# E6312 Homework 3
# Spring 2009
# Due $8^{th}$ March 2009

March 2, 2009

## 1 Problem 1

In this problem, we explore different kinds of feedback networks. Feedback networks are usually analysed using the regular loop gain based approach. Though this approach gives us more intuition in to the functioning of the network, it does not help much when it comes to design and synthesis of new networks. Another way of analysing feedback networks, particularly circuits, is to use two port networks. If you think you know most of these, go ahead and skip the intial introduction to two port parameter based analysis of feedback networks.

Consider the network shown in figure 1. It is a two port network and is defined by two currents $I_1$ and $I_2$ going into the network and two voltages $V_1$ and $V_2$ at the two ports. Using superposition of linear networks, the two currents can be written as a linear combination of $V_1$ and $V_2$. In other words, there exist parameters $y_{11}, y_{12}, y_{21}, y_{22}$ such that

$$\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned} \tag{1}$$

$$\Rightarrow \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = Y \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \text{ where } Y = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \tag{2}$$

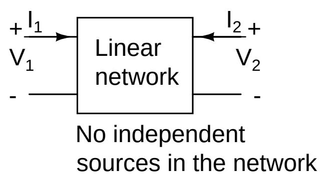



Figure 1: A generic two port network

This matrix Y, is called the y-parameter matrix and gives us the currents going in to the network when excited by two voltages $V_1$ and $V_2$. In a similar way, a matrix can be defined when we use a current excitation and measure the voltages at the two ports.

$$\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned} \tag{3}$$

$$\Rightarrow \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = Z \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \text{ where } Z = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}$$

There are other matrices called hybrid matrices where the excitations are mixed i.e., current and voltage. H-parameters are one of them and are defined by

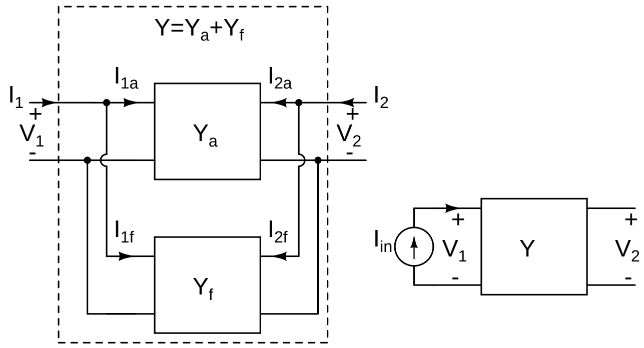



Figure 2: Two port analysis of shunt-shunt feedback

$$\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned} \tag{4}$$

$$\Rightarrow \begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = H \begin{pmatrix} I_1 \\ V_2 \end{pmatrix} \text{ where } H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

From these equations, it can be seen that all these matrices are related. For example, $Z = Y^{-1}$. Similar to the above definitions, a parameter set called G-parameters can be defined and are related to these as $G = H^{-1}$. Obviously, G and H can be related to Y and Z but are a little more complex than the relations between Z and Y or G and H.

The first of kind of feeback we look at is called shunt-shunt feedback. This is shown in figure 1. As can be seen from the figure, the voltages $V_1$ and $V_2$ are common to both the two ports (marked $Y_a$ and $Y_f$) by virtue of the way they are connected (shunt/ parallel connection at both ends). By using Kirchoff's current laws at the two ports, the Y parameter matrix for the entire network can be derived as follows:

$$\begin{pmatrix} I_{1a} \\ I_{2a} \end{pmatrix} = Y_a \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}, \quad \begin{pmatrix} I_{1f} \\ I_{2f} \end{pmatrix} = Y_f \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} I_{1a} \\ I_{2a} \end{pmatrix} + \begin{pmatrix} I_{1f} \\ I_{2f} \end{pmatrix} = (Y_a + Y_f) \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Thus the Y parameter matrix for the entire network is $Y = Y_a + Y_f$. If we excite the network by a current source $I_{in}$, as shown in figure 1 and look at the second port voltage $V_2$, we can write

$$\begin{aligned} I_{in} &= y_{11}V_1 + y_{12}V_{out} \\ 0 &= y_{21}V_1 + y_{22}V_{out} \end{aligned} \Rightarrow \frac{V_{out}}{I_{in}} = \frac{y_{21}}{y_{12}y_{21} - y_{11}y_{22}} = \frac{1}{y_{12}} \frac{\frac{-y_{12}y_{21}}{y_{11}y_{22}}}{1 + (\frac{-y_{12}y_{21}}{y_{11}y_{22}})} \equiv \frac{1}{f} \frac{Af}{1 + Af}$$

From the above equation, we can identify the term $LG = \frac{-y_{12}y_{21}}{y_{11}y_{22}}$ with the loop gain of the system and $\frac{1}{y_{12}}$ with the ideal gain of the system. Note that $y_{12} = y_{12a} + y_{12f}$ and $y_{21} = y_{21a} + y_{21f}$. In practice, the forward amplifier ($Y_a$) is unilateral and has a large gain. Because of unilateral character of the amplifier, $y_{12a} = 0 \Rightarrow y_{12} = y_{12f}$ (Also called reverse isolation, this should be preferably small for stability of the system). Also, the forward gain of the forward amplifier, $y_{21a}$ is very large. The loop gain, $LG = \frac{-y_{12}y_{21}}{y_{11}y_{22}}$ has to be large for ideal operation and this analysis gives us the measure for the term "large". This indicates that $y_{21} \approx y_{21a} \gg \frac{y_{11}y_{22}}{y_{12f}}$ for ideal operation of the feedback system.

Now to the more important input and output impedance characteristics. The output impedance of the system is calculated by disabling the input excitation (which in this case is the current source $I_{in}$) and measure the impedance looking in from the port 2 of the circuit. In other words,

$$Z_{out} = \left. \frac{V_2}{I_2} \right|_{I_{in} = I_1 = 0}$$

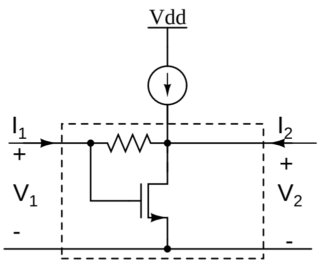



Figure 3: An example of Y-feedback in application

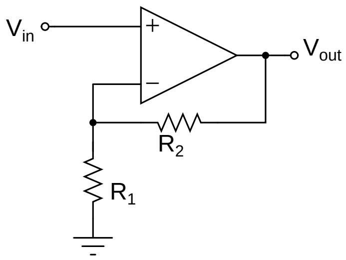



Figure 4: Circuit for Problem 1

From the definition of y-parameters in equation 1,

$$\begin{aligned} 0 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned} \quad \Rightarrow \; I_2 = \big(y_{22} - \frac{y_{21}y_{12}}{y_{11}}\big)V_2$$

$$\Rightarrow Z_{out} = \frac{y_{11}}{y_{11}y_{22} - y_{12}y_{21}} = \frac{1}{y_{22}}\frac{1}{1+LG}$$

Note that the term $\frac{1}{y_{22}}$ is the output impedance of the **open loop** system. In other words, this means that when the system is put in feedback, the output impedance of the system is **diminished** by the factor $1 + LG$. The input impedance of the system can be calculated by calculating $Z_{in} = \frac{V_1}{I_{in}}\big|_{I_2=0}$. Again from the definition of y-parameters in equation 1,

$$\begin{aligned} I_{in} &= y_{11}V_1 + y_{12}V_2 \\ 0 &= y_{21}V_1 + y_{22}V_2 \end{aligned} \quad \Rightarrow I_{in} = \big(y_{11} - \frac{y_{12}y_{21}}{y_{22}}\big)V_1$$

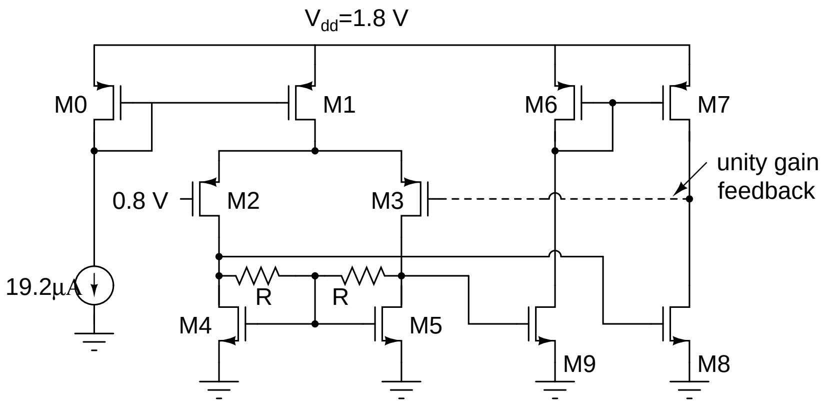
$$Z_{in} = \frac{V_1}{I_{in}}\Big|_{I_2=0} = \frac{y_{22}}{y_{11}y_{22} - y_{12}y_{21}} = \frac{1}{y_{11}}\frac{1}{1+LG}$$

Again, $y_{11}$ is the input admittance of the system in open loop. Thus the closed loop looking in impedance of the system is **reduced** by the factor $1 + LG$. In the ideal case when $LG \to \infty$, the input and output impedances of a network in shunt-shunt feedback (as it is called due to shunt connection of the two networks at both the ends) go to 0. Thus, the network is best suited for operation as a current controlled voltage source (CCVS). Because of the characteristic that the y-parameters of the total network is the sum of the individual networks' y-parameters, it is also called y-feedback. One example of a system in y-feedback is shown in figure 3. Here, the transistor is the error amplifier and the resistor in feedback is the feedback "network".

Now consider the network shown in figure 4. This is the popular non-inverting amplifier with the opamp as the error amplifier. Identify the kind of feeback this network is in, derive the transfer function *using the appropriate describing two-port parameters* and describe what happens to the input and output impedances of the network as compared to the open loop impedances. Based on these characteristics, what kind of a controlled source is this network? A more detailed description of this theory can be found here [1].

# 2 Problem 2

Consider the opamp architecture shown in figure 5. It is a simple two stage single ended opamp. Assuming that $\mu_n C_{ox} = 320\ \frac{\mu A}{V^2}$ for the NMOS transistors and $\mu_p C_{ox} = 160\ \frac{\mu A}{V^2}$ for the PMOS transistors and $V_{thn} = |V_{thp}| = 0.45\text{V}$, derive the bias points of all nodes in the circuit when the opamp is placed in unity gain feedback. Explain what sets each node voltage in the circuit.

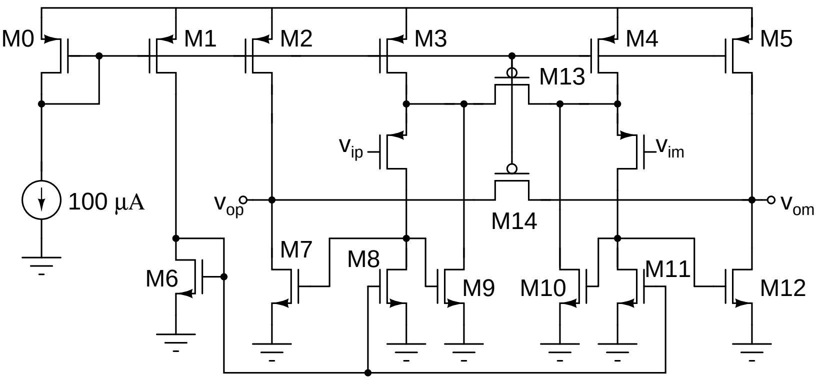



| | |
|---|---|
| M0 | 4(0.27μ/0.18μ) |
| M1 | 20(0.27μ/0.18μ) |
| M2,3 | 10(0.27/0.18μ) |
| M4,5 | 10(0.27μ/0.36μ) |
| M6,7 | 20(0.27μ/0.18μ) |
| M8,9 | 10(0.27μ/0.18μ) |

Figure 5: Two stage opamp for Problem 2

# 3 Problem 3

Consider the circuit shown in figure 6. Calculate the input to output gain **without** going into small signal circuit analysis at very low frequencies. Explain. Identify the dominant poles of the transfer function by inspection and provide reasoning. Confirm your answers by doing small signal circuit analysis as well.




| | |
|---|---|
| M0,1,13 | X |
| M2,5 | 2X |
| M3,4,14 | 4X |
| M6 | Y |
| M7-12 | 2Y |

Figure 6: Circuit for Problem 3

# References


[1] Hurst, P.J., "A comparison of two approaches to feedback circuit analysis," IEEE Transactions on education, vol.35, no.3, pp.253-261, Aug 1992.