Internet Appendix for "Cash Flow or Discount Risk? Evidence from the Cross Section of the Present Values"

Bingxu Chen

This appendix provides details of constructing and estimating the present-value model adapted to the overlapping data. In addition, I report the estimates of the state factors and parameters not reported in the main body of the paper.

I.A List of Variables

In this paper, I use the convention of denoting the unconditional mean of series of state variables $\{x_t\}$ as $\bar{x}$ and the demeaned state variables as $\{\hat{x}_t\}$. The superscript M indicates the parameters or state variables are on market portfolio level and superscript P indicates the parameters or variables are for a specific Portfolio P. All the returns, dividend and price variables are in real term. The variables used in the article are the following:

$P_t^P(P_t^M)$ = price of asset P (market portfolio M) at the end of the t-th quarter;

$D_{t-4,t}^P(D_{t-4,t}^M)$ = annual dividends paid out by asset P (market portfolio) between the $t-4$-th and t-th quarter.

$r_{t-4,t}^f$ = log real risk-free rate over the period between $t-4$ and t.

$\omega_{t,t+4}^P(\omega_{t,t+4}^M)$ = log real expected growth of portfolio P (market portfolio).

$$\omega_{t,t+4} = \ln \mathrm{E}_t[D_{t,t+4}/D_t]$$

$\Delta d_{t,t+4}^P(\Delta d_{t,t+4}^M)$ = log dividend growth of portfolio P (market portfolio) in real term.

$$\Delta d_{t,t+4} = \ln D_{t+4,t}/D_{t,t-4}$$

$g_{t,t+4}^M$ = expected annual log real dividends growth of market portfolio.

$$\Delta d_{t,t+4}^M = g_t^M + \sigma_{g,t}^M u_{t,t+4}^d$$

γ_t^P =cash flow exposure: the portfolio P's expected dividend growth loading on the contemporaneous market portfolio's expected dividend growth.

c^P = Alpha in the portfolio expected dividend growth that cannot be explained by the time-varying cash-flow exposure.

$$\omega_t^P = c^P + \gamma_t^P \omega_t^M$$
$$\mathrm{E}_t(\Delta d_{t,t+4}^P) + \frac{1}{2}\mathbf{Var}_t(\Delta d_{t,t+4}^P) = c^P + \gamma_t^P(g_{t,t+4}^M + \frac{1}{2}(\sigma_{g,t}^M)^2)$$

$\mu_{t,t+4}^P(\mu_{t,t+4}^M)$ = log real expected return of portfolio P (market portfolio).

$$\mu_{t,t+4} = \ln \mathrm{E}_t[(P_{t+4} + D_{t,t+4})/P_t]$$

$r_{t,t+4}^P(r_{t,t+4})$ = realized log return over the period between t and $t+4$.

$z_{t,t+4}$ = Risk premium (expected log excess market return)

$$r_{t+1}^M \equiv \log \frac{P_{t+1}+D_{t+1}}{P_{t+1}} - r_t^f - \pi_t = z_t + \sigma_{z,t}^M u_{t+1}^r$$

β_t^P = Market beta of portfolio P during the period $[t, t+4]$.

α^P = Conditional errors in discounted rate that cannot be explained by time-varying β.

$$\mu_t^P = c^P + \gamma_t^P \mu_t^M$$
$$\mathbf{E_t}(r_{t+1}^P) + \frac{1}{2}\mathbf{Var}_t(r_{t+1}^P)^2 - r_t^f = \alpha^P + \beta_t^P(z_t + \frac{1}{2}(\sigma_{z,t}^M)^2)$$

I.B Model with Overlapping Observation

In the empirical analysis, monthly and quarterly dividends display a pattern of seasonality, we measure the dividends paid over a period of one year following Cochrane (1992). In consistent with the dividend growth, the discounted rate variables also have horizons of one year. While these state variables are observed quarterly for fully exploitation of the data, the observation is constructed as overlapping. In this section, I describe the model adapted to the overlapping observations structure in details. As specified in section 2 and the previous appendix, the state vector composes seven state variables.

$$X_t = [\hat{r^f}_{t,t+4}\ \hat{g}_{t,t+4}^M\ (\hat{\sigma}_{g,t}^M)^2\ \hat{z}_{t,t+4}\ (\hat{\sigma}_{g,t}^M)^2\ \hat{\gamma}_t^P\ \hat{\beta}_t^P]' \tag{I.B .1}$$

The first five state variables capture the the market level environment, also are denoted as X_t^M, while the last two capture the state of a certain asset. Among the variables in the state vectors, the first factor is the real risk-free rate, assumed being an exogenous macro variable, and the others are latent variables cannot be observed directly.

Presumedly, the state vector subjects to a VAR process.

$$X_{t+1} = \Phi X_t + \Sigma^{\frac{1}{2}} \epsilon_{t+1} \tag{I.B .2}$$

where $\epsilon_{t+1} = [\epsilon^r_{t+1,t+5}\ \epsilon^g_{t+1,t+5}\ \epsilon^{\sigma_g}_{t+1,t+5}\ \epsilon^z_{t+1,t+5}\ \epsilon^{\sigma_z}_{t+1,t+5}\ \epsilon^\gamma_{t+1}\ \epsilon^\beta_{t+1}]'$.

$$\Phi = \begin{bmatrix} \Phi_r & & & & & & \\ & \Phi_g & & & & & \\ & & \Phi_{\sigma_g} & & & & \\ & & & \Phi_z & & & \\ & & & & \Phi_{\sigma_z} & & \\ & & & & & \Phi_\gamma & \\ & & & & & & \Phi_\beta \end{bmatrix}, \tag{I.B .3}$$

$$\Sigma = \begin{bmatrix} \Sigma_r & \Sigma_{rg} & \Sigma_{r\sigma_g} & \Sigma_{rz} & \Sigma_{r\sigma_z} & \Sigma_{r\gamma} & \Sigma_{r\beta} \\ \Sigma_{gr} & \Sigma_g & \Sigma_{g\sigma_g} & \Sigma_{gz} & \Sigma_{g\sigma_z} & \Sigma_{g\gamma} & \Sigma_{g\beta} \\ \Sigma_{\sigma_g r} & \Sigma_{\sigma_g g} & \Sigma_{\sigma_g} & \Sigma_{\sigma_g z} & \Sigma_{\sigma_g\sigma_z} & \Sigma_{\sigma_g\gamma} & \Sigma_{\sigma_g\beta} \\ \Sigma_{zr} & \Sigma_{zg} & \Sigma_{z\sigma_g} & \Sigma_z & \Sigma_{z\sigma_z} & \Sigma_{z\gamma} & \Sigma_{z\beta} \\ \Sigma_{\sigma_z r} & \Sigma_{\sigma_z g} & \Sigma_{\sigma_z\sigma_g} & \Sigma_{\sigma_z z} & \Sigma_{\sigma_z} & \Sigma_{\sigma_z\gamma} & \Sigma_{\sigma_z\beta} \\ \Sigma_{\gamma r} & \Sigma & \Sigma_{\gamma\sigma_g} & \Sigma_{\gamma z} & \Sigma_{\gamma\sigma_z} & \Sigma_\gamma & \Sigma_{\gamma\beta} \\ \Sigma_{\beta r} & \Sigma_{\beta g} & \Sigma_{\beta\sigma_g} & \Sigma_{\beta z} & \Sigma_{\beta\sigma_z} & \Sigma_{\beta\gamma} & \Sigma_\beta \end{bmatrix} \tag{I.B .4}$$

In the variance covariance matrix, we allow interaction between shocks of each pair of state variables.This specification allows the potential channel that the contemporaneous covariance between the risk premium and the beta helps explaining the unconditional pricing error, as documented in Jagannathan and Wang (1996) and among others. I also notate the top left 5×5 blocks of Φ and Σ as Φ_M and Σ_M for market portfolio pricing.

Since the observations are overlapping by construction, the errors ϵ_t are not i.i.d $N(0, I)$, but having structural autocorrelation. The annual dividend growth and the discounted rate, as well as their volatility, are compounded from the quarterly corresponding variables.

$$r^f_{t,t+4} = \sum_{k=0}^{3} r^f_{t+k,t+k+1} \tag{I.B .5}$$

$$g_{t,t+4} = \sum_{k=0}^{3} g_{t+k,t+k+1} \tag{I.B .6}$$

$$\sigma^2_{g,t,t+4} = \sum_{k=0}^{3} \sigma^2_{g,t+k,t+k+1} \tag{I.B .7}$$

$$z_{t,t+4} = \sum_{k=0}^{3} z_{t+k,t+k+1} \tag{I.B .8}$$

$$\sigma^2_{z,t,t+4} = \sum_{k=0}^{3} \sigma^2_{z,t+k,t+k+1} \tag{I.B .9}$$

Therefore, the annual errors of those variables are also compounded from the quarterly error by the same token.

$$\epsilon^s_{t,t+4} = \sum_{k=0}^{3} \epsilon^s_{t+k,t+k+1} \quad s \in \{f, g, \sigma_g, z, \sigma_z\} \tag{I.B .10}$$

Assume that the quarterly errors are i.i.d. normal with identity variance covariance matrix. We have the following auto-covariance of the annual error from the data structure.

$$\mathrm{E}_t(\epsilon^s_{t,t+4}\epsilon^s_{t-k,t+4-k}) = \frac{4-k}{4} \quad k = 0, 1, 2, 3; \quad s \in \{f, g, \sigma_g, z, \sigma_z\} \tag{I.B .11}$$

The errors of the market beta and the asset characteristic are assumed to be i.i.d. since they are not compoundable rate of return or growth, but variables represent the status.

In the rest of the section, I show how to infer the latent variables from observable price and dividend given the above setup. According to the horizon of the observations, the price of an asset is the present value of all annual dividends discounted by the annual discounted rate.

$$P_t = \sum_{s=1}^{\infty} \mathrm{E}_t[\prod_{k=0}^{s-1} \exp(-\mu_{t+4k-4,t+4k} - \pi_{t+4k,t+4k+4}) D_{t+4s-4,t+4s}] \tag{I.B .12}$$

For the market portfolio, the discounted rate in real term

$$\mu^M_{t,t+4} = \phi_M + \xi'_M X_t \tag{I.B .13}$$

where $\phi_M = \bar{r^f} + \bar{z} + \frac{1}{2}(\bar{\sigma}^M_{z,t})^2$ and $\xi_M = [1\ 0\ 0\ 1\ \frac{1}{2}]'$

As for a specific piece of asset P, the discount rate is a quadratic form of the state vectors,

$$\mu^P_{t,t+4} = \phi_P + \xi'_P X_t + X'_t \Omega X_t \tag{I.B .14}$$

where $\phi_P = \alpha^P + \bar{r^f} + \bar{z}\bar{\beta}$ and $\xi_P = [1\ 0\ 0\ \bar{\beta}\ \frac{1}{2}\bar{\beta}\ 0\ \bar{z}]'$. The matrix of the quadratic form is

$$\Omega = \frac{1}{2}(e_{47} + \frac{1}{2}e_{57} + e_{74} + \frac{1}{2}e_{75}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 0 & 0 \end{bmatrix} \tag{I.B .15}$$

On the other hand, recall that realized dividend growth in real term equals to the expected dividend growth plus an orthogonal shock:

$$\Delta d^{P(M)}_{t,t+4} = g^{P(M)}_{t,t+4} + \sigma^{P(M)}_d u^{P(M)}_t \tag{I.B .16}$$

The expected dividend growth can be put into a quadratic form of the state vectors, too.

$$g^M_{t,t+4} = \psi_M + \zeta'_M X_t \tag{I.B .17}$$

where $\psi_M = \bar{g}^M$,$\zeta_M = [0\ 1\ \frac{1}{2}\ 0\ 0\ 0\ 0]'$, and the expected dividend growth is of quadratic form of the state variables.

$$\omega^P_{t,t+4} = \psi_P + \zeta^{'}_P X_t + X'_t W X_t \tag{I.B .18}$$

where $\psi_P = c^P + \bar{\gamma}\bar{g}^M$ and $\zeta_P = [0\ \bar{\gamma}\ \frac{1}{2}\bar{\gamma}\ 0\ 0\ \bar{g}\ 0]'$. The matrix W is

$$W = \frac{1}{2}(e_{26} + \frac{1}{2}e_{36} + e_{62} + \frac{1}{2}e_{63}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{I.B .19}$$

In this framework, the price-dividend ratio of the market portfolio and the specific asset P is therefore a function of the state vectors. The following two proposition provide the results.

Proposition 4 *Let $X^M_t = [\hat{r^f}_{t,t+4}\ \hat{g}^M_{t,t+4}\ (\hat{\sigma}^M_{g,t})^2\ \hat{z}_{t,t+4}\ (\hat{\sigma}^M_{g,t})^2]'$ denote the state vector describing the market. Suppose the market state vector follow the VAR process as specified as equation (I.B .2) and the market log expected return follows equation (I.B .13),and the market expected dividend growth is defined as in equation (I.B .16) and follows equation (I.B .17). The price of the market portfolio is then given by*

$$\frac{P^M_t}{D^M_{t-4,t}} = \sum_{s=1}^{\infty} \mathrm{E}_t[\prod_{k=0}^{s-1} \exp(-\mu^M_{t+4k,t+4k+4} - \pi_{t+4k,t+4k+4}) \frac{D^M_{t+4s-4,t+4s}}{D^M_{t-4,4}}] \tag{I.B .20}$$

$$= \sum_{n=1}^{\infty} \exp(a^M_n + b^M_n X^M_t) \tag{I.B .21}$$

where a^M_n, b^M_n follows the following iteration rules.

$$\begin{cases} a^M_{n+1} &= -\phi_M + \psi_M + a^M_n + \frac{1}{2} b^{M'}_n \tilde{\Sigma}^M b^M_n \\ b^M_{n+1} &= -\xi_M + \zeta_M + \tilde{\Phi}^{'}_M b^M_n \end{cases} \tag{I.B .22}$$

The initial specifications are

$$\begin{cases} a^M_1 &= -\phi_M + \psi_M \\ b^M_1 &= -\xi_M + \zeta_M \end{cases} \tag{I.B .23}$$

The parameters in the equations here are

$$\begin{cases} \tilde{\Phi}_M &= \Phi^4_M \\ \tilde{\Sigma}_M &= (I\ \Phi_M\ \Phi^2_M\ \Phi^3_M)\Sigma_M \otimes O(I\ \Phi_M\ \Phi^2_M\ \Phi^3_M)' \end{cases} \tag{I.B .24}$$

where

$$O=\begin{bmatrix} 1 & 3/4 & 1/2 & 1/4 \\ 3/4 & 1 & 3/4 & 1/2 \\ 1/2 & 3/4 & 1 & 3/4 \\ 1/4 & 1/2 & 3/4 & 1 \end{bmatrix} \qquad \text{(I.B .25)}$$

Proposition 5 *Let X_t^P be the full state vector of Portfolio P defined as in Equation (I.B .1) describing the market and an asset P, following the VAR process specified as in Equation (I.B .2). Suppose the log expected return of the market portfolio and the asset P follows equation (I.B .14), and the expected real dividend growth g_t^P is defined as in equation (I.B .16) and follow equation (I.B .18) .Then the prices of the portfolio P are given by*

$$\begin{aligned} \frac{P_t^P}{D_{t-4,t}^P} &= \sum_{s=1}^{\infty} \mathrm{E}_t[\prod_{k=0}^{s-1} \exp(-\mu_{t+4k,t+4k+4}^P - \pi_{t+4k,t+4k+4}) \frac{D_{t+4s-4,t+4s}^P}{D_{t-4,4}^P}] & \text{(I.B .26)} \\ &= \sum_{n=1}^{\infty} \exp(a_n^P + b_n^{P'} X_t^P + X_t^{P'} H_n^P X_t^P) & \text{(I.B .27)} \end{aligned}$$

where a_n^P, b_n^P and H_n^P follows the following iteration rules.

$$\begin{cases} a_{n+1}^P & = -\phi_P + \psi_P + a_n^P - \frac{1}{2}\ln\det(I - 2\tilde{\Sigma}^P H_n^P) + \frac{1}{2}(b_n^P)'(\tilde{\Sigma^P}^{-1} - 2H_n^P)^{-1} b_n^P \\ b_{n+1}^P & = -\xi_P + \zeta_P + \tilde{\Phi}'(b_n^P) + 2\tilde{\Phi}' H_n^P(\tilde{\Sigma^P}^{-1} - 2H_n^P)^{-1} b_n^P \\ H_{n+1}^P & = -\Omega + W + \tilde{\Phi}' H_n \tilde{\Phi} + 2\tilde{\Phi}' H_n^P(\tilde{\Sigma^P}^{-1} - 2H_n^P)^{-1} H_n^P \tilde{\Phi} \end{cases} \qquad \text{(I.B .28)}$$

The initial specifications are

$$\begin{cases} a_1^P & = -\phi_P + \psi_P \\ b_1^P & = -\xi_P + \zeta_P \\ H_1^P & = -\Omega + W \end{cases} \qquad \text{(I.B .29)}$$

The parameters in the equations here are

$$\begin{cases} \tilde{\Phi} & = \Phi^4 \\ \tilde{\Sigma} & = (I\ \Phi\ \Phi^2\ \Phi^3)\Sigma \otimes O(I\ \Phi\ \Phi^2\ \Phi^3)' \end{cases} \qquad \text{(I.B .30)}$$

The proof of Proposition 4 and Proposition 5 are documented in the appendix of Ang and Liu (2006) in detail.

I.C Estimating the Model with MCMC Gibbs Sampler

I estimate the model described by equations (I.B .2), (I.B .16), (I.B .21) and (I.B .27) by Gibbs sampling and MCMC. Other similar models involving latent state variables are estimated by Jacquier, Polson and Rossi (1994,2004), and Ang and Chen(2007).Particularly, the processes of the expectation and variance of real market dividend growth ((g_t^M) and $(\sigma_{g,t}^M)^2$), expectation and variance of market excess return (z_t and $\sigma_{z,t}^M)^2$), the cash-flow exposure (γ_t^P) and the market beta (β_t^P) are estimated using Forward Filtering Backward Sampling (FFBS) algorithm of Carter and Kohn (1994). A textbook exposition of Gibbs sampling is provided by Robert and Casella (1999).

I.C .1 log-linearization of the observation equation

Notice that the price-dividend ratio as signal is a non-linear function of the state vector. I first perform log-linearization on the signal equations (I.B .21) and (I.B .27) such that the log price-dividend ratio of market and specific assets can be approximated by linear function of state vectors.It expedites drawing of the latent state variables by allowing us to use the linear Kalman filter.

The measurement equation of $lnpd^M$ can be linearized around $X_t = 0$, because each exponents in equation (I.B .21) are linear in $X_t = 0$.

$$\ln \frac{P_t^M}{D_{t-4,t}^M} = A^M + B^{M\prime} X_t \qquad \text{(I.C .1)}$$

where

$$A^M = \ln \sum_{n=1}^{\infty} \exp(a_n^M)$$

$$B^M = \frac{\sum_{n=1}^{\infty} \exp(a_n^M) b_n^M}{\sum_{n=1}^{\infty} \exp a_n^M}$$

Unlike the case of $lnpd^M$, the exponents in equation (I.B .27) have quadratic forms of X_t, therefore $lnpd^P$ has quadratic form of X_t around $X_t = 0$. If using Taylor expansion around the state vector X_t as well as their quadratic terms, one can get the *lnpd* as a quadratic form of X_t.

$$\frac{P_t^P}{D_{t-4,4}^P} = A^P + B^{P\prime} X_t + X_t' Q^P X_t \tag{I.C .2}$$

where

$$A^P = \ln \sum_{n=1}^{\infty} \exp(a_n^P)$$

$$B^P = \frac{\sum_{n=1}^{\infty} \exp(a_n^M) b_n^P}{\sum_{n=1}^{\infty} \exp a_n^M}$$

$$Q^P = \frac{\sum_{n=1}^{\infty} \exp(a_n^M) H_n^P}{\sum_{n=1}^{\infty} \exp a_n^M}$$

As $lnpd^P$ works separately as observation when drawing both $\{\hat{\gamma}_t\}$ and $\{\hat{\beta}_t\}$, it suffices to get *lnpd* as a linear form of each single state variable separately. This purpose can be achieved by reorganizing Equation (I.C .2). For each single state variable, one can obtain *lnpd* being linear in it with time-varying coefficients.

Taking the case of $\hat{\beta}$ for example, I rewrite Equation (I.C .2) as the form of

$$\ln \frac{P_t^P}{D_{t-4,t}^P} = A_t^{P,\beta} + (B_t^{P,\beta})' \hat{\beta}_t \tag{I.C .3}$$

The time-varying coefficients are

$$A_t^{P,\beta} = A^P + B_{-\beta}^P{}' X_{t,-\beta} + X_{t,-\beta}' Q_{-\beta,-\beta}^P X_{t,-\beta}$$

$$B_t^{P,\beta} = B_\beta^P + X_{t,-\beta}' Q_{-\beta,\beta}^P$$

The subscript $-\beta$ means the sub-vector or sub-matrix excluding entries related to β, while the subscript β indicates the entries related to β.

By the same token, the log-linearization of $lnpd^P$ around $\hat{\gamma} = 0$ is

$$\ln \frac{P_t^P}{D_{t-4,t}^P} = A_t^{P,\gamma} + (B_t^{P,\gamma})' \hat{\gamma}_t \tag{I.C .4}$$

In general, I combine the two equations in one general notation

$$\ln \frac{P_t^P}{D_{t-4,t}^P} = A_t^P + B_t^{P\prime} X_t \tag{I.C .5}$$

The linearization has been proposed in particular by King, Plosser and Rebelo (1987) and Campbell (1994). It would increase the computational efficiency of the estimation. As log-linearization is a first-order Taylor series approximation, with some errors induced, I model the approximation with normal errors as following

$$\ln \frac{P_t^M}{D_{t-4,t}^M} = A^M + B^{M\prime} X_t + \sigma_v^M v_t^M \tag{I.C .6}$$

$$\ln \frac{P_t^P}{D_{t-4,t}^P} = A_t^P + B_t^{P\prime} X_t + \sigma_v^P v_t^P \tag{I.C .7}$$

where $v_t^M \sim N(0,1)$ and $v_t^P \sim N(0,1)$.

Simulation results validate this log-linearization algorithm. I simulate the market model choosing the parameters as

	r^f	g^M	$(\sigma_g^M)^2$	z	$(\sigma_z^M)^2$
Average	0.01	0.01	0.005	0.05	0.03
VAR coeff. Φ	0.9	0.95	0.7	0.98	0.70
Shock Variance Σ	0.08	0.04	0.007	0.01	0.45

The correlation between shocks are assumed as zeroes. With this specification, the summary statistics of the observables are comparable with the realistic numbers given by the CRSP value-weighted portfolio.

	Cashflow growth:g_t			Price Dividend ratio			Return		
	Mean	Stdev	auto	Mean	Stdev	auto	Mean	Stdev	auto
CRSP	0.049	0.207	0.037	31.91	15.84	0.92	0.11	0.22	0.045
simulated	0.051	0.212	-0.0031	30.53	12.14	0.91	0.06	0.20	-0.02

Using this parameters specifications, I simulate the present-value model. I calculate the *lnpd* using the exact formulae as sum of exponential functions of X_t like as in equation (I.B .21), called it the simulated *lnpd*. I also calculate *lnpd* using the log-linearized formulae as in equation (I.C .6), and call it linearized *lnpd*.

In Figure A.1, I simulate the model for a time period of $T = 1000$, and show the scatter plot of the simulated *lnpd* and linearized *lnpd*. In the plot, one can see that the linearization approximation works very well in the range where most data points fall into.

I.C .2 Identifying Strategy

At this point, I integrate both conditional cash flow exposure and discount rate risk into a state-space model via a present-value approach.

$$\begin{cases} X_{t+1} = \Phi X_t + \Sigma^{\frac{1}{2}}\epsilon_{t+1} \\ \Delta d^M_{t,t+4} = \bar{g}^M + e_2' X^M_t + \sigma^M_{g,t} u^M_{d,t,t+4} \\ r^M_{t,t+4} - r^f_t - \pi_{t,t+4} = z_t + \sigma^M_{z,t} u^M_{z,t,t+4} \\ \ln \frac{P^M_t}{D^M_{t-4,t}} = A^M + B^{M\prime} X^M_t + \sigma^M_v v^M_t \\ \Delta d^P_{t,t+4} = \tilde{\psi}_P + \zeta_P' X_t + X_t' W X_t + \sigma^P_d u^P_{d,t,t+4} \\ r^P_{t,t+4} - r^f_{t,t+4} = \tilde{\phi}_P + \xi_P' X_t + X_t' \Omega X_t + \sigma^P_z u^P_{z,t,t+4} \\ \ln \frac{P^P_t}{D^P_{t-4,t}} = A^P_t + B^{P\prime}_t X_t + \sigma^P_v v^P_t \end{cases} \tag{I.C .8}$$

In the model, I need identify the latent state variables g^M_t,$(\sigma^M_{g,t})^2$,z_t,$(\sigma^M_A z, t^M)^2$, γ_t and β_t in the state vectors and the following listed parameters in the model by simulating their distributions using MCMC.

$$\Theta = (\bar{r}^f, \bar{g}^M, (\bar{\sigma}^M_g)^2, \bar{z}, (\bar{\sigma}^M_z)^2, \bar{\gamma}^P, \bar{\beta}^P, \Phi, \Sigma, (\sigma^M_v)^2, (\sigma^P_v)^2, (\sigma^P_z)^2, (\sigma^P_d)^2) \tag{I.C .9}$$

I first group the parameters by equations relating to the parameters, and estimate the parameters group by group. The parameters groups are

$$\begin{aligned} \Theta_1 &= \{\bar{r}^f, \Phi_{rr}, \Sigma_{rr}\} \\ \Theta_2 &= \{\bar{g}^M, \Phi_{gg}, \Sigma_{gg}\} \\ \Theta_3 &= \{(\bar{\sigma}^M_g)^2, \Phi_{\sigma_g\sigma_g}, \Sigma_{\sigma_g\sigma_g}\} \\ \Theta_4 &= \{\bar{z}, \Phi_{zz}, \Sigma_{zz}\} \\ \Theta_5 &= \{(\bar{\sigma}^M_z)^2, \Phi_{\sigma_z\sigma_z}, \Sigma_{\sigma_z\sigma_z}\} \\ \Theta_6 &= \{\Sigma_{rg}, \Sigma_{r\sigma_g}, \Sigma_{rz}, \Sigma_{r\sigma_z}, \Sigma_{g\sigma_g}, \Sigma_{gz}, \Sigma_{g,\sigma_z}, \Sigma_{\sigma_g,z}, \Sigma_{\sigma_g,\sigma_z}, \Sigma_{z,\sigma_z}\} \\ \Theta_7 &= \{(\sigma^M_v)^2\} \\ \Theta_8 &= \{\bar{\gamma}, c, \bar{\beta}, \alpha\} \\ \Theta_9 &= \{\Phi_{\gamma\gamma}, \Sigma_{\gamma\gamma}\} \\ \Theta_{10} &= \{\Phi_{\beta\beta}, \Sigma_{\beta\beta}\} \\ \Theta_{11} &= \{\Sigma_{r\gamma}, \Sigma_{g\gamma}, \Sigma_{\sigma_g\gamma}, \Sigma_{z\gamma}, \Sigma_{\sigma_z\gamma}, \Sigma_{r\beta}, \Sigma_{g\beta}, \Sigma_{\sigma_g\beta}, \Sigma_{z\beta}, \Sigma_{\sigma_z\beta}\} \\ \Theta_{12} &= \{(\sigma^P_v)^2, (\sigma^P_z)^2, (\sigma^P_d)^2\} \end{aligned}$$

As shown before, $\Theta_1 - \Theta_7$, g^M_t, $(\sigma^M_{g,t})^2$, z_t and $(\sigma^M_{z,t})^2$ describe the market portfolio and can be inferred from the information about the market returns and dividends, denoted as $Y^M = \{y^M_t\} = \{\ln(P_t/D_{t-4,t})^M, \Delta d^M_{t-4,t}, r_{t-4,t}\}$. While the rest parameters $\Theta_8 - \Theta_{12}$, also γ_t and β_t are relevant to the specific assets and therefore in need of both market dividend and price information Y^M and also portfolio specific price and dividend information, denoted as $Y^P = \{y^P_t\} = \{\ln(P_t/D_{t-4,t})^P, \Delta d^P_{t-4,t}, r_{t-4,t}\}$. With this information hierarchy, we would like to draw $\Theta_1 - \Theta_6$, g^M_t and z_t given Y^M first. Provided with the estimated market state variables, we then estimate the cash-flow risk γ_t and the discounted rate risk β_t with Y^M and Y^P for 21 tertile test portfolios separately. This hierarchy structure of estimation bases on the following proposition.

Proposition 6 *The posterior distribution of market risk premium z_t conditional on the market and specific portfolio* *information is the same as the posterior distribution of it conditional on the market information only.*

$$\mathbf{P}(z_t|Y_{t+1}^M, Y_{t+1}^i) = \mathbf{P}(z_t|Y_{t+1}^M)$$

Proof.

$$\begin{aligned} \mathbf{P}(z_t|Y_{t+1}^M, Y_{t+1}^i) &= \frac{\mathbf{P}(z_t, Y_{t+1}^M, Y_{t+1}^i)}{\mathbf{P}(Y_{t+1}^M, Y_{t+1}^i)} \\ &= \frac{\mathbf{P}(z_t)\mathbf{P}(Y_{t+1}^M, Y_{t+1}^i|z_t)}{\mathbf{P}(Y_{t+1}^i|Y_{t+1}^M)\mathbf{P}(Y_{t+1}^M)} \\ &= \frac{\mathbf{P}(z_t)\mathbf{P}(Y_{t+1}^M|z_t)\mathbf{P}(Y_{t+1}^i|Y_{t+1}^M, z_t)}{\mathbf{P}(Y_{t+1}^i|Y_{t+1}^M)\mathbf{P}(Y_{t+1}^M)} \\ &= \frac{\mathbf{P}(z_t)\mathbf{P}(Y_{t+1}^M|z_t)}{\mathbf{P}(Y_{t+1}^M)} \\ &= \mathbf{P}(z_t|Y_{t+1}^M) \end{aligned} \tag{I.C .10}$$

■

In our estimations, we use a burn-in period of 15,000 draws and draw 5,000 observations to represent the posterior distribution. With this number of sampling, our estimation converges by passing the convergence test in Geweke (1992).

The Gibbs Sampler involves the iterations of the following sampling of posterior distribution conditional on other parameters and state vectors.

The loop for market risk premium and market parameters.

$$(P1)\ \{\hat{g}_t^M\}|\Theta, \{Y_t^M\}, \{X_{-g,t}^M\}$$

$$(P2)\ \{(\hat{\sigma}_g^M)^2|\Theta, \{Y_t^M\}, \{X_{-(\sigma_g^M)^2}\}$$

$$(P3)\ \{\hat{z}_t\}|\Theta, \{Y_t^M\}, \{X_{-z,t}^M\}$$

$$(P4)\ \{(\hat{\sigma}_z^M)^2|\Theta, \{Y_t^M\}, \{X_{-(\sigma_z^M)^2}\}$$

$$(P5)\ \Theta_1|\{\Theta_i\}_{\{1\leq i\leq 7, i\neq 1\}}, \{Y_t^M\}, \{X_t^M\}$$

$$(P6)\ \Theta_2|\{\Theta_i\}_{\{1\leq i\leq 7, i\neq 2\}}, \{Y_t^M\}, \{X_t^M\}$$

$$(P7)\ \Theta_3|\{\Theta_i\}_{\{1\leq i\leq 7, i\neq 3\}}, \{Y_t^M\}, \{X_t^M\}$$

$$(P8)\ \Theta_4|\{\Theta_i\}_{\{1\leq i\leq 7, i\neq 4\}}, \{Y_t^M\}, \{X_t^M\}$$

$$(P9)\ \Theta_5|\{\Theta_i\}_{\{1\leq i\leq 7, i\neq 5\}}, \{Y_t^M\}, \{X_t^M\}$$

$$(P10)\ \Theta_6|\{\Theta_i\}_{\{1\leq i\leq 7, i\neq 6\}}, \{Y_t^M\}, \{X_t^M\}$$

$$(P11)\ \Theta_7|\{\Theta_i\}_{\{1\leq i\leq 7, i\neq 7\}}, \{Y_t^M\}, \{X_t^M\}$$

where the notation $X_{-x,t}^M$ (for example $X_{-g,t}^M$) denotes the entries in the vector X_t^M excluding x_t (g_t^M). The loop for a specific portfolio's market beta and relevant parameters.

$$(P12)\ \{\hat{\gamma}_t\}|\{\Theta_i\}, \{y_t^P\}, \{X_{-\hat{\gamma},t}\}$$

$$(P13)\ \{\hat{\beta}_t\}|\{\Theta_i\}, \{y_t^P\}, \{X_{-\hat{\beta},t}\}$$

$$(P14)\ \Theta_8|\{\Theta_i\}_{i\neq 8}, \{y_t^P\}, \{X_t\}$$

$$(P15)\ \Theta_9|\{\Theta_i\}_{i\neq 9}, \{y_t^P\}, \{X_t\}$$

$$(P16)\ \Theta_{10}|\{\Theta_i\}_{i\neq 10}, \{y_t^P\}, \{X_t\}$$

$$(P17)\ \Theta_{11}|\{\Theta_i\}_{i\neq 11}, \{y_t^P\}, \{X_t\}$$

$$(P18)\ \Theta_{12}|\{\Theta_i\}_{i\neq 12}, \{y_t^P\}, \{X_t\}$$

Drawing the Market Expected Dividend Growth(P1)

I draw the expected dividend growth g_t^M using the forward filtering backward sampling (FFBS) algorithm (Carter and Kohn 1994). We first run a Kalman filter (Hamilton 1994) forward with the state equations and the measurement equations (I.C .8). There are two measurement equations for the dividend growth.

$$\begin{cases} X_{t+1}^M = \Phi_M X_t^M + \Sigma_M^{\frac{1}{2}} u_{t+1}^M \\ \ln \frac{P_t^M}{D_{t-4,t}^M} = A^M + B^{M\prime} X_t^M + \sigma_v^M v_t^M \\ \Delta d_{t,t+4} = \bar{g^M} + \hat{g}_{t,t+4}^M + \sigma_{g,t}^M u_{g,t,t+4}^M \end{cases} \tag{I.C .11}$$

After we run the Kalman filter, we sample backward following Carter and Kohn (1994).

Drawing $(\sigma_{g,t}^M)^2$ (P2)

I draw the conditional variance of dividend growth $(\sigma_{g,t}^M)^2$ following FFBS algorithm as the previous draw of g_t^M. There are two measurement equations of the log price-dividend ratio and expected dividend growth.

$$\begin{cases} X_{t+1}^M = \Phi_M X_t^M + \Sigma_M^{\frac{1}{2}} \epsilon_{t+1}^M \\ \ln \frac{P_t^M}{D_{t-4,t}^M} = A^M + B^{M\prime} X_t^M + \sigma_v^M v_t^M \\ (\Delta d_{t,t+4} - \bar{g^M} - \hat{g}_{t,t+4}^M)^2 = (\sigma_{g,t}^M)^2 + u_{\sigma_g,t,t+4} \end{cases} \tag{I.C .12}$$

Drawing the Market Risk Premium Process (P3)

I draw the risk premium z_t following the same FFBS algorithm as the previous draw of g_t^M. There are two measurement equations of the log price-dividend ratio and log excess return for z_t.

$$\begin{cases} X_{t+1}^M = \Phi_M X_t^M + \Sigma_M^{\frac{1}{2}} \epsilon_{t+1}^M \\ \ln \frac{P_t^M}{D_{t-4,t}^M} = A^M + B^{M\prime} X_t^M + \sigma_v^M v_t^M \\ r_{t,t+4}^M - r_t^f - \pi_{t,t+5} = \bar{z} + \hat{z}_t + \sigma_{z,t}^M u_{z,t,t+4}^M \end{cases} \tag{I.C .13}$$

Drawing $(\sigma_{g,t}^M)^2$ (P4)

The conditional variance of excess returns $(\sigma_{z,t}^M)^2$ is drawn using FFBS algorithm by the same token. There are two measurement equations of the log price-dividend ratio and expected excess return.

$$\begin{cases} X_{t+1}^M = \Phi_M X_t^M + \Sigma_M^{\frac{1}{2}} \epsilon_{t+1}^M \\ \ln \frac{P_t^M}{D_{t-4,t}^M} = A^M + B^{M\prime} X_t^M + \sigma_v^M v_t^M \\ (r_{t,t+4} - r_t^f - \pi_{t,t+4} - \bar{z} - \hat{z}_{t,t+4})^2 = (\sigma_{z,t}^M)^2 + u_{\sigma_z,t,t+4} \end{cases} \tag{I.C .14}$$

Drawing Θ_1(P5)

We estimate $\bar{r}^f$ by the sample mean of the real risk-free rate for simplification. Since the risk-free rate r_t^f is assumed as exogenous, we draw Φ_{rr} and Σ_{rr} simply by Bayesian Regression of r_t^f on its lagged value, (see Carlin and Louis 2000, Lancaster 2004 and Geweke 2005). The standard Bayesian normal regression model is $y = Xb + e$, where y is column vector of n observations,X is fixed regressors matrix, b is the coefficients, and the residual e subjected to zero-mean multivariate normal distribution $e \sim N(0, V_e)$. Given V_e and a normal prior of b, $b \sim N(\mu_b, V_b)$. The posterior is $b \sim N(\mu_b^*, V_b^*)$, where $\mu_b^* = (X'V_e^{-1}X + V_b^{-1})^{-1}(X'V_e^{-1}y + V_b^{-1}\mu_b)$, where $V_b^* = (X'V_e^{-1}X + V_b^{-1})^{-1}$. Assume that $V_e = \sigma^2 I$, and if the parameter prior is inverse-gamma distribution denoting as $\sigma^2 \sim IG(A/2, B/2)$, the posterior distribution is conjugate inverse-gamma distribution $\sigma^2 \sim IG(A^*/2, B^*/2)$, where $A^* = (A + n)$ and $B^* = B + e'e$, (see Kim and Nelson 2000). The priors of $\Phi_{\pi\pi}$ and Σ_{rr} are uninformative.

$$\Phi_{\pi\pi} \sim N(0, 100^2)$$
$$\Sigma_{\pi\pi} \sim IG(1, 0.001)$$

Drawing Θ_2(P6)

For simplicity, we fixed $\bar{g}^M$ by the sample mean of the market growth of dividend in real term for each draw.
I draw the VAR coefficient and the variance using the independent Metropolis-Hasting reject-accept sampling (see

Robert and Casella 1999). The posterior distribution is

$$P(\Phi_{gg},\Sigma_{gg}|\Theta_{-2},X_t^M,\ln(\frac{P_t}{D_t})^M)\propto P(\ln(\frac{P_t}{D_t})^M|\Theta)P(g_t^M|\Phi_{gg},\Sigma_{gg})P(\Sigma_{gg}|\Phi_{gg})P(\Phi_{gg})$$

$P(\Phi_{gg})$ is the prior distribution of the autocorrelation of g_t^M in the VAR. Since $P(\ln(\frac{P_t}{D_t})^M|\Theta)$ is not trivial, as the linear coefficients A^M, B^M are functions of Θ_2, I cannot use Bayesian Regression to draw Θ only using the state equation as done in the last two steps. However, I can propose the draw using a Bayesian regression from the state equation, and determine reject or accept with likelihood of measurement equation.

I first draw $\Phi_{gg}^{'}$ using Bayesian regression described aforementioned, the prior of $\Phi_{gg}^{'}$ is flat and uninformative. Provided with Φ_{gg}, we then draw Σ_{gg}, with a conjugate prior $IG(\frac{b}{2},\frac{B}{2})$ where $b=1, B=0.001$. This prior is uninformative and the result is not sensitive to the choice of b and B. Conditional on the VAR coefficient sampled and the state variables, we have the posterior distribution of Σ_{gg} being $IG(\frac{b_1}{2},\frac{B_1}{2})$ where

$$\begin{cases} b_1 &= b+T-1\\ B_1 &= B+(Y-\Phi X)'(Y-\Phi X)\end{cases}$$

The proposed $\Sigma_{gg}^{'}$ has a lower bound of 10^{-8}, which is small amount. Its upper bound is the sample variance of Δd_t^M. Using the truncated prior, the posterior has the same parameter in the unrestricted case, but truncated to the same interval of as the prior (see Gewewke 2005; Hasbrouck 2009).

The proposed $(\Phi_{gg}', \Sigma_{gg}^{'})$ is going to replace the original (Φ_{gg},Σ_{gg}) with probability α

$$\alpha=\min\{1,\frac{P(Y_t^M|\Phi_{gg}^{'},\Sigma_{gg}^{'},\Theta_{-2},X_t)}{P(Y_t^M|\Phi_{gg},\Sigma_{gg},\Theta_{-2},X_t)}\}$$

Drawing Θ_3(P7)

The parameters about the state equation of $(\sigma_{g,t}^M)$ is drawn using Metropolis-Hasting algorithm. The posterior distribution is

$$\begin{aligned} P((\bar{\sigma}_g^M)^2,\Phi_{\sigma_g\sigma_g},\Sigma_{\sigma_g\sigma_g}|\Theta_{-3},X_t,Y_t) \quad &\propto \quad P(Y_t^M|\Phi_{\sigma_g\sigma_g},\Sigma_{\sigma_g\sigma_g},(\bar{\sigma}_g^M)^2,\Theta_{-3},X_t)P((\bar{\sigma}_g^M)^2|\Phi_{\sigma_g\sigma_g},\Sigma_{\sigma_g\sigma_g})\\ &\quad\quad P(\Sigma_{\sigma_g\sigma_g}|\Phi_{\sigma_g\sigma_g})P(\Phi_{\sigma_g\sigma_g})P(\bar{\sigma}_g^M)\end{aligned}$$

Firstly, I propose the draw the mean of $(\sigma_g^M)^2$ using Random-walk draw around the previous draw of it. Secondly, I propose to draw the $\Phi_{\sigma_g\sigma_g}$ using Bayesian regression, with the prior again conjugate uninformative. After drawing the Φ, the $\Sigma_{\sigma_g\sigma_g}$ is then attempted drawn using Inverse-Gamma distribution provided in previous subsection, the prior is conjugate $IG(\frac{b}{2},\frac{B}{2})$ with $b=1, B=0.001$. The proposal is going to replace the original draw with probability α

$$\alpha=\min\{1,\frac{P(Y_t^M|(\bar{\sigma}_g^M)'^2,\Phi_{\sigma_g\sigma_g}^{'},\Sigma_{\sigma_g\sigma_g}^{'},\Theta_{-3},X_t)}{P(Y_t^M|(\bar{\sigma}_g^M)^2\Phi_{\sigma_g\sigma_g},\Sigma_{\sigma_g\sigma_g},\Theta_{-3},X_t)}\}$$

Drawing Θ_4(P8)

I draw the parameters about the state equation of z_t using the independent Metropolis-Hasting reject-accept sampling,too. The posterior distribution is

$$P(\bar{z},\Phi_{zz},\Sigma_{zz}|\Theta_{-4},X_t,Y_t)\propto P(Y_t^M|\Phi_{zz},\Sigma_{zz},\bar{z},\Theta_{-4},X_t)P(z_t|\Phi_{zz},\Sigma_{zz})P(\Sigma_{zz}|\Phi_{zz})P(\Phi_{zz})P(\bar{z})$$

I first draw $\bar{z}$ as the sample mean of the log excess return. The proposal drawing of Φ_{zz} and Σ_{zz} is from the Bayesian regression of the state equation of z_t by the same token as in the last step. Presumably, a prior distribution for Σ_{zz} as $IG(\frac{b}{2},\frac{B}{2})$ where $b=1, B=0.001$, and the prior of Φ_{zz} as $N(0,100^2)$. The proposed $(\bar{z}',\Phi_{z\cdot}',\Sigma_{zz}^{'})$ is going to replace the original $(\bar{z},\Phi_{z\cdot},\Sigma_{zz})$ with probability α

$$\alpha=\min\{1,\frac{P(Y_t^M|\Phi_{zz}^{'},\Sigma_{zz}^{'},\bar{z}',\Theta_{-4},X_t)}{P(Y_t^M|\Phi_{zz},\Sigma_{zz},\bar{z},\Theta_{-4},X_t)}\}$$

Drawing Θ_5(P9)

The parameters about the state equation of $(\sigma_{z,t}^M)$ is drawn using Metropolis-Hasting algorithm. The posterior distribution is

$$\begin{aligned}P((\bar{\sigma}_z^M)^2, \Phi_{\sigma_z\sigma_z}, \Sigma_{\sigma_z\sigma_z}|\Theta_{-5}, X_t, Y_t) \quad &\propto \quad P(Y_t^M|\Phi_{\sigma_z\sigma_z}, \Sigma_{\sigma_z\sigma_z}, (\bar{\sigma}_z^M)^2, \Theta_{-5}, X_t)P((\bar{\sigma}_z^M)^2|\Phi_{\sigma_z\sigma_z}, \Sigma_{\sigma_z\sigma_z}) \\ &\quad\quad P(\Sigma_{\sigma_z\sigma_z}|\Phi_{\sigma_z\sigma_z})P(\Phi_{\sigma_z\sigma_z})P(\bar{\sigma}_z^M)\end{aligned}$$

Firstly, I propose the draw the mean of $(\sigma_z^M)^2$ using Random-walk draw around the previous draw of it. Secondly, I propose to draw the $\Phi_{\sigma_z\sigma_z}$ using Bayesian regression, with conjugate uninformative prior. The $\Sigma_{\sigma_z\sigma_z}$ is then proposed to draw from Inverse-Gamma distribution posterior distribution with uninformative prior $IG(\frac{b}{2}, \frac{B}{2})$ with $b = 1, B = 0.001$. The proposal is going to replace the original draw with probability α

$$\alpha = \min\{1, \frac{P(Y_t^M|(\bar{\sigma}_z^M)'^2, \Phi'_{\sigma_z\sigma_z}, \Sigma'_{\sigma_z\sigma_z}, \Theta_{-5}, X_t)}{P(Y_t^M|(\bar{\sigma}_z^M)^2\Phi_{\sigma_z\sigma_z}, \Sigma_{\sigma_z\sigma_z}, \Theta_{-5}, X_t)}\}$$

Drawing Θ_6P(10)

I sample the off-diagonal entries in the variance covariance matrix of the VAR using the random-walk Metropolis-Hasting algorithm (Metropolis 1953).

I first propose a draw of correlation between innovations of state variables, say ρ_{gz} as an illustration, using a random-walk sampling around the previous draw. For each correlations, the prior is uniform on [-1,1]. The proposed covariance Σ'_{gz} is therefore $\rho'_{gz}\Sigma_{gg}^{1/2}\Sigma_{zz}^{1/2}$.After we have the proposed Σ', we replace the original Σ with probability α

$$\alpha = \min\{1, \frac{P(Y_t^M|\Theta_{-6}, \Sigma', X_t)}{P(Y_t^M|\Theta_{-6}, \Sigma, X_t)}\}$$

Drawing Θ_7(P11)

I would draw $(\sigma_v^M)^2$ from an Inverse Gamma Distribution, $IG(\frac{b_1}{2}, \frac{B_1}{2})$. The parameters in the distribution are determined by

$$u = (lnpd_t^M - A_M - B_M'(X_t - \bar{X}))$$
$$B_1 = B + u_1'u_1 \quad b_1 = b + T - 1$$

where b and B are uninformative prior parameters, which I choose as 1 and 0.001.

By iterating the market loop, I can get posterior distribution as estimation of the market state variables and the VAR parameters in the market state equations. For disaggregate level information, I can next implement the Gibbs Sampling on each test portfolio by iterating the portfolio loop combined with the market loop from above.

I.C .2.1 Drawing $\{\hat{\gamma}_t\}$(P12)

By the same token as in the case of drawing latent factor g_t^M, I draw the latent factor $\{\hat{\gamma}_t\}$ using FFBS. I have one state equation in VAR for γ_t and two measurement equations from real dividend growth and log price-dividend ratio of portfolio, listed as following

$$\begin{cases} X_{t+1}^P & = \Phi^P X_t^P + \Sigma_P^{\frac{1}{2}}\epsilon_{t+1} \\ \ln(\frac{P_t}{D_t})^P & = A^P + B^{P'}X_t^P + \sigma_v^P v_t^P \\ \Delta d_{t,t+4}^P & = \tilde{\psi}_P + \zeta_P' X_t + X_t'WX_t + \sigma_d^P u_{d,t,t+4}^P \end{cases}$$

As I relax rational expectation in growth for individual stocks, I use only the demeaned version of the last observation equation to extract series of demeaned series of γ. The level of γ and other level parameters are estimated using information of $lnpd^P$. Since W does not have diagonal entries, the last measurement equation is linear in $\hat{\gamma}$.

Drawing $\{\hat{\beta}_t\}$ (P13)

Drawing this latent state variable $\{\hat{\beta}_t\}$, we apply the same FFBS algorithm as before. The observed signals only come from measurement of log price-dividend ratio of test portfolios.

$$\begin{cases} X^P_{t+1} & = \Phi^P X^P_t + \Sigma_P^{\frac{1}{2}} \epsilon_{t+1} \\ \ln \frac{P^P_t}{D^P_{t-4,t}} & = A_{P,t} + B'_{P,t} X_t + \sigma^P_v v^P_t \\ r^P_{t,t+4} - r^f_t & = \tilde{\phi_P} + \zeta'_P X_t + X'_t \Omega X_t + \sigma^P_z u^P_{z,t,t+4} \end{cases}$$

As I relax rational expectation in return for individual stocks, I use only the demeaned version of the last observation equation to extract series of demeaned series of β.

Drawing Θ_8(P14)

Like as when drawing $\bar{z}$, we draw the level parameters using random walk Metropolis-Hasting algorithm. First, we propose a draw of Θ'_8 with a random walk of the last draw Θ_8.

The proposed drawing of Θ'_8 will replace Θ_8 with probability α

$$\alpha = \min\{1, \frac{P(Y^P_t|\Theta'_8, \Theta_{-8})}{P(Y^P_t|\Theta_8, \Theta_{-8})}\}$$

Drawing Θ_9(P15)

Like as when drawing Θ_2, I use independent Metropolis-Hasting algorithm. The posterior distribution is

$$P(\Phi_{\gamma\gamma}, \Sigma_{\gamma\gamma}|\Theta_{-9}, X_t, Y^P, Y^M) \propto P(Y^P|\Theta)P(\hat{\gamma}_t|\Phi_{\gamma\gamma}, \Sigma_{\gamma\gamma})P(\Sigma_{\gamma\gamma}|\Phi_{\gamma\gamma})P(\Phi_{\gamma\gamma}) \quad \text{(I.C .15)}$$

The proposed distribution is $P(\hat{\gamma}_t|\Phi_{\gamma\gamma}, \Sigma_{\gamma\gamma})P(\Sigma_{\zeta\zeta}|\Phi_{\gamma\gamma})P(\Phi_{\gamma\gamma})$ which is Bayesian OLS regression posterior with robust overlapping errors. The prior of VAR coefficient is flat and we restrict the eigenvalue of Φ less than 0.999.The prior of $\Sigma_{\gamma\gamma}$ is $IG(b/2, B/2)$, and $b = 1, B = 0.001$. The proposed Θ'_9 is going to replace the original Θ_9 with probability α

$$\alpha = \min\{1, \frac{P(Y^P_t|\Theta'_9, \Theta_{-9})}{P(Y^P_t|\Theta_9, \Theta_{-9})}\}$$

Drawing Θ_10(P16)

I use independent Metropolis-Hasting algorithm by the same token as in last subsection. The posterior distribution is

$$P(\Phi_{\beta\beta}, \Sigma_{\beta\beta}|\Theta_{-10}, X_t, Y^P, Y^M) \propto P(Y^P|\Theta)P(\hat{\beta}_t|\Phi_{\beta\beta}, \Sigma_{\beta\beta})P(\Sigma_{\beta\beta}|\Phi_{\beta\beta})P(\Phi_{\beta\beta}) \quad \text{(I.C .16)}$$

I first propose a draw of $\Theta_1 0'$ with the Bayesian OLS posterior with robust overlapping errors, $P(\hat{\beta}_t|\Phi_{\beta\beta}, \Sigma_{\beta\beta})P(\Sigma_{\beta\beta}|\Phi_{\beta\beta})P(\Phi_{\beta\beta})$ The prior of VAR coefficient is flat and we restrict the eigenvalue of Φ less than 0.999.The prior of $\Sigma_{\beta\beta}$ is the 0.001uninformative $IG(b/2, B/2)$, and $b = 1, B = 0.001$. The proposed Θ'_{10} is going to replace the original Θ_{10} with probability α

$$\alpha = \min\{1, \frac{P(lnpd^P|\Theta'_{10}, \Theta_{-10})}{P(lnpd^P|\Theta_{10}, \Theta_{-10})}\}$$

Drawing Θ_{11}(P17)

The methodology we use is random walk Metropolis-Hasting algorithm,the same as drawing the covariance entries of the market level innovations.We first draw a new correlation between innovations subject to a random walk around the original correlation. The proposed covariance is therefore the proposed correlation times the product of the standard errors of the innovations. After we have the proposed Σ', we replace the original Σ with probability α

$$\alpha = \min\{1, \frac{P(Y^P_t|\Theta_{-11}, \Sigma', X_t)}{P(Y^P_t|\Theta_{-11}, \Sigma, X_t)}\}$$

Drawing Θ_{12}(P18)

I would draw $(\sigma_v^P)^2$ from an Inverse Gamma Distribution, $IG(\frac{b_3}{2}, \frac{B_3}{2})$. where

$$u_3 = (lnpd_t^P - A_{P,t} - B'_{P,t}(X_t - \bar{X}))$$
$$B_3 = B + u_3' u_3, \quad b_3 = b + T - 1$$

where b and B are uninformative prior parameters, which I choose as 1 and 0.001. I confine the $(\sigma_v^P)^2$ less than the $var(lnpd^P)$, to make sure the MCMC algorithm converge.

Similarly, drawing of $(\sigma_d^P)^2$ and $(\sigma_z^P)^2$ are like as drawing of $(\sigma_v^P)^2$ from the inverse gamma distribution $IG(\frac{b_4}{2}, \frac{B_4}{2})$ and $IG(\frac{b_5}{2}, \frac{B_5}{2})$. where

$$u_4 = (\Delta d_{t,t+4} - \psi_P + \bar{\pi} - (\zeta_P - e_1)' X_t - X_t' W X_t)$$
$$B_4 = B + u_4' u_4, \quad b_4 = b + T - 1$$

and

$$u_5 = (r_{t,t+4}^P - r_t^f - \pi_{t,t+4} - \phi_P - \xi_P' X_t - X_t' \Omega X_t)$$
$$B_5 = B + u_5' u_5, \quad b_5 = b + T - 1$$

I.D Complete Estimation of the Model

In this section, I will show the additional estimation results of the state factors and parameters in the models featuring different risks M_0, M_{CF} and M_{DR}.

Again, I report the results in the order from the market to the individual level. At market level, I have shown all parameters and filtered series of g_t^M and z_t. In Figure A.2, I plot the filtered series of stochastic volatility in CF and DR as $(\sigma_{g,t}^M)^2$ and $(\sigma_{z,t}^M)^2$. Under both M_0 and M_{CF}, I can identify large volatility in both CF and DR during the most recent crisis. In Model M_0, the volatility in CF is averagely low before 2000, and the stock market crashes due to the energy crisis in 1973 and the recession in the early 1980's correspond to spikes in volatility of DR. In contrast, under M_{CF}, because the volatility in DR and CF are restricted as collinear, the recessions induce peaks in both DR volatility and CF volatility.

At the individual portfolio level, I first compare the filtered value of γ estimated in all three models, M0, MCF and MDR. The left three panels of Figure A.3 cover the value stocks (BM5) and growth stocks (BM1), and the right three panels cover the extreme portfolios in momentum group MOM1 and MOM5. First, the CF exposures under all three models display similar time-varying patterns. All estimated CF exposures are persistent. The γ of BM5 is of higher level and higher volatility than that of BM1, while the of MOM1 and MOM5 are at the same level with similar volatility. Second, the CF exposure can covary with the aggregate CF factors. Under model M_0 and M_{DR}, the γs of BM1 and MOM1 move together with aggregate CF, while these CF factors of BM5 and MOM5 covary negatively. As an illustration, the γs of BM5 and MOM5 climb up when the aggregate expected CF decreases during the early 1970s, 1980s and the most recent recessions. However, I dont find this covariance between CF exposure and CF factor related to the level of returns in other test portfolios.

After examination of the CF exposure , I turn to focus on the DR exposure β. Figure A.4 illustrates the time-serial pattern of estimated under all three specifications M_0, M_{DR} and M_{CF}. In the left three panels, I compare the β of value stocks (BM5) with that of growth stocks (BM1) under all three models. The right three panels shows the β of past winner (MOM5) and past loser (MOM1) portfolios when sorting on momentum.

At the first glance of the estimated values of β, they are different under each settings. Take the two B/M sorted portfolios for example, the volatility of β estimated under M_{CF} is much smaller than that under M_0 or M_{DR}. In addition, the β estimated in M_{CF} and M_0 are more persistent than that in M_{DR}. Notice that β under the M_{DR} is quite close to the restricted value calculated use daily realized returns. With a closer observation, the time-series pattern of in unconstrained model M_0 integrates the traits of the same β under the other two constrained settings. Take the β of BM5 under the three settings as an illustration. The β under M_0 peaks during the early 70s recession and the recent 2008 crisis, and declines sharply at the end of 2011, just like under M_{DR}. On the other hand, the β under M_0 reaches a trough in the middle of the booms in 1990s, and climbs up after the burst of dot-com bubble, like as under M_{CF}. The underlying reason is the following. The β under the DR risk model M_{DR} reflects the DR risk, describing covariance between the portfolio and market DR, while the β under M_{CF} reflects the co-movement in the CF. In the unconstrained model M_0, the contains information from both sides. These observations and arguments also apply to other portfolios, for instance, the momentum portfolios.

In the end, I report the estimated parameters at the individual stock level. I have reported the VAR coefficients of γ and β, and the correlation between the their shocks. I will not repeat them in this table, and only covers the mean of CF and DR exposure ($\bar{\gamma}$ and $\bar{\beta}$), the variance of the VAR shocks (Σ_γ and Σ_β), the correlation between the shocks of CF and DR exposures and the shocks of aggregate CF and DR factors ($\rho_{\beta z}$, $\rho_{\beta\sigma_z}$, $\rho_{\gamma g}$ and $\rho_{\gamma\sigma_g}$). I estimate each parameter under all three specifications of models. In Table A.1, I report estimates under M_0, and the results of M_{CF} and M_{DR} can be found in Table A.2 and Table A.3.

Compare these tables, one can draw the following conclusions. (1) The estimates of the average CF and DR exposures are similar. (2) The conditional errors in DR and CF one-factor models are both insignificant. (3) The correlation between β and z, and the correlation between β and σ_z are high for portfolios with high returns, which is consistent with the conditional CAPM literature.

Table A.1: Estimation of the Parameters at the Individual Portfolio Level under General Model M_0

	$m(\bar{\gamma})$					$s(\bar{\gamma})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.946	0.917	1.160	1.469	1.291	0.026	0.027	0.028	0.028	0.028
BM	0.851	0.737	0.699	0.966	1.474	0.001	0.025	0.026	0.019	0.028
MOM	1.371	0.653	0.967	0.900	0.959	0.025	0.022	0.016	0.028	0.025
VOL	2.670	1.826	1.228	0.847	0.822	0.313	0.137	0.025	0.021	0.027
ACC	0.755	1.333	0.873	0.685	0.607	0.000	0.026	0.025	0.028	0.028
CI	0.912	0.590	1.600	0.946	0.594	0.026	0.000	0.000	0.027	0.028
LIQ	0.822	0.701	1.496	1.358	0.901	0.025	0.028	0.027	0.029	0.028
	$m(\bar{\beta})$					$s(\bar{\beta})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.943	1.047	1.077	1.112	1.178	0.026	0.027	0.028	0.028	0.027
BM	1.000	0.922	0.868	0.848	0.926	0.001	0.025	0.024	0.028	0.029
MOM	1.230	0.919	0.894	0.907	1.093	0.027	0.018	0.026	0.027	0.029
VOL	1.557	1.504	1.295	1.030	0.830	0.027	0.027	0.025	0.021	0.026
ACC	1.060	0.977	0.901	0.921	1.064	0.000	0.018	0.025	0.028	0.028
CI	0.994	0.884	0.937	0.969	1.096	0.027	0.000	0.000	0.018	0.028
LIQ	0.888	0.944	0.971	1.031	1.089	0.024	0.028	0.018	0.020	0.028
	$m(c)$					$s(c)$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.001	0.011	0.005	-0.006	0.008	0.002	0.003	0.003	0.006	0.005
BM	0.007	0.007	0.003	-0.006	-0.020	0.000	0.002	0.002	0.003	0.006
MOM	-0.001	0.005	-0.001	0.005	0.018	0.001	0.001	0.002	0.003	0.005
VOL	-0.085	-0.011	0.010	0.011	0.000	0.037	0.010	0.006	0.001	0.002
ACC	0.002	-0.005	0.002	0.011	0.023	0.000	0.003	0.001	0.003	0.004
CI	0.009	0.005	0.000	0.005	0.027	0.002	0.000	0.000	0.003	0.004
LIQ	0.000	0.012	-0.013	-0.004	0.018	0.000	0.003	0.005	0.006	0.004
	$m(\alpha)$					$s(\alpha)$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.000	0.000	0.000	0.003	0.002	0.000	0.003	0.004	0.007	0.005
BM	-0.005	-0.001	-0.002	0.000	0.000	0.002	0.003	0.003	0.003	0.006
MOM	0.007	-0.001	0.000	0.000	0.000	0.006	0.001	0.002	0.003	0.004
VOL	0.022	0.015	-0.001	-0.002	0.000	0.030	0.013	0.002	0.002	0.003
ACC	-0.017	0.000	-0.001	0.000	0.000	0.004	0.001	0.003	0.003	0.005
CI	0.000	-0.001	0.006	0.000	0.001	0.002	0.000	0.000	0.003	0.005
LIQ	0.000	0.000	0.003	0.001	0.001	0.001	0.003	0.006	0.006	0.004
	$m(\Sigma_\gamma) \times 1000$					$s(\Sigma_\gamma) \times 1000$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.320	6.538	4.110	5.168	9.832	0.101	1.970	1.306	2.078	3.726
BM	0.910	2.142	2.394	10.940	25.372	0.269	0.650	0.770	2.611	5.511
MOM	48.863	0.653	0.384	30.140	270.496	11.390	0.201	0.106	7.436	56.563
VOL	43.749	20.349	10.463	2.189	2.414	17.687	5.863	2.490	0.712	0.726
ACC	25.936	1.941	1.552	14.753	61.546	6.507	0.533	0.458	4.196	14.894
CI	4.889	0.167	0.318	10.919	47.065	1.261	0.000	0.000	3.002	12.479
LIQ	0.091	8.197	6.964	25.813	66.167	0.030	2.114	2.398	7.907	13.475

	$m(\Sigma_\beta) \times 1000$					$s(\Sigma_\beta) \times 1000$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.344	1.011	1.711	3.496	4.449	0.092	0.300	0.364	0.571	0.683
BM	0.701	0.593	0.705	1.627	4.271	0.183	0.159	0.139	0.351	1.015
MOM	8.477	2.795	2.192	2.182	5.701	1.747	0.607	0.623	0.576	1.516
VOL	16.182	7.301	3.256	0.683	0.487	2.268	1.394	0.842	0.189	0.135
ACC	4.427	1.179	1.469	1.712	2.056	1.142	0.323	0.423	0.489	0.537
CI	1.129	0.293	0.737	0.789	1.431	0.277	0.000	0.063	0.200	0.375
LIQ	0.522	0.928	1.769	2.821	3.817	0.147	0.155	0.270	0.384	0.406

	$m(\rho_{\gamma g})$					$s(\rho_{\gamma g})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.162	-0.280	-0.266	-0.249	-0.251	0.308	0.185	0.190	0.180	0.179
BM	0.526	0.235	-0.254	-0.302	-0.328	0.202	0.274	0.173	0.200	0.206
MOM	0.302	0.421	0.280	-0.291	-0.283	0.187	0.239	0.180	0.189	0.188
VOL	0.410	0.324	0.326	0.326	-0.290	0.200	0.187	0.189	0.279	0.190
ACC	0.458	0.178	0.090	-0.253	-0.259	0.210	0.130	0.326	0.174	0.172
CI	0.334	0.822	-0.848	0.060	-0.277	0.196	0.021	0.002	0.310	0.190
LIQ	0.294	-0.248	-0.272	-0.285	-0.264	0.292	0.180	0.188	0.184	0.188

	$m(\rho_{\gamma\sigma_g})$					$s(\rho_{\gamma\sigma_g})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.045	0.034	0.065	0.012	0.062	0.325	0.311	0.307	0.324	0.320
BM	-0.001	0.020	0.047	0.015	0.100	0.311	0.308	0.325	0.349	0.303
MOM	-0.034	-0.040	0.045	0.059	0.026	0.288	0.321	0.304	0.348	0.356
VOL	0.053	-0.036	0.003	0.083	0.038	0.322	0.325	0.319	0.288	0.320
ACC	-0.009	-0.105	0.078	0.003	-0.004	0.292	0.302	0.297	0.326	0.306
CI	-0.022	0.504	0.327	-0.019	0.050	0.337	0.087	0.025	0.296	0.290
LIQ	0.055	0.088	-0.047	-0.021	0.006	0.317	0.301	0.326	0.300	0.323

	$m(\rho_{\beta z})$					$s(\rho_{\beta z})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	-0.163	0.421	0.410	0.403	0.418	0.310	0.147	0.146	0.139	0.143
BM	-0.424	0.033	0.398	0.379	0.397	0.143	0.344	0.133	0.129	0.139
MOM	-0.432	-0.115	-0.419	0.412	0.425	0.150	0.350	0.144	0.142	0.143
VOL	-0.427	-0.424	-0.424	-0.047	0.397	0.148	0.143	0.143	0.323	0.134
ACC	-0.419	-0.428	-0.045	0.418	0.427	0.144	0.147	0.324	0.140	0.141
CI	-0.441	0.348	0.458	-0.016	0.416	0.150	0.048	0.026	0.324	0.141
LIQ	-0.195	0.419	0.397	0.414	0.422	0.321	0.140	0.133	0.143	0.149

	$m(\rho_{\beta\sigma_z})$					$s(\rho_{\beta\sigma_z})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	-0.184	0.052	0.000	-0.032	-0.065	0.288	0.341	0.327	0.302	0.316
BM	-0.282	-0.067	0.046	-0.072	-0.021	0.292	0.313	0.280	0.320	0.302
MOM	-0.146	-0.452	-0.063	0.077	0.065	0.329	0.248	0.307	0.279	0.322
VOL	-0.225	-0.162	-0.136	-0.029	0.004	0.289	0.295	0.310	0.292	0.315
ACC	-0.064	0.139	-0.109	0.041	0.130	0.309	0.328	0.312	0.331	0.303
CI	-0.123	-0.806	0.928	-0.038	0.062	0.297	0.027	0.011	0.330	0.308
LIQ	-0.294	0.114	0.076	0.019	0.119	0.274	0.327	0.319	0.330	0.338

Note to Table A.1 I report the estimates of the parameters at the individual portfolio level under M_0 which are not reported in the main body of the paper. The parameters include the mean of CF and DR exposures ($\bar{\gamma}$ and $\bar{\beta}$), the conditional errors in the one-factor models for CF and DR (c and α), the variance of the shocks of CF and DR exposure (Σ_γ and Σ_β), and the correlations describing the stability of the CF and DR exposures ($\rho_{\gamma g}$, $\rho_{\gamma\sigma_g}$), $\rho_{\beta z}$ and $\rho_{\beta\sigma_z}$.

Table A.2: Estimation of the Parameters at the Individual Portfolio Level under CF Model M_{CF}

	$m(\bar{\gamma})$					$s(\bar{\gamma})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.948	0.915	1.158	1.464	1.289	0.027	0.029	0.029	0.028	0.028
BM	0.820	0.737	0.687	0.968	1.471	0.026	0.028	0.028	0.019	0.029
MOM	1.375	0.628	0.968	0.900	0.955	0.027	0.028	0.016	0.029	0.025
VOL	2.484	1.834	1.221	0.828	0.821	0.379	0.139	0.028	0.029	0.027
ACC	0.708	1.341	0.863	0.691	0.606	0.035	0.028	0.029	0.028	0.029
CI	0.910	0.547	1.638	0.947	0.595	0.029	0.028	0.030	0.029	0.028
LIQ	0.813	0.701	1.487	1.356	0.904	0.029	0.029	0.027	0.029	0.029

	$m(\bar{\beta})$					$s(\bar{\beta})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	1.028	0.960	0.948	0.946	0.803	0.001	0.002	0.003	0.003	0.003
BM	1.101	0.977	0.900	0.880	0.940	0.002	0.002	0.002	0.002	0.003
MOM	1.075	0.944	0.917	0.961	1.143	0.003	0.002	0.002	0.002	0.003
VOL	1.219	1.308	1.232	1.070	0.899	0.005	0.004	0.003	0.001	0.001
ACC	1.114	0.975	0.937	0.978	1.102	0.003	0.002	0.002	0.002	0.003
CI	1.044	0.999	0.963	1.009	1.076	0.002	0.002	0.002	0.002	0.002
LIQ	1.010	0.912	0.839	0.788	0.688	0.001	0.002	0.003	0.003	0.003

	$m(c)$					$s(c)$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.001	0.004	-0.003	-0.011	-0.010	0.003	0.005	0.006	0.007	0.008
BM	0.003	0.005	0.001	-0.009	-0.022	0.003	0.004	0.002	0.005	0.007
MOM	0.000	0.001	-0.001	0.003	0.013	0.001	0.004	0.002	0.007	0.009
VOL	-0.069	-0.008	0.006	0.002	0.000	0.039	0.011	0.011	0.007	0.002
ACC	0.000	-0.002	0.000	0.011	0.013	0.001	0.005	0.002	0.005	0.013
CI	0.002	0.000	0.000	0.003	0.021	0.007	0.003	0.000	0.006	0.009
LIQ	0.000	0.007	-0.017	-0.019	-0.008	0.000	0.005	0.005	0.009	0.013

	$m(\alpha)$					$s(\alpha)$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.000	0.000	0.001	0.002	0.001	0.000	0.005	0.006	0.007	0.006
BM	-0.002	-0.001	0.000	0.000	0.001	0.004	0.004	0.005	0.005	0.007
MOM	0.004	0.000	0.000	0.001	0.001	0.007	0.001	0.002	0.006	0.006
VOL	0.008	0.011	0.000	-0.001	0.000	0.016	0.013	0.002	0.003	0.004
ACC	0.001	0.000	0.000	0.000	0.000	0.015	0.001	0.004	0.006	0.006
CI	0.000	0.000	0.001	0.000	0.000	0.002	0.000	0.003	0.005	0.006
LIQ	0.000	0.000	0.005	0.001	0.000	0.001	0.005	0.008	0.007	0.005

	$m(\Sigma_\gamma) \times 1000$					$s(\Sigma_\gamma) \times 1000$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.310	5.696	3.669	5.273	10.609	0.074	0.740	0.445	1.002	1.766
BM	0.968	1.799	2.179	10.478	23.254	0.235	0.163	0.228	1.613	4.905
MOM	46.729	0.705	0.345	28.849	231.590	8.272	0.119	0.070	4.936	42.088
VOL	132.708	19.493	10.423	2.053	2.121	20.740	4.745	1.070	0.397	0.299
ACC	28.923	1.656	1.361	13.157	58.895	6.209	0.154	0.189	1.815	7.100
CI	4.857	0.385	0.601	10.023	45.417	0.972	0.092	0.085	1.878	4.755
LIQ	0.106	7.932	6.415	24.420	68.357	0.033	0.911	1.534	6.126	12.714

	$m(\Sigma_\beta) \times 1000$					$s(\Sigma_\beta) \times 1000$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	66.390	22.480	22.257	8.700	28.152	30.737	5.810	10.862	9.039	32.601
BM	133.464	38.761	23.246	5.421	8.023	33.133	4.779	9.035	3.307	8.119
MOM	37.823	485.123	14.303	59.264	63.418	54.076	124.874	6.832	34.165	71.068
VOL	49.440	17.130	436.972	399.058	10.537	42.080	21.109	189.878	88.069	7.577
ACC	4865.922	171.662	161.425	18.090	465.740	3601.202	46.418	44.962	7.118	195.480
CI	1756.372	786.320	643.178	59.334	79.946	813.054	250.794	110.855	36.388	26.598
LIQ	81.784	9.728	5.401	12.232	82.300	29.228	3.014	6.806	8.313	48.985

	$m(\rho_{\gamma g})$					$s(\rho_{\gamma g})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	-0.089	0.007	-0.087	-0.061	-0.068	0.368	0.350	0.401	0.398	0.414
BM	-0.001	0.099	0.015	0.036	-0.188	0.400	0.357	0.380	0.397	0.408
MOM	-0.142	0.025	-0.059	0.024	0.184	0.409	0.414	0.412	0.368	0.397
VOL	0.071	0.098	-0.097	-0.034	0.166	0.387	0.390	0.338	0.354	0.385
ACC	0.162	0.106	-0.051	0.186	0.101	0.333	0.373	0.394	0.359	0.349
CI	0.138	-0.145	0.305	0.093	0.256	0.382	0.359	0.327	0.367	0.343
LIQ	-0.049	0.157	-0.191	-0.076	0.030	0.369	0.390	0.395	0.414	0.434

	$m(\rho_{\gamma\sigma_g})$					$s(\rho_{\gamma\sigma_g})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	-0.128	0.229	0.246	0.180	0.285	0.215	0.364	0.338	0.389	0.364
BM	-0.377	0.220	0.064	-0.049	0.275	0.221	0.316	0.333	0.428	0.465
MOM	0.117	0.001	-0.072	-0.140	-0.090	0.322	0.175	0.300	0.335	0.328
VOL	-0.170	0.087	-0.022	-0.137	-0.192	0.444	0.392	0.257	0.231	0.352
ACC	-0.216	0.265	0.105	-0.042	0.050	0.296	0.198	0.240	0.357	0.306
CI	-0.115	0.046	0.480	0.108	-0.104	0.165	0.141	0.134	0.351	0.381
LIQ	-0.117	-0.021	0.205	0.011	-0.088	0.203	0.400	0.376	0.442	0.416

	$m(\rho_{\beta z})$					$s(\rho_{\beta z})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	-0.128	0.229	0.246	0.180	0.285	0.215	0.364	0.338	0.389	0.364
BM	-0.377	0.220	0.064	-0.049	0.275	0.221	0.316	0.333	0.428	0.465
MOM	0.117	0.001	-0.072	-0.140	-0.090	0.322	0.175	0.300	0.335	0.328
VOL	-0.170	0.087	-0.022	-0.137	-0.192	0.444	0.392	0.257	0.231	0.352
ACC	-0.216	0.265	0.105	-0.042	0.050	0.296	0.198	0.240	0.357	0.306
CI	-0.115	0.046	0.480	0.108	-0.104	0.165	0.141	0.134	0.351	0.381
LIQ	-0.117	-0.021	0.205	0.011	-0.088	0.203	0.400	0.376	0.442	0.416

	$m(\rho_{\beta\sigma_z})$					$s(\rho_{\beta\sigma_z})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	-0.128	0.229	0.246	0.180	0.285	0.215	0.364	0.338	0.389	0.364
BM	-0.377	0.220	0.064	-0.049	0.275	0.221	0.316	0.333	0.428	0.465
MOM	0.117	0.001	-0.072	-0.140	-0.090	0.322	0.175	0.300	0.335	0.328
VOL	-0.170	0.087	-0.022	-0.137	-0.192	0.444	0.392	0.257	0.231	0.352
ACC	-0.216	0.265	0.105	-0.042	0.050	0.296	0.198	0.240	0.357	0.306
CI	-0.115	0.046	0.480	0.108	-0.104	0.165	0.141	0.134	0.351	0.381
LIQ	-0.117	-0.021	0.205	0.011	-0.088	0.203	0.400	0.376	0.442	0.416

Note to Table A.2 I report the estimates of the parameters at the individual portfolio level under M_{CF} which are not reported in the main body of the paper. The parameters include the mean of CF and DR exposures ($\bar{\gamma}$ and $\bar{\beta}$), the conditional errors in the one-factor models for CF and DR (c and α), the variance of the shocks of CF and DR exposure (Σ_γ and Σ_β), and the correlations describing the stability of the CF and DR exposures ($\rho_{\gamma g}$, $\rho_{\gamma\sigma_g}$), $\rho_{\beta z}$ and $\rho_{\beta\sigma_z}$.

Table A.3: Estimation of the Parameters at the Individual Portfolio Level under DR Model M_{DR}

$m(\bar{\gamma})$	Low Return	2	3	4	High Return	$s(\bar{\gamma})$ Low Return	2	3	4	High Return
SIZE	0.965	0.900	1.141	1.456	1.248	0.023	0.028	0.027	0.028	0.017
BM	0.852	0.745	0.705	0.967	1.474	0.000	0.024	0.025	0.019	0.028
MOM	1.345	0.667	0.969	0.900	0.959	0.008	0.012	0.016	0.028	0.025
VOL	2.533	1.817	1.226	0.861	0.833	0.406	0.132	0.026	0.012	0.025
ACC	0.755	1.333	0.879	0.687	0.608	0.000	0.027	0.024	0.028	0.029
CI	0.918	0.591	1.600	0.946	0.596	0.026	0.000	0.000	0.028	0.028
LIQ	0.856	0.705	1.486	1.321	0.867	0.000	0.029	0.027	0.021	0.025

$m(\bar{\beta})$	Low Return	2	3	4	High Return	$s(\bar{\beta})$ Low Return	2	3	4	High Return
SIZE	0.954	1.024	1.049	1.028	0.990	0.088	0.091	0.116	0.124	0.235
BM	1.101	0.976	0.900	0.880	0.940	0.001	0.002	0.002	0.002	0.003
MOM	1.220	0.943	0.917	0.961	1.143	0.118	0.002	0.002	0.002	0.002
VOL	1.273	1.341	1.232	1.070	0.898	0.111	0.065	0.003	0.001	0.001
ACC	1.113	0.975	0.936	0.978	1.101	0.002	0.002	0.002	0.002	0.003
CI	1.044	0.997	0.964	1.004	1.076	0.002	0.002	0.001	0.005	0.002
LIQ	1.000	0.912	0.918	0.962	0.944	0.000	0.002	0.081	0.164	0.253

$m(c)$	Low Return	2	3	4	High Return	$s(c)$ Low Return	2	3	4	High Return
SIZE	0.002	0.010	0.006	-0.004	0.006	0.002	0.003	0.004	0.005	0.007
BM	0.007	0.008	0.003	-0.004	-0.019	0.000	0.002	0.001	0.004	0.006
MOM	-0.001	0.005	-0.001	0.007	0.018	0.001	0.001	0.002	0.003	0.005
VOL	-0.070	-0.012	0.009	0.011	0.001	0.038	0.009	0.005	0.001	0.001
ACC	0.002	-0.005	0.002	0.013	0.024	0.000	0.003	0.001	0.004	0.005
CI	0.009	0.005	0.000	0.006	0.026	0.002	0.000	0.000	0.003	0.004
LIQ	0.000	0.010	-0.014	-0.005	0.012	0.000	0.004	0.004	0.006	0.010

$m(\alpha)$	Low Return	2	3	4	High Return	$s(\alpha)$ Low Return	2	3	4	High Return
SIZE	0.000	0.000	0.000	0.001	0.002	0.000	0.003	0.003	0.004	0.005
BM	-0.009	-0.002	-0.002	0.000	0.001	0.000	0.004	0.004	0.004	0.007
MOM	0.004	-0.001	0.000	0.000	0.001	0.005	0.001	0.002	0.003	0.004
VOL	0.018	0.011	-0.001	-0.002	-0.001	0.027	0.011	0.002	0.002	0.003
ACC	-0.019	0.000	-0.001	0.000	0.001	0.005	0.001	0.003	0.004	0.005
CI	0.000	-0.001	0.006	0.000	0.001	0.002	0.000	0.000	0.003	0.005
LIQ	-0.002	0.000	0.001	0.001	0.000	0.000	0.004	0.005	0.004	0.004

$m(\Sigma_\gamma) \times 1000$	Low Return	2	3	4	High Return	$s(\Sigma_\gamma) \times 1000$ Low Return	2	3	4	High Return
SIZE	0.321	6.406	4.103	5.788	10.259	0.097	1.931	1.247	1.808	3.368
BM	0.550	2.068	2.353	10.654	24.355	0.173	0.630	0.780	2.397	5.760
MOM	50.253	0.622	0.383	29.677	262.321	11.009	0.196	0.103	7.698	53.657
VOL	45.829	20.831	10.545	2.151	2.455	17.359	5.598	2.483	0.660	0.769
ACC	25.567	1.913	1.522	14.159	62.044	6.320	0.552	0.451	4.465	14.911
CI	4.757	0.195	0.171	10.878	46.593	1.266	0.000	0.000	3.073	12.348
LIQ	0.045	8.033	7.286	25.704	64.135	0.001	2.079	2.005	5.075	13.467

	$m(\Sigma_\beta) \times 1000$					$s(\Sigma_\beta) \times 1000$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.156	0.903	1.860	4.037	5.318	0.013	0.111	0.321	0.820	1.179
BM	0.477	0.580	0.773	2.267	5.674	0.116	0.071	0.132	0.428	1.387
MOM	10.621	3.450	2.408	2.298	7.415	2.674	0.490	0.336	0.436	1.331
VOL	18.329	7.429	2.956	0.649	0.404	5.498	1.782	0.565	0.088	0.053
ACC	4.308	0.992	1.229	1.635	1.681	0.872	0.101	0.160	0.227	0.223
CI	0.981	0.690	0.592	0.606	1.175	0.083	0.110	0.056	0.054	0.143
LIQ	0.364	0.994	1.866	3.592	4.429	0.021	0.149	0.369	1.041	1.335

	$m(\rho_{\gamma g})$					$s(\rho_{\gamma g})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.122	-0.265	-0.308	-0.293	-0.274	0.311	0.180	0.193	0.189	0.182
BM	0.918	0.245	-0.243	-0.275	-0.277	0.029	0.282	0.173	0.186	0.187
MOM	0.273	0.535	0.286	-0.273	-0.245	0.175	0.197	0.176	0.192	0.180
VOL	0.384	0.322	0.320	0.437	-0.261	0.197	0.189	0.186	0.222	0.178
ACC	0.398	0.161	0.185	-0.259	-0.224	0.209	0.122	0.308	0.182	0.165
CI	0.415	0.828	-0.849	0.053	-0.240	0.207	0.020	0.001	0.304	0.166
LIQ	0.806	-0.260	-0.304	-0.257	-0.236	0.034	0.185	0.187	0.177	0.169

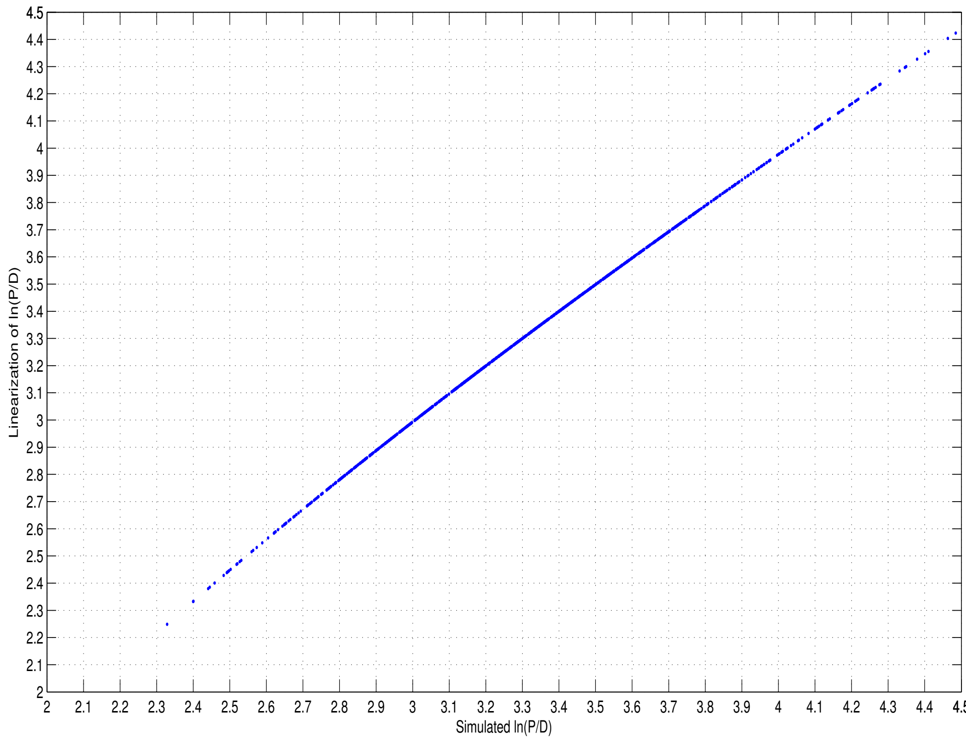
	$m(\rho_{\gamma \sigma_g})$					$s(\rho_{\gamma \sigma_g})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	0.122	-0.265	-0.308	-0.293	-0.274	0.311	0.180	0.193	0.189	0.182
BM	0.918	0.245	-0.243	-0.275	-0.277	0.029	0.282	0.173	0.186	0.187
MOM	0.273	0.535	0.286	-0.273	-0.245	0.175	0.197	0.176	0.192	0.180
VOL	0.384	0.322	0.320	0.437	-0.261	0.197	0.189	0.186	0.222	0.178
ACC	0.398	0.161	0.185	-0.259	-0.224	0.209	0.122	0.308	0.182	0.165
CI	0.415	0.828	-0.849	0.053	-0.240	0.207	0.020	0.001	0.304	0.166
LIQ	0.806	-0.260	-0.304	-0.257	-0.236	0.034	0.185	0.187	0.177	0.169

	$m(\rho_{\beta z})$					$s(\rho_{\beta z})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	-0.077	0.406	0.413	0.418	0.403	0.321	0.137	0.138	0.145	0.141
BM	-0.445	0.070	0.400	0.390	0.398	0.130	0.356	0.136	0.130	0.134
MOM	-0.399	-0.175	-0.419	0.433	0.420	0.133	0.304	0.145	0.143	0.144
VOL	-0.438	-0.438	-0.427	-0.096	0.410	0.150	0.151	0.147	0.321	0.137
ACC	-0.393	-0.398	-0.007	0.417	0.414	0.130	0.136	0.316	0.139	0.142
CI	-0.409	-0.628	0.493	-0.068	0.408	0.141	0.083	0.014	0.335	0.143
LIQ	-0.737	0.412	0.412	0.423	0.418	0.040	0.134	0.143	0.148	0.141

	$m(\rho_{\beta \sigma_z})$					$s(\rho_{\beta \sigma_z})$				
	Low Return	2	3	4	High Return	Low Return	2	3	4	High Return
SIZE	-0.110	0.102	-0.024	-0.035	0.065	0.293	0.291	0.308	0.294	0.333
BM	-0.608	-0.026	0.058	0.037	-0.125	0.088	0.313	0.312	0.291	0.291
MOM	-0.085	-0.504	-0.127	0.053	0.043	0.305	0.248	0.283	0.290	0.335
VOL	-0.221	-0.163	-0.130	-0.159	0.026	0.296	0.319	0.297	0.325	0.321
ACC	-0.145	0.102	-0.125	0.066	0.048	0.306	0.305	0.309	0.290	0.323
CI	-0.102	-0.902	0.905	-0.026	0.033	0.300	0.037	0.013	0.353	0.307
LIQ	-0.940	0.056	0.008	0.043	0.117	0.021	0.271	0.336	0.300	0.289

Note to Table A.3 I report the estimates of the parameters at the individual portfolio level under M_{DR} which are not reported in the main body of the paper. The parameters include the mean of CF and DR exposures ($\bar{\gamma}$ and $\bar{\beta}$), the conditional errors in the one-factor models for CF and DR (c and α), the variance of the shocks of CF and DR exposure (Σ_γ and Σ_β), and the correlations describing the stability of the CF and DR exposures ($\rho_{\gamma g}$, $\rho_{\gamma \sigma_g}$), $\rho_{\beta z}$ and $\rho_{\beta \sigma_z}$.

Figure A.1: Validation of Log-linearizations



The figure shows the scatter plot of the real log of price-dividend ratio (*lnpd*) as simulated by

$$lnpd_t^M = \log \sum_{n=1}^{\infty} \exp(a_n^M + b_n^M X_t^M)$$

, with the approximated *lnpd* calculated with the log-linearization

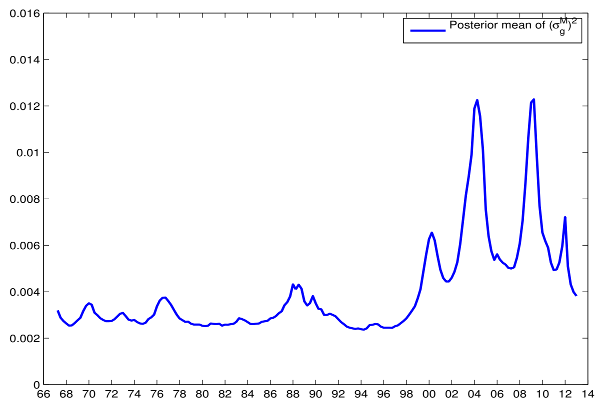
$$lnpd_t^M = A^M + B^{M\prime} X_t$$

in a simulation with $T = 1000$. The parametric specification of the simulation is documented in Appendix I.C .1

Figure A.2: Stochastic Volatility of Dividends and Excess Returns

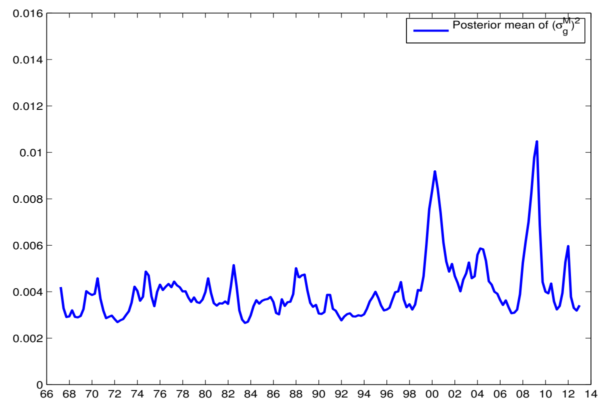
$M_0(M_{DR})$

Stochastic Volatility of Dividend Growth

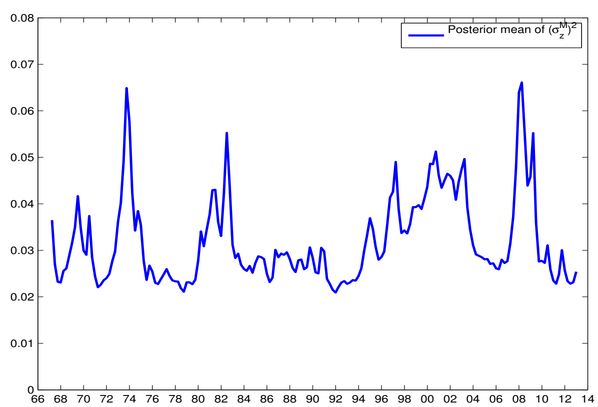


M_{CF}

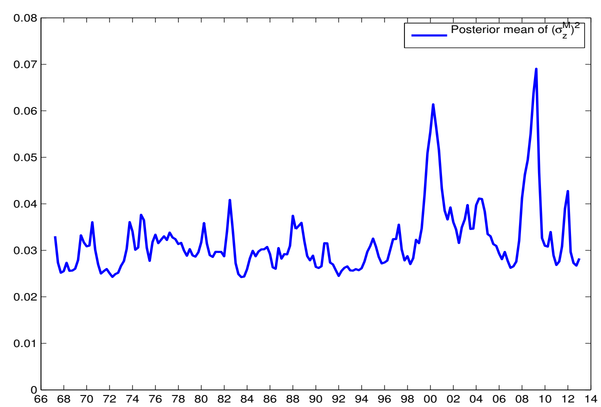
Stochastic Volatility of Dividend Growth



Stochastic Volatility of Excess Return



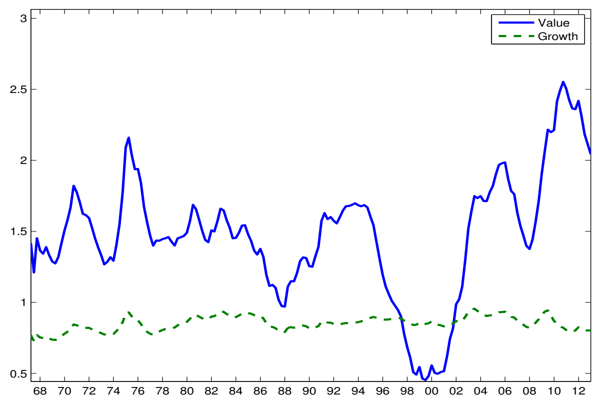
Stochastic Volatility of Excess Return



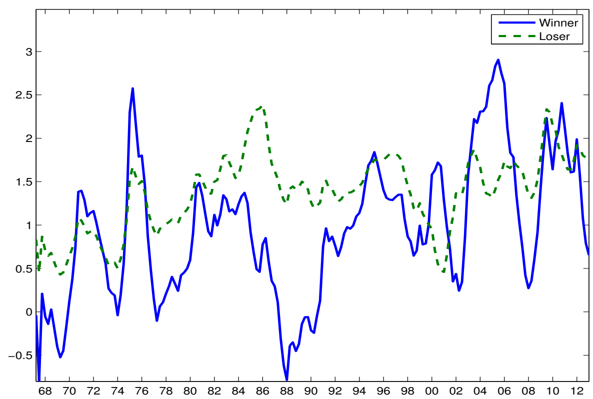
The plots show the filtered series of stochastic volatility in dividend growths and excess returns under two models with different specifications of risks. The upper two panels shows the stochastic volatility $(\sigma_{g,t}^M)^2$ in dividend growth, while the lower two panels report the stochastic volatility $(\sigma_{z,t}^M)^2$in excess returns. The left two panels reflect estimations in the unconstrained market M_0, and the right panel represent the CF risk model M_{CF}.

Figure A.3: Cash Flow Exposure γ

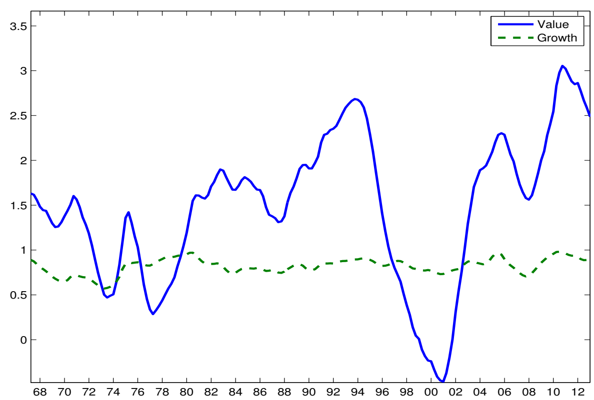
γ of Value and Growth Portfolios in M_0



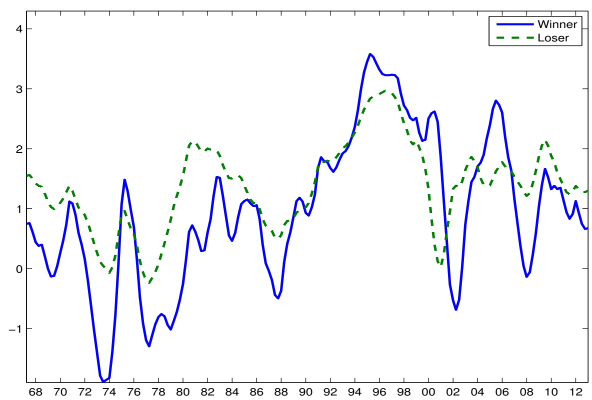
γ of Winners and Losers Portfolios in M_0



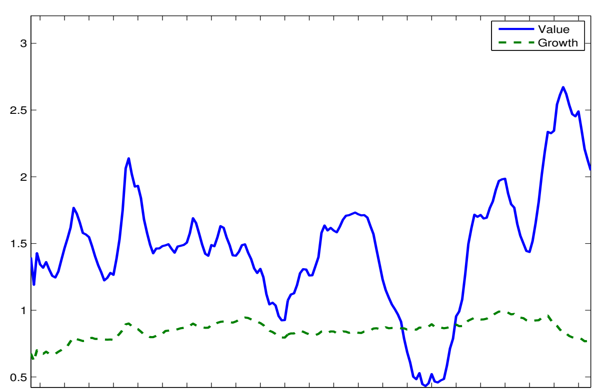
γ of Value and Growth Portfolios in M_{CF}



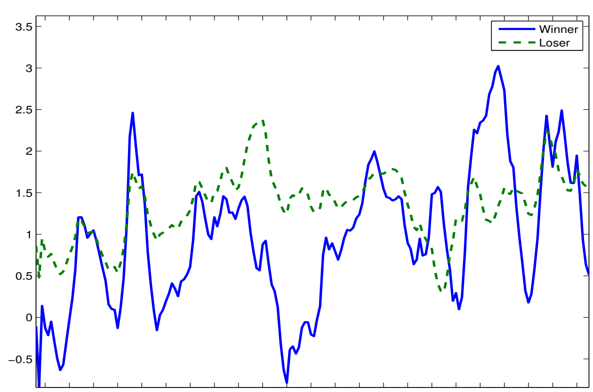
γ of Winners and Losers Portfolios in M_{CF}



γ of Value and Growth Portfolios in M_{DR}

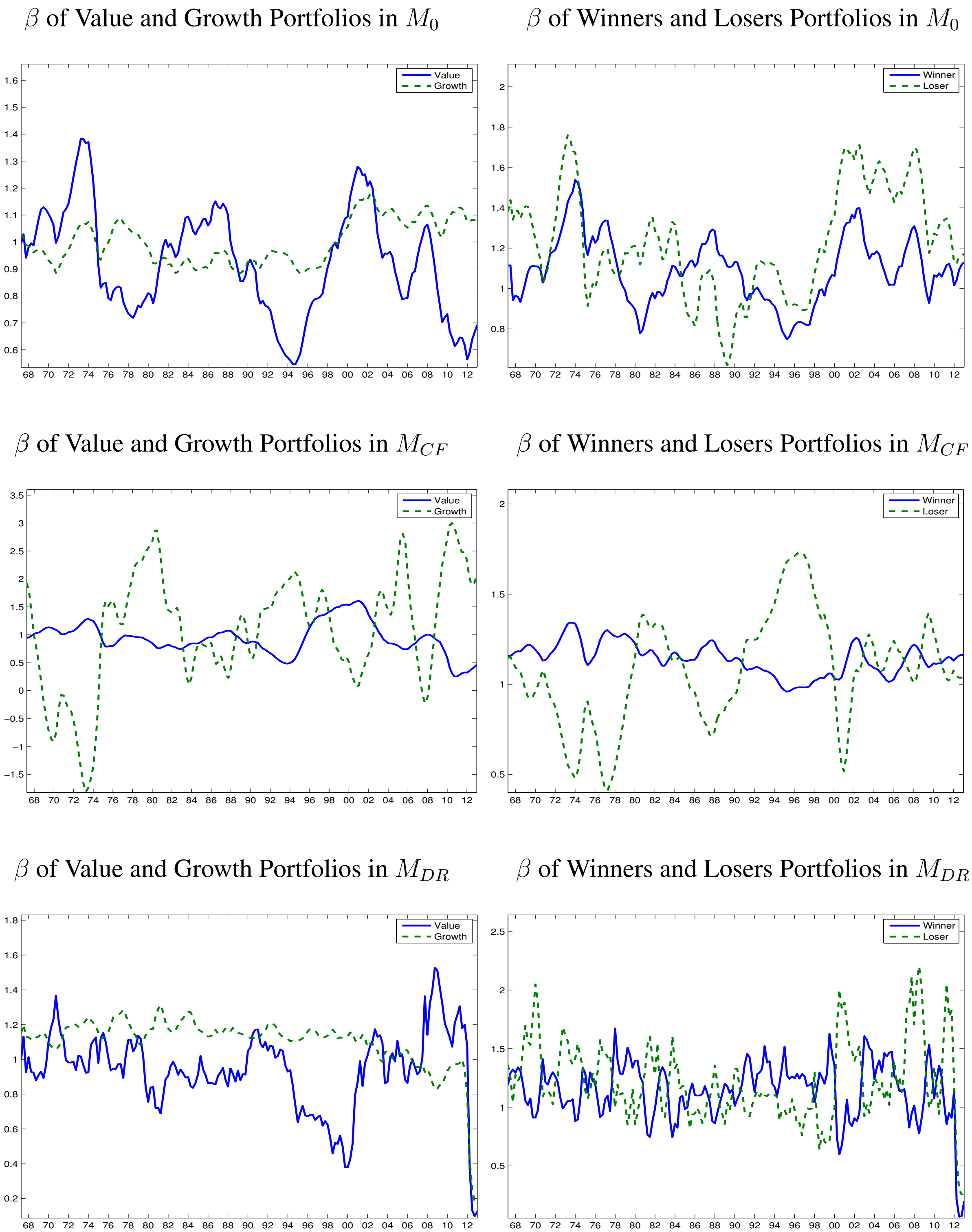


γ of Winners and Losers Portfolios in M_{DR}



The plots show the filtered series of γ, the exposure to the aggregate CF factor. The left panels are for the test portfolios $BM1$ and $BM5$, which stand for growth and value firms, while the right panels are for the test portfolios $MOM1$ and $MOM5$ which are the past losers and past winners portfolio in momentum sorting. The upper two panels are estimates from the unconstrained model M_0, the middle two are from model with CF risk M_{CF} and the lower two are from model with DR risk M_{DR}.

Figure A.4: Discount Rate Exposure β



The plots show the filtered series of β, the exposure to the aggregate DR factor. The left panels are for the test portfolios $BM1$ and $BM5$, which stand for growth and value firms, while the right panels are for the test portfolios $MOM1$ and $MOM5$ which are the past losers and past winners portfolio in momentum sorting. The upper two panels are estimates from the unconstrained model M_0, the middle two are from model with CF risk M_{CF} and the lower two are from model with DR risk M_{DR}.