaluation goes back to van Fraassen (1966), who used it to provide a semantics for free logic, but the basic idea can be found already in Mehlberg (1956), § 29, interestingly with reference to geographic vagueness.

¹⁶ See e.g. Fodor and Lepore (1996).

¹⁷ Lewis (1993), p. 22.

¹⁸ The four objections are to be found scattered through the literature, though not necessarily in the form given here. For instance, objection 1 can be found in Tye (1990), objection 2 in Burns (1991) and Williamson (1994), and objection 3 in Unger (1980) and Lowe (1995). Objection 4, in the more general version, is already discussed at length in Fine (1975).

¹⁹ This section has some overlap with the sixth section of Varzi (2001).

²⁰ This objection has an analogue in the general case where, say, vague predicates are said to be unsharpenable on the grounds that every precisification would entail a change in the *sense* of the relevant predicate. (See Fodor and Lepore, 1996). I think that the solution offered below can be extended to cover such cases, though I will not elaborate on this. On the other hand, other arguments against predicate sharpenability, such as the case examined in Collins and Varzi (2001), do not seem to have an analogue as we move from predicates to (geographic) singular terms.

²¹ The *locus classicus* is Smullyan (1948).

²² We can also formulate this analysis with the help of an analogy. Think of precisifications as possible worlds, with super-truth playing a role analogous to necessary truth—truth in every possible world. The analogy between the operators 'it is contin-

gent that' and 'it is indeterminate whether' is then immediate and the diagnosis of the equivalence in (14) extends directly to (16). See Lewis (1988).

²³ See Evans (1978).

²⁴ Austin (1962), p. 126.

²⁵ See Sorensen (1985).

²⁶ See Deas (1989).

²⁷ Fine (1975) introduces this notion only with respect to predicates, but its generalization to singular terms is obvious.

²⁸ These matters are dealt with at length in Smith and Brogaard (2001).

²⁹ See Fine [1975] for details.

³⁰ See Wright (1975). Of course one could formulate variants of the argument in which the second premise has a different form. Such alternatives, however, do not introduce any significant difference from the case considered here.

³¹ I think the first good defense is in McGee and McLaughlin (1995). My own account is in Varzi (1999).

³² Some would insist that there are no such solutions. See e.g. Schiffer (1998).

³³ Many thanks to David Mark, Barry Smith, and an anonymous referee for helpful comments. An early version of this paper was read at the Society for Philosophy and Geography during the 74th Annual Meeting of the Pacific Division of the American Philosophical Association (Albuquerque, April 2000). Earlier portions were delivered also at the 6th National Conference of the Italian Society for Logic and Philosophy of Science (Urbino, February 1999) and at a workshop on Semantic Approximation, Granularity, and Vagueness held during the 7th International Conference on Principles of Knowledge Representation and Reasoning (Breckenridge, April 2000).

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