Screening Risk-Averse Agents Under Moral Hazard: Single-crossing and the CARA Case[1]

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Abstract

Principal-agent models of moral hazard have been developed under the assumption that the principal knows the agent's risk-aversion. This paper extends the moral hazard model to the case when the agent's risk-aversion is his private information, so that the model also exhibits adverse selection. We characterize the optimal menu of contracts; while its detailed properties depend on the setting, we show that some of them must hold for all environments. In particular, the power of incentives always decreases with risk-aversion. We also characterize the relationship between the outside option and the optimal contracts. We then apply our results to testing for asymmetric information in insurance markets.

Introduction

The traditional literature on moral hazard emphasizes the trade-off between risk-sharing and the provision of incentives. It derives optimal contracts whose shape closely depends on the agent's risk-aversion. One may then wonder how the principal knows the agent's preferences so precisely; and it seems likely that the agent will try to manipulate the principal's perception of his risk-aversion. We here extend the moral hazard model to the case when there is adverse selection on the agent's risk-aversion. While such an extension is of theoretical interest, we argue that it may also help solve some empirical puzzles that have been uncovered in recent years. Several papers (e.g. Chiappori-Salanié (2000)) indeed have shown that the standard models of insurance under asymmetric information predict correlations that often cannot be found in the data. Thus it seems that we need a richer model to account for the empirical evidence.

We analyze a two-outcome/two-type model of moral hazard with adverse selection on the agent's risk-aversion. We first show that when the agent's risk-aversion is public information (the public risk-aversion model), very little can be said on the link between risk-aversion and the optimal contract. In particular, intuition suggests that more risk-averse agents should face lower-powered incentives. In fact, we find that the power of incentives may be a non-monotonic function of risk-aversion.

Surprisingly, this anomaly disappears when the agent's risk-aversion is his private information (the private risk-aversion model). We indeed show that in the contract space, the traditional Spence-Mirrlees condition is verified under natural assumptions. This implies that adverse selection imposes more structure on the design of optimal contracts: more risk-averse agents must opt for contracts with lower-powered incentives, as intuition suggests.

While the single-crossing property greatly simplifies the analysis of the private risk-aversion model, it still turns out to allow for a variety of configurations. In the "regular" case in which more risk-averse agents face lower-powered incentives in the public risk-aversion model, the private risk-aversion optimum always separates types. In the "non-responsive" case in which more risk-averse agents face higher-powered incentives in the public risk-aversion model, the private risk-aversion optimum may involve separation or bunching.

A crucial parameter allowing to sort out these cases is the power of the incentives implicit in the outside option. This power is high in insurance models, since the outside option is no insurance; it is low in labor economics models, when the outside option is associated to unemployment. We find that whether there is overprovision or underprovision of incentives relative to the public risk-aversion model, and which type of agent benefits from an informational rent, depends in a monotonic way on the characteristics of the outside option. In fact, a novel feature of the private risk-aversion model is that the power of incentives in the optimal contract increases (weakly) with that of the outside option, contrary to the public risk-aversion model in which it is independent of that of the outside option. This implies that *observed probabilities of success should be higher for agents who face more powerful incentives in their outside options.*

Note however that while more risk averse agents always face lower-powered incentives, they may well provide more effort since this reduces the risk they face. *Hence the relationship between the incentives displayed by the optimal contract and the observed probability distribution of outcomes is ambiguous.* As announced above, this important result allows for a better understanding of the links between incentives, performance and risk, and may be of interest in many economic activities. Our model indeed seems well-suited to the study of car insurance markets for instance, where insurers see heterogeneity in risk attitudes as a crucial element of insurees' behavior. Thus our results may explain the empirical findings by Chiappori and Salanié (2000) who find no correlation between risk and coverage for car insurance.

On a theoretical level, our paper's contribution is both to introduce heterogeneous risk-aversions and to solve a model with both moral hazard and adverse selection (what Myerson (1982) calls a generalized agency model). Most papers so far have studied adverse selection or moral hazard separately (as done by Pauly (1974) for insurance). Adverse selection with a risk-averse agent was treated by Salanié (1990) for vertical contracting and Laffont-Rochet (1998) for the regulation of firms. Landsberger and Meilijson (1994) studied competitive insurance markets when adverse selection bears on the insurees' risk-aversion; Smart (2000) and Villeneuve (2003) extended their analysis to two-dimensional adverse selection, where both risk-aversion and risk are privately known by the insuree. Adverse selection on risk-aversion creates new difficulties when there is moral hazard, as the degree of risk-aversion affects the agent's behavior and thus the principal's expected utility from a given contract. The existing papers on the generalized agency model (Baron and Besanko (1987), Chassagnon and Chiappori (1997), Faynzilberg and Kumar (1997, 2000), Stewart (1994)) have focussed on the case where the agent's private characteristic affects the technology. In that case, effort is a monotonic function of type under some regularity conditions. Our model is much richer in that higher risk-aversion typically leads to higher effort but also to lower-powered incentives (and thus to lower effort), so that the relationship between effort and type is fundamentally ambiguous[1].

To the best of our knowledge, very little is known on optimal contracting under moral hazard when the agent's risk-aversion is his private information. Related work by de Meza and Webb (2000) studies competitive equilibrium in an insurance market. However, their model makes a number of simplifying assumptions: one of the two types is risk-neutral, he makes no effort, and more risk-averse agents always make more effort (which may not be true, see Jullien, Salanié and Salanié (1999)). They also introduce some administrative costs which justify public intervention. On the other hand, our model is a Principal-Agent model; it applies to insurance but also to labor economics situations for instance; and we resort to none of the special assumptions used by de Meza and Webb.

Section 1 analyses the agent's choice of effort and proves the Spence-Mirrlees condition. We then specialize the model to CARA utility functions, and study the public (resp. private) risk-aversion model in Section 2 (resp. Section 3). Section 4 concludes by applying our results to managerial compensation, corporate finance, and especially insurance markets. All proofs are gathered in an Appendix.

1 The General Model and the Single-crossing Property

Consider a risk-neutral principal offering a contract to a risk-averse agent facing a binomial income risk: success or failure. The agent can exert a costly effort to reduce the probability of failure $p \in [0, p_0]$, at a monetary cost[2] $e = c(p)$. For the moment we only assume that $c(p)$ is decreasing

[1] Hemenway (1990, 1992) identified the first effect in the insurance context and called it "propitious selection".

[2] Since we focus on the effects of a change in risk-aversion on effort, it is important to ensure that whatever their risk-aversion, all types of agents have access to the same technology. In other words, the marginal rate of substitution between effort and wealth should not depend on the agent's risk-aversion. This is only possible with the so-called

convex, that failure is always a concern ($c(0) = +\infty$), and we normalize $c(p_0)$ to zero.

The principal can observe the realization of the risk but not the level of effort. A contract specifies transfers contingent on success or failure. Letting W and $W - \Delta$ be the final incomes (gross of the cost of effort), we refer to $C = (W, \Delta)$ as a contract, and to Δ as the power of incentives.

The agent has an increasing concave Von Neumann-Morgenstern utility function, unknown to the principal. We assume that the utility of the agent belongs to some smooth one-dimensional family of utility functions $\mathcal{F} = \{u_\sigma\}$ that is ranked according to risk aversion: for any wealth level x, $-\frac{u''_\sigma(x)}{u'_\sigma(x)}$ is increasing with σ. Thus σ defines the type of the agent.

Given a contract C, the type σ agent's problem writes as

$$\mathcal{U}_\sigma(W, \Delta) \equiv \max_p \quad (1-p)u_\sigma(W - c(p)) + pu_\sigma(W - \Delta - c(p)). \qquad (1)$$

We denote by $p_\sigma(W, \Delta)$ a solution to this program.

1.1 The single-crossing property

It is well-known (see Ehrlich-Becker (1972), Dionne-Eeckhoudt (1985), and Jullien-Salanié-Salanié (1999)) that this apparently simple problem displays complex properties. In particular, the effect of a change in risk-aversion is ambiguous: even when the Arrow-Pratt index of risk-aversion is a constant (the CARA case), the probability of failure $p_\sigma(W, \Delta)$ may be non-monotonic with respect to risk-aversion. Hence, and contrary to what intuition suggests, more risk-averse agents do not necessarily exert more effort. The reason is that effort reduces the income in case of failure, so that a more risk-averse agent facing a high probability of failure may opt for increasing his worst-case income instead of reducing the probability of failure[3]. As a consequence, the effect of varying W also is ambiguous if wealth effects are present.

Nevertheless this subsection proves an important result that will make tractable the analysis of contracts under adverse selection. The value $\mathcal{U}_\sigma(W, \Delta)$ of the agent's program (1) defines his indifference curves in the space (W, Δ) of contracts. The Spence-Mirrlees property orders these indifference curves:

"monetary cost" formulation of effort.

[3]Jullien-Salanié-Salanié (1999) shows that for a given incentive contract (W, Δ) and a given technology c, the optimal probability of failure (locally) decreases in risk aversion if and only if this probability is small enough.

Single-crossing property: *the slope of the indifference curves* $\left(\frac{\partial W}{\partial \Delta}\right)_{\mathcal{U}_\sigma}$ *increases with* σ.

As a consequence, if an agent is indifferent between two contracts, then a more risk-averse agent prefers the lower-powered contract. In that sense, "more risk-averse agents prefer lower-powered contracts". This is what intuition suggests; but as shown by the fact that more risk-averse agents may expend less effort, intuition may be a poor guide in these models. The difficulty, of course, is that p is endogenous, and varies when risk-aversion is changed. Despite the complexity of the comparative static of risk aversion, the single crossing condition holds under simple and natural regularity conditions. We offer below two distinct sets of sufficient conditions.

To start with, consider the following property:

Property *(Q)*: *if* $\mathcal{U}_\sigma(W_1, \Delta_1) = \mathcal{U}_\sigma(W_2, \Delta_2)$ *and* $\Delta_1 < \Delta_2$ *then, denoting* $e_k^\sigma = c(p_\sigma(W_k, \Delta_k))$,

$$W_2 - \Delta_2 - e_2^\sigma < W_1 - \Delta_1 - e_1^\sigma < W_1 - e_1^\sigma < W_2 - e_2^\sigma.$$

This property is easily understood. Since $\Delta_1 < \Delta_2$ and the agent is indifferent between both contracts, then it must be that $W_1 < W_2$. But this in turn implies that $W_1 - \Delta_1 > W_2 - \Delta_2$, because otherwise contract C_2 would dominate contract C_1. Summarizing:

$$W_2 - \Delta_2 < W_1 - \Delta_1 < W_1 < W_2.$$

In other words, the second contract is riskier in the sense that the range of possible wealth levels is enlarged. Property *(Q)* requires that this ordering of contracts remain valid in utility space, that is once the agent has adjusted his choice of effort. Clearly *(Q)* holds if effort is set at the same level for both contracts, and it also holds if effort does not vary too much with incentives.

We are now ready to state our first result:

Proposition 1 *If (Q) holds, then the single-crossing property holds.*

Another set of conditions obtains by strenghtening the concept of increasing risk aversion, in particular by assuming that agents can be ordered in the Ross (1981) sense, where v is Ross-more risk-averse than u if and only if for all x and y in the relevant range, $-v''(x)/v'(x) > -u''(y)/u'(y)$.

Proposition 2 *The single-crossing condition holds for two types $\sigma' > \sigma$ if $u_{\sigma'}$ is more risk-averse than u_σ in the sense of Ross, and $pc'(p)$ is non-decreasing in p.*

To interpret the condition on $c(p)$, consider a risk-neutral agent, who chooses effort to minimize $(p\Delta + c(p))$, so that $\Delta = -c'(p)$. The agent's expected revenue from the Principal thus is $(W + pc'(p))$ and it is a non-increasing function of Δ if $pc'(p)$ is non-decreasing in p (p is a non-increasing function of Δ since c is convex). To rephrase this, the condition on the cost function holds if the expected payment from the Principal to the risk-neutral agent is reduced when the penalty in case of failure increases, which seems natural. It is satisfied by several simple classes of technologies, for instance

$$c(p) = \frac{1}{a}(p^{-a} - p_0^{-a}) \quad (a > 0)$$

which includes as a limiting case the logarithmic function $c(p) = \log(p_0/p)$.

Finally, note that the assumption requires that $c(p)$ be convex enough, so that effort does not vary too much with incentives. Hence a link may be drawn between our two propositions, as the following lemma illustrates :

Lemma 1 *(Q) holds if the utility functions are CARA and $pc'(p)$ is increasing in p.*

Thus in the CARA case, the two propositions point to the same condition for the single crossing-condition to hold. From now on we shall focus on this case as it abstracts from wealth effects on risk-aversion that would make the analysis much more complicated.

1.2 The Model

Given these preliminary results, we may now specify more precisely our Principal-Agent model with moral hazard and adverse selection on the agent's risk-aversion. Our CARA assumption writes

Assumption 1 *$\mathcal{F}$ is the set of CARA utility functions: $u_\sigma(x) = -\exp(-\sigma x)$.*

As we have just seen, the single-crossing property obtains under

Assumption 2 *$pc'(p)$ is non-decreasing in p.*

It is also easy to see that it is a natural condition to guarantee the quasi-concavity of the agent's program (1), so that the solution is unique. In the absence of wealth effects, we denote it $p_\sigma(\Delta)$, which is continuous and non-increasing[4] with respect to Δ.

Define the certainty equivalent of a contract as

$$CE_\sigma(W,\Delta) = u_\sigma^{-1}(\mathcal{U}_\sigma(W,\Delta)).$$

Given that the agent would exert no effort under a sure wealth, it can be interpreted as the level of wealth W that the agent would exchange for a full insurance contract $\Delta = 0$, and is given by

$$CE_\sigma(W,\Delta) = W - c(p_\sigma(\Delta)) - \frac{1}{\sigma}\log(1 - p_\sigma(\Delta) + p_\sigma(\Delta)\exp(\sigma\Delta)).$$

From now on, we shall focus on the two-type case:

Assumption 3 *$\sigma \in \{L, H\}$, with $L < H$.*

We denote by $\mu \in]0,1[$ the probability of the more risk-averse H type. Note that the difference between the certainty equivalents of the two types is a function of Δ only, that we denote

$$\phi(\Delta) = CE_L(W,\Delta) - CE_H(W,\Delta).$$

Moreover, the single-crossing condition exactly says that ϕ is increasing.

Denote S the total wealth created in case of success, and $(S - D)$ the wealth created in case of failure, with $0 < D < S$. The principal's profit writes

$$\pi_\sigma(W,\Delta) = S - W - p_\sigma(\Delta)(D - \Delta).$$

We define total surplus as

$$B_\sigma(\Delta) = \pi_\sigma(W,\Delta) + CE_\sigma(W,\Delta).$$

Note that it does not depend on W.

Finally we assume that the agent has a reservation option leading to wealth W_0 in case of success and $(W_0 - \Delta_0)$ in case of failure, with $0 \le \Delta_0 < W_0$. This formulation encompasses both the case of insurance models, for which $\Delta_0 = D$ for the outside option of no-insurance, and labor economics models for which the outside option is that of unemployment or of fixed wage, and Δ_0 is low.

[4]This is shown in the proof of Lemma 1 in the Appendix.

2 The Optimal Contract with Public Risk-aversion

Let us now consider the case where the type σ is observed by the principal. The situation thus only involves moral hazard. Maximizing

$$\pi_\sigma(W, \Delta)$$

under the participation constraint

$$CE_\sigma(W, \Delta) \geq CE_\sigma(W_0, \Delta_0)$$

boils down, since both objective functions are linear in W, to maximizing over Δ the total surplus

$$B_\sigma(\Delta)$$

and adjusting W to bring the agent on his reservation utility curve. We show in the Appendix that this program has a solution, and that all such solutions must belong to $]0, D[$. Note that solutions are independent from the outside option (W_0, Δ_0). For reasons which will be made clear later on, we shall define Δ_L^1 as the largest solution to max B_L, and Δ_H^1 as the smallest solution to max B_H. We have:

Lemma 2 *Both Δ_L^1 and Δ_H^1 belong to $]0, D[$, and are non-decreasing with respect to D.*

The public risk-aversion case thus presents some regularity, since the increase D in surplus from failure to success is shared between the principal and the agent. Nevertheless comparative statics with respect to risk-aversion are less conclusive. One would expect that $\Delta_H^1 < \Delta_L^1$, since more risk-averse agents prefer lower-powered incentive schedules. However, the derivative of Δ_σ^1 in σ depends *inter alia* on the cross derivative of $p_\sigma(\Delta)$ in (σ, Δ), on which little is known in general. It is in fact easy to find examples in which Δ_σ^1 is non-monotonic in σ.

Example 1: Assume that the technology is given by

$$c(p) = -\frac{1}{\lambda} \log \frac{p}{p_0} ,$$

which satisfies Assumption 2. Then easy calculations show that the optimum of the agent's program is equal to p_0 if $\sigma > \lambda$ and to

$$\min\left(p_0, \frac{\sigma}{\lambda-\sigma}\frac{1}{e^{\sigma\Delta}-1}\right) \tag{2}$$

otherwise. As we have a closed form for $p_\sigma(\Delta)$, it is easy to maximize B_σ numerically[5] and to see how its maximizer Δ^1_σ depends on σ. We ran such a simulation for $D = \lambda = 10$ and $p_0 = 0.1$. We found that while Δ^1_σ decreases in σ for $\sigma < 8.4$, it starts increasing afterwards. The behavior of $p_\sigma(\Delta^1_\sigma)$ is even more complicated, as it increases with σ until we reach $\sigma = 0.3$, then decreases, and starts increasing again after $\sigma = 8$. This counterintuitive behavior takes place without the constraint $p \leq p_0$ becoming binding in this region.

3 The Optimal Contract with Private Risk-aversion

We now introduce adverse selection on the agent's risk-aversion. The principal's problem then is to choose a pair of contracts (W_H, Δ_H) and (W_L, Δ_L) to maximize his expected profit, which writes

$$\mu(B_H(\Delta_H) - CE_H(W_H, \Delta_H)) + (1-\mu)(B_L(\Delta_L) - CE_L(W_L, \Delta_L))$$

given the two incentive constraints

$$CE_L(W_L, \Delta_L) \geq CE_L(W_H, \Delta_H)$$
$$CE_H(W_H, \Delta_H) \geq CE_H(W_L, \Delta_L)$$

and the two participation constraints

$$CE_L(W_L, \Delta_L) \geq CE_L(W_0, \Delta_0)$$
$$CE_H(W_H, \Delta_H) \geq CE_H(W_0, \Delta_0).$$

The problem is fairly standard except for two specificities. First, as the properties of the optimal contract with adverse selection depend on the comparison between Δ^1_H and Δ^1_L, we will have to distinguish between the "regular" case when $\Delta^1_H < \Delta^1_L$ and the "non-responsive"[6] case when $\Delta^1_H > \Delta^1_L$.

[5]With this cost function, $B_\sigma(\Delta)$ turns out to be strictly quasi-concave.

[6]We borrow the term "non-responsive" from Guesnerie-Laffont (1984) and the survey by Caillaud-Guesnerie-Rey-Tirole (1988).

Second, the agent has an outside option (W_0, Δ_0) that affects its participation decision and thus the minimal utility under the offered contract.[7] As we show below, this implies that depending on the value of Δ_0, any of the two incentive constraints can bind. Nevertheless, we are able to take advantage of the simple two-type structure to provide general results, without having to discuss several cases from the beginning.

3.1 General Properties of Optimal Contracts

Let us define the informational rent of type σ as

$$U_\sigma = CE_\sigma(W_\sigma, \Delta_\sigma) - CE_\sigma(W_0, \Delta_0).$$

We will eliminate W_L and W_H from the Principal's program and replace them with U_L and U_H. Because the certainty equivalent is increasing with W for given Δ, this is a well-defined change of variables. Recall also that

$$\phi(\Delta) = CE_L(W, \Delta) - CE_H(W, \Delta).$$

We may now rewrite our problem in a simpler form. We want to find (U_L, U_H, Δ_L, Δ_H) to maximize

$$\mu(B_H(\Delta_H) - U_H) + (1 - \mu)(B_L(\Delta_L) - U_L) \tag{3}$$

under the participation constraints

$$U_L \geq 0 \qquad U_H \geq 0 \tag{IR}$$

and the incentive compatibility constraints

$$U_H \geq U_L + \phi(\Delta_0) - \phi(\Delta_L) \qquad U_L \geq U_H + \phi(\Delta_H) - \phi(\Delta_0). \tag{IC}$$

Now adding both constraints in (IC) we get

$$\Delta_H \leq \Delta_L. \tag{4}$$

since ϕ is increasing. Also (IC) and (IR) together imply

$$U_H \geq \max(0, \phi(\Delta_0) - \phi(\Delta_L)) \qquad U_L \geq \max(0, \phi(\Delta_H) - \phi(\Delta_0)). \tag{5}$$

[7] As the value of this minimal utility depends on risk-aversion, this is usually referred to as "type-dependent" participation constraints (see Jullien (2000)).

It turns out that rewriting the problem in this way considerably simplifies things. The objective function (3) clearly increases when the constraints in (5) are binding. Reciprocally, if a pair of contracts (U_L, U_H, Δ_L, Δ_H) is such that the constraints in (5) are binding and (4) holds, then it is easy to check that this pair of contracts verifies the initial constraints (IR) and (IC)[8].

Consequently, we have obtained two important results. First, given Δ_L and Δ_H, optimal contracts must allocate informational rents according to

$$U_H = \max(0, \phi(\Delta_0) - \phi(\Delta_L)) \qquad U_L = \max(0, \phi(\Delta_H) - \phi(\Delta_0)). \quad (6)$$

Secondly, we may replace these values in the objective (3) and rearrange terms to obtain a simple additive program :

$$\max_{\Delta_H, \Delta_L} \quad \mu V_H(\Delta_H) + (1-\mu) V_L(\Delta_L) \qquad (7)$$

under the constraint $\Delta_H \leq \Delta_L$, where we define the virtual social surpluses as

$$V_H(\Delta) = B_H(\Delta) - \frac{1-\mu}{\mu} \max(0, \phi(\Delta) - \phi(\Delta_0))$$

and

$$V_L(\Delta) = B_L(\Delta) - \frac{\mu}{1-\mu} \max(0, \phi(\Delta_0) - \phi(\Delta)).$$

We immediately get

Proposition 3 *In the private risk-aversion case, the optimal powers of incentives (Δ_L, Δ_H) are such that*

$$\Delta_H \leq \Delta_L \qquad \Delta_H \leq \Delta_H^1 \qquad \Delta_L^1 \leq \Delta_L.$$

Moreover, both Δ_L and Δ_H are non-decreasing with respect to D and Δ_0. As a consequence, probabilities of success also increase (weakly) with respect to D and Δ_0.

The first inequality simply recalls the Spence-Mirrlees condition. The next two inequalities assert that compared to the public risk-aversion case, Δ_H is distorted downward while Δ_L is distorted upward, so as to relax incentive

[8]This can be seen by inspection by examining the three cases $\Delta_H \leq \Delta_L \leq \Delta_0$, $\Delta_H \leq \Delta_0 \leq \Delta_L$, and $\Delta_0 \leq \Delta_H \leq \Delta_L$.

constraints. The monotonicity of incentives with respect to the stake D simply extends Proposition 2 to the private risk-aversion case. More interesting is the fact that the incentives given by the optimal contract are monotonic with respect to the incentives Δ_0 in the outside option. This feature is characteristic of adverse selection: in the public risk-aversion case, Δ^1_σ did not depend on Δ_0.

To go further, we define Δ^2_L as the largest solution[9] to $\max_\Delta V_L(\Delta)$. Note that in any case $\Delta^2_L \geq \Delta^1_L$, and that both values are equal if and only if $\Delta^1_L \geq \Delta_0$. Define also Δ^2_H as the smallest solution to $\max_\Delta V_H(\Delta)$. Similarly, we have $\Delta^2_H \leq \Delta^1_H$, and both values are equal if and only if $\Delta^1_H \leq \Delta_0$.

A natural candidate for the solution would be (Δ^2_H, Δ^2_L), provided the constraint $\Delta_H \leq \Delta_L$ holds. This is why we distinguish two cases in the following.

3.2 The Regular Case

The regular case is defined by $\Delta^1_H < \Delta^1_L$, so that we get

$$\Delta^2_H \leq \Delta^1_H < \Delta^1_L \leq \Delta^2_L.$$

Then (Δ^2_H, Δ^2_L) indeed is the solution to the private risk-aversion case. We now discuss its properties as Δ_0 varies, and we summarize the results on Figure 1, which plots the power of incentives as a function[10] of Δ_0.

First suppose $\Delta_0 < \Delta^1_H$. Then $\Delta^2_L = \Delta^1_L$, so that there is no distortion for the L type. Conversely we have $\Delta^2_H < \Delta^1_H$, and type H enjoys no informational rent. Type-L enjoys a rent if $\Delta^2_H > \Delta_0$.

Intuitively this case is associated with low outside incentives Δ_0, as in labor economics model where the outside option is unemployment. Then the Principal would like to transfer some risk to the agent, in order to increase the agent's effort. This distortion in risk-sharing is less costly for the less risk-averse type L, who here is the "good" type. Type H allocation is distorted downward, so that type L does not find it interesting to lie.

[9] Such a solution exists, since anyway V_L is decreasing for $\Delta > \max(D, \Delta_0)$.

[10] For simplicity the figure focusses on the case when both V_H and V_L are strictly quasi-concave; otherwise the optimal contracts may switch between local maxima as Δ_0 varies.

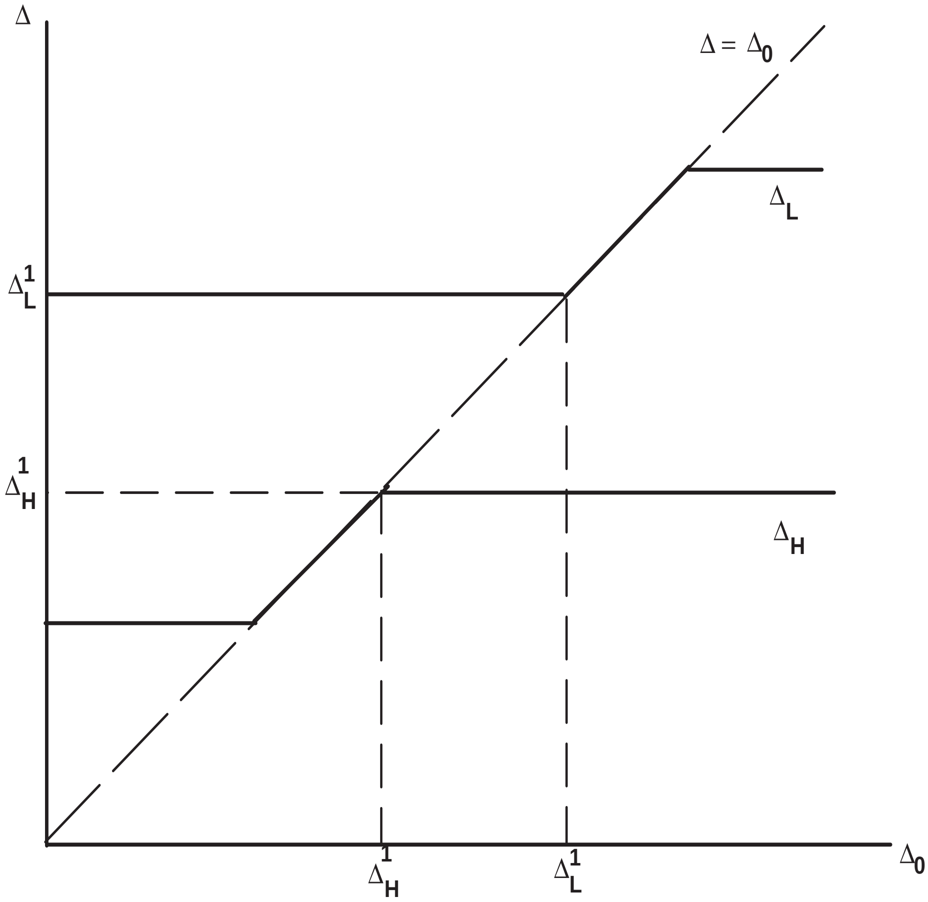

Figure 1: Optimal Incentives in the Regular Case

The case when $\Delta^1_H \leq \Delta_0 \leq \Delta^1_L$ is an intermediate case in which the public risk-aversion allocation is incentive compatible, so that agents enjoy no rent and get Δ^1_σ. The importance of this result should not be overstated, as it is an artefact of the two-type case.

Finally the case $\Delta_0 > \Delta^1_L$ is symmetrical to the first case. Now agents would like to transfer some risk to the risk-neutral Principal, who behaves as an insurer. The "good" type now is the H type, as he is ready to pay more for more insurance. Type H gets the undistorted incentives $\Delta^2_H = \Delta^1_H$, while type L enjoys no rent and receives over-optimal incentives $\Delta^2_L > \Delta^1_L$, so as to deter type H from lying. Type H enjoys an informational rent if $\Delta^2_L < \Delta_0$.

3.3 The Non-responsive Case

Now assume that $\Delta^1_H > \Delta^1_l$. This case is more intricate as the first-best contract is not incentive-compatible anymore. Nevertheless our second-best candidate (Δ^2_H, Δ^2_L) may still be incentive-compatible if the distorsions due to imperfect information are important enough; otherwise intuition suggests that bunching is likely. However, multiple local maxima of V_H and V_L may play an important role in the absence of further regularity conditions. We give below an overview of what may happen, depending on the values of Δ_0 and μ.

Let us first focus on the extreme case when the proportion μ of H-type is close to one[11]. Then the principal would like to give the first-best level of incentives Δ^1_H to type H, since it appears with probability close to one. Moreover, to reduce the H-type rent, the Principal will want to dramatically distort upwards the incentives of type L (this effect appears in the second term of the virtual social surplus V_L). Hence a natural candidate in this case is (Δ^1_H, Δ^2_L). Even though $\Delta^1_L < \Delta^1_H$, the distortion in Δ_L may be big enough to ensure that Δ^1_H is smaller than Δ^2_L, so that (Δ^1_H, Δ^2_L) would indeed be the solution.

For general values of μ, it may be that $\Delta^2_H > \Delta^2_L$, so that (Δ^2_H, Δ^2_L) is not incentive-compatible. The optimal pair of contracts should then be solution to

$$\max_{\Delta_H \leq \Delta_L} \quad \mu V_H(\Delta_H) + (1-\mu) V_L(\Delta_L)$$

If the solution has $\Delta_H < \Delta_L$ and the virtual social surpluses V_H and V_L are

[11] The case when μ is close to zero is symmetrical.

strictly quasi-concave, then Δ_σ must maximize V_σ for $\sigma = H, L$. But this would lead us back to (Δ_H^2, Δ_L^2), which we know is not incentive compatible. This leads to a contradiction, so that there must be bunching $\Delta_H = \Delta_L$ in this case. Then the solution (Δ_B, Δ_B) is such that Δ_B maximizes $(\mu V_H + (1-\mu)V_L)$.

Of course, V_H and V_L needn't be strictly quasi-concave. To give a more precise mapping from the primitives of the model to the optimal pair of contracts, we must delve further into the problem. This is done in the Appendix and leads to the following

Proposition 4 *If $\Delta_0 > \Delta_H^1 > \Delta_L^1$, then*

- *if μ is close to one, then $\Delta_H^1 < \Delta_L^2$ and the optimal pair of contracts is the separating one (Δ_H^1, Δ_L^2).*
- *if μ is close to zero and the social surpluses B_L and B_H are strictly quasi-concave, there is bunching in Δ_B.*

If $\Delta_0 < \Delta_L^1 < \Delta_H^1$, then

- *if μ is close to zero, then $\Delta_H^2 < \Delta_L^1$ and the optimal pair of contracts is the separating one (Δ_H^2, Δ_L^1).*
- *if μ is close to one and the social surpluses B_L and B_H are strictly quasi-concave, there is bunching in Δ_B.*

Finally, if $\Delta_L^1 < \Delta_0 < \Delta_H^1$, and if the social surpluses B_H and B_L are strictly quasi-concave, then there is bunching in Δ_B.

Proposition 4 clearly does not exhaust all possibilities; still, it gives a general idea of the properties of the solution in the non-responsive case.

4 Concluding Remarks

Extending the moral hazard model to the case when the agent privately knows his risk-aversion is quite natural and has several interesting consequences for standard applications of the pure moral hazard model. First consider the issue of managerial pay for performance. This is a case in which the outside option has lower-powered incentives than the optimal contract: Δ_H^1 and Δ_L^1 are larger than Δ_0. Then our model implies that all managers except the least risk-averse one should be given lower-powered incentive pay than the pure moral hazard model predicts. Thus it may help explain why actual managerial pay schemes appear to depend so little on performance (as documented for instance by Jensen and Murphy (1990)).

Going to the field of corporate finance, consider an entrepreneur who looks for funds to start a project. Our results predict that more risk-averse entrepreneurs will rely less on equity and more on debt. More interestingly, Proposition 3 tells us that the probability of success of the project increases with Δ_0. But in this application, Δ_0 is the payoff of the entrepreneur when he relies on inside funding for his project; thus we predict that projects with more inside funding are more successful.

The most interesting application of our framework is insurance; private conversations with insurers indeed suggest that in many cases, the risk attitude of insurees and its effect on their behaviour are their main concern[12]. The standard insurance model considers a monetary loss D, which is insured with a deductible F against a premium P. It fits within our general model, with

$$\begin{cases} \Delta_0 = D \\ W = W_0 - P \\ \Delta = F \end{cases}$$

In this case the optimal menu of contracts is (Δ_H^1, Δ_L^2) when it is separating. Thus low risk-aversion insurees have a lower coverage. As we saw, the optimal contract can also be bunching in the non-responsive case.

Chiappori-Salanié (2000) have recently given evidence of an empirical puzzle. In the Rothschild-Stiglitz (1976) model of competitive insurance markets, equilibrium, when it exists, has higher-risk insurees getting better coverage. Thus their model predicts a positive correlation between risk and coverage. Using data on the French car insurance market, Chiappori-Salanié find that the correlation of risk and coverage is in fact close to zero[13]. It turns out that our model is rich enough that depending on the parameters, it may imply positive, negative, or (approximately) zero correlation between risk and coverage, even in the most regular case when the optimal contracts

[12]Finkelstein and McGarry (2003) indeed find support for the mechanisms described in this paper in a study of the long-term care insurance market.

[13]Their result has been replicated by several papers on car insurance data. Cawley-Philipson (1999) also find no evidence for asymmetric information, using health insurance data.

are separating[14]. The point is that while less risk averse agents choose higher deductible levels and thus have less coverage, they could also take fewer precautions, so that they may well end up with a higher probability of accident. Indeed, this fits well with the folklore of insurers.

To see this, focus on the regular case. First assume that there is little unobserved heterogeneity in risk-aversions: $H \simeq L$. Then it is easy to check that risk and coverage are positively correlated[15]. Now turn to the case when μ is small: then Δ_L^2 is close to Δ_L^1. But we know from Example 1 that the probability of an accident may be smaller for more risk-averse types in the public risk-aversion case. Thus it is possible that risk and coverage be negatively correlated.

We should emphasize two points here. First, this explanation assumes that one cannot fully control for risk-aversion, which seems reasonable enough. Second, Chiappori et al. (2002) prove that the positive correlation result must hold for a large class of models, including competitive models and models where risk-aversion is public. In this sense, our model with private risk-aversion plus insurer's market power is the only model that allows for a negative correlation when insurees have superior information.

[14]As explained in the introduction, de Meza-Webb (2000) investigate a somewhat similar model. They show that bunching may occur at a competitive equilibrium; empirically, this may be interpreted as zero correlation between risk and coverage. However, in their model it is due to an explicit assumption that the single-crossing condition does not hold, related to their assumption of a non-monetary cost of effort (see also footnote 2); while in our model the single-crossing property was shown to hold.

[15]Indeed the functions p_H and p_L then are close to each other, while Δ_L^2 and Δ_H^1 stay away from each other.

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Appendix: proofs.

Proof of Proposition 1: Suppose that agent σ is indifferent between contract C_1 with optimal effort e_1 and contract C_2 with optimal effort e_2. Assume that $\Delta_1 < \Delta_2$. For an agent type $\sigma' > \sigma$, let $e_2^{\sigma'}$ be an optimal effort level for agent σ' under C_2. Let σ' approach σ from above. As effort is upper hemi-continuous in risk-aversion, $e_2^{\sigma'}$ must approach some effort level that is optimal for σ under C_2; choose e_2 to be that effort level. Property (Q) applied at (C_1, e_1) and (C_2, e_2) implies that for σ' close enough to σ,

$$W_2 - \Delta_2 - e_2^{\sigma'} < W_1 - \Delta_1 - e_1 < W_1 - e_1 < W_2 - e_2^{\sigma'}.$$

Now let F_1 (resp. F_2) be the cumulative distribution function of outcomes under (C_1, e_1) (resp. under $(C_2, e_2^{\sigma'})$):

- for $R < W_2 - \Delta_2 - e_2^{\sigma'}$, $F_1(R) = F_2(R) = 0$;
- on $[W_2 - \Delta_2 - e_2^{\sigma'}, W_1 - \Delta_1 - e_1)$, $F_1 = 0$ and $F_2 = p(e_2^{\sigma'})$;
- on $[W_1 - \Delta_1 - e_1, W_1 - e_1)$, $F_1 = p(e_1)$ and $F_2 = p(e_2^{\sigma'})$;
- on $[W_1 - e_1, W_2 - e_2^{\sigma'})$, $F_1 = 1$ and $F_2 = p(e_2^{\sigma'})$;
- for $R \geq W_2 - e_2^{\sigma'}$, $F_1 = F_2 = 1$.

Hence $(F_2 - F_1)$ is positive then negative. A result of Jewitt (1989) (see also Jullien-Salanié-Salanié(1999)) then implies that the more risk-averse agent σ' must prefer (C_1, e_1) to $(C_2, e_2^{\sigma'})$. A fortiori, σ' must prefer C_1 to C_2.

Now consider the indifference curve of agent σ going through C_1 in the $(\Delta,\ W)$ plane. We have just shown that when σ increases, the indifference curve moves counterclockwise. Since this is true for any σ, we get the result.

Proof of Proposition 2: for a given contract (W, Δ) and a given utility function u we have

$$\left(\frac{\partial W}{\partial \Delta}\right)_{\mathcal{U}} = \frac{pu'(W - \Delta - c(p))}{pu'(W - \Delta - c(p)) + (1-p)u'(W - c(p))}$$

and we want to prove that this term is non-decreasing with risk-aversion. So let us consider $\sigma < \sigma'$, and define p (resp. p') as an optimal choice of σ (resp. σ') under (W, Δ). We want to show that

$$A \equiv \frac{1-p}{p}\frac{u'_\sigma(W-c(p))}{u'_\sigma(W-\Delta-c(p))} \geq A' \equiv \frac{1-p'}{p'}\frac{u'_{\sigma'}(W-c(p'))}{u'_{\sigma'}(W-\Delta-c(p'))}.$$

Suppose first that $p' \geq p$. Then

$$\frac{1-p}{p} \geq \frac{1-p'}{p'}$$

Now note that under the Ross ordering, the function

$$w_{\sigma'}(x) = u_{\sigma'}(x-c(p'))$$

is more concave than the function

$$w_\sigma(x) = u_\sigma(x-c(p)).$$

In particular, the ratio

$$\frac{w'_{\sigma'}(x)}{w'_\sigma(x)}$$

must be decreasing in x. By comparing the values of this ratio in $x_0 = W$ and $x_1 = W - \Delta$, it follows that

$$\frac{u'_\sigma(W-c(p))}{u'_\sigma(W-\Delta-c(p))} \geq \frac{u'_{\sigma'}(W-c(p'))}{u'_{\sigma'}(W-\Delta-c(p'))}$$

which allows us to conclude.

Suppose now that $0 < p' < p$. Then p' must be an interior solution to (1), so that

$$\begin{aligned} &u_{\sigma'}(W-\Delta-c(p')) - u_{\sigma'}(W-c(p')) \\ &= c'(p')\left(p'u'_{\sigma'}(W-\Delta-c(p')) + (1-p')u'_{\sigma'}(W-c(p'))\right). \end{aligned}$$

This yields

$$A' = -\frac{1}{p'c'(p')}\frac{w_{\sigma'}(x_0) - w_{\sigma'}(x_1)}{w'_{\sigma'}(x_1)} - 1.$$

Similarly for agent σ, for whom we only know that the derivative at p is non-negative (as p may equal p_0):

$$u_\sigma(W - \Delta - c(p)) - u_\sigma(W - c(p))$$
$$\geq c'(p)\left(pu'_\sigma(W - \Delta - c(p)) + (1-p)u'_\sigma(W - c(p))\right).$$

This yields

$$A \geq -\frac{1}{pc'(p)}\frac{w_\sigma(x_0) - w_\sigma(x_1)}{w'_\sigma(x_1)} - 1$$

Therefore it is sufficient to prove that

$$\frac{1}{-pc'(p)}\frac{w_\sigma(x_0) - w_\sigma(x_1)}{w'_\sigma(x_1)}$$

is greater than

$$\frac{1}{-p'c'(p')}\frac{w_{\sigma'}(x_0) - w_{\sigma'}(x_1)}{w'_{\sigma'}(x_1)}$$

Under our assumption and $p' < p$, we have $p'c'(p') \leq pc'(p)$. Besides, defining a function k by $w_{\sigma'} = k \circ w_\sigma$, we claim that

$$w_{\sigma'}(x_1) \leq w_{\sigma'}(x_0) + (w_\sigma(x_1) - w_\sigma(x_0))\frac{w'_{\sigma'}(x_0)}{w'_\sigma(x_0)}.$$

This follows from the concavity of k (which is again a consequence of the Ross ordering) and from $k'(w_\sigma(x)) = w'_{\sigma'}(x)/w'_\sigma(x)$. This allows us to conclude.

Proof of lemma 1: In the CARA case $u(x) = -\exp(-\sigma x)$, with $\sigma > 0$. As a preliminary step, let us study the agent's program (1), which here becomes

$$\max_p \left(-p\exp(-\sigma(W - \Delta - c(p))) - (1-p)\exp(-\sigma(W - c(p)))\right).$$

Denote $\gamma = \exp(\sigma\Delta) - 1 > 0$. Then the program reduces to

$$\max_p W - c(p) - \frac{1}{\sigma}\log(1 + \gamma p). \quad (8)$$

The derivative with respect to p is

$$-c'(p) - \frac{1}{\sigma}\frac{\gamma}{1+\gamma p} \tag{9}$$

which is a decreasing function of γ and thus of Δ. Therefore the optimal p is a decreasing function of Δ. Finally setting the derivative to zero, we get

$$1 + \gamma p = \frac{1}{1 + \sigma p c'(p)} \tag{10}$$

which shows the quasi-concavity of the objective, under our assumption on $pc'(p)$. To summarize, we have shown that the agent's program admits a unique solution, continuous and non-increasing with respect to Δ.

We now prove Lemma 1. We know that $W_2 - \Delta_2 < W_1 - \Delta_1 < W_1 < W_2$. Given that $e_1 < e_2$ with CARA utility functions, we get $W_2 - \Delta_2 - e_2 < W_1 - \Delta_1 - e_1$.

The proof of $W_1 - e_1 < W_2 - e_2$ requires some computations. The utility for agent σ of contract (W, Δ) is given in (8), and using (10) we get

$$W - c(p) + \frac{1}{\sigma}\log(1 + \sigma p c'(p)).$$

Besides the agent is indifferent between C_1 and C_2; therefore

$$W_1 - c(p_1) + \frac{1}{\sigma}\log(1 + \sigma p c'(p_1)) = W_2 - c(p_2) + \frac{1}{\sigma}\log(1 + \sigma p c'(p_2)).$$

Finally $p_1 > p_2$ and our assumption on $pc'(p)$ yield the result.

Proof of Lemma 2: We have

$$B_\sigma(\Delta) = [S - p_\sigma(\Delta)(D - \Delta)] - \min_p \, [c(p) + \frac{1}{\sigma}\log(1 + \gamma p)].$$

For $\Delta \geq D$, the first term is decreasing since $p_\sigma(\Delta)$ is decreasing with respect to Δ. And the second term strictly decreases with γ, and thus with Δ. This shows that $\Delta_\sigma^1 < D$.

For Δ small, (9) simplifies to

$$c'(p_\sigma(\Delta)) \simeq -\Delta$$

and under Assumption 1, it implies $\frac{\partial p}{\partial \Delta} < 0$ in $\Delta = 0$. Therefore

$$B'_\sigma(0) = -D\frac{\partial p}{\partial \Delta} + p - \frac{p(\gamma+1)}{1+\gamma p} = -D\frac{\partial p}{\partial \Delta} > 0$$

because $\gamma = \exp(\sigma\Delta) - 1$ is zero at $\Delta = 0$.

The second part of the Lemma follows directly from the fact that the derivative of B with respect to D is $-p_\sigma(\Delta)$, which is increasing with respect to Δ.

Proof of Proposition 3: recall that

$$V_H(\Delta) = B_H(\Delta) - \frac{1-\mu}{\mu}\max(0, \phi(\Delta) - \phi(\Delta_0)).$$

It is clear that the slope of V_H is everywhere below the slope of B_H, and thus $\Delta_H \leq \Delta_H^1$. Similarly, the slope of V_H never decreases when Δ_0 increases; and the same property holds when D increases (see the proof of Proposition 2). The same line of reasoning may be followed for Δ_L. Overall Δ_L and Δ_H must be non-decreasing with Δ_0 and D.

Proof of Proposition 4: First consider the insurance case defined by $\Delta_L^1 < \Delta_H^1 < \Delta_0$, so that we have $\Delta_H^2 = \Delta_H^1$. Now recall that Δ_L^2 maximizes

$$V_L(\Delta) = B_L(\Delta) - \frac{\mu}{1-\mu}\max(0, \phi(\Delta_0) - \phi(\Delta)). \quad (11)$$

Let μ go to one. Then the term $\mu/(1-\mu)$ goes to infinity and we must have $\liminf(\phi(\Delta_0) - \phi(\Delta_L^2)) \leq 0$, or $\Delta_0 \leq \liminf \Delta_L^2$. Therefore we have obtained $\Delta_H^2 = \Delta_H^1 < \Delta_0 \leq \liminf \Delta_L^2$, so that (Δ_H^2, Δ_L^2) is incentive-compatible for μ large enough, and it must be the solution.

Now let μ go to zero. We know from the discussion in the text that if V_H and V_L are strictly quasi-concave, then the optimal pair of contracts is bunching. But for μ close to zero, V_L is close to B_L, which is strictly quasi-concave by assumption. As for V_H, let us look at

$$\frac{\mu}{1-\mu}V_H(\Delta) = \min(0, \phi(\Delta_0) - \phi(\Delta)) + \frac{\mu}{1-\mu}B_H(\Delta)$$

The first term is zero for $\Delta < \Delta_0$ and decreasing afterwards. The second term is strictly quasi-concave by assumption and has a unique maximum in $\Delta_H^1 < \Delta_0$. Overall V_H must be strictly increasing until Δ_H^1 and then strictly decreasing; thus it is strictly quasi-concave.

The case when $\Delta_H^1 > \Delta_L^1 > \Delta_0$ is entirely symmetrical.

Finally, consider the intermediate case when $\Delta_L^1 < \Delta_0 < \Delta_H^1$ and assume that the social surpluses B_H and B_L are strictly quasi-concave.

Note that Δ_H must solve $\max_{\Delta \leq \Delta_L} V_H(\Delta)$. Since $\Delta_0 < \Delta_H^1$, then from our quasi-concavity assumption on B_H we get that V_H is strictly increasing below Δ_0. This shows that the solution Δ_H must be above Δ_0, or equal to Δ_L if $\Delta_L < \Delta_0$.

By the same line of reasoning, Δ_L must be solution to $\max_{\Delta \geq \Delta_H} V_L(\Delta)$. Since $\Delta_0 > \Delta_L^1$, V_L is strictly decreasing above Δ_0. This shows that the solution Δ_L must be below Δ_0, or equal to Δ_H if $\Delta_H > \Delta_0$.

Together these two conclusions imply that bunching must happen; indeed, if $\Delta_H < \Delta_L$, then both $\Delta_H \geq \Delta_0$ and $\Delta_L \leq \Delta_0$ must hold, a contradiction.