Asymmetric Information in Insurance: General Testable Implications*

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Abstract

Several recent papers on empirical contract theory and insurance have tested for a positive correlation between coverage and ex post risk, as predicted by standard models of pure adverse selection or pure moral hazard. We show here that the positive correlation property can in fact be extended to much more general set-ups: competitive insurance markets, and also cases where risk aversion is public. We also provide an empirical test of our results on a new French dataset. Our tests confirm that the estimated correlation is not significantly positive; they suggest that the explanation can be found in the combination of market power and privately known risk aversions.

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Introduction

Asymmetric information is present on all markets. Whatever the product or service sold, the seller almost never knows the buyer's preferences, nor the maximum price she would be willing to pay to acquire it. Similarly, the buyer is in general unlikely to have much information about the seller's production technology or marginal costs. Most of the time, however, this asymmetry is irrelevant. In a perfectly competitive setting, the seller would not benefit from a detailed knowledge of the buyer's willingness to pay, because he has to charge the competitive price; and the buyer needs no information about the technology, since again all the information she needs is contained in the price. Hence, asymmetric information is in general both paramount and inconsequential.

A common feature of the examples above is that the value of the hidden information is private (in the sense that the payoff of the uninformed party does not depend on it for a given contract). In a recent paper, Chiappori, Salanié and Salanié (2004) prove, under weak assumptions, that competition in the private values case always leads to an equilibrium, which is moreover efficient. For want of a better term, we will call such a case "irrelevant asymmetric information" in the present paper. The main innovation of the literature on asymmetric information, as pioneered by Akerlof (1970), Rothschild and Stiglitz (1976) and many others, was to exhibit cases in which, on the contrary, asymmetric information was indeed relevant—and actually had important consequences for the existence and efficiency of competitive equilibrium. The key property driving this conclusion is the presence of "common values", in the form of a link between an agent's hidden information and the other party's payoff. In the market for lemons for instance, the

buyer's payoff depends on the quality of the car, which is only known by the seller. Similarly, an insurer's profit depends on the risk of the insurees who buy contracts from him.

When considering empirical applications of such models, the previous remarks have important consequences. One is that evidence of information asymmetries, while relatively easy to produce, are often of little interest unless the asymmetries are of the relevant type. To give only one example, agents are often faced with menus of contracts. Menus of contracts are indeed suggestive of asymmetric information. Most of the time, however, this asymmetry is irrelevant. New cars are offered in different colors, which indeed reflects the seller's ignorance about the buyer's taste. Still, market equilibrium will typically exist and be efficient as usual, as the buyer's taste does not directly affect the seller's payoff. Different levels of insurance coverage may be proposed to insurees, reflecting asymmetric information about risk aversion. Insofar as differences in risk aversion have no impact on the insurer's profit, however, the Akerlof-Rothschild-Stiglitz conclusions do not apply, and standard analysis is still valid. This simply reflects the fact that, in a competitive setting, the insuree's true risk matters to the insurer, even conditional on the insuree's contract choice, while risk aversion does not. The former is a case of common values, and the latter a case of private values.

Clearly, one should primarily be interested in testing for asymmetric information in the "relevant" case. The main purpose of the paper is precisely to propose *robust* empirical tests of *relevant* information asymmetries. Throughout the paper, we concentrate on the particular case of insurance contracts, both because the main theoretical contributions to competition under adverse selection (starting with Rothschild and Stiglitz's seminal paper) used this framework, and because a large fraction of existing empirical

literature deals with insurance contracts. However, our conclusions are general, and the methodology developed here could be useful in other cases.

The Positive Correlation Property In the literature on insurance, the general notions just sketched lead to a well-known property, on which recent empirical work has largely focussed¹. Under both moral hazard and "relevant" adverse selection, one should observe a positive correlation (conditional on observables) between risk and coverage: if different insurance contracts are actually sold to observationally identical agents, then the frequency of accidents among the subscribers of a contract should increase with the coverage it offers. In the Rothschild and Stiglitz (1976) model of competition under adverse selection, where riskiness is an exogenous and unobservable characteristic of agents, the correlation stems from the fact that "high risk" agents are ready to pay more than "low risk" ones for additional coverage, and will therefore choose contracts with higher coverage. Under pure moral hazard, as in Arnott and Stiglitz (1988), an opposite causality generates the same correlation: an agent who, for any unspecified (and exogenous) reason, switches to a contract with greater coverage makes less effort and thus becomes riskier.

Popular as the "positive correlation" prediction may be, its robustness may however be questioned—a standard problem facing any empirical work on the topic. Theoretical models of asymmetric information typically use oversimplified frameworks, that can hardly be directly transposed to real life

¹See for instance Chiappori and Salanié (2000, hereafter CS) and the references in Chiappori (2000). This type of property has actually been tested in different contexts. For example, one of the first papers to test the Stiglitz and Weiss (1981) theory of credit markets is Ausubel (1999), in the context of credit cards. Ausubel finds convincing evidence of adverse selection, through a similar test of correlation: customers who accept higher interest rates are more likely to default.

situations. Rothschild and Stiglitz's model assumes that accident probabilities are exogenous (which rules out moral hazard), that only one level of loss is possible, and more strikingly that agents have identical preferences which are moreover perfectly known to the insurer. The theoretical justification of these restrictions is straightforward: analyzing a model of "pure", one-dimensional adverse selection is an indispensable first step. But their empirical relevance is dubious, to say the least. In real life, moral hazard can hardly be discarded a priori (and interacts with adverse selection in a non-trivial way, as precaution depends on risk and preferences²); losses are continuous variables, often ranging from small amounts to hundreds of thousands of dollars; and preference heterogeneity is paramount and largely unobserved.

The first part of our paper is devoted to a theoretical analysis of this issue. We show that the positive correlation property derived from Rothschild-Stiglitz extends to much more general models, as already conjectured by CS, although its form and robustness vary with the type of competition at stake. Specifically, we extend the property in two directions. First, we consider the case of competitive markets, and show that relevant asymmetric information (with any combination of adverse selection and moral hazard that generates common values) indeed implies a positive correlation between risk and coverage, for suitably defined such notions. This result is a direct extension of Rothschild-Stiglitz's initial idea to a very general framework (entailing heterogeneous preferences, multiple level of losses, multidimensional adverse selection plus possibly moral hazard, and even non-expected utility). Secondly, we study the case of imperfect competition, and we underline the key

 $^{^2 \}mathrm{See}$ Chassagnon and Chiappori (1997)), de Meza and Webb (2001), Jullien, Salanié and Salanié (2001).

role of the agent's risk aversion. If it is public information, then some form of positive correlation must hold. In particular, with only one level of loss and expected utility, contracts with higher coverage must exhibit a larger frequency of accidents. Conversely, if risk-aversion is private information, the property does not necessarily hold: this was shown in Jullien-Salanié-Salanié (2001). Risk aversion thus is a key parameter whose informational status drives the testable implications of simple models in the presence of market power.

In the second part of the paper, we illustrate the theoretical analysis by testing the predictions it generates on a dataset collected by a large French car insurer. We first test the general relevance of our setting, and in particular of the assumption that agents correctly assess their accident probability. Our test uses a revealed preference argument that is robust to any assumption on the information structure or the nature of competition. We find that the data strongly corroborate the predicted property, which validates our approach. We then test for the positive correlation property, and we only find marginally significant evidence of a positive (generalized) correlation. This finding is consistent with the results of CS, who however tested a much simpler and less robust variant of the theory developed in this paper. A closer examination of the data suggests that the insurer's profits are probably higher for contracts with a higher coverage, contrary to the predictions of competitive models. This suggests that more work should be devoted to analyzing imperfectly competitive models of insurance markets.

Section 1 builds a general model of insurance under asymmetric information. In Section 2, we apply a revealed preference argument to obtain a first testable implication, that relates the premium differential to expected indemnities. Section 3 analyses the robust version of the correlation property; we show that it holds both when competition drives profits to zero and when risk aversion is public information. Section 4 tests the properties derived in Sections 2 and 3. Section 5 concludes.

1 The Model

1.1 The General Framework

Suppose that we observe a population of insurance policy holders, their insurance policies and their insurance claims. Since we want to derive testable predictions about the insured agents' choices, we must deal with heterogeneity across agents. Throughout this paper, we denote by X the characteristics of the insured which are observable by insurers, and we assume that the econometrician observes X (from the insurers' files). A group of agents sharing the same values for X are ex-ante indistinguishable from the insurers' viewpoint, and so must have access to the same set of insurance contracts. To derive predictions on their choice of a particular contract, we focus on such a group, which simply means that we work conditionally on the value of X; for convenience we often omit the variable X.

Our theoretical framework thus focusses on unobservable heterogeneity. Agents face the risk of an accident, which we assume to be equivalent to a monetary loss $L \geq 0$. Each agent is characterized by a parameter $\theta \in \Theta$, which is his private information and may affect his preferences and/or his risk. An agent of type θ may secretly choose the distribution of losses G in some set \mathcal{G}^{θ} . This set may be a singleton, as in pure adverse selection models, or include more than one choice, as when agents choose prevention efforts in moral hazard models.

Each agent can also buy an insurance contract C = (R(.), P), which

specifies a premium P and an indemnity $R(L) \geq 0$ for every possible claim $L \geq 0$. By definition, R(0) = 0. In some situations the agent may also choose not to report a loss, for example if the loss is smaller than the deductible in the contract. For conciseness in notation, we identify claims and losses here, but our predictions extend to more complex settings (see Appendix 2). Hence the final wealth of the agent is $W(L) = W_0 + R(L) - L - P$, where W_0 denotes initial wealth. Throughout the paper, we concentrate on reimbursement schedules such that a higher loss is always bad news for the agents, in the sense that the net loss L - R(L) is non-decreasing with L. Not only is this property satisfied empirically, but it also relies on compelling theoretical arguments³.

To compare two contracts C_1 and C_2 proposed on the market, we rely on the following simple definition:

Definition 1 Contract C_2 covers more than contract C_1 if $R_2(L) - R_1(L)$ is non-decreasing.

In the oft-treated case of two events $L \in \{0, \bar{L}\}$, the condition reduces to $R_2(\bar{L}) \geq R_1(\bar{L})$. In the empirically prevalent case of contracts with straight deductibles $R_i(L) = \max\{L - d_i, 0\}$, the definition amounts to $d_2 \leq d_1$. Similarly, for contracts offering constant copayment $R_i(L) = \alpha_i L$, our criterion is equivalent to $\alpha_2 \geq \alpha_1$. As it turns out, most real life contracts (including the contracts considered in our empirical application) belong to these classes.

From a more theoretical viewpoint, the definition requires that C_2 offer relatively greater indemnities when L is higher; one implication is that a

 $^{^{3}}$ Contracts for which R(L) increases faster than L are almost systematically ruled out because of usual risk-sharing arguments, and because of their perverse incentives properties. Indeed, under such contracts, the agent gains in *exaggerating* the loss, a type of fraud that is extremely difficult to detect. This point has been widely emphasized by the litterature on insurance fraud (see Picard (2000)).

contract with more coverage reduces the risk on final wealth, in a precise sense which is discussed below. Obviously, the "order" induced by our notion of coverage is only partial; i.e., for two arbitrary contracts, it may well be the case that none covers more than the other. Then an agent's decision (and the empirical tests based on it) cannot be analyzed without a precise knowledge of the agent's preferences.

1.2 Behavioral assumptions

Within this very general setup, we state the following assumptions:

- A1 Each agent's preferences can be represented by a state-independent preference ordering over the distribution of final wealth⁴, monotonic with respect to first order stochastic dominance.
- **A2** Agents are risk-averse in the sense that they are averse to mean-preserving spreads on wealth.
- A3 "Realistic Expectations": When agents choose a contract, they correctly assess their accident probability and loss distribution; i.e., they use the true loss distribution G.

The first two assumptions are very weak. Assumption 3 is stronger, but indispensable for empirical applications, since the agents' subjective assessments of their risk are not observable. Our first task will be to provide a test allowing to check the empirical relevance of this assumption.

⁴This implicitly rules out situations in which the agents' choices are partly driven by extrinsic, unobserved characteristics such as the reliability of the insurer, its location, etc. It fits well empirical studies involving choices between different contracts offered by the *same* company, as is the case in the application below; it may however be more problematic when different companies are considered.

Before describing such a test, one should first emphasize that the class of models we consider encompasses most existing contributions, including the following works which all assume a von Neumann Morgenstern utility function $u^{\theta}(W, G)$:

- Pure adverse selection (Rothschild and Stiglitz (1976) or Stiglitz (1977)): here \mathcal{G}^{θ} is a singleton. The Von Neumann-Morgenstern utility function u^{θ} does not depend on G, but it may depend on θ as in the multidimensional model of Landsberger and Meilijson (1999).
- Pure moral hazard (e.g., Arnott and Stiglitz (1988)): here Θ is a singleton, since agents are ex ante identical, while \mathcal{G}^{θ} has at least two elements.
- Moral hazard plus adverse selection on prevention cost (Chassagnon and Chiappori (1997)): here $u^{\theta}(W,G) = v(W) c^{\theta}(G)$, where v is common to all types of agents.
- Moral hazard plus adverse selection on risk aversion. In de Meza and Webb (2001), utility takes the form $u^{\theta}(W,G) = v^{\theta}(W) c(G)$; in Jullien, Salanié and Salanié (2001), $u^{\theta}(W,G) = v^{\theta}(W-c(G))$. In both models, c is common to all types of agents, which differ only through their utility of wealth v^{θ} .

Lastly, it is important to stress what our results do *not* require. Although we allow for a general form of adverse selection (including multidimensional characteristics) plus possibly moral hazard, we do not impose any single-crossing condition. We do not restrict the number of types, nor their distribution. Neither do we assume expected utility maximization; our results hold in a non-expected utility framework as well, provided agents are risk-averse.

2 Testing the Realistic Expectations Assumption

A test of Assumption A3 above must check that the agent's subjective assessment of her own risk is not at odds with her true loss distribution. Since subjective representations are not directly accessible, they must be inferred from the agent's decision, i.e. contract choice. Hence, ultimately, the test must boil down to some consistency property between agent's choices and true underlying probabilities. Ideally, the test should be valid irrespectively of the information structure (i.e., whether information is symmetric or not) and the competitive context. We now present a general test that meets these criteria.

2.1 A Revealed Preference Argument

The test relies on a simple but robust revealed preference property. Assume that an agent prefers a contract C_1 to a contract C_2 which covers more. First notice that as $R_2(L)$ is larger than $R_1(L)$ for all L, the premium must be higher for contract C_2 ($P_2 - P_1 > 0$), for otherwise choosing C_1 would violate first order stochastic dominance. Risk aversion allows to strengthen this bound on the premia differential:

Lemma 1 Assume that an agent prefers contract C_1 to C_2 , and C_2 covers more than C_1 . Let G be the distribution of claims as anticipated by the agent under C_1 . Then under Assumptions A1 and A2

$$P_2 - P_1 \ge \int [R_2(L) - R_1(L)] dG(L).$$
 (1)

The proof is given in Appendix 1, and it relies on a standard argument. Under Definition 1, for any given distribution G the final wealth under C_1 is riskier than the final wealth under C_2 . Since the insured dislikes meanpreserving spreads, it must then be that the final wealth under C_1 has a higher expectation, to compensate for its higher riskiness; otherwise the agent would gain by buying C_2 while keeping the same distribution of claims.

In words, if an agent chooses one contract over another with better coverage, the decrease in premium must be sufficient for the expected income of the agent to increase at unchanged behavior. As this result only uses revealed preference, it is very general. For instance, it still holds if there is some compulsory insurance, as it only involves the comparison between two available contracts, conditional on the fact that the agent buys a contract. Also, it is valid with symmetric or asymmetric information, and it does not require perfect competition—the property holds under monopoly or oligopoly as well.

Lemma 1 was obtained under the assumption that the agent reports all losses. However, it turns out that it can be extended to more complex settings. We refer the reader to Appendix 2 for a proof that Lemma 1 still holds when this assumption is relaxed.

2.2 A Testable Prediction

Lemma 1 was phrased in terms of a revealed preference argument, valid at the agent's level. To obtain a testable prediction, let us consider a group of indistinguishable agents, with the same values for the observable variables X. Assume that the insurers' data show that some of them bought contract C_1 while others bought contract C_2 . The premia P_2 and P_1 are known, and the data also allow to compute an empirical distribution of claims $F_1(L)$ for contract C_1 . We then have the following result: **Proposition 1** Suppose that contract C_2 covers more than contract C_1 , and that both contracts are sold to indistinguishable agents with realistic expectations. Then under assumptions A1, A2 and A3

$$P_2 - P_1 \ge \int [R_2(L) - R_1(L)] dF_1(L).$$
 (2)

In most cases, insurers set the levels of premia according to observables X but leave indemnities unchanged for each contract. Then the prediction must hold for any group of indistinguishable agents, that is for any value of X, and one may test the following family of inequalities:

$$\forall X, P_2(X) - P_1(X) \ge \int [R_2(L) - R_1(L)] dF_1(L|X).$$

To illustrate the prediction, let us apply it to the case when there are only two events $L \in \{0, \bar{L}\}$. In this case let q_i be the empirical probability of a claim under contract C_i . If contract 2 covers more than contract 1, then $R_2(\bar{L}) > R_1(\bar{L})$, and (2) obviously gives

$$P_2 - P_1 \ge q_1(R_2(\bar{L}) - R_1(\bar{L})). \tag{3}$$

The result may also be applied to the case of two contracts with straight deductibles $R_i(L) = \max\{L - d_i, 0\}$ with $d_1 \geq d_2$. From the data we can compute the probability $q_1 = Prob(L > d_1)$ that a positive indemnity is transferred under C_1 . Then $(R_2 - R_1)$ is zero in the absence of claim or when $L < d_2$, is non-negative for $L \in [d_2, d_1]$, and is equal to $(d_1 - d_2)$ otherwise. We get

Corollary 1 Suppose that C_2 and C_1 are two straight deductible contracts, and C_2 covers more than C_1 ($d_1 \ge d_2$). Let q_1 be the probability that L is

above d_1 under C_1 . Then

$$P_2 - P_1 \ge q_1(d_1 - d_2). \tag{4}$$

Proposition 1 provides a test of the realistic expectations assumption, in the sense that the property should hold under the null that agents perfectly know their distribution of losses. Its power should however be qualified, for the following reason. Assume that agents are subject to biases in their risk perceptions. Then (1) only applies to the distribution G as perceived by each agent, and not to the agent's true distribution of claims F. Suppose however that agents are pessimistic and overestimate the risk, in the sense that $G \leq F$. Then

$$\int [R_2(L) - R_1(L)][dG(L) - dF(L)] = \int \frac{d}{dL} [R_2(L) - R_1(L)][F(L) - G(L)]dL \ge 0$$

and (1) remains valid when replacing G by F. Hence Proposition 1 and the above applications still hold when agents are pessimistic.

Technically, we thus test the null of realistic expectations against the assumption that agents are optimistic, in the sense that they systematically underestimate their risk. Note, however, that optimism is frequently assumed in the literature. For instance, in a model analyzed by Koufopoulos (2002), competitive equilibria are such that optimistic agents choose a low coverage but do not make any prevention effort, while pessimistic agents choose a high coverage and exert some effort to reduce their probability of accident. Then both (1) and the positive correlation property below may be violated, if some agents are optimistic enough.

3 The Positive Correlation Property

The result in Proposition 1 provides a test that does not rely on the market structure, but which does not translate obviously into a correlation structure between risk and coverage. This is not surprising. In contrast with the previous results, the positive correlation property cannot be expected to hold independently of the competitive context or the information structure. We study below two cases in which the property indeed holds. Once again, we omit the observable variables X, although it should be clear that all results are conditioned on it.

3.1 A Prediction in a Competitive Environment

As is well known, the mere definition of a competitive equilibrium under asymmetric information is a difficult task, on which it is fair to say that no general agreement has been reached. For the moment, we only assume that competition, whatever its particular form, leads to profits that are non-increasing in coverage. Also, we assume away proportional costs; moreover, while fixed administrative costs are allowed for, we assume they are identical accross contracts.⁵ Technically, let F_i be the empirical distribution of claims under C_i , and let $\pi(C_i)$ be the profit the insurer makes on contract C_i . Then

$$\pi(C_i) = P_i - \int R_i(L)dF_i(L) - \Gamma$$

where Γ denotes fixed costs associated with the contract. We then assume the following:

⁵Note that this assumption cannot be satisfied in the special case when contract C_1 is the no-insurance contract: then administrative costs are not incurred for C_1 , and this changes the expression for the actuarial premium. de Meza-Webb (2001) indeed offer a model in which agents choose between insurance and no insurance, and in which insured agents may be riskier ex post than uninsured agents.

Non-Increasing Profit assumption (NIP): if C_2 covers more than C_1 , then $\pi(C_2) \leq \pi(C_1)$.

This assumption holds trivially in the Rothschild-Stiglitz model and in fact in most theories of competitive equilibrium that have been proposed in the literature, where competition drives profits to zero on every contract. It is however more general than a standard zero-profit assumption, since also holds in the cross-subsidies model of Miyazaki (1977), where the losses made on the full coverage contract (chosen by high risk agents) are subsidized by the profits stemming from the alternative, partial coverage contract (that attracts low risk agents). Of course, it need not hold in non-competitive models such as Stiglitz (1977) or Jullien, Salanié and Salanié (2001).

Now the NIP assumption may be rewritten as

$$P_2 - P_1 \le \int R_2(L)dF_2(L) - \int R_1(L)dF_1(L)$$

and replacing in (2) we get:

Proposition 2 Assume that assumptions A1, A2, A3 and NIP hold. If two contracts C_1 and C_2 are bought in equilibrium, and C_2 covers more than C_1 , then

$$\int R_2(L)dF_2(L) \ge \int R_2(L)dF_1(L). \tag{5}$$

The result states that the empirical risk is larger for the contract with the higher coverage, in the sense that the average indemnity would be smaller with the distribution of claims of the other contract. The general insight can be summarized as follows. First assume that competition leads to actuarially fair contracts and yet our result does not hold: at least two contracts C_1 and

 C_2 are sold at equilibrium, and C_1 covers less than C_2 but has ex post riskier buyers. Since C_1 has higher ex post risk, its "unit price" (i.e., the ratio of premium to coverage) will be larger. But this leads to a contradiction, as under fair pricing, rational agents will never choose a contract entailing less coverage at a higher unit price.

Note that in models of pure adverse selection on preferences (risk aversion for instance) the distribution of claims is identical across contracts. Just as in models with symmetric information, (5) then is obvious, as F_1 and F_2 are identical. This is the "irrelevant" case of private values discussed in the introduction. Agents do have some private information, but this information is of no relevance for the insurer: it is clear from the expression of profits that only hidden information that affects the distribution of risk matters to him.

Of course, the interesting part of Proposition 2 is that whenever information asymmetry on the distribution of risk is involved and a menu of different contracts are offered and sold at equilibrium, a *strict* inequality must hold. In that case, the asymmetry is relevant, in the sense that the insuree's information matters for the insurer's profit. All the specific features derived in the literature (cream-skimming, non-existence or inefficiency of equilibrium, etc.) can only occur in this case. This obviously includes adverse selection on risk, but also adverse selection on risk aversion in the presence of moral hazard.

It is easy to derive consequences of (5). Of particular theoretical interest is the case in which contracts specify a fixed level of reimbursement for any accident. Denote again q_i the probability of a claim under contract C_i . Then the empirical riskiness must be positively correlated with the coverage, which is the test performed in Chiappori and Salanié (1997) and CS:

Corollary 2 Assume that assumptions A1, A2, A3 and NIP hold and that $L \in \{0, \overline{L}\}$. If two contracts C_1 and C_2 are bought in equilibrium, and C_2 covers more than C_1 , then $q_1 \leq q_2$.

In the case of general loss distributions, let C_1 and C_2 be straight deductible contracts, assume that losses smaller than the value of the deductible are not reported, and define the expected claims under contract C_i as $E_i[L] = \int L dF_i(L)$. Then we obtain

$$E_2[L] - E_1[L] \ge d_2(q_2 - q_1).$$
 (6)

Thus if contract C_2 leads to a higher probability of a claim, it must also generate larger expected claims. In particular, a contract with full insurance, if available, must generate larger expected claims than any other straight deductible contract (apply (6) with $d_2 = 0$).

3.2 Testing the Prediction

Testing Proposition 2 or its corollaries only requires observing the variables X in the insurer's files, two contracts, one of which has higher coverage, and being able to estimate the conditional distributions of claims. In particular it does not require knowing the premia under the two contracts. Rejecting the prediction would thus tend to indicate that our first prediction (2) or/and the NIP assumption do not hold.

In practice the definition of profits may need to be refined. Taxation of premia (or indemnities) may have to be taken into account. Similarly it is often reported that insurers compute premia by applying a constant "loading factor" λ to the expectation of indemnities. In such cases, one has to rewrite the definition of profits accordingly:

$$\pi(C_i) = \frac{P_i}{1+t} - (1+\lambda) \int R_i(L) dF_i(L).$$

Then the NIP assumption still allows to compute an upper bound for the premia differential $P_2 - P_1$. Replacing in (1) then yields a testable prediction:

$$\int R_2(L)dF_2(L) - \int R_2(L)dF_1(L) \ge K(E_1[L] - E_2[L]),\tag{7}$$

denoting $K = (1+t)(1+\lambda) - 1$. This prediction still does not depend on premia, and can be tested given reasonable values of t and λ , as discussed in the following Section.

Finally, if fixed costs vary between contracts, the condition may still apply, but under a different form. Namely, if

$$\pi(C_i) = P_i - \int R_i(L)dF_i(L) - \Gamma_i$$

with $\Gamma_1 \neq \Gamma_2$, then (NIP) must be replaced by the following property: if C_2 covers more than C_1 , then $\pi(C_2) + \Gamma_2 \leq \pi(C_1) + \Gamma_1$. Under zero profit, it follows that $\Gamma_1 \geq \Gamma_2$; this implies, for instance, that a contract involving less coverage has a higher fixed cost. This property may however fail to hold. For example, a contract with higher coverage may also be more comprehensive⁶, so that costs per claim may be higher. Under general contracts and costs $c_i(L)$ which may differ across contracts, the result in Proposition 2 becomes

$$\int R_2(L)[dF_2(L) - dF_1(L)] \ge \int c_1(L)dF_1(L) - \int c_2(L)dF_2(L)$$

and whether the left-hand-side remains positive now becomes an empirical question. Clearly more information is needed on costs per claim to provide a fully convincing test; but one may still estimate the left-hand side from the data and compare its value to a reasonable estimate of the right-hand side.

⁶Consider for example automobile insurance, for which the basic contract only covers damages to third parties. Extending the coverage to the damages incurred by the insuree requires that the insurer devote resources to estimating these damages (we thank David de Meza and David Webb for this remark).

The main conclusion of this discussion, thus, is that whatever the cost function of the insurers, if we have enough information about it then we can test some well-defined implication of asymmetric information (which may not look like a positive correlation property any more⁷).

3.3 Expected Utility with Public Risk Aversion

While the previous section was dealing with competitive environments, we now allow for market power and imperfect competition. This generalization comes at a cost. In order to keep the positive correlation property, we need to assume that (i) the agent has a von Neumann-Morgenstern utility function $u^{\theta}(W, F)$, and (ii) observationally identical agents exhibit the same risk aversion⁸, the latter thus being independent of the distribution F. Under this assumption the utility function is determined up to an affine transformation:

There exist functions v(W), a^{θ} and c^{θ} such that, for any θ , one can write

$$u^{\theta}(W, F) = a^{\theta}(F)v(W) - c^{\theta}(F)$$

with
$$a^{\theta}(F) > 0$$
.

The class of models satisfying this assumption includes the standard models of pure adverse selection à la Stiglitz (1977) and pure moral hazard à la Arnott and Stiglitz (1988), as well as more complex frameworks.

⁷Assume for instance that the payment R(L) actually costs $\kappa R(L)$ to the insurer, where $\kappa > 1$, and that competition drives profits to zero. Then any contract offered at equilibrium must satisfy $P_i = \kappa \int R_i(L) dF_i(L)$. It is well-known that, in this case, the optimal contract entails only partial coverage (actually, a fixed deductible). With homogenous risk but different risk aversions, more risk averse individuals will optimally choose a contract entailing more coverage (a smaller deductible). Now, let us introduce some infinitesimal difference in risk that is perfectly correlated with risk aversion; namely, assume that more risk averse agents are slightly less risky. Then the correlation between risk and coverage is negative—but we can still test (7).

 $^{^8}$ This is equivalent to assuming that risk aversion is publicly observable, hence is included in the observables X. Notice that when risk-aversion depends on wealth, the assumption requires that wealth be observable—or at least that enough information about wealth may be inferred from observables.

Let two contracts C_1 and C_2 be bought in equilibrium by some individuals within the population at stake. For i = 1, 2, denote $w_i(L) = v(-L + R_i(L) - P_i)$ the utility under contract i after a loss L. For any agent buying C_1 and choosing a distribution of claims G_1 under C_1 , we must have (after simplifying the a and c terms)

$$\int w_1(L)dG_1(L) \ge \int w_2(L)dG_1(L).$$

If agents have realistic expectations, by aggregating over the types buying C_1 we find that

$$\int w_1(L)dF_1(L) \ge \int w_2(L)dF_1(L)$$

where F_1 is the empirical distribution of losses under C_1 . A similar argument applied on C_2 yields

$$\int w_2(L)dF_2(L) \ge \int w_1(L)dF_2(L).$$

Now substract these inequalities to get

$$\int (w_2(L) - w_1(L))(dF_2(L) - dF_1(L)) \ge 0.$$
 (8)

Notice that this result is valid whatever the contracts. If we moreover assume that C_2 covers more than C_1 , then it can be shown that $w_2 - w_1$ is negative, then positive, when L increases. (8) then looks like a positive correlation property: in a weighted sense, F_2 puts more weight than F_1 on high losses and less weight on low losses. Nevertheless the result involves the utility function v, which is unknown to the econometrician. Its implication is clearest for the case of two events, accident and no accident:

Corollary 3 Assume that risk aversion is public, expectations are realistic, and $L \in \{0, \overline{L}\}$. If two contracts C_1 and C_2 are bought in equilibrium, and C_2 covers more than C_1 , then $q_1 \leq q_2$.

This result was already known in the Rothschild-Stiglitz case. Our contribution here is to highlight the key role played by the assumption of identical risk-aversion. In particular, once agents have chosen their preventive efforts they can be ordered according to their riskiness; and then the assumption guarantees that agents which are *ex-post* riskier indeed prefer contracts with higher coverage.

The assumption of identical risk-aversion is necessary for the result to hold. The underlying intuition is simple, and can be described in the polar case of an insurance monopoly. Start with the benchmark situation where agents have identical risks, but different risk aversion. Then in the optimal monopoly contract, partial coverage is used to screen agents according to their risk aversion, exploiting the fact that more risk averse individuals are willing to pay more for additional coverage; typically, more risk averse agents are fully covered, while less risk averse clients reveal their type (and benefit from a lower premium) by accepting partial coverage. Now, introduce an infinitesimal difference in risk that is fully correlated with risk aversion; specifically, the more risk averse agents have a (slightly) smaller accident probability. The optimal contract will still offer more coverage for the more risk averse individuals, at a higher price, despite the fact that the aggregate risk for that population is (slightly) smaller - a pattern that creates a negative correlation between risk and coverage.

⁹Of course, the situation just described is somewhat specific, because it relies on a strong, *exogenous* negative correlation between risk and coverage. Assuming, for instance, a positive correlation (risk averse agents are riskier) would revalidate the positive correlation property. A more interesting approach would *endogenize* the correlation. For instance, Jullien, Salanié and Salanié (2001) consider a model where risk-aversion is the agent's private information and agents secretly choose some prevention effort (moral hazard). Then the correlation between preferences and realized risk is endogeneized; the authors show that a monopoly may optimally propose two contracts that involve a violation of the positive correlation property.

4 An Empirical Test

Tests of the positive correlation between risk and coverage on insurance contracts have provided mixed results. Most papers on automobile insurance (see, e.g., CS) cannot reject the no-correlation null: there in fact appears to be no correlation between the coverage of a contract and the ex post riskiness of its subscribers. Puelz and Snow (1994) was an early exception; but Dionne, Gouriéroux and Vanasse (2001) attribute their result to the spurious effect of a linear specification. Cawley and Philipson (1999) find no evidence of a positive correlation in their study of life insurance contracts. On the other hand, the market for annuities seems to be plagued by adverse selection problems, as documented by Brugiavini (1993) and more recently Finkelstein and Poterba (2000); Bach (1998) reaches similar conclusions in her study of mortgage-related unemployment insurance contracts.

Since all of these papers rest on a simplified analysis of the insurance market, it is interesting to see whether the more general predictions we obtained in this paper fare better when taken to the data. Note that the maintained assumptions of the theory are different for each of our results. Proposition 1 only relies on a revealed preference argument, while Proposition 2 adds a non-increasing profit condition and Corollary 3 assumes that risk-aversion is public and losses are 0-1.

We use a dataset on car insurance that was given to us by a large French insurer. This dataset covers the two years between October 1, 1987 and September 30, 1989 and contains information on insuree characteristics, contract variables and claims. We only kept those individuals that are observed for at least a full year, and we normalized the data so that our figures correspond to exactly one year. The dataset we use in the application comprises

Table 1: Deductibles for Comprehensive Policies

Deductible (francs) %	Number of policies	Average q	Average claim
0	11,218	0.063	7,820
500	7,247	0.045	7,130
1,000	14,044	0.061	8,980
2,000	1,779	0.066	11,840

69,892 policies. About half (34,288) of these policies entail comprehensive coverage. We focus on these policies because they are straight deductible contracts, with varying deductible levels. As shown in Table 1, the most common deductible levels are 0 franc and 1,000 francs¹⁰. We will use the 1,000 francs contract as our contract C_1 and the 0 franc contract as our C_2 .

We start by testing Proposition 1 from Corollary 1, which tells us that we should have

$$P_2 - P_1 \ge q_1(d_1 - d_2)$$

where here $d_1 = 1,000$ and $d_2 = 0$. A first difficulty is to define claims, as q_i is the probability of a claim under contract C_i . For every claim in France, insurance experts assign responsibilities to the policyholders involved. A policyholder that is deemed not to be responsible is fully reimbursed; a responsible policyholder is only reimbursed if (s)he has a comprehensive policy, and then only up to the deductible. Thus we should clearly focus on claims in which the policyholder was responsible and incurred some damage; there are 2,007 such claims in the data¹¹, so that the average q (over all contracts) is about 0.059. The third column of Table 1 gives the average value of q for each of the four comprehensive contracts.

¹⁰One franc was about 16 US cents in 1989.

 $^{^{11}}$ Some policyholders have more than one responsible claim; we neglect these multiple occurrences in the following.

Remember that our predictions are conditional on all variables X that are observed by the insurer (and hopefully by the econometrician). There are a large number of such variables in the data. As in CS, our approach is to define "cells" of policyholders with identical values of those X variables that prior studies have identified as the most relevant. We choose six 0-1 X variables:

- whether the policyholder has the best experience rating (a 50% bonus)
- whether (s)he is a man^{12}
- whether his/her car is relatively powerful
- whether his/her car is relatively expensive
- whether the driver is young or old
- whether the car is driven in an urban area.

This defines $2^6 = 64$ cells. Each of them holds about 1,000 policyholders on average, but some are much smaller; thus we drop from the analysis the 16 cells that contain fewer than 200 insurees.

Within each cell X, we first estimate nonparametrically the probability $q_1(X)$ and compute the quantity

$$T_1(X) = P_2 - P_1 - q_1(X)(d_1 - d_2)$$

then we compute a Student statistic by dividing $T_1(X)$ by its standard error. This yields a collection of 48 = 64 - 16 numbers. Under the null hypothesis that $T_1(X) = 0$ for all X, these numbers should be distributed as a N(0,1)

 $^{^{12}}$ Premium discrimination based on sex is legal in France, although the European Commission is planning to forbid it.

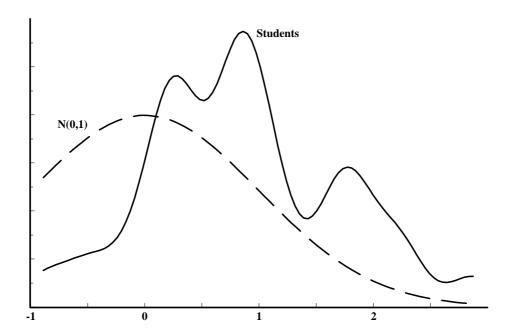


Figure 1: Studentized Estimates of q_2-q_1

normal distribution. It turns out that *all* of these numbers are positive (the smallest one is 1.63), which gives very strong corroborating evidence for Proposition 1 at any reasonable level of significance.

Within each cell X, we then test whether $q_2(X) \geq q_1(X)$, as done by CS. We first estimate $q_1(X)$, $q_2(X)$ and their estimated standard errors, and compute the Studentized version as above. Figure 1 shows the estimated nonparametric density of these numbers (weighted by cell sizes), along with the N(0,1) density. Contrary to CS, we find clear evidence here that $q_2(X) > q_1(X)$, as the distribution of the Students lies to the right of the normal curve¹³.

 $[\]overline{\ \ }^{13}$ A Kolmogorov-Smirnov test gives a value of 0.381, way above the 1% critical value for 48 observations.

Of course, this is not conclusive, since we only derived this theoretical prediction (in corollaries 2 and 3) when there is only one loss level. Our dataset gives us a rather detailed breakdown of the costs of each claim for the insurer, for each type of guarantee. What matters to us is the total cost of the claim, which is easily reconstructed from the data. Figure 2 plots the estimated nonparametric density of the costs for all responsible claims. Clearly, it is very dispersed: some claims are very costly and some very cheap¹⁴. Thus the corollaries may not apply, and it is of some interest to test the generalized positive correlation property, as given in Proposition 2. It is indeed possible that while the contract with the 1,000 francs deductible has more claims, these are less costly than under the contract with no deductible. The last column of Table 1 gives some evidence on the size of the average claim under the comprehensive guarantee.

Comprehensive coverage premia were taxed at a rate of t=18% in 1989. Moreover, we need to account for the cost structure of insurers. If we knew the value of the loading factor λ , we could directly test prediction (7). Since we do not know the precise value of λ , we ran the test for values of λ from 0 to 100%, which is a rather wide interval (a value of 20% is a reasonable guesstimate). For each of these values, we compute the Kolmogorov-Smirnov statistic for the null hypothesis that the Studentized statistics are distributed as N(0,1). We find that the value of the test statistic increases rather slowly with λ . It reaches a p-value of 10% between $\lambda=0.10$ and $\lambda=0.15$, which seems low, and a p-value of 5% between $\lambda=0.40$ and $\lambda=0.45$, which seems high. Our conclusion is that for reasonable values of λ , the test gives weak evidence for the positive correlation property: the null of zero correlation

¹⁴There are indeed a few negative costs. This is due to payment rules among insurers: small claims are settled by the insurer of each policyholder involved, and lump-sum payments are sometimes exchanged.

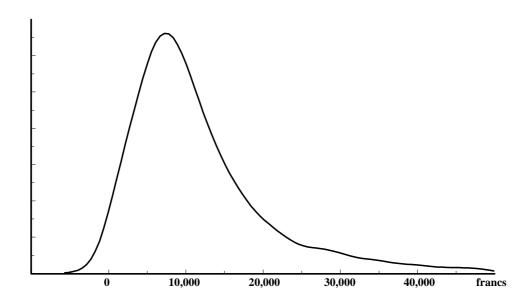


Figure 2: The Distribution of Costs of Claims

cannot be rejected conclusively. This confirms the evidence given in CS, on a much more solid basis since we take into account both the dispersion of claims and the cost structure of the insurer.

Since we have very strong evidence for Proposition 1 and weak evidence for Proposition 2, an auxiliary hypothesis must be violated. The obvious suspect is the NIP assumption. To test this, we compute profits

$$\pi_i(X) = \frac{P_i(X)}{1+t} - (1+\lambda) \int R_i(L) dF_i(L|X)$$

for λ between 0 and 100% and we test $\pi_1(X) \geq \pi_2(X)$. Not surprisingly given the tests above, we find very strong evidence to the contrary: the data suggest that in fact, $\pi_1 < \pi_2$ in most cells and so the NIP assumption is rejected. This is an interesting finding per se, as it points towards the presence of imperfect competition in the French car insurance market.

Another possibility is that in testing Proposition 2, we erred in assuming that the cost structure of the insurer is well captured by a loading factor. A fixed cost per insuree would not change anything if it is independent of coverage. On the other hand, higher administrative costs per claim for the contract with higher coverage might restore the validity of the NIP assumption; still, an easy calculation shows that it would take a very large difference of 5,000 francs in administrative costs per claim. As this is unlikely, a violation of NIP due to imperfect competition remains the most credible culprit for our failure to reject a zero correlation of risk and coverage.

Finally, let us consider experience rating. It is a legal feature of French car insurance, but we neglected it so far in these empirical tests. It is easy to see that (provided both insuree and insurer have the same discount rates) introducing experience rating amounts to shifting the reimbursement function $R_i(L)$ by the discounted sum of increase in premia $c_i(L)$ in the objective func-

tions of both insuree and insurer. Since insurees are free to switch contracts after a responsible claim, it seems reasonable to assume that $c_1 = c_2 = c$. Then Proposition 1 is unchanged, while the right hand-side of the inequality in Proposition 2 becomes

$$\int c(L)(dF_2(L) - dF_1(L))$$

As this is just $(q_2 - q_1)c$ and we have strong evidence that $q_2 > q_1$, it follows that taking experience rating into account would leave our evidence for Proposition 1 unchanged and weaken our weak evidence for Proposition 2 even more. We can go a bit further. To the extent that insurees do not change contracts easily, there may be a case for assuming that $c_2 > c_1$ (as the experience rating system in France is a multiplicative bonus-malus system, so that a responsible claim brings a higher increase in premia for the more expensive contract). It is easily seen that this introduces a term

$$\int (c_1(L) - c_2(L))dF_1(L)$$

in the right-hand side of the inequality in Proposition 1. But since this new term is negative, this would strengthen even more our corroborating evidence for Proposition 1.

5 Conclusion

A first lesson stemming from this paper is that in an asymmetric information context, a positive correlation between coverage and risk properly restated seems to be a natural and robust consequence of the competitive assumption. In that sense, our paper provides (somewhat a posteriori) a theoretical foundation for many existing empirical papers, although it points to the fact that the comparison of risk is not unambiguous and that a proper measure

of risk must be used. Proposition 2 is characteristic of a competitive setting. However, Proposition 2 must be reformulated with proportional loading or taxation, experience rating, or administrative costs of processing a claim.

Under imperfect competition, the non-increasing profit assumption may not hold, and the correlation need not be positive. Indeed, the insurance companies extract rent from the policyholders, and optimal rent extraction may be such that more profit is extracted on contracts entailing more coverage. However, if risk aversion is public, which encompasses many frameworks (e.g. Rothschild and Stiglitz (1976) or Chassagnon and Chiappori (1997)) then at least with a single claim size, the positive correlation property also holds. Notice however that public risk aversion is not a natural assumption in the context of insurance, as it eliminates any unobserved heterogeneity on a key determinant of the demand for insurance. Risk aversion clearly affects both the choice of an insurance policy and the precautionary attitude. Moreover it is an intrinsic property of preferences that cannot easily be observed by insurers.

Empirically, most data sets on automobile insurance (including the one studied in this paper) do not reject the null of zero correlation. The simplest explanation is probably the absence of significant asymmetric information in automobile insurance, although more complex stories can be evoked (see de Meza and Webb (2001) and CS for a detailed discussion). Finding a significant, negative correlation would raise a more serious challenge to the theory. Our paper, together with previous findings by Jullien, Salanié and Salanié (2001), suggests that the explanation should be grounded into market power and adverse selection on risk-aversion¹⁵. In fact, the theoretical results

 $^{^{15}}$ Using subjective assessments by insurees, Finkelstein and McGarry (2003) find evidence for such a model in long-term care insurance.

in this paper strongly suggest that there is a crying need for such models. An alternative is to turn the asymmetric information model on its head, by assuming that the insurer actually knows more than the insured. This is done by Villeneuve (2000) within an otherwise standard hidden information model; he indeed finds that the correlation may be reversed, at least in a principal-agent framework. The competitive case however is more tricky, since competition tends in general (but not always) to result in full revelation.

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Appendix 1: Proof of Lemma 1

The proof is based on the following well-known result:

Lemma 2 Suppose that $W_2(L)$ and $W_1(L)$ are two non-increasing functions of a random variable L, and that $W_2(L) - W_1(L)$ is non-decreasing with L. Then $X_1(L) = W_1(L) - EW_1(L)$ is a mean-preserving spread of $X_2(L) = W_2(L) - EW_2(L)$.

Indeed denote by $H_i(x) = Prob(X_i(L) < x)$ the c.d.f. for $X_i(L)$. The assumptions in Lemma 2 imply that the difference $(H_2(x) - H_1(x))$ is positive then negative when x increases. Consequently the function

$$D(X) = \int_{-\infty}^{X} [H_2(x) - H_1(x)] dx$$

is increasing then decreasing. Moreover, $D(-\infty) = 0$, and by integrating by parts it is easily seen that

$$D(+\infty) = E[X_2(L) - X_1(L)] = 0.$$

Thus D is non-negative everywhere, and in fact is positive as soon as $W_1(L)$ differs from $W_2(L)$ with positive probability. The result follows, by definition of a mean-preserving spread.

We can now prove Lemma 1. We know that the agent prefers contract C_1 to contract C_2 , and thus he must prefer C_1 under G to C_2 under G. Let $W_i(L) = R_i(L) - L - P_i$ be the resulting wealth under contract C_i . Notice that under our assumptions on R_1 and R_2 Lemma 2 applies. Since the agent dislikes mean-preserving spreads, it must be that the expectation of W_1 is above the expectation of W_2 . This yields the result.

Appendix 2: On Claims and Losses

First consider the case when the agent incurs some observable costs c(L) when the loss L is realized. These costs may represent a monetary value for health disease or uncovered damages, or some transaction costs incurred when reporting a loss; they may also represent the anticipated increase in premia in subsequent periods, due to experience rating. This amounts to replacing the indemnity $R_i(L)$ under contract C_i with $R'_i(L) = R_i(L) - c(L)$. Definition 1 trivially extends, so that (1) still holds, provided that all assumptions hold for R'_i instead of R_i .

Let us now show that Lemma 1 extends to the case when the agent may decide not to report a loss if the indemnity is below c(L). In such a case¹⁶ the distribution of claims F may differ from the distribution of losses G: the latter is determined by the agent's precautionary behavior and characteristics, while the former may depend on each contract chosen. Suppose that the agent has chosen the distribution G, and that he reports a loss only if he finds it advantageous, whatever the contract chosen. Everything is as if the insured was facing an indemnity

$$\hat{R}_i(L) = \max(R_i(L) - c(L), 0)$$

and would declare a loss only if $\hat{R}_i(L)$ is positive. Under a weak assumption¹⁷, the property in Definition 1 is preserved by this transformation, and higher losses are still bad news. Then the proof of Lemma 1 still applies: we get

$$P_2 - P_1 \ge \int [\hat{R}_2(L) - \hat{R}_1(L)] dG(L).$$

Now recall that $R_2(L)$ is above $R_1(L)$. Denote A the set of losses L which

 $^{^{-16}}$ An obvious illustration is that of a contract with a deductible d_i , in which case the agent will not report losses smaller than d_i .

¹⁷The slope of the function $R_2(L) - c(L)$ must lie between zero and one when its value is positive.

are reported as claims under C_1 , and thus such that $R_1(L) > c(L)$. Observe that since $R_1(L) > c(L)$ implies $R_2(L) > c(L)$, the bracketed term in the above inequality is equal to $(R_2(L) - R_1(L))$ if $L \in A$, and is non-negative otherwise. Therefore its integral is larger than

$$\int_{A} [R_2(L) - R_1(L)] dG(L).$$

Now G does not differ from the empirical distribution of losses F on A; and for $L \notin A$, either L > 0 and then dF(L) = 0, or L = 0 and then $R_2(0) = R_1(0) = 0$. Overall we have shown that

$$P_2 - P_1 \ge \int_A [R_2(L) - R_1(L)] dG(L) = \int [R_2(L) - R_1(L)] dF(L)$$

and Lemma 1 extends to the case when the agent may choose not to report a loss.