The Econometrics of Matching Models*

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1 Introduction

In October 2012 the Nobel prize was attributed to Al Roth and Lloyd Shapley for their work on matching. Both the seminal Gale-Shapley (1962) paper and most of Roth's work were concerned with allocation mechanisms when prices or other transfers cannot be used—what we will call non-transferable utility (NTU) in this survey. Gale and Shapley used college admissions, marriage, and roommate assignments as examples; and Roth's fundamental work in market design has led to major improvements in the National Resident Matching Program (Roth and Peranson 1999) and to the creation of a mechanism for kidney exchange (Roth, Sönmez and Ünver 2004.)

While these are important economic applications, matching problems are much more pervasive. Market and non-market mechanisms such as auctions match agents with goods, and buyers with sellers; agents match to each other in production teams, and production tasks are matched with workers; and in trade theory, countries are matched with goods or varieties. Yet while the basic theory of matching was in place forty years ago, only recently has there been an explosion of empirical work in this area. Several developments

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have concurred to bring it to the attention of applied researchers.

On the theoretical front, the path-breaking contributions of Koopmans-Beckmann (1957), Gale-Shapley (1962), Shapley-Shubik (1972), Becker (1973, 1974) and Kelso-Crawford (1982) were followed by extensive investigations in the 1980s; these culminated in the classic monograph by Roth and Sotomayor (1992). Important contributions renewed interest in matching models more recently. Hatfield-Milgrom (2005) exploited the analogy of matching with contract theory, auctions and general equilibrium. Their paper encompassed NTU and TU matching in a general framework, and it also opened the way to new results on many-to-one and many-to-many matching. Several authors explored models of matching with frictions (e.g. Shimer-Smith 2000 and Eeckhout-Kircher 2010), with the aim of enriching equilibrium models of unemployment in particular. One-to-one matching models have been revisited to take into account imperfectly transferable utility (Chiappori-Reny 2007, Legros-Newman 2007.) Finally, another strand of the theoretical literature on TU models has built on advances in the mathematical theory of optimal transportation, whose application to several fields of economic theory has proved quite successful[1]; we will give an example later in this survey.

The resulting insights have been applied to a host of issues, including the allocation of students to schools, the marriage market with unbalanced gender distributions, the role of marital prospects in human capital investment decisions, the social impact of improved birth control technologies and many others. Finally, and perhaps even more interestingly, the econometrics of matching models have recently been reconsidered, from different and equally innovative perspectives. The goal of the present project will be to survey these methodological advances. We shall describe the main difficulties at stake, the various answers provided so far, and the issues that remain open.

[1]The original optimal transportation model dates back to Gaspard Monge (1781); as became apparent much later, optimal transportation and matching under transferable utility are very tightly connected. See Ekeland 2010, Chiappori-McCann-Nesheim 2010 and other papers in that special issue of *Economic Theory*.

1.1 TU and NTU

Any presentation of recent contributions in this booming area must be structured around a small number of basic distinctions. The first, and arguably the most important, is between transferable (TU) and non transferable (NTU) utility models. While recent theoretical advances have shown that these two settings can be analyzed using similar tools (see Hatfield-Milgrom (2005) and the subsequent literature), their areas of practical relevance are largely disjoint. This crucial point is often misunderstood. In some situations (e.g., the allocation of students to public schools), transfers are simply excluded, and a TU framework would make little sense. In many other applications (on the job market, and also within the household), explicit or implicit transfers are paramount and can hardly be ignored.

This would not matter if these two classes of models had similar testable predictions; but the "market clearing" mechanisms are different in the two contexts (with transfers playing the role of market clearing prices in TU models), leading to significantly different comparative statics. For instance, take Becker's famous result that with one-dimensional characteristics, positive complementarities in joint surplus imply positive assortative matching (PAM). Becker also showed in that paper (Becker 1973, pp. 835-6) that without transfers, the condition for PAM is that preferences on each side be increasing in types. Neither condition implies the other. Smith (2011) and Lee-Yariv (2014) give simple examples in which the unique NTU stable matching is PAM, while the unique TU stable matching exhibits negative assortative matching.

This does not mean that it is easy to discriminate between the two models empirically when only matching patterns can be observed. As we will see in section 5.1, *any* matching is rationalizable under TU (and a fortiori under NTU) once we allow for within-type variation in preferences. Echenique et al (2013) explore the testable predictions of TU and NTU stable matchings when the analyst observes *all* payoff-relevant characteristics of the agents[2]. They first show that NTU matching is testable in this setting: there exist

[2]They use the term "aggregate matchings"; but their assumption is really about the econometrician having highly *disaggregated* information.

matchings that cannot be stable NTU matchings for any profile of preferences. They then prove that TU matching is strictly more restrictive than NTU matching; in fact, any matching that is rationalizable by some profile of preferences under TU is also rationalizable by a men-preferred (or a women-preferred) NTU stable matching. This implies that it is impossible to test TU versus NTU using only information about observed matches. Clearly, however, the assumption that all matching-relevant information is observed by the econometrician is excessively strong. It implies for instance that observationally identical should always have observationally identical matches. Most of this survey deals precisely with the ways this assumption can be relaxed.

Introducing the possibility of transfers (given quasi-linear utilites) in a NTU market clearly enhances the total joint surplus, since that is by construction maximal under TU. Lee-Yariv (2014) show that in large one-to-one matching markets, transfers are actually not always necessary for efficiency purposes: for some classes of preferences, stable NTU matchings are asymptotically efficient. On the other hand, for other specifications allowing transfers has a large effect on efficiency. This is an area that cries for more research.

Even in contexts in which transfers cannot be ignored, the standard TU framework relies on a strong assumption—namely, that utility can be transferred between partners at a constant "exchange rate". This has testable consequences that may or may not be acceptable. Take, for instance, the case of households who match on the market for marriage. The TU assumption is only valid under specific individual preferences[3]; these in turn imply that the household's demand for goods is the same for all Pareto efficient allocations. Therefore changes affecting the matching game (say, variations in the composition of the populations of men and women), and generally variations in the spouses' respective weights cannot possibly affect such household's decisions as the amount spent on health care, education or

[3]Necessary and sufficient conditions for TU, that generalize previous contributions by Bergstrom and Varian (1984) and Bergstrom and Cornes (1983), are provided in Chiappori and Gugl (2014). Technically, the conditional indirect utility must be affine in the (conditional) sharing rule.

children expenditures. These restrictions may be excessive in some contexts. Then it is necessary to generalize the basic model by allowing for a nonlinear utility frontier. We will briefly describe such "imperfectly transferable utility" (ITU) models, although to the best of our knowledge they have not yet been taken to data.

1.2 Data and Theory

A second, recurrent theme of this survey will be that the econometrics of matching models needs to combine data with theory judiciously. Unlike single-agent models, matching models by definition involve at least two parties; as a result they can give rise to much richer observable patterns, which makes it much harder to identify parameters of interest without the help of a well-defined theoretical structure.

To illustrate this, consider changes in assortative matching on the marriage market over the past fifty years. Many social scientists have documented an increase in educational homogamy using descriptive statistics—see for instance Schwartz and Mare (2005)[4]. But what lies behind this increased homogamy? How much of it is due to changes in preferences, how much to changes in the "supply" of partners by skill, how much to changes in the returns to education on the labor market? Going beyond the causes, which categories of men and women benefited from the resulting matching patterns? As we will explain in more detail in section 3, the two-sided nature of matching problems makes it particularly hard to interpret estimates from descriptive techniques; a theoretical framework is essential to begin to answer these questions.

This structure need not be very tightly specified, if the analyst can observe enough data. We will assume throughout that matching patterns ("who matches with whom") are observed, either in one large market or in many markets that share some characteristics. In some data sets, information about the dynamics of the matching process (e.g., on the various offers made by each agent) is also available. Sometimes, as in employment

[4]In a similar vein, researchers have used duration data models for instance to describe the increase in ages at marriage.

relationships, transfers between agents (here the wage) are also recorded. This is typically not the case in household economics, if only because transfers are mostly implicit (e.g., they operate through changes in the structure of household expenditures); however, in collective models of household behavior, transfers may often be at least partially identified when individual consumption or labor supply functions are observable. The ability to recover the intrahousehold distribution of resources and welfare indeed is a major asset of matching models in family economics.

In some settings the data also contain useful proxies for "match output" or "joint surplus." When students are matched with schools, for instance, their educational outcomes can often be observed. To some degree, wage increases or separations also give information on match output in employment relationships; and one could argue that divorces and children outcomes also proxy for marital output. Once again, observing such data allows the analyst to relax theoretical restrictions and therefore the specification of the model.

1.3 Empirical Approaches

Two broad types of empirical strategies have been followed so far in testing and estimating matching models. On the one hand, some approaches explicitly introduce a stochastic structure at the level of individual matches; a standard justification, that we shall discuss in more detail, is that the corresponding random terms reflect some unobserved heterogeneity among agents. These are interpreted as characteristics that all individuals observe, and along which they match, but which are not available to the econometrician. Obtaining a useful characterization of the solutions of a matching game explicitly involving random payoffs in its most general form is an extremely difficult problem. However, a complete characterization may obtain under additional hypotheses regarding the stochastic process.

In the TU framework, identification can be achieved under a separability property introduced by Choo-Siow (2006). Depending on the context, identification may obtain from data relative to a single market; or it may instead require the observation of several markets sharing structural characteristics, as discussed by Chiappori, Salanié and Weiss (2014) in the TU context and by Hsieh (2011) in the NTU framework. Chiappori, Oreffice and Quintana-Domeque (2012) have proposed a different approach, which reduces the individual variation in preferences for partners: all individuals agree on an "attractiveness index" that aggregates the traits of potential partners. All these issues will be considered below.

Alternatively, some work relies on regularity conditions that all stable matchings are assumed to satisfy. The rank-order properties introduced by Fox (2010a, 2010b) belong to that family. We will discuss the underlying theory, a related result by Graham (2011, 2013), and recent applications.

1.4 Scope of the survey

To keep our task manageable, we have had to make some difficult choices. Much of our discussion bears on one-to-one matching, where the link between theory and empirics is the most straightforward. We will not cover matching markets in which match output is observed, since the recent survey by Graham (2011) does this very well; we refer the reader to his section 3. On the other hand, we will explain how observed transfers can be taken into account.

A common feature of all the matching models we study is the absence of frictions: any participant in a matching game is supposed to have perfect information about all possible mates, even in large markets. This sharply contrasts with search models, in which frictions are explicitly modeled and play a key role. We see the vast literature on matching in a search context and in particular its more recent advances in labor economics[5] as complementary to the frictionless view. Each approach exhibits specific advantages and limitations, and the choice of one or another should primarily be driven by the nature of the issue under consideration. For instance, models aimed at explaining unemployment can hardly afford ignoring frictions; if, however, the key issue under investigation is the matching of firms and top executive, a frictionless benchmark may make perfect sense. Using the dynamic fea-

[5]In particular Postel-Vinay and Robin (2002) and the contributions that followed from it.

tures of search models is also more appealing when the data does indeed follow agents matching over time; as we will explain in our conclusion, in cross-sectional data the two approaches are essentially equivalent.

Clarifying the relationship between matching and search models from an explicitly empirical perspective, is an important challenge to be faced by future research. At any rate, we believe that a review of the recent literature on search would go far beyond the scope of the present survey. This literature, by its size, its scope, and its specificities, amply justifies an independent presentation.

In the next section, we give a brief exposition of bilateral matching, focusing on the elements of the theory that are necessary to approach empirical work. Section 3 gives an overview of the various empirical philosophies that have guided the contributors to this literature. We describe work on NTU models in section 4; and we move to TU models in section 5. We conclude this survey with some of the most important challenges empirical work on matching still faces.

2 Theoretical Background

2.1 Common structure

The theoretical frameworks that underpin most empirical work on matching share a few basic features. First, they consider bipartite matching. We start with two sets I and J of agents, whom we will refer to from now on as "men" and "women". Each of these sets can be endowed with a measure (resp. μ and ν), which can be discrete or continuous but must be finite. Any individual $i \in I$ may be matched with an individual $j \in J$ or remain single, and conversely; to accommodate single men, we add two dummy populations of null agents, denoted $\emptyset$, to J and I respectively, and we extend the probabilities μ and ν accordingly (so that a Dirac mass equal to the total mass of J is put on the dummy population added to I and conversely.)

It is important to note at this point that very few restrictions are imposed on the sets of male and female characteristics, I and J. In particular, we do *not* restrict their dimension to be one. Multidimensional matching received scant attention until recently, but this trend is now being reversed; we will cover some of the major contributions in this new direction.

Second, a *matching* defines who is matched with whom, or remains unmatched. Technically, a matching is defined by a measure η on the product space $I \times J$; intuitively, $\eta(i,j)$ is the '"probability" that Mr i is matched with Ms j. Obviously, such a measure must satisfy a feasibility constraint, reflecting the fact that any given individual can be matched to one person at most; formally, the marginals of η must therefore equal μ and ν respectively. If for instance the sets I and J are finite, the feasibility constraint for Mr i is simply

$$\sum_{j \in J} \eta(i,j) + \eta(i,\emptyset) = \mu(i). \tag{1}$$

Finally, the standard equilibrium concept is *stability.* We will return to its definition, which is slightly different in the TU and the NTU cases. Broadly speaking, a matching is stable if

(i) no matched individual would rather be single, and

(ii) no pair of individuals would *both* rather be matched together than remain in their current situation.

The stability concept therefore involves robustness against deviations by individuals and couples.

2.2 Bilateral matching under NTU

The last ingredient of a matching game are the payoffs; they are defined in quite different ways with or without transfers. We start with the NTU case. Here, matching Mr i with Ms j generates some utility for each of them; in other words, the game is defined by *two* exogenous functions, $U(i,j)$ and $V(i,j)$. By assumption, these utilities are *fixed*; agents are not able, through further trade, to increase a person's utility while reducing the others (what transfers would typically allow). They are primitives of the problem, and may in principle be econometrically recovered if the model is identified.

Stability has a very direct translation in the NTU context. Let $u(i)$ and $v(j)$ denote the utility levels respectively reached by Mr i and Ms j at a stable matching η. First, for any (i, j) belonging to the support of a stable matching η (i.e., for any man-woman pair who marries with positive probability), we require that:

$$u(i) = U(i, j) \text{ and } v(j) = V(i, j)$$

Moreover, stability requires that:

$$u(i) = \max_{k \in J} \{U(i, k) \mid V(i, k) \geq v(k)\}$$

and

$$v(j) = \max_{k \in I} \{V(k, j) \mid U(k, j) \geq u(k)\}.$$

The first equation, for instance, simply states that

$$\text{if } U(i, k) > u(i) = U(i, j), \text{ then } V(i, k) < v(k);$$

that is, any woman k whom i would strictly prefer to his current match j must be strictly better off in her current situation than if she were matched with i.

2.3 Bilateral matching under TU

Under TU, things are quite different. The primitive of the problem now is a *single* function, $s(i, j)$, usually called the (joint) surplus. The surplus generated by any matched couple must be shared between the spouses; however, this sharing is now *endogenous*, and is typically determined (or at least constrained) by the stability conditions. In practice, therefore, the matching game is defined by the two sets I and J, together with the associated measures μ and ν, and the function s.

A solution (a "matching") is now defined by a measure η on the product space *and* by two functions, $u(i)$ and $v(j)$, which describe the payoffs to partners. In particular, these functions are such that for any couple matched with positive probability—that is, for any (i, j) in the support of η—we have:

$$u(i) + v(j) = s(i, j)$$

Stability has a simple translation; namely, for any (not necessarily matched) pair (i, j), it must be that:

$$u(i) + v(j) \geq s(i, j)$$

Indeed, if we had $u(i) + v(j) < s(i, j)$ for some (i, j), then i and j could match together and split the surplus in a way that gives i more than $u(i)$ and j more than $v(j)$, contradicting the definition of u and v as equilibrium payoffs.

The main theoretical result for the TU framework is the well-known equivalence between stability and surplus maximization. Specifically, let us forget for a minute the notion of stability and consider the following problem: find a measure η on the product space $I \times J$ such that

(i) the marginals of η equal μ and ν respectively[6]

(ii) η maximizes total surplus

$$S = \int_{I \times J} s(i, j)\, d\eta(i, j).$$

Note that the problem just described is *linear* in its unknown, namely the measure η. This linear programming problem admits a dual program, which can be written as:

$$\min_{u,v} \left(\int_I u(i)\, d\mu(i) + \int_J v(j)\, d\nu(j) \right)$$

under the constraints

$$u(i) + v(j) \geq s(i, j) \quad \text{for all } i, j,$$

[6]That is, feasibility constraints like (1) must hold.

which are exactly the stability constraints stated above. It follows that if a stable match exists, then the corresponding measure maximizes total surplus; conversely, for any solution to the surplus maximization problem, one can find the functions u and v by simply solving the (linear) dual problem—which has a solution by standard duality results.

2.4 Bilateral matching under Imperfectly Transferable Utility (ITU)

Finally, the ITU case can be seen as a direct generalization of the TU case. The equation of the Pareto frontier generated by a couple (i, j) is no longer linear; therefore the surplus function, $s(i, j)$, is replaced with a function $F(i, j, v)$ that defines the maximum utility reachable by i when matched with j, if j receives (at least) a utility equal to v. Again, the intra-couple sharing (defined by the pair (u, v)) is endogenous, and is typically determined (or at least constrained) by the stability conditions. The matching game is thus defined by the two measurable sets I and J, together with the associated measures μ and ν, and the function F. As in the TU case, a solution (a "matching") is defined by a measure η on the product space and by two functions, $u(i)$ and $v(j)$. These functions are such that for any couple matched with positive probability—that is, for any (i, j) in to the support of η—we have:

$$u(i) = F(i, j, v(j))$$

As before, stability has a simple translation; namely, for any (not necessarily matched) pair (i, j), it must be that:

$$u(i) \geq F(i, j, v(j))$$

The main advantage of the ITU model is its generality. In particular, it does not imply that couples always behave like individuals—a property characteristic of the TU framework (see Section 1.1). The price to pay is that we lose the equivalence between stability and surplus maximization; in fact, the mere notion of aggregate surplus can no longer be defined in that context.

2.5 Hedonic models as matching models

Hedonic models study markets for goods and services that can be decomposed into a vector of attributes. An equilibrium then is characterized by a price function, which describes the relationship between the attributes of the good and the price at which it is traded (see for instance Heckman, Matzkin and Nesheim 2010.) The general structure of an hedonic model consists of three sets: a set I of "buyers" (together with a measure μ), a set J of "sellers" (together with a measure ν) and a set K of "products"[7]. Each product k has a price $P(k)$, which is endogenously determined, as are the matches between buyers and sellers. To put it concisely, equilibrium determines who buys what from whom, and at which price.

We assume that each buyer i has quasi linear preferences of the form $U(i,k) - P(k)$; similarly, seller j maximizes her profit $P(k) - c(j,k)$, where $c(j,\cdot)$ is a seller-specific cost function. An equilibrium is defined by a price function $P(k)$ such that when each buyer maximizes utility and each seller maximizes profit, market clearing obtains for all products in K. Technically, an equilibrium consists of a function P *and* a measure α on the product set $I \times J \times K$ such that

(i) the marginal of α on I (resp. J) coincides with μ (resp. ν), and

(ii) for all (i,j,k) in the support of α,

$$
\begin{aligned}
U(i,k) - P(k) &= \max_{k' \in K} \left(U\left(i,k'\right) - P\left(k'\right) \right) \\
\text{and} \quad P(k) - c(j,k) &= \max_{k' \in K} \left(P\left(k'\right) - c\left(j,k'\right) \right).
\end{aligned}
$$

In words, (i,j,k) belong to the support of α if, with positive probability, buyer i consumes product k and seller j supplies product k.

As shown by Chiappori, McCann and Nesheim (2010), there exists a canonical correspondance between hedonic models of the type just described

[7]Different interpretations are obviously possible; e.g. I could be the set of employers, J the set of employees, and K the set of characteristics of potential jobs.

and matching models under TU. Specifically, consider a hedonic model, and define the surplus function s by:

$$s(i,j) = \max_{k\in K}(U(i,k) - c(i,k))$$

Let η be the marginal of α over $I \times J$ and define $u\left(i\right)$ and $v\left(j\right)$ by

$$u\left(i\right) = \max_{k\in K}\left(U\left(i,k\right) - P\left(k\right)\right) \text{ and } v\left(j\right) = \max_{k\in K}\left(P\left(k\right) - c\left(j,k\right)\right)$$

Then one can readily check that (η, u, v) defines a stable matching for the matching problem defined on $I \times J$ by the surplus function s. Conversely, starting from a stable matching (η, u, v), we know that

$$u(i) + v(j) \geq s\left(i,j\right) \geq U\left(i,k\right) - c\left(j,k\right) \text{ for all } \left(i,j,k\right)$$

which implies that

$$c\left(j,k\right) + v\left(j\right) \geq U\left(i,k\right) - u(i) \text{ for all } \left(i,j,k\right).$$

Any $P\left(k\right)$ such that

$$\inf_{j\in J}\left\{c\left(j,k\right) + v\left(j\right)\right\} \geq P\left(k\right) \geq \sup_{i\in I}\left\{U\left(i,k\right) - u\left(i\right)\right\}$$

for all k is an equilibrium price function for the hedonic model.

The theory of (quasi-linear) hedonic models therefore has close ties with that of matching models under TU. From an empirical point of view, though, a key difference is that transfers between agents are much more likely to be observed in hedonic models, via the price function P.

3 The Econometrics of Matching: Introductory Remarks

Deterministic matching models tend to yield stark predictions, such as positive assortative matching. This obtains in models with one-dimensional characteristics when each agent's utility is increasing in the partner's type (in the NTU case) or when the joint surplus is supermodular (for TU models—see Becker 1973). If the relevant variable is, say, income, then the model predicts that the richest man will marry the richest woman, the second richest man will marry the second richest woman, and so on. While we do observe a positive correlation between spouses' income, perfect assortativeness of this type is of course counterfactual. Moreover, deterministic models predict that observationally equivalent agents should have identical matching outcomes—again not an empirically appealing feature.

To reconcile the highly restrictive predictions derived by this barebones matching model with reality, two paths can be followed. One is to invoke frictions. With imperfect and costly information and sequential meetings, the wealthiest woman may well settle for a man who is high enough in the income distribution, rather than wait for an hypothetical meeting with an even wealthier mate. In addition, the randomness of the meetings process guarantees that similar agents will have different types of partners in equilibrium. Following the seminal contribution of Shimer and Smith (2000), several authors have started to combine the search and the matching frameworks[8]; for lack of space, we shall not cover this work in this survey.

Alternatively, one may maintain the frictionless context but enrich the model by considering a multidimensional setting. While this is a first step towards realism, it is not enough: to accommodate the dispersion in matching outcomes of observationally equivalent agents, some of the relevant traits must be unobservable to the econometrician. In other words, agents also differ by some unobservable but matching-relevant heterogeneity, which is modeled as a stochastic term. All structural models we consider below follow this second direction, sometimes implicitly.

In all studies we know of (except some versions of hedonics models), the stochastic term enters additively. Take the NTU context first: the utilities generated by the matching of Mr i (with observable characteristics x_i) and

[8] See Jacquemet and Robin (2012) and Goussé (2014).

Ms j (with observable characteristics y_j) take the respective forms:

$$u_{ij} = U(x_i, y_j) + \varepsilon_{ij}^m \quad \text{and} \quad v_{ij} = V(x_i, y_j) + \varepsilon_{ij}^w \tag{2}$$

where the shocks $\varepsilon_{ij}^{m,w}$ reflect the impact of unobserved heterogeneity on match quality.

Similarly, structural TU approaches postulate that the joint surplus generated by such a match can be written as

$$s_{ij} = s(x_i, y_j) + \varepsilon_{ij}. \tag{3}$$

Different studies have imposed very contrasted assumptions on the joint distribution of these random terms. Some authors assume full independence; in the TU model for instance, ε_{ij} would be independent of x_i, of y_j, and of all ε_{kl} unless $i = k$ and $j = l$. Other postulate specific covariance structures. To understand the issues at stake, it is useful to consider a special but widely used example in which the observable traits x and y are discrete. Let for instance x (resp. y) denote the wife's (husband's) education, with values in some finite set. If say Mr i is particularly fond of educated partners, then the distribution of ε_{ij} will move to the right as y_j increases. If moreover such preferences are common among educated men, the distribution of ε_{ij} will also vary with x_i. By contrast, independence imposes that Mr i's idiosyncratic preferences cannot be related to any of the observable characteristics of her potential spouses. Of course, if independence is not imposed, then the *unconditional* correlation structure of the ε_{ij}'s may exhibit specific patterns. For instance, ε_{ij} and ε_{ik} will typically be correlated if i and k belong to the same education class; and the identification strategy will have to take this correlation structure into account.

Note that there are often strong theoretical arguments for *not* assuming independence. Consider, for instance the matching model of Chiappori, Iyigun and Weiss (2009), in which agents first invest in education and then match on the marriage market. Agents differ ex ante by two idiosyncratic characteristics, both unobservable to the econometrician: their willingness to marry and their cost of acquiring education. The authors show that both aspects influence educational choices. In particular, agents with a high preference for marriage are more likely to invest in education, since they are more likely to reap returns from their education on the marriage market (a better educated spouse, for instance.) In this context, modeling the marriage market under the assumption that idiosyncratic preferences are independent of education would be incorrect, since education is in fact endogenous to the realization of preferences for marriage.

Another important simplification that much (but not all of) this literature relies on is that the markets considered are large. One can show that in large matching markets, nothing is lost for inference[9] by looking only at the matching patterns conditional on the μ and ν's marginal distributions.[10]

In a nutshell, assumptions regarding the joint distribution of the random terms are far from innocuous; we shall explicitly discuss them in what follows. Testing between different stochastic specifications, however, is not an easy task. As we will see, some empirical work uses data on (one or several) large matching markets, while other work relies on exclusion restrictions across markets to identify the primitives of the model[11].

Sometimes the data restricts what can be done. For instance, it is often easier to get data on realized matches than on unmatched agents. Then only some patterns of the utilities or surplus can be identified. In the TU case for instance, it is easy to see that with data on realized matches only, we can only hope to identify the joint surplus up to a sum of an arbitrary function of the man's type and an arbitrary function of the woman's type. Intuitively, these functions describe the expected utility of marriage for the various types of agents; and it stands to reason that we need data on unmatched agents

[9]Menzel (2015b) gives a rigorous argument proving that under reasonable conditions, in large markets correlations across agents play a vanishing role in the likelihood function.

[10]In large markets, assuming independence of the match-specific random shocks has another drawback. If the distribution of the random shock is unbounded (as is usually assumed in empirical models), then, when the size of the market increases, the expected utility of any given individual tends to become very large and mostly driven by the stochastic component. The intuition is that utilities, in this context, are related to the *maximum* of the shocks over all possible partners.

[11]We shall not consider here the empirical contributions that rely on calibration or simulation, and do not offer an econometric analysis in the strict sense (for instance the recent work on CEO compensation by Gabaix-Landier 2008 and Edmans et al 2009.)

to identify them.

Before we proceed, it is important to mention that there is a wealth of descriptive empirical work by demographers and sociologists on assortative matching. Their empirical strategy is often based on the analysis of variance framework. They assume that men and women belong to a finite (and usually small) set of types; these can be income or age brackets, education levels or others. They define an index of homogamy H_{xy} for each pair of observed types (x, y) of men and women; and they run a regression of the form

$$H_{xy} = a_x + b_y + \xi_{xy},$$

where a_x and b_y can be flexible functions of observables relating to men of type x and to women of type y, respectively. Then they interpret the contributions to the explained variance of the a, b and ξ terms.

The problem with this approach is that it negates the equilibrium effects—what Choo and Siow (2006) call "spillover effects". Whether the relevant model allows for transfers or not, the number of matches between types x and y is a function of *all* proportions of types: $\mu(x)$ and $\nu(y)$ certainly, but also all $\mu(x')$ and $\nu(y')$ for $x \neq x', y \neq y'$. If for instance the proportion of men of a given type increases, it is likely to increase the proportion of single men of all similar types. As a consequence, these regressions omit many relevant variables and their results are very hard to interpret.[12] The key conclusion, here, is that since the equilibrium number of matches between types x and y depends in a very nonlinear and asymmetric manner on all $\mu(x')$ and $\nu(y')$, this methodological issue is not easy to solve: without a structural model, it is very hard to guess which of these many variables should be added to the regression, and how.

[12] Chiappori and Salanié (2014) give a telling example. They generate data from a Choo-Siow model for different distributions of men and women across the various types, keeping structural preferences for homogamy unchanged. ANOVA regressions on such data conclude that the contributions of the various terms to total variance have changed, which the applied literature typically (and mistakenly) interprets as a change in preferences for homogamy.

4 The NTU Case

A host of applied theory contributions have been devoted to matching problems in a NTU framework. Following Roth's (1984) seminal contribution, which studies the market for new doctors in the U.S., many papers have considered various markets for medical interns and residents, but also the allocation of children to public schools (Abdulkadiroglu and Sönmez 2003, Abdulkadiroglu, Pathak and Roth 2005, Abdulkadiroglu et al. 2005) or the organ exchange programs between hospitals (Ashlagi and Roth 2012), to name just a few. This body of literature often adopts a normative approach: it aims at constructing an algorithm that would solve the matching problem under specific requirements (stability, incentive compatibility, etc.). These contributions are not covered by our survey, which concentrates on the econometrics of matching.

Still, several recent articles explicitly address estimation and testing of a NTU matching model. Referring to the distinction made earlier between "structural" and "reduced form" approaches, these contributions generally follow the structural path. In this context, two types of econometric works can be found, depending on the data that are available. In some cases, the econometrician only observes the final matching (or the corresponding contingency table, indicating, for discrete characteristics, the size of the sample for each combination of male and female traits). However, some authors observe not only the final match, but also the entire dynamics of the matching game. For instance, data from online dating sites typically include the set of potential partners an individual has considered (by clicking on their file). Clearly, such data have a stronger empirical content; in particular, they may allow to directly estimate agents' preferences, independently of the realized match.

4.1 Direct identification of preferences

Hitsch, Hortacsu and Ariely (2010—from now on HHA) consider matching on online dating sites. This is a context in which an NTU approach makes sense, since the corresponding technology does not allow for transfers of any type. Specific to their data is the fact that they can observe not only the final matching, but also *all the potential partners whom any given agent contacts* ("clicks"). They rely on a search-and-matching model by Adachi (2003), in which agents optimally fix an "attractiveness threshold" and propose to (here click) all potential mates above the threshold. Adachi shows that the outcome of the search model converges to a stable matching when frictions vanish. Assuming away strategic behavior (e.g., not contacting one's favorite choice because (s)he is considered as "out of reach"), an agent's clicking strategy thus gives a direct indication of his/her preferences, which can be recovered using standard, discrete choice approaches.

In practice, HHA consider stochastic utilities of the type (2); in addition, they assume that

$$\varepsilon_{ij} = \alpha_i + \tilde{\varepsilon}_{ij} \text{ and } \eta_{ij} = \beta_j + \tilde{\eta}_{ij}$$

where α, β are individual fixed effects and the random shocks $\tilde{\varepsilon}_{ij}$ and $\tilde{\eta}_{ij}$ are iid and extreme value distributed. As explained earlier, this assumption of independence (across partners, and from observed characteristics) is very strong; but it makes it very easy to estimate the model, using fixed effects logit. Finally, having estimated preferences, HHA can apply a Gale-Shapley algorithm to recover the predicted, stable matchings. They find that the predicted matches are similar to the actual matches achieved by the dating site, and that the actual matches are approximately efficient.

Banerjee et al (2013) exploit matrimonial advertisements in a major Indian newspaper to study the relative importance of in-caste preferences and preferences for other characteristics. To do so, their paper imposes strong symmetry assumptions on preferences. They document a strong preference for in-caste marriage; interestingly, this does not seem to interfere much with preferences over education for instance. This may help explain the persistence of castes in India.

4.2 Using matching patterns only

Other approaches use data relative to realized matches only. In some cases, however, information is available about behavior, which can be directly used for the estimation process. One of the first contributions along this line is due to del Boca and Flinn (2014, from now on DBF.) The basic insight can be summarized as follows. DBF assume that household allocation decisions are made according to a rule which is exogenously given—in sharp contrast to a standard TU framework, in which sharing rules are an equilibrium outcome. DBF consider either non cooperative Nash equilibrium or the maximization of a weighted sum of individual utilities, with the weights as parameters to be empirically identified.

For any given rule, the observation of household behavior allows DBF to recover the parameters characterizing the preferences and (household) productivities of both spouses. In turn, this allows to construct preference orderings for each male over all possible females and conversely; note that these preferences are recovered *conditionally on the decision rule*, and entail a random component as described above. Finally, they apply the Gale and Shapley algorithm to determine a stable matching of the game thus defined, and compare the correspondence between predicted and observed matches using a likelihood-based metric, that can be used to determine the decision rule and the relevant parameter.

A different path is followed by Boyd et al. (2013), who analyze the matching of public school teachers to jobs over several years. Their approach is more explicitly structural. They start from stochastic utilities of the type (2) with a parametric representation of the deterministic components, and assume that the random shocks are independent. A first remark is that, for any random draw of the stochastic shocks, the Gale Shapley algorithm generates a stable matching, for which a set of descriptive statistics (in terms of correlations between attributes) can be computed. This suggests using a method of simulated moments in order to select the values of parameters that fit best the moments observed in the real data. While remarkably powerful, such simulation-based approaches are computationally cumbersome, and may become impractical when the number of players becomes large.

To circumvent the computational problem, Hsieh (2011) uses a modified version of the Gale -Shapley algorithm that allows to directly compute the contingency table of marriage types without explicitly solving for the stable matching. He considers a model in which women and men can be categorized into M and N types respectively: $x_i \in \{1, ..., M\}$ and $y_j \in \{1, ..., N\}$. As above, an individual's spousal preference is the sum of a deterministic term that depends on the agent's and the potential partner's type, and of a random term; but the latter only depends on the partner's type:

$$u_{ij} = V(x_i, y_j) + \varepsilon_i(y_j) \quad \text{and} \quad v_{ij} = V(x_i, y_j) + \eta_i(x_i)$$

In particular, each agent equally values his/her potential partners who belong to the same category.

Unlike in DBF, no behavior is observed; identification comes only from the observation of matching patterns. Hsieh shows that stability is not testable from the observation of a single market: any contingency table can be rationalized as a stable matching for well chosen functions. However, when we observe several markets with the same deterministic functions and the same stochastic distributions, stability generates testable predictions. Moreover, the model is parametrically identified.

4.3 Extensions

Agarwal (2014) notes that sometimes it is reasonable to assume that one side of the market is only vertially differentiated. For instance, in the resident matching program hospitals seem to agree on their ranking of potential residents. This considerably reduces the scope for deviations and makes an estimator based on pairwise stability conditions quite manageable. Agarwal uses this insight to criticize the argument that the resident matching program unfairly reduces the salaries of residents. He shows that low salaries can in fact be rationalized as the price residents pay for valuable training in the better hospitals.

Finally, a wide-ranging contribution by Menzel 2015a obtains very strong results on large matching markets. Following Dagsvik 2000, Menzel assumes that preferences on each side of the market are a function of observed characteristics and of an unobserved shock; for instance,

$$U_{ij} = U(x_i, y_j) + \zeta_{ij}$$

represents the utility man i derives from a match with woman j. In large markets, any man has many potential matches; and it is intuitively clear that the maximum of these utilities over willing partners will play an important role in the theory. Now in statistics, extreme value theory shows that the (properly standardized) maximum of a large number of independent and identically distributed random variable can only converge to one of three parameter-free distributions. One of these is the type I extreme value distribution that figures prominently in discrete choice econometrics, as well as in this survey.

Menzel assumes that the ζ's (and the corresponding shocks for women) are iid across i and j, and that the tail distribution of these shocks belongs to the class that yields convergence of the maximum to the type I EV distribution. He then takes the "very large market" limit[13]. He shows that in every stable matching, the number of matches between partners of characteristics x and y satisfies

$$\frac{\eta(x,y)}{\eta(x,0)\eta(0,y)} = \exp(U(x,y) + V(x,y)). \tag{4}$$

Menzel's analysis therefore yields a remarkably simple formula, that only relies on a fairly weak restriction on the form of the distribution of errors—but a strong one on their iid character. It also shows that the observed matches are informative only about the quasi-surplus[14] $(U+V)$. Data on the observed matches is not enough to separate the preferences U and V of both sides of the market. On the other hand, it is possible to compute expected utilities at a stable matching: in the large market limit, they coincide with

[13]This involves much work and technical details that we cannot discuss here.

[14]It may seem surprising that adding U and V makes sense in an NTU world; but note that the assumption that shocks are identically distributed for men and women introduces an implicit normalization of utility scales.

minus the logarithm of the share of unmatched agents, as in the Choo and Siow paper discussed in 5.1.

5 Matching under TU

The structural approaches to TU matching all consider surplus functions entailing an additive, random shock as in (3). Even under this restricted form, identifying both the function s and the distribution of ε_{ij} conditional on (x_i, y_j) is a difficult task. To understand why, consider an alternative model of marriage for a moment: we observe women j with observable characteristics y_j choosing men i with observable characteristics i; men have no say in the matter, and get zero surplus in their imposed marriage.

This alternative, one-sided specification is simply a discrete choice model in which woman j chooses a man i by maximizing $s(x_i, y_j) + \varepsilon_{ij}$. While this model is identified under theoretically reasonable conditions (Berry and Haile 2010), it is still a very hard model to estimate without strong parametric assumptions. The reason is simple: even if we assume that the ε terms are distributed independently of x_i and y_j, and identically and independently across women, if observable characteristics of men take M values then the joint distribution of the ε's is M-dimensional. Its variance-covariance matrix, for instance, has $M(M-1)/2-1$ degrees of freedom after the standard normalizations are applied.

The two-sided matching model is an order of magnitude more complex of course. From section 2.3, the individual utilities u_i and v_j associated with the stable matching solve the *coupling equations*:

$$u_i = \max_{j \in J} \left(s\left(x_i, y_j\right) + \varepsilon_{ij} - v_j \right)$$

and

$$v_j = \max_{i \in I} \left(s\left(x_i, y_j\right) + \varepsilon_{ij} - u_i \right).$$

Each of these two systems of equations has the structure of a one-sided choice model; but the key difference is that the choice models of men and of women are coupled through the equilibrium constructs u_i and v_j. This is a specific feature of the TU framework, as opposed to NTU models. This, along the fact that we cannot safely assume that the unobservable ε's are independent across i or j, greatly complicates identification.

Since fully nonparametric identification is out of the question, we must impose restrictions on the function s, which we will call the "mean surplus"[15], and on the conditional distribution of the unobservable ε's. A useful way of contrasting existing empirical approaches to matching models under TU is in whether they choose to impose stronger restrictions on s or on ε.

A first group of methods, pioneered by Choo and Siow (2006), does not impose any restriction on the mean surplus s; on the other hand, it restricts the distribution of the unobserved heterogeneity ε_{ij} and derives implications for matching patterns. By contrast, Fox (2010a) has proposed and used an approach which does not explicitly specify the distribution of the unobserved heterogeneity. Instead it directly postulates a "rank-order property" that imposes restrictions on the relationship between matching patterns and the surplus function.

Chiappori, Oreffice and Quintana-Domeque (2012) use a different semiparametric idea: they assume that the joint surplus has an "index" structure, in the sense that it is weakly separable into two one-dimensional functions of female and male characteristics respectively. As we will see, this allows them to run simple regressions to estimate the index, and to test the index assumption.

Finally, Fox, Hsu and Yang (2015) take the opposite approach to Choo and Siow: they restrict the specification of the mean surplus s in order to identify the distribution of the unobservables ε. The simplest version of their model assumes that the mean surplus s is observed for every possible match and that the ε's are distributed independently of s. They show that if the analyst observes many markets with different mean surplus but the same distribution for the unobservables, then the distribution of the complementarities across unobservables is identified.

[15] We use the term "mean" by analogy with discrete choice models, although, as we will see, s is not equal to the sum of average equilibrium utilities.

We now present these four approaches in more detail.

5.1 Separable Surplus

Remember that in equation (3), we associated to each potential match of a man i and a woman j a joint surplus

$$s_{ij} = s(x_i, y_j) + \varepsilon_{ij};$$

and we now normalize the mean surplus s to be zero for singles[16].

Now consider the stability problem from the perspective of aggregate surplus maximization. For each realization of the ε's, we have a well-posed maximization problem, which has a solution: a measure η which gives matching patterns, and associated dual variables u, v that describe equilibrium utilities[17]. These dual variables are now random, as for any given (x, y), different realizations of the ε's generate different dual variables. Knowing the distribution of the ε's, is it possible to infer the distribution of the dual functions u and v?

To the best of our knowledge, little is known about this question, except in a specific case, initially considered by Choo-Siow (2006), and on which much of the relevant literature is based. It relies on two crucial assumptions:[18]

1. The observable characteristics $x \in X$ and $y \in Y$ are discrete; they define a finite number of categories, each of which contains a continuum of individuals.

2. The surplus function is additively *separable* in the unobserved components of both partners. That is, for any match of a man i and a woman j, the surplus generated can be written

$$s(x_i, y_j) + \varepsilon_{ij} = s(x_i, y_j) + \alpha_i^{y_j} + \beta_j^{x_i}. \tag{5}$$

[16] Formally, we let $s(x, \emptyset) = s(\emptyset, y) = 0$ for all x and y.

[17] With finite populations, the solution η is generically unique but the u and v are not uniquely defined. In large markets they converge to a unique solution.

[18] In addition, Choo and Siow assume that the random terms follow an extreme value distribution. However, this assumption can be dispensed of—see below.

The separability assumption allows for any type of complementarity between observable characteristics; but it rules out any complementarity between unobservable characteristics. The term α_i^y, for instance, could reflect an idiosyncratic preference of man i for women of category y, or a preference of women of type y for man i, or some interaction in domestic production.

Chiappori, Salanié and Weiss (2014) showed that given these assumptions, there exists a decomposition

$$s(x,y) \equiv U(x,y) + V(x,y)$$

such that, at any stable matching a man i will be matched with a partner[19] whose category y maximizes $U(x_i,y)+\alpha_i^y$; moreover, i's utility at equilibrium is equal to the value of this maximum. To see this, remember the coupling equation

$$u(i) = \max_{j\in J}\left(s(x_i,y_j) + \alpha_i^{y_j} + \beta_j^{x_i} - v(j)\right);$$

and note that we can decompose symbolically

$$\max_{j\in J} = \max_{y\in Y} \max_{j \text{ s.t. } y_j=y}$$

(first choose the category of your partner, then a partner within that category.) We can pull out $\alpha_i^{y_j}$ from the maximization over j, to get

$$u(i) = \max_{y\in Y}\left(s(x_i,y) + \alpha_i^y + \max_{j \text{ s.t. } y_j=y}\left(\beta_j^{x_i} - v(j)\right)\right).$$

Now define

$$V(x_i,y) = \min_{j \text{ s.t. } y_j=y}\left(v(j) - \beta_j^{x_i}\right) \text{ and } U(x_i,y) = s(x_i,y) - V(x_i,y)$$

so that

$$u(i) = \max_{y\in Y}\left(U(x_i,y) + \alpha_i^y\right),$$

as announced; and by the same token, $v(j) = \max_{x\in X}\left(V(x,y_j) + \beta_j^x\right)$.

[19]Again, we are allowing for unmatched agents—then $y = \emptyset$.

This result is important for two reasons. First, it gives an exact characterization of the stochastic distribution of the dual variables, which are a major outcome of interest in many applications (including household economics.) Second, and from a more practical viewpoint, it leads to a simple characterization of individual choices that links them directly to standard, discrete choice models.

5.1.1 Identification

This setting can be analyzed from a general perspective, as in Galichon-Salanié (2014). Take a man i whose category is x; and suppose that the distribution of the vector of shocks (α_i^y) for all values of $y \in Y$ is P_x. Note that P_x is a multidimensional distribution, and that it may vary with x. Similarly, denote Q_y the distribution of the vector of shocks (β_j^x) for all values of $x \in X$. Galichon and Salanié prove that the distribution of matching patterns η solves a convex program

$$\max_{\eta} \left(\sum_{x,y} \eta(x,y) s(x,y) + \mathcal{E}(\eta) \right) \tag{6}$$

where the *generalized entropy* $\mathcal{E}$ is a concave function whose specific form depends on the distributions P_x and Q_y.

As a consequence the matching patterns η are linked to the unknown mean surplus s by a simple formula; and so is the distribution of utilities. Galichon and Salanié also provide an algorithm to solve for the optimal matching η that is very fast in leading examples.

Econometricians are in fact interested in the inverse problem: given the observation of the marginals μ and ν and the matching patterns η, what can we infer on the mean surplus s and the distributions P_x and Q_y?

First assume that we only observe data from one large matching market—say, marriages in America in 2013. Galichon and Salanié prove that for *any* possible choice of the distributions P_x and Q_y, the relationship between matching patterns η and mean surplus s is one-to-one; that is, any matching pattern can be rationalized by one and only one surplus function. In other terms, given exact knowledge of the P_x and Q_y, the mean surplus s is nonparametrically just identified.

Take for instance the most common assumption, namely that the α's and β's follow standard type-I extreme value distributions; then the choice of a spouse can be modeled using a standard multilogit model. This was the path followed by Choo-Siow (2006). Then the generalized entropy is the standard measure of entropy; and we obtain the formula

$$s(x,y) = \ln \frac{\eta(x,y)^2}{\eta(x,\emptyset)\,\eta(\emptyset,y)} \tag{7}$$

(remember that $\eta(x,\emptyset)$ is the probability of an unmatched man of type x.) This expression can also be written as:

$$\ln \eta(x,y) = \frac{1}{2}s(x,y) + \frac{1}{2}\ln \eta(x,\emptyset) + \frac{1}{2}\ln \eta(\emptyset,y) \tag{8}$$

Note the similarity with equation (4) of Menzel (2015a). The only difference is the “square” exponent—but that is an important one as it involves the scaling properties of the equilibrium. It is fair to say that at this stage, the underlying differences are not well-understood.

The multilogit model fixes all P_x and Q_y distributions to be one and the same: no heteroskedasticity, and no correlation in preferences over partners. For instance, it imposes both that the dispersion of preferences of men of type x be independent of x, and that the idiosyncratic preferences of any such man over women of types y and z be independent.

These are extremely strong assumptions; one of their consequences, as pointed out in Siow (2009), is that we can test for complementarities just by looking at the total positivity of observed matching patterns. Moreover, they imply specific predictions in terms of comparative statics of the model—see Decker et al (2012). As shown by Galichon and Salanié, only some of these predictions hold for all separable surplus functions.

Yet the results of Galichon-Salanié show that any more flexible choice of the distribution is underidentified from cross-sectional data, and the just identification result implies that we cannot test any assumption on the distributions P_x and Q_y. There are only three ways out of this dilemma:

1. we can use data on several markets, using well-chosen exclusion restrictions
2. we can impose parametric or semiparametric assumptions on the mean surplus s and the distributions P_x and Q_y
3. using data on transfers would also give useful information that allows the researcher to identify more features of the surplus function s and the distributions P's and Q's.

Using observations on transfers is discussed in Section 6. In section 5.1.5, we will describe how Chiappori, Salanié and Weiss (2014) combine the first two approaches to estimate and test a heteroskedastic model.

Finally, a recent contribution by Mourifié and Siow (2014) generalizes the basic Choo-Siow version by replacing (8) with:

$$\ln \eta (x, y) = \frac{1}{2} s (x, y) + a \ln \eta (x, \emptyset) + b \ln \eta (\emptyset, y)$$

where a and b are parameters that can be interpreted as representing peer and/or scale effects. The authors note that this formulation encompasses, as particular cases, not only the standard Choo-Siow framework (which correspond to $a = b = 1/2$), but also the Dagsvik-Menzel setting ($a = b = 1$), the heteroskedastic version of Chiappori, Salanié and Weiss (2014), and possibly other models as well. The price to pay is that the model has no natural, structural interpretation. For instance, it does not necessarily belong to either the TU or the NTU framework.

5.1.2 Sign-based Identification

The results by Galichon and Salanié (2014) nonparametrically identify the mean surplus when the distributions of the unobservables are separable and known. Now assuming perfect knowledge of the distribution of the unobservables is quite strong; what can we identify if we only make weaker assumptions? Graham (2011, 2013) gives such a result under separability, using only information on matches. His Theorem 4.1 shows that if the analyst only assumes that unobservables are independently and identically distributed, then the sign of the complementarities

$$C(x, y, x', y') = s(x, y) + s(x', y') - s(x, y') - s(x', y) \tag{9}$$

is identified.

To understand Graham's result, it is useful to consider the simpler discrete choice model in which man i of type x chooses among women of type y by maximizing

$$U(x, y) + \alpha_i^y.$$

Now if the α_i^y are iid over y, the monotonicity result of Manski (1975) shows that the probability $\eta(x, y)$ that such a man chooses a woman of type y is an increasing function of the mean utility $U(x, y)$. In particular, the sign of $U(x, y') - U(x, y)$ is that of $\eta(x, y') - \eta(x, y)$ and is therefore identified.

The same argument applies to women, with $V(x, y) = s(x, y) - U(x, y)$; and expanding the complementarity in (9), it is easy to see that if

$$\min\left(\eta(x, y), \eta(x', y')\right) > \max\left(\eta(x', y), \eta(x, y')\right)$$

then the complementarity $C(x, y, x', y')$ is positive. Graham's more refined result follows from similar arguments.

While identifying the sign of the complementarities may not sound that exciting, complementarities lie at the heart of matching as we have known since Becker (1973). It also gives a rigorous foundation for Fox's rank-order based approach (2010a), which we describe in section 5.2. The price to pay is the assumption that unobserved heterogeneities are identically and independently distributed, which seems very strong.

5.1.3 Continuous Separable Models

The general framework of separable models described above need not be restricted to discrete characteristics such as diploma or race. In fact, the general ideas in Galichon and Salanié (2014) can be extended to continuously distributed characteristics like income. Several technical difficulties must be faced, however. The first, obvious one is that standard discrete choice models such as the multinomial logit behave badly when taken to the continuous limit. Suppose for instance that mean utilities (U_j) are iid; then if J is a continuous set, $\max_{j \in J} (U_j + \varepsilon_j)$ is infinite unless the (ε_j)'s have a bounded support.

A first way of solving this problem, pioneered by Dagsvik (1994), is to assume that each agent is only confronted by a countable set of choices—technically, a random draw from a Poisson process whose intensity is governed by a standard type-I extreme value distribution. Dupuy and Galichon (2014) show how this can be extended to the matching framework. In this setting, each woman for instance can only match with men whom she met; and this meeting process is random and governed by a Poisson process, much as it is in the job search literature. The great appeal of this approach is that the probability density that a woman of type y matches with a man of type x is a direct continuous analog of the logit formula:

$$\eta(x|y) = \frac{\exp(U(x,y))}{\int \exp(U(z,y))dz}.$$

Dupuy and Galichon also describe a version of this model in which the mean joint surplus of an (x, y) pair is bilinear in types:

$$\Phi(x, y) = x'Ay$$

where A is an "affinity matrix." This yields very simple formulæ and a direct way of selecting relevant characteristics by a singular value decomposition of A.

One can also restrict the stochastic specification of joint surplus so that the infinity problem disappears. To take a very simple example, suppose that x and y are the incomes of man i and woman j. Men and women have different "bliss points" for the income gap within the couple. This could be described by a joint surplus like

$$-a(x-y-\alpha_i)^2 - b(x-y-\beta_j)^2,$$

which allows for variation in the intensity of preferences across genders, and for idiosyncratic variation in the bliss points. This is clearly separable, since there is no interaction term between the α's and β's. Moreover, the joint surplus is bounded above and so must expected utilities in every matching. Chiappori, Galichon and Salanié (in progress) studies the conditions under which the discrete separable approach can be extended to continuous characteristics.[20]

5.1.4 ITU Models

The techniques just described may in some cases be extended to the ITU case. While such an extension raises specific difficulties, due in particular to the non linearities introduced by the ITU framework, these difficulties are not insuperable; see Galichon, Kominers and Weber (2014) for recent advances on this topic.

5.1.5 Empirical applications and results

In the paper that introduced separable matching, Choo-Siow (2006a) explored the welfare effects of the 1973 legalization of abortion in the US. Given their specification, the change in the expected utility of men of age x is directly related to the change in their probability of staying single. Choo and Siow rely on the variation on the legal status of abortion across US states before 1973 to obtain a difference-of-differences estimate of the welfare effects. As expected, these effects are concentrated on women of childbearing age and the men who marry them. They estimate that 20 to 30 percent of the observed fall in the marriage rates of young men can be attributed to Roe vs Wade; and that it also contributed to delaying marriage.

[20] Finally, a continuum economy can also be represented as the limit of a discrete economy. This is done in a non-separable NTU setting by Menzel (2015a), which we discussed in Section 4.3.

Choo-Siow (2006b) used the same framework, augmented to allow for cohabitation, to examine the effects of the baby boom in Canada. They find that men benefited from the baby boom, while women did not. The baby boom was gender-neutral in its effect on birth rates; but preferences of men and women differ, on age at marriage in particular. Women prefer earlier marriage, while men can more easily wait. The baby boom benefited men born in its early years by making young women more plentiful. Choo and Siow's estimates confirm that the baby boom increased the net gains of marriage to men born between 1940 and 1955 and lowered them for women born before 1960.

The simplicity of the Choo and Siow model, in its "logit" implementation, yields predictions that are very stark. For instance, in their setting, just as in the original Becker model, positive assortative matching is exactly equivalent to positive complementarity in the joint surplus. Siow (2009) exploits this property to provide strong evidence for educational complementarities in surplus, especially in cities.

Several papers have gone beyond the separable logit. Galichon and Salanié (2014) revisit Choo and Siow's results using their more flexible approach; in particular, they estimate a model with random coefficients that has quite different comparative statics outcomes than the Choo and Siow model. Chiappori, Salanié and Weiss (2014) pool data from thirty cohorts of American men and women in order to explore changes in the returns to education on the marriage market. By restricting the variation in the joint surplus across cohorts, they are able to identify the changes in the "marital college premium" over the post-WWII period. They find that having a college (or higher) education has benefited women more and more over time, partly by reducing the likelihood that they stay single and partly by improving their bargaining position within the couples they form.

Finally, Dupuy-Galichon (2014) applied their model of matching on continuous types to data from the DNB Household Survey. They focussed on several groups of characteristics: education, health, physical measurements, and personality traits (the "big five", and attitudes towards risk.) Their estimates and tests on the affinity matrix show that sorting occurs on several dimensions: personality traits matter as well as education. Preferences over personality traits vary across genders, and the matching patterns of individuals with different traits also differ markedly.

5.2 The Rank-order Approach

The results obtained by Choo and Siow (2006a) and their subsequent extension by Galichon and Salanié (2012) rely on a parametric specification of the stochastic terms in the joint surplus, while possibly leaving the non-stochastic part fully flexible. By the standards of the semiparametric literature, this is an unusual modeling choice: much of semiparametric econometric theory has dedicated itself to relaxing assumptions on the distribution of error terms. Several important papers have followed this alternative route. The most influential approach is that proposed by Jeremy Fox, which he has articulated in several papers (Fox 2010a,b, Fox and Bajari 2013.)

A common feature of these papers is that they rely on a "rank-order property;" but this property differs across papers. For notational simplicity, we will continue to confine our discussion to one-to-one, bipartite matching—the marriage problem. It is worth stressing here that because Fox's approach only relies on pairwise stability, it can be applied more widely, to many-to-one or even many-to-many matching.

Unlike Choo-Siow (2006a), who work with one large market, Fox (2010a) seeks to identify the surplus function s by comparing matchings from a collection of independent finite-size markets. The term "independent" here means that individuals cannot match across market boundaries. Fox assumes that the mean surplus function $s(x, y)$ is the same in all of these markets; his aim is to recover estimates of $s(x, y)$ while imposing few restrictions on the stochastic part.

Denote C^n the list of observable characteristics on market n, and A^n the observed matching (the list of (x_i, y_j) matches and of single men and women.) If on each market the observed matching is stable given the list of characteristics on that market, it follows that A^n maximizes the total surplus given the constraints imposed by the draw of observable characteristics C^n and of unobserved characteristics ε on market n. Now take any possible list of observed characteristics C. Given a large number of markets, there will be many whose list of characteristics is close to C—with very similar numbers of white college-educated men, of Hispanic high-school graduate women, etc. The data therefore identify $\Pr(A|C)$, the probability that such a market has given matching patterns over observable characteristics—say, a given number of marriages between a white college-educated man and a Hispanic high-school graduate woman. Note that the reason that $\Pr(A|C)$ is not concentrated over one particular matching is that even markets with exactly the same distribution of observed characteristics have different distributions of unobserved characteristics. This is a consequence of the assumption that each of these markets is "small."

Now the knowledge of $\Pr(A|C)$ is clearly not enough to completely characterize the joint surplus function. But sometimes we are only interested in those of its features that only depend on observed characteristics, that is in the mean surplus $s(x, y)$. Suppose that for given C, $\Pr(A|C)$ is an increasing function of $\sum_{(i,j)\in A} s(x_i, y_j)$, so that

$$\Pr(A_1|C) > \Pr(A_2|C) \text{ iff } \sum_{(i,j)\in A_1} s(x_i, y_j) > \sum_{(i,j)\in A_2} s(x_i, y_j).$$

Fox (2010a) shows that under this *rank-order property*, if the distribution of observed characteristics has continuous support then comparing markets with the same C but different A identifies the function s up to a monotonic transformation.

The rank-order property at first seems to be a natural extension of the monotonicity in the usual single-agent discrete choice models. Such models have a 0-1 variable y determined by whether some index $F(x)$ is large enough:

$$y = 1 \text{ iff } F(x) > \varepsilon.$$

If the distribution of ε is independent of x, then clearly $\Pr(y = 1|x)$ is an increasing function of $F(x)$.

Unfortunately, such monotonic behavior is rare in matching models: the intution above just does not carry over to two-sided markets. Fox (2010a) argues, on the basis of simulations, that the rank-order property holds approximately if ε_{ij} is iid across matches (i, j). But take separable models for instance: the rank-order property only applies if we can neglect the generalized entropy term in equation (6), so that the optimal matching maximizes $\sum_{x,y} \eta(x, y)s(x, y)$. Since the entropy term scales like the dispersion of the unobserved heterogeneity term, the rank-order property can only be a good approximation if these "fixed effects" are negligible.

When it applies, the rank-order property lends itself very well to an estimation approach based on inequalities. Suppose for instance that the matchings A_1 and A_2 above only differ in that the partners w and w' of two men m and m' have been switched. Then the rank-order property, if it holds, implies that

$$\Pr(A_1|C) > \Pr(A_2|C) \text{ iff } s(x_i, y_j) + s(x_{i'}, y_{j'}) > s(x_{i'}, y_j) + s(x_i, y_{j'}).$$

Any such inequality generates information on the function s; given enough such inequalities, the function s can be identified. Moreover, its unknown parameters β_0 can be estimated using maximum score (Manski 1975.) To see this, take any 4-tuple (x, y, x', y'). The rank-order property implies that we are more likely to observe matches of x with y and of x' with y' if

$$C(x, y, x', y', \beta_0) \equiv s(x, y, \beta_0) + s(x', y', \beta_0) - s(x, y', \beta_0) - s(x', y, \beta_0)$$

is positive. Now consider the following objective function:

$$F(\beta) = \sum_n \sum_{i \neq j} \mathbb{1}\left(C(x^i, y^i, x^j, y^j, \beta) > 0\right),$$

where the sums extend over the different markets n and over the matches $i = (x^i, y^i)$ and $j = (x^j, y^j)$ observed in each market. The set of values of β that maximize this *score function* converges to β_0 as the number of markets becomes large. The function F is discontinuous, but the score can be smoothed (Horowitz 1992.) Moreover, the analyst need not use all matches i and j on each market; she only needs to select enough of them that a unique estimate of β_0 is obtained.

Note that the validity of a maximum score estimator only requires that the sign of the complementarities C in the mean surplus s can be inferred from the matching patterns $\eta(x, y)$. Graham's (2013) result of section 5.1.2 in particular shows that this holds in separable models when the distributions of heterogeneity terms are iid for each gender; this gives a primitive justification for the large market rank order property in both Fox and Bajari (2013) and Fox (2010b), and therefore a rigorous justification for the use of a maximum score approach in these context.

Fox-Bajari (2013) also propose a maximum score estimator based on a rank-order property. Their paper, however, assumes data on only one large matching market. Their version of the rank-order property is that of Fox (2010b, section 3.1). For simplicity again, assume as in section 5.1 that the observed characteristics x and y can only take a finite number of values, and denote $\eta(x, y)$ the number of matches between men of type x and women of type y on the market. The "one large market" version of the rank-order property states that

$$s(x, y) + s(x', y') > s(x, y') + s(x', y)$$

if and only if

$$\eta(x, y)\eta(x', y') > \eta(x, y')\eta(x', y).$$

Unlike the "many small markets" version, this rank-order property can be derived from more primitive assumptions. For simplicity, assume that x and y are continuously distributed one-dimensional attributes. Then it is easy to see that the mean surplus functions s that rationalize observed matching patterns η satisfy

$$\frac{\partial^2 s}{\partial x \partial y}(x, y) = F\left(\frac{\partial^2 \log \eta}{\partial x \partial y}(x, y)\right) \tag{10}$$

for some increasing function F such that $F(0) = 0$. In particular, choosing $F(x) = 2x$ gives

$$s(x, y) = 2 \log \eta(x, y) + a(x) + b(y);$$

but as we saw in section 5.1, this is precisely the equation that identifies the surplus in the Choo and Siow (2006) specification. Therefore the "one large market" rank-order property applies in the Choo and Siow model. We do not know which other primitive assumptions yield (10); it does not apply in the heteroskedastic Choo and Siow specification of Chiappori, Salanié and Weiss (2014) for instance.

Finally, it bears repeating that the techniques described in this subsection can readily be extended to many to-one or many-to-many matching. Fox and Bajari 2013 illustrates the former by estimating a structural model of the FCC spectrum auctions; and Fox 2010b takes this approach to the (many-to-many) relationships between car producers and seller of car parts.

5.3 Index-based Approaches

One approach investigates the conditions under which such a model can actually be analyzed using one-dimensional tools. This is the case, for instance, if the various components of x and y only enter payoffs through one-dimensional indices, $A(x)$ and $B(y)$. Such a property in turn has empirically testable consequences that are analyzed by Chiappori, Oreffice and Quintana-Domeque (2012, from now on COQ). Specifically, they consider a model in which each potential wife, say $i \in I$, is characterized by a vector $x_i = \left(x_i^1, ..., x_i^K\right) \in R^K$ of observable characteristics, and by some vector of unobservable characteristics $\varepsilon_i \in R^N$; similarly, man $j \in J$ is defined by a vector of observable variables $y_j = \left(y_j^1, ..., y_j^L\right) \in R^L$ and some unobservable characteristics $\nu_j \in R^N$, where the random components ν and ε are drawn from continuous and atomless distributions.

COQ assume that both the surplus function and the distribution of unobservables only depends on the observable characteristics through two one-dimensional indices (one for men and one for women). An immediate consequence is that whenever two males, j and j', have different vectors of observable characteristics but the same index, their spouses must be drawn from the same distribution; technically, there exists, for each gender, an (unknown) function of observable characteristics that is a sufficient statistic for the spouse's distribution. COQ show that the index can be (ordinally) identified from data on matching patterns; moreover, the index assumption can be tested non parametrically. Note that this approach can be generalized beyond the TU framework; it applies to NTU or ITU models, and also to search frameworks.

5.4 Identifying Complementarities on Unobservables

Fox, Hsu and Yang (2015) suggest an approach to identification in marriage markets that in some sense is the polar opposite of that of Choo and Siow (2006): instead of restricting the distribution of errors to learn aboaut the mean surplus, they restrict the mean surplus function and they go after the distribution of the unobservables ε.

While Fox et al discuss several specifications, all of them rely on the analyst observing many small markets in which the mean surplus function is identical. Their baseline result (which is relaxed later in the paper) in fact assumes that the mean surplus of every possible match is observed by the analyst. Therefore the joint surplus of a match between a man i and a woman j is

$$S(i,j)+\varepsilon_{ij}$$

and the function S is known, while the ε's are drawn from an unknown distribution G. All markets have N men and N women, and all agents must be matched on each market. Markets share the same distribution G, but each has its own draw of the ε's; and each market has a different function S.

A simple way of representing this problem is to write the surpluses of all possible matches in a given market as an $N\times N$ matrix of the form $(S+E)$. Then the optimal matching is the bistochastic matrix[21] Δ that maximizes the total joint surplus

$$\sum_{i,j=1}^{N}\Delta_{ij}(S_{ij}+E_{ij})=\mathrm{Tr}\left(\Delta'(S+E)\right).$$

[21] A bistochastic matrix has non-negative elements, and each row and each column sum up to one.

The data identify the distribution of Δ conditional on S, which is driven by the unknown distribution G of E. Fox et al show that if there are enough markets that all possible matrices S are represented, then the distribution G is identified.

This is a remarkable result, but it requires very strong assumptions. Moreover, its implementation is likely to be very challenging as the number of markets required to get a reliable estimate of a non-restricted distribution G will probably be very large. Nevertheless, combining these ideas with those in section 5.1 seems to be a very promising research direction.

6 Observable surplus or transfers

In all models discussed so far, we constantly assumed that the econometrician can only observe matching patterns. Quite obviously, this restriction strongly limits the scope of any empirical work. It is easy to get a first intuition of these limitations. Forget unobservable characteristics for a moment, and assume one characteristic only matters for matching; assume moreover that we observe a perfectly assortative matching. From this observation, we can certainly infer that the surplus function is supermodular. But, conversely, any supermodular function would generate the same matching; therefore, matching patterns tell us exactly nothing about the precise nature of the surplus function within the (rather large) set of supermodular mappings. While this example is highly specific, it conveys the main message: when only matching patterns are observed, we should either limit our expectations to very partial identification (e.g., sign-based identification), or be willing to accept strong (and probably parametric) assumptions.

6.1 Hedonic models

The situation is much more favorable when other aspects of the matching outcomes are observed as well. In some cases, for instance, the equilibrium transfers are also observable. A typical example is provided by hedonic models (which are canonically equivalent to matching models, as discussed above). Indeed, available data generally include not only matching patterns (which buyer purchases which product from which seller), but also equilibrium prices. In that case, one could expect much stronger identification results to obtain.

The econometrics of hedonic models has until recently concentrated on a very specific model, initially analyzed by Tinbergen (1956). For expositional purposes, it is useful to briefly recall its main characteristics[22]. Buyers' preferences and producers' profits are quadratic:

$$\begin{aligned} U(x,z) &= xz - \frac{a}{2}z^2 - P(z) \\ \Pi(y,z) &= P(z) - yz - \frac{b}{2}z^2 \end{aligned}$$

where $P(z)$ is the price of product z, a, b are parameters, and x and y are the buyer's and seller's respective idiosyncratic characteristic; the latter are assumed one dimensional and normally distributed with respective mean and variance $\left(m_a, \sigma_a^2\right), a = x, y$. In that case, one can show that matching is negative assortative:

$$x = \alpha y + \beta, \alpha < 0$$

and the equilibrium price is also quadratic in z:

$$P(z) = p_1 z + \frac{p_2}{2} z^2$$

where α, β, p_1, p_2 are parameters that can in principle be econometrically identified. The identification problem, in that case, boils down to the following question: can we, from the sole observation of matching patterns and prices (that is, α, β, p_1, p_2), recover the parameters of the model (two means, two variances and the structural parameters a and b)? The answer is clearly negative: equilibrium prices, together with matching patterns, provide four equations which cannot pin down the six unknowns.[23] Moreover, the identification process, initially suggested by Rosen (1974) and discussed by Brown and Rosen (1982), raises difficult endogeneity problems.

[22]Our presentation exactly follows that of Ekeland, Heckman and Nesheim (2002).

[23]In fact, one of the variances can be normalized to one, but one of the equations is redundant, and the indetermination remains.

Recent advances, however, have shown that the normal-quadratic example is misleading: the non identification result that characterizes that case is highly specific. The intuition is that, in general, we can observe much more than the mean and (co)variances of the distributions at stake. That a distribution can be entirely defined by these few parameters (as is the case under normality assumptions) is a highly peculiar ('non generic') case. In general, we should think of the identification problem in non parametric terms: both the observables and the unknowns are functions and distributions. In a systematic analysis, Ekeland, Heckman and Nesheim (2004) actually show that, under the assumption of additive separability of the random component, hedonic models of this kind are non parametrically identified in a generic sense. The extension of these results to the non separable case is discussed in Heckman, Matzkin and Nesheim (2010) and Chernozhukov, Galichon and Henry (2014). Recently, Nesheim (2013) has proposed a multidimensional extension of these results.

6.2 Observable behavior

In the (many) cases in which transfers are not observed, additional identification power can in principle be gained by observing the behavior of the matched partners. While this approach has not yet been fully developed, we describe here a recent contribution of Chiappori, Costa-Dias and Meghir (2015). They consider a model in which agents first invest in education, then enter the marriage market and match based on their human capital; during a third stage ("productive life"), they consume, save and supply labor. Gains from marriage arise from two sources: the joint consumption of a public good and the sharing of the risk generated by random shocks affecting wages and human capital. The authors adopt a TU framework; therefore, once married, the couple maximizes the sum of individual utilities. This makes it possible to use standard, dynamic models of savings and labor supply. The main parameters (and in particular the distribution of individual preference parameters) can therefore be estimated from labor supply. Crucially, this implies that the value of the surplus generated by each type of marriage can be directly recovered from observed behavior.

The analysis of the matching stage, in turn, determines the equilibrium allocation of the surplus between spouses; ultimately, this pins down the return on investments in human capital, which can be used to estimate the first stage.

The model is estimated using the British HPS. From a technical viewpoint, a parametric version can be identified using moment estimators. Unlike the Choo-Siow framework, however, the model is testable, because the matching patterns must be compatible with the surplus estimations derived from the analysis of savings and labor supply. Clearly, more work is needed in this promising direction.

Conclusion

Although the empirical literature on frictionless matching has made spectacular progress over the last decade, many questions remain open. For one thing, the majority of existing applications consider a model of one-to-one, bilateral matching; but there is a host of other types of matching models ripe for empirical analysis. These include "roommate" problems, in which people matching to form a pair belong to the same population[24]; many-to-one and many-to-many matching, which bring up thorny theoretical issues[25]; and more general approaches of the "matching in contracts" type (Hatfield and Milgrom 2005), which unify TU and NTU at the theoretical level but have yet to be taken to data in a rigorous way.

Another set of open issues are related to what could be called "prematching investments". In the analysis described above, the distributions of agents' characteristics were considered as exogenously given. In practice, however, characteristics on which agents match are often the product of some investment decision that was made before the matching game. The (prospective) outcome of the future matching game then typically influences the investment decisions. For instance, when choosing a level of education, or more generally deciding on a human capital investment, agents presum-

[24]See Chiappori, Galichon and Salanié (2013) for a recent investigation.
[25]See Fox and Bajari 2013 and Fox 2010b.

ably consider all returns to this investement, including those perceived on the marriage market. After all, a higher stock of human capital may affect a person's marriage probability and the "quality" (say, the education or income) of their potential spouses—but also the size of the surplus generated and its allocation between spouses. This intuition is formalized by Chiappori, Iyigun and Weiss (2009), who argue that features of the marriage market can explain why investments in higher education has been remarkably asymmetric between genders over the recent decades; Chiappori, Salanié and Weiss (2013) provide an empirical investigation of this view. In the same vein, Low (2014) argues that a shift in the nature of matching equilibria, itself generated by structural changes in both desired fertility and returns to human capital investments, has dramatically reduced the cost of female investement in higher education. While reduced form evidence supports these views, structural estimates (possibly in the direction initiated by Chiappori, Costas Dias and Meghir 2015) still need to be developed.

A last and promising avenue for future investigation is the relationship between models based on frictionless matching and models related to search approaches. We argued in our Introduction that given only data about "who matches with whom", it will be hard to distinguish the predictions of matching models with unobserved heterogeneity and those of matching models with frictions. To be more precise, take the pioneering Shimer–Smith (2000) model. This is somehat different from our framework in that the two sides of the market are treated symmetrically; but it makes our points more transparently.

Shimer and Smith describe a market in which types $i, j \in [0, 1]$ meet randomly, consider how they can share their surplus $s(i, j)$, and decide to match or to wait for a better partner. The primitives are the distribution of types μ and the joint surplus function s, along with discount rate r, rate of random meetings ρ, and rate of destruction of matches δ. Given various restrictions, there is a unique steady state equilibrium; this yields numbers of singles $\eta(i, 0)$ and matching patterns $\eta(i, j)$. Now let the econometrician observe some aggregate matching patterns, say

$$\tilde{\eta}(x,0) = \int \mathbb{1}(x_i = x) d\mu(i) \text{ and } \tilde{\eta}(x,y) = \int \mathbb{1}(x_i = x, x_j = y) d\eta\,(i,j)\,.$$

The econometrician could neglect frictions and fit a Choo and Siow model to this data, with separable unobserved heterogeneity of the type I EV form. We know from Section 5 that this model is just identified nonparametrically; that is, the econometrician could rationalize the data perfectly with a model of the form

$$s(x,y) + \alpha_i^y + \beta_j^x$$

with

$$s(x,y) = \log \frac{\tilde{\eta}(x,y)^2}{\tilde{\eta}(x,0)\tilde{\eta}(y,0)}.$$

Conversely, can data generated by a model of frictionless matching with unobserved heterogeneity also be rationalized by a model of matching with frictions but no unobserved heterogeneity? In the Shimer–Smith model, the answer turns out to be negative. Take a type i. When unmatched, she will meet all other types with a probability proportional to their frequency in the unmatched population. This applies in particular to all types that belong to i's "matching set" (that is, types that are acceptable to her.) Take two such types j and k; then in the notation above,

$$\frac{\eta(i,j)}{\eta(i,k)} = \frac{\eta(j,0)}{\eta(k,0)}.$$

With frictions, matching sets are nondegenerate; and given data that is disaggregated enough, the equality above will impose restrictions on the observed matching patterns. It follows that the model of Choo and Siow can rationalize matching patterns that are inconsistent with the framework of Shimer and Smith.

This can be seen as a positive or a negative, depending on one's inclination. In any case, it is very specific to assumptions that Shimer and Smith make mostly for simplicity. If for instance meetings were driven by directed search, the relationship above would break. There are many variants of models with frictions and it is hard to make a general statement. We conjecture, however, that given only data about matching patterns in a cross-section, it is impossible to distinguish between models with frictions and models with unobserved heterogeneity. Explaining the dispersion of observed outcomes by frictions, in that sense, is an a priori not a data-driven choice.[26]

[26] Of course, models with frictions are most often used in settings in which a time dimension is available and/or transfers can be observed. On labor markets for instance, wage transitions and the dynamics of employment offer rich information to identify the model. A recent contribution by Hagedorn et al (2014) shows how all components of the Shimer-Smith model can be identified, assuming that the surplus function is constant over time.

References

Abdulkadiroglu, Atila and Tayfun Sönmez. 2003. "School Choice: A Mechanism Design Approach." *American Economic Review* 93, 729–47.

Abdulkadiroglu, Atila, Parag Pathak and Alvin Roth. 2005. "The New York City High School Match." *American Economic Review* 95, 364–67.

Abdulkadiroglu, Atila, Parag Pathak, Alvin Roth and Tayfun Sönmez. 2005. "The Boston Public School Match." *American Economic Review* 95, 368–71.

Adachi, Hiroyuki. 2003. "A Search Model of Two-Sided Matching under Nontransferable Utility.". *Journal of Economic Theory*, 113, 182–98.

Agarwal, Nikhil. 2014. "An Empirical Model of the Medical Match". Mimeo MIT.

Ashlagi, Itai and Alvin Roth. 2012. New Challenges in Multi-hospital Kidney Exchange. *American Economic Review* 102, 354–359.

Banerjee, Abhijit, Esther Duflo, Maitreesh Ghatak and Jeanne Lafortune. 2013. "Marry For What? Caste and Mate Selection in Modern India". *American Economic Journal: Microeconomics*, 5, 33—72.

Becker, Gary. 1973. "A Theory of Marriage: Part I". *Journal of Political Economy*, 81, 813–846.

Becker, Gary. 1974. "A Theory of Marriage: Part II". *Journal of Political Economy*, 82, S11–S26.

Bergstrom, Theodore and Richard Cornes. 1983. "Independence of Allocative Efficiency from Distribution in the Theory of Public Goods". *Econometrica*, 51, 1753–1765.

Bergstrom, Theodore and Hal Varian. 1985. "When do market games have transferable utility?". *Journal of Economic Theory* 35, 222-33.

Berry, Steven and Philip Haile. 2010. "Nonparametric Identification of Multinomial Choice Demand Models with Heterogenous Consumers", Cowles DP 1718.

Botticini, Maristella and Aloysius Siow. 2008. "Are there Increasing Returns in Marriage Markets?," University of Toronto mimeo.

Boyd, Donald, Hamilton Lankford, Susanna Loeb and James Wyckoff. 2013. "Analyzing the Determinants of the Matching of Public School Teachers to Jobs: Estimating Compensating Differentials in Imperfect Labor Markets". *Journal of Labor Economics*, 31, 83–117.

Brown, James and Harvey Rosen. 1982. "On the Estimation of Structural Hedonic Price Models." *Econometrica*, 1982, 765-69.

Chernozhukov, Victor, Alfred Galichon and Marc Henry. 2014. "Identifying Multi-Attribute Hedonic Models," mimeo, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2392549

Chiappori, Pierre-André, Monica Costa-Dias and Costas Meghir. 2015. "The Marriage Market, Labor Supply and Education Choice", NBER 21004.

Chiappori, Pierre-André and E. Gugl. 2014. "Necessary and sufficient conditions for transferable utility". mimeo, Columbia University

Chiappori, Pierre-André, Alfred Galichon and Bernard Salanié. 2013. "The Roommate Problem Is More Stable Than You Think", Mimeo, Columbia University.

Chiappori, Pierre-André, Alfred Galichon and Bernard Salanié. 2015. "Separable Matching with Discrete and Continuous Types.", in progress.

Chiappori, Pierre-André, Murat Iyigun and Yoram Weiss. 2009. "Investment in Schooling and the Marriage Market". *American Economic Review*, 99, 1689-1714.

Chiappori, Pierre-André, Robert McCann and Lars Nesheim. 2010. "Hedonic Price Equilibria, Stable Matching, And Optimal Transport: Equivalence, Topology, And Uniqueness". *Economic Theory*, 42, 317–354.

Chiappori, Pierre-André, Sonia Oreffice, and Climent Quintana-Domeque. 2012. "Fatter Attraction: Anthropometric and Socioeconomic Characteristics in the Marriage Market." *Journal of Political Economy*, 120, 659-695.

Chiappori, Pierre-André, and Phil Reny. 2007. "Matching to Share Risk.", mimeo, University of Chicago.

Chiappori, Pierre-André and Bernard Salanié. 2015. "Identifying Assortative Matching: Methods and Pitfalls". in progress.

Chiappori, Pierre-André, Bernard Salanié and Yoram Weiss. 2014. "Partner Choice and the Marital College Premium". mimeo, Columbia University.

Choo, Eugene and Aloysius Siow. 2006a. "Who Marries Whom and Why". *Journal of Political Economy*, 114, 175–201.

Choo, Eugene and Aloysius Siow. 2006b. "Estimating a Marriage Matching Model with Spillover Effects". *Demography*, 43, 463-490.

Dagsvik, John. 1994. "Discrete and Continuous Choice, Max-Stable Processes, and Independence from Irrelevant Attributes". *Econometrica*, 62, 1179—1205.

Dagsvik, John. 2000. "Aggregation in Matching Markets.". *International Economic Review*, 41, 27–57.

Decker, Colin, Elliott Lieb, Robert McCann, and Benjamin Stephens. 2012. "Unique Equilibria and Substitution Effects in a Stochastic Model of the Marriage Market," *Journal of Economic Theory*, 148, 778–792.

Del Boca, Daniela and Christopher Flinn. 2014. "Household Behavior and the Marriage Market". *Journal of Economic Theory*, 150, 515–550.

Dupuy, Arnaud and Alfred Galichon. 2014. "Personality Traits and the Marriage Market". *Journal of Political Economy,* 122, 271–1319.

Echenique, Federico, Sangmok Lee, Matthew Shum, and Bumin Yenmez. 2013. "The Revealed Preference Theory of Stable and Extremal Stable Matchings". *Econometrica*, 81, 153–171.

Ekeland, Ivar, James Heckman and Lars Nesheim. 2002. "Identifying Hedonic Models". *American Economic Review*, 92, 304–309.

Ekeland, Ivar, James Heckman and Lars Nesheim. 2004. "Identification and Estimation of Hedonic Models." *Journal of Political Economy*, 112, S60–S109.

Fox, Jeremy. 2010a. "Identification in matching games". *Quantitative Economics*, 1, 203–254.

Fox, Jeremy. 2010b. "Estimating matching games with transfers". Mimeo.

Fox, Jeremy and Patrick Bajari. 2013. "Measuring the Efficiency of an FCC Spectrum Auction ". *American Economic Journal: Microeconomics*, 5, 100–146.

Fox, Jeremy, David Hsu and Chengyu Yang. 2015. "Unobserved Heterogeneity in Matching Games with an Application to Venture Capital", mimeo University of Michigan.

Galichon, Alfred, Scott Kominers and S. Weber. 2014. "Costly Concessions: Estimating The Unintended Consequences Of Policy Intervention In Matching Markets". mimeo, Sciences Po.

Galichon, Alfred and Bernard Salanié. 2014. "Cupid's Invisible Hand: Social Surplus and Identification in Matching Models ". mimeo.

Gordon, Nora and Bryan Knight. 2009. "A Spatial Merger Estimator with an Application to School District Consolidation". *Journal of Public Economics*, 93, 752–765.

Goussé, Marion. 2014. "Who should you marry? A multidimensional search and matching model". Mimeo Sciences Po.

Graham, Bryan. 2011. "Econometric Methods for the Analysis of Assignment Problems in the Presence of Complementarity and Social Spillovers". in J. Benhabib, M. Jackson and A. Bisin eds: *Handbook of Social Economics*, North-Holland, Vol. 1B, 965–1052.

Graham, Bryan. 2013. "Errata in 'Econometric Methods for the Analysis of Assignment Problems in the Presence of Complementarity and Social Spillovers'". Mimeo, UC Berkeley.

Hagedorn, Marcus, Tzuo Hann Law, and Iourii Manovskii. 2014. "Identifying Equilibrium Models of Labor Market Sorting", mimeo Penn.

Hatfield, John and Paul Milgrom. 2005. "Matching with Contracts". *American Economic Review*, 913-35.

Heckman, James, Rosa Matzkin and Lars Nesheim. 2010. "Nonparametric Identification And Estimation of Nonadditive Hedonic Models". *Econometrica*, 78, 1569–1591.

Hitsch, Günther, Ali Hortaçsu and Dan Ariely. 2010. "Matching and Sorting in Online Dating". *American Economic Review*, 100, 130–163.

Horowitz, Joel. 1992. "A Smoothed Maximum Score Estimator for the Binary Response Model". *Econometrica*, 60, 505–531.

Hsieh, Yuwei. 2011. "Understanding Mate Preferences from Two-Sided Matching Markets: Identification, Estimation and Policy Analysis". mimeo, New York University.

Jacquemet, Nicolas and Jean-Marc Robin. 2012. "Assortative Matching and Search with Labor Supply and Home Production". Mimeo Sciences Po.

Koopmans, Tjalling and Martin Beckmann. 1957. "Assignment Problems and the Location of Economic Activities", *Econometrica*, 25, 53–76.

Lee, Sangmok and Leeat Yariv. 2014. "On the Efficiency of Stable Matchings in Large Markets". Mimeo Caltech.

Manski, Charles. 1975. "Maximum Score Estimation of the Stochastic Utility Model of Choice", *Journal of Econometrics*, 3, 205–228.

Menzel, Konrad. 2015a. "Large Matching Markets as Two-sided Demand Systems", forthcoming in *Econometrica*.

Menzel, Konrad. 2015b. "Inference for Games with Many Players". NYU mimeo.

Monge, Gaspard. 1781. "Mémoire sur la théorie des déblais et des remblais". *Histoire de l'Académie Royale des Sciences*, 666–704.

Mourifié, I., and A. Siow, 2014, "Cohabitation versus marriage: Marriage matching with peer effects", mimeo, University of Toronto

Nesheim, Lars, 2012, "Identification in multidimensional hedonic models", presentation, Conference 'Matching Problems: Economics Meets Mathematics', Chicago

Postel-Vinay, F., and J.M. Robin (2002), "Equilibrium Wage Dispersion with Heterogeneous Workers and Firms", *Econometrica*, vol. 70, 1295-1350.

Rosen, Sherwin. 1974. "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition." *Journal of Political Economy* 82, 34–55.

Roth, Alvin. 1984. "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory". *Journal of Political Economy* 92, 991–1016.

Roth, Alvin and Elliott Peranson. 1999. "The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design". *American Economic Review* 89, 748–79.

Roth, Alvin and Marilda Sotomayor. 1992. *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis.* Cambridge University Press.

Schwartz, Christine and Robert Mare. 2005. "Trends in Educational Assortative Marriage from 1940 to 2003". *Demography*, 42, 621–646.

Shapley, Lloyd and Martin Shubik. 1972. "The Assignment Game I:

The Core". *International Journal of Game Theory*, 1, 111–130.

Shimer, Robert and Lones Smith. 2000. "Assortative Matching and Search", *Econometrica*, 68, 343–369.

Siow, Aloysius. 2009. "Testing Becker's Theory of Positive Assortative Matching". Discussion paper, University of Toronto.

Smith, Lones. 2011. "Frictional Matching Models". *Annual Review of Economics*, 3, 319–338.

Sorensen, Mørten. 2007. "How Smart Is Smart Money? A Two-Sided Matching Model of Venture Capital". *Journal of Finance*, 62, 2725–62.

Tinbergen, Jan. 1956. "On the Theory of Income Distribution." *Weltwirtschaftliches Archiv*, 77, 155-73.

Yang, Yupin, Mengze Shi and Avi Goldfarb. 2009. "Estimating the Value of Brand Alliances in Professional Team Sports". *Marketing Science*, 28, 1095–1111.