Central Bank Commitment under Imperfect Information*

Chengcheng Jia†

November 9, 2017

LINK TO THE LATEST VERSION

Abstract

I study optimal monetary policy when both the central bank and the private sector have imperfect information about the underlying economy. I model forward guidance as providing the central bank's own forecast on optimal policy conditional on its own imperfect information. When the private sector has rational expectation, it is able to infer the imperfect information held by the central bank from the forward guidance policy. The central bank can either commit to the forward guidance policy, or re-optimize when accurate information becomes available in later stage. I demonstrate the policy trade-off for central bank commitment under imperfect information: re-optimization closes the output gap, but also makes the aggregate price level deviate further away from zero, as re-optimization leads to additional uncertainty in firms' pricing decisions, whose effect is amplified through higher order beliefs.

*I am deeply indebted to Michael Woodford for his invaluable guidance and support over the course of this project. I would also like to thank Andres Drenik, Jennifer La'O, and Jon Steinsson for their insights and helpful advice. I also greatly benefited from comments and discussion with Patrick Bolton, Frederic Mishkin, Jose Scheinkman, Stephanie Schmitt-Grohe, Martin Uribe, and other seminar participants in the Economics Department of Columbia University.. All errors are mine.

†Department of Economics, 420 W. 118th St., New York, NY 10027 Email: chengcheng.jia@columbia.edu

1 Introduction

What should central bank do when both itself and the private sector have imperfect information about the underlying economy? The empirical relevance of this question becomes more important in the in the recent practice of forward guidance policy after the Great Recession. Through announcing the forward guidance policy, the central bank provides information about its forecast on the future paths of policy, conditional on its own imperfect information. Besides announcing the forward guidance policy, central bank also faces the choice of whether it should commit to the forward guidance policy.

Feroli et al. (2017) call forward guidance with commitment as "time-based" forward guidance, as this type of forward guidance specifies a target rate based on calendar time. In their paper, "time-based" forward guidance has two disadvantages. First, it ignores future macroeconomic news, and second, it reduces flexibility in monetary policy decisions. After all, why should the central bank tie its hands while facing an uncertain future? They argue that instead of "time-based", forward guidance should be "data-dependent", under which central bank only communicates what monetary policy would react to future macroeconomic news, but allows the actual future policy to depend on more accurate information when later available.

In this paper, I argue that forward guidance without commitment may be sub-optimal than forward guidance with commitment. Although re-optimizing monetary policy conditional on accurate information in later stage can close the output gap, re-optimization drives fluctuations in the aggregate price level. This happens when firms have imperfect information and extract information from the forward guidance policy. Under forward guidance without commitment, firms need to guess how the central bank will re-optimize in later stage. I show that the strategic complementarities in price behaviors and the higher order beliefs amplify this effect, which results in higher deviations in the aggregate price level.

I model a flexible-price economy where firms maximize profit in every period. Firms set prices before household makes consumption and labor supply decisions. Prices become rigid one set. I assume that the technology shock is the only source of uncertainty in the private sector, which composites an idiosyncratic component and an aggregate component. Firms set prices in the beginning of period where they only observe their firm-specific technology, without having information on the aggregate technology or decisions by other firms. The representative household makes consumption decisions when all information is revealed in the end of the period.

I study three types of central bank. First, a "perfect opaque" central bank does not provide information on forward guidance. It sets nominal aggregate demand in the end of period when all information becomes accurate. In this case, the optimal monetary policy becomes a linear function of the aggregate technology. I assume that the private sector has rational expectation and correctly understands the central bank's reaction function. When information is perfect, the real dichotomy holds: prices fully adjust to the change in aggregate nominal demand, and output is determined only by the aggregate technology. In other words, monetary policy has no real effect, and the optimal monetary policy stabilizes price level. In comparison, when firms have only imprecise private signals, they set prices conditional on their estimates on the monetary policy. Although firms correctly understand how central bank will react as a function of the aggregate technology, as they have partial information on the aggregate technology, they underestimate the change in monetary policy when making pricing decisions. In this situation, monetary policy has little impact on the aggregate price level. I show that optimal monetary policy changes from output gap stabilization to price level stabilization when private signals become more imprecise.

The second type of central bank does forward guidance with commitment. As the private sector has rational expectation and correctly understands the central bank's reaction function, they can correctly infer the information held by the central bank from the announcement of forward guidance policy. [1] Consequently, firms get a public signal on the aggregate technology shock in addition to their private signals of their firm-specific technology. Under forward guidance with commitment, firms can set prices to fully absorb the change in monetary policy, which leaves monetary policy having no effect on real consumption. Consequently, the optimal monetary policy stabilizes price level.

The precision of public signals (central bank's information) and the precision of private signals (firms' information) have different implication on the aggregate price level. After technology shocks, when the central bank has more precise information, announcing the forward guidance policy also reveals more precise information to the private sector. As firms having more precise information about the aggregate technology shock and the response of monetary policy, the aggregate price level moves closer to zero. In comparison, when the private sector has more accurate information on the aggregate technology shock, each firm correctly understands that central bank will commits to a wrong forward guidance policy which will drive the aggregate price level away from zero. Since price setting is strategic complement, all firms increase their prices as a response.

Different from the case under perfect opacity, the imperfect information which the forward guidance policy conditions on introduces the source of monetary policy error. After a policy error, when the the public signal is more precise, the aggregate price level deviates further away from zero, as firms put higher weight on the public signal when making pricing decisions.

The third type of central bank does forward guidance without commitment. The information that the private sector extracts from the announcement of forward guidance is the same as the second type, which is the imperfect information held by the central bank on the aggregate tech-

[1] One assumption is that forward guidance of nominal aggregate demand reacts linearly to only one variable. My job market paper analyze the case where monetary policy is regarded as one signal about two shocks.

nology. Different from the forward guidance with commitment, the private sector under forward guidance without commitment also needs to forecast the future monetary policy, as central bank will re-optimize in later stage. The private sector correctly understands that future monetary policy will be a function of the actual aggregate technology and the noise in the central bank's imperfect information. Consequently, each firm needs to guess the two state variables. As prices are strategic complements, firms also need to guess the beliefs formed by other firms. The higher order beliefs amplify the effect of this re-optimization on the aggregate price level. Central bank faces a trade-off between forward guidance with and without commitment. Re-optimization fully closes the output gap, but makes the aggregate price level deviate further away from zero.

I show that as precision of public signal increases, the aggregate price level deviates further away from zero after both aggregate technology shocks and policy shocks. As the precision of private signal increases, output gap approaches to efficient level after both shocks.

Related Literature

Past literature has extensively discussed the effect of forward guidance. Del Negro, Giannoni and Patterson (2012) provide empirical evidence that standard DSGE models tend to overestimate the effect of forward guidance. Angeletos and Lian (2016) answer this puzzle by introducing imperfect information, and argue that imperfect common knowledge predicts that the attenuated effect of forward guidance. Besides Feroli et al. (2017), Campbell et al. (2012) also characterize two types of forward guidance as whether forward guidance is accompanied with or without commitment. In their paper, "Odyssean Forward Guidance" is defined as making explicit commitment to future policy actions, whereas "Delphic Forward Guidance" is defined as forecasting economic conditions without commitment of future policy actions.

In particular, I study the welfare gains from policy commitment in forward guidance. There has been a long history in studying the gains from monetary policy commitment. Classical literature include Kydland and Prescott (1977) and Barro and Gordon (1983), which show the inflationary bias in the central bank's objective function leads to higher inflation when the private sector has rational expectation. Clarida, Gali and Gertler (2000) study how commitment to a future path of policy rates reduces current stabilization bias between output gap stabilization and inflation stabilization which is induced by ad-hoc cost-push shocks. Woodford (1999) studies how history-dependent policy can be achieved by having interest-smoothing included in the central bank's objective function. Eggertsson et al. (2003) show optimal commitment to delayed response can mitigate the distortion under zero lower bound of interest rates.

This paper studies how imperfect information leads to gains from monetary policy commitment, which builds on the abundant literature on optimal monetary policy under imperfect information. This field is revived since Woodford (2001), which show how higher order beliefs lead to persistent effect of monetary policy, following the assumption of imperfect information in Phelps (1970) and Lucas (1972). Since then there have been many papers studying optimal monetary policy under imperfect information. Papers in this field can be divided by their assumption of whether monetary policy has informational effect.

The majority of papers which characterize optimal monetary policy under information frictions assume monetary policy has no informational effect, and thus assume the beliefs formed in the private sector are independent of policy decisions. Ball, Mankiw and Reis (2005) study optimal monetary policy with sticky information, and conclude that optimal monetary policy should be described as elastic price standard: the central bank should allow price to deviate from target when output deviates from natural rate. Adam (2007) models partial information economy, and allows the precision of private signals to be endogenous. He argue that as signals get more precise, optimal monetary policy changes from output gap stabilization to price level stabilization. Lorenzoni (2010) assumes the central bank has no superior information, and points out announcing monetary policy has trade-off between aggregate stabilization and cross-sectional efficiency. Angeletos and La'O (2011) consider both nominal and real frictions that caused by information friction, and describe optimal monetary policy seeks negative correlation between price level and output.

Recent papers have started to investigate the situation where people extract information on the underlying economy from monetary policy decisions. Baeriswyl and Cornand (2010) emphasize the signaling effect of policy actions, and conclude that the central bank distorts its policy in order to optimally control the information it conveys. Central bank alters optimal policy response in order to reduce information revealed on cost push shock through policy decisions. Berkelmans (2011) demonstrates that with multiple shocks, tightening policy may initially increase inflation. Tang (2013) shows that with rational expected private sector, the stabilization bias is reduced when monetary policy has information effect.

My paper deals with the higher order beliefs problem, and past literature has demonstrated how higher order beliefs amplify the real effect of monetary policy. (see Woodford (2001) for example) In addition, many papers have show different solution methods, including Melosi (2016), Huo and Takayama (2015) and Nimark (2017), as examples.

My paper is also motivate by the empirical evidence on the informational effect of monetary policy and the policy discussion on forward guidance. Romer and Romer (2000), Romer and Romer (2003) show the information asymmetry between central bank and the private sector by providing evidence on inflation forecast changes after FOMC announcement. Faust, Faust, Swanson and Wright (2004) further confirm that forecasts by private sector respond to monetary policy changes. Nakamura and Steinsson (2013) use high frequency trading data to identify the informational effect of monetary policy news.

The rest of paper is organized as follows: section 2 sets up the private sector of my model. Section 3 defines equilibrium under three strategies of communication and commitment. Section 4 compares the gains of communication and the gains from commitment. Section 4 concludes the paper and points out further steps.

2 The Private Sector

The private sector consists of a representative household and a continuum of firms. Each firm i specializes in producing one differentiated good i to sell in a monopolistic competitive market. Firms set prices before producing goods. After prices are set, production takes place to meet demands of the representative household.

Following Phelps (1970), Woodford (2001), and Angeletos and La'O (2010), I model firms are separated by a continuum of islands whose boundaries create geographical isolation of information. In each time t, events take the following sequence:

- Technology shock realizes, and each firm i learns its own technology, A_i, but not the technology of others, $A_{j,\ j\neq i}$.
- All firms set prices based on its own information set ω_i.
- All information is revealed, and the the representative household sends one labor to each island. The household makes consumption decisions across differentiated goods. Firms produce until product markets clear.

2.1 Household

The representative household consists of a consumer and a continuum of workers. As specified in the timing of events, I assume the representative household has perfect information, and chooses consumption and labor supply to each island to maximize its utility:

$$u(C,\,N) = \frac{C^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{N_i^{1+\psi}}{1+\psi} di \tag{1}$$

subjected to its budget constraint:

$$PC \leq \int_0^1 W_i N_i + \Pi \tag{2}$$

where Π stands for all lump-sum income including profits from firms. C is final good consumption that composites individual goods. N_i is labor supply to individual firms. I assume good market to be monopolistic competitive, and the composition function of aggregate good follows:

$$C = \left[\int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where P denotes the aggregate price index after household optimally allocates individual good consumption to minimize expenditure

$$min_{\{C_i\}} \int_0^1 P_i C_i di \tag{3}$$

which yields the demand of individual good to be

$$C_i = \left(\frac{P_i}{P} \right)^{-\varepsilon} C \tag{4}$$

2.2 Firms

Prices are assumed to be perfectly flexible, and each firm sets its price to maximize expected profit conditional on its own information set, ω_i which will be specified in section 3.

$$max_{\{P_i\}} E\left\{ P_i Y_i - W_i N_i | \omega_i \right\} \tag{5}$$

subjected to its production function $Y_i = A_i N_i$ and individual good demand function $Y_i = \left(\frac{P_i}{P}\right)^{-\varepsilon} Y$

2.3 States and Signals

In the private sector, the aggregate technology is the only aggregate state variable. Firms have two signals about the aggregate technology, which are their firm-specific technology and the information from the central bank's forward guidance policy.

States

I assume that the aggregate technology shock is i.i.d. with log-normal distribution, i.e.,

$$\bar{a} \sim N(0,\ \sigma_a^2)$$

Information

I assume that information is fully revealed after stage 2, so that the representative household is perfectly informed. Before stage 2, both firms and the central bank are subjected to partial information. Each firm learns its own technology, a_i, which becomes its private signal of $\bar{a}$.

$$a_i \equiv log(A_i) = \bar{a} + s_i \quad s_i \sim N(0, \sigma_s^2) \tag{6}$$

The central bank surveys a sample of firms, and gets a measurement of $\bar{a}$. If the central bank signals its measurement to firms through forward guidance, firms get a public signal of aggregate technology shock, which I denote as m. I assume the central bank's measurement error is normally distributed with mean 0:

$$m = \bar{a} + \upsilon \quad \upsilon \sim N(0, \sigma_\upsilon^2) \tag{7}$$

2.4 Price Setting with Higher Order Beliefs

To solve for the equilibrium, we work backwards: first find the household demand function of goods and supply function of labor, plug these two constraints into individual firm expected profit function, and then derive optimal price setting for each firm.

In stage 3, household utility maximization over labor and consumption sets real wage as the marginal rate of substitution between consumption and leisure:

$$W_i P = \frac{N_i^\psi}{C^{-\sigma}} \tag{8}$$

Household expenditure minimization problem, together with market clearing condition, determines individual good demand as a function of aggregate demand:

$$Y_i = \left(\frac{P_i}{P}\right)^{-\varepsilon} Y \tag{9}$$

In stage 2, each firm sets price to maximizes expected current period profit:

$$E\left\{P_i Y_i - W_i N_i | \omega_i\right\} = E\left\{\left(\frac{P_i}{P}\right)^{-\varepsilon} Y P_i - P_i^{-\varepsilon(1+\psi)} A_i^{-(1+\psi)} Y^{1+\psi+\sigma} P^{\varepsilon(1+\psi)} | \omega_i\right\} \tag{10}$$

Taking first order condition on P_i results in the optimizing choice of price for each individual firm i:

$$P_i^{1+\varepsilon\varphi} = \frac{\varepsilon(1+\varphi)}{\varepsilon - 1} E\left\{P^{1+\varepsilon\varphi} Y^{\varphi+\sigma} A_i^{-(1+\varphi)} | \omega_i\right\} \tag{11}$$

which is approximated in log-linear form as:

$$p_i = E_i\left[p + \alpha y\right] - \beta a_i \tag{12}$$

where $\alpha = \frac{\varphi+\sigma}{1+\varepsilon\varphi}$, and $\beta = \frac{1+\varphi}{1+\varepsilon\varphi}$. The operator E_i represents the conditional expectation of firm i on its own information set, ω_i, and

Higher Order Beliefs

The above equation states the optimal pricing strategy as conditional expectation on aggregate price and output. The aggregate price is defined to be the average of all p_i, and thus average over all individual expectation on p, which consequently makes p_i depend on others belief, and others belief on others belief, and so on. We obtain the expression for this higher order belief in price through successively substituting y by the nominal demand equation, $y = n - p$ (see Appendix for detailed derivation)

$$p_i = \alpha \Sigma_{j=1}^{\infty} (1-\alpha)^{j-1} E_i \bar{E}^{j-1} n - \beta \Sigma_{j=1}^{\infty} (1-\alpha)^{j} E_i \bar{E}^{j-1} \bar{a} - \beta a_i \tag{13}$$

where $\bar{E}\left[\cdot\right]$ denotes the average expectations operator, given by

$$\bar{E}\left[\cdot\right] = \int E_i \left[\cdot\right] di \bar{E}^{j}\left[\cdot\right] = \int E_i \bar{E}^{j-1}\left[\cdot\right] di = \bar{E}\bar{E}^{j-1}\left[\cdot\right] \tag{14}$$

3 Monetary Policy under Imperfect Information

Under the assumption that household has perfect information, the central bank only changes the information sets for firms. The central bank does two decisions. First, the central bank decides whether or not to reveal its information on aggregate technology, m, before firms make pricing decisions. If the central bank decides not to reveal its information, I call it "perfect opacity". If the bank decides to reveal its information, it provides information through forward guidance. The second decision is whether to commit to its forward guidance policy. If the central bank commits its policy conditional on its imperfect information, I call it forward guidance with commitment. If the central bank decides to re-optimize in later period, I call it "forward guidance without commitment". The target of the central bank is to minimize the price deviation and output gap by choosing aggregate nominal demand. I denote the central bank's objective function as:

$$f = max\left\{ -E\left[\left(y - y^{eff} \right)^2 + \tau p^2 \right] \right\} \tag{15}$$

3.1 Perfect Opacity

Perfect opacity allows the central bank to set nominal aggregate demand when it has perfect information in later stage. In this situation, the information set of each firm consists only its firm-specific technology, $\omega_i = \{a_i\}$. The optimal price depends on firm's expectation on both both

$\bar{a}$ and n. Firms use their own technology as private signals to form conditional expectation on aggregate technology:

$$E_i\bar{a} = \frac{\kappa_s}{\kappa_s+\kappa_a}a_i + \frac{\kappa_a}{\kappa_s+\kappa_a}\mu_a = \frac{\kappa_s}{\kappa_s+\kappa_a}a_i \tag{16}$$

where $\kappa_s = 1/\sigma_s^2$denotes the precision of private signals, and $\kappa_a = 1/\sigma_a^2$ denotes the precision of prior.

As optimal prices are strategic complements, firms need to form expectation on the aggregate nominal demand, n, as this will affect the aggregate consumption in later stage. When central bank acts in stage 3 with perfect information, it observes $\bar{a}$ perfectly. I guess and verify that in equilibrium, the optimal monetary policy of aggregate nominal demand takes linear form with $\bar{a}$: $n = \gamma_{po}\bar{a}$. Under this assumption, the decision of money supply is equivalent as the choice of γ_{po}. Assuming that firms have rational expectation, they know the value of γ_{po}, so firm i's expectation on n becomes:

$$E_i n = \gamma_{po}E_i\bar{a} = \frac{\kappa_s}{\kappa_s+\kappa_a}\gamma_{po}a_i \tag{17}$$

Equilibrium:

For every realization of $\bar{a}$, there exists an unique Nash Equilibrium where $\left(\vec{P},\ \gamma_{po}^*\right)$ is the best response of firms and the central bank such that:

$$E\left[\pi_i\left(p_i,\ p_{-i},\ \gamma_{po}^*\right)|\omega_i\right] \geq E\left[\pi_i\left(p_i',\ p_{-i},\ \gamma_{po}^*\right)|\omega_i\right] \quad \forall p_i',\ \forall i$$
$$f(\gamma_{po}^*,\ \vec{P}) \geq (\gamma_{po}',\ \vec{P}) \quad \forall\gamma_{po}'$$

where $\vec{P}$ denotes the vector of all p_i.

The equilibrium of optimal prices and optimal monetary policy, $\left(\vec{P},\ \gamma_{po}^*\right)$, is found to be: (see Appendix for derivation)

$$p_i = \frac{(\alpha\gamma_{po}^* - \beta)\kappa_s - \beta\kappa_a}{\kappa_a+\alpha\kappa_s}a_i \tag{18}$$

$$\gamma_{po}^* = \left(\frac{\kappa_a^2+\tau\alpha^2\kappa_s^2}{\kappa+\alpha\kappa_s}\right)^{-1}\left(\frac{\beta(\kappa_a+\kappa_s)(\alpha\kappa_s\tau-\kappa_a)}{\kappa_a+\alpha\kappa_s}+\frac{\beta}{\alpha}\kappa_a\right) \tag{19}$$

The equilibrium aggregate price level and output under perfect opacity are:

$$p = \frac{(\alpha\gamma_{po}^* - \beta)\kappa_s - \beta\kappa_a}{\kappa_a+\alpha\kappa_s}\bar{a} \tag{20}$$

$$y = \frac{\beta\kappa_s + (\gamma_{po}^*+\beta)\kappa_a}{\kappa_a+\alpha\kappa_s}\bar{a} \tag{21}$$

Discussion:

Infinite precision of private signals

First consider when the private signals are infinitely precise, $\kappa_s \to \infty$. Equation (21) shows that real output goes to efficient level as defined in perfect information benchmark, and the monetary policy affects price only. The economy approaches to real dichotomy.

$$y \to \frac{\beta}{\alpha}\bar{a} = y^{eff} \quad p \to \frac{\alpha\gamma_{po} - \beta}{\alpha}$$

Zero precision of private signals

When $\kappa_s \to 0$, firms do not update beliefs on aggregate technology, and thus how monetary policy reacts to aggregate technology shock does not play a role in firms' pricing decisions. From equation (20) and (21), we see the monetary policy affects only output. $p \to -\beta\bar{a} \quad y = (\gamma_{po} + \beta)\bar{a}$

Intermediate precision with fixed γ_{po}

A more realistic situation is when private signals has finite precision, in which case both real output and price level depends on γ_{po}. Figure 1 plots the dynamics of aggregate price and output with fixed γ_{po} when after an aggregate technology shock such that $\bar{a} = 0.1$. I assign values to other parameters as $\sigma = 0.2$, $\psi = 0.5$, $\varepsilon = 2$, under which $\alpha = 0.35$, and $\beta = 0.75$.

We see that as the precision of κ_s increases, output under different value of γ_{po} starts from different values but all goes to efficient level. On the other hand, prices all start from same level when $\kappa_s = 0$, but go to different levels depending on the value of γ_{po}. This figure confirms the real dichotomy that monetary policy only affects nominal variables. When $\gamma_{po} = \frac{\beta}{\alpha}$, price asymptotically goes to zero.

Intermediate precision with optimal γ^*_{po}

Under finite precision of private signals, optimal monetary policy is set to weigh between price level stabilization and output gap stabilization. In Figure 2, I compare the optimal monetary policy that which is defined in equation (19) with two alternative policy, together with the equilibrium dynamics of the aggregate price level and output under optimal monetary policy. The two alternative monetary policy are price stabilization policy and output gap stabilization policy. First, if the central bank were to consider only output gap stabilization, γ_{po} is set to make $y = \frac{\beta\kappa_s + (\gamma^*_{po}+\beta)\kappa_a}{\kappa_a + \alpha\kappa_s}\bar{a} = y^{eff}$, which results in:$\gamma_{po}^{output\ stab} = \frac{\beta}{\alpha} - \beta$. Second, if central bank were to consider only price stabilization, it achieves so by making $p = \frac{(\alpha\gamma^*_{po}-\beta)\kappa_s - \beta\kappa_a}{\kappa_a + \alpha\kappa_s}\bar{a} = 0$, which results in $\gamma_{po}^{price\ stab} = \frac{\beta\kappa_a}{\alpha\kappa_s} + \frac{\beta}{\alpha}$.

Proposition 1: *With perfect opaque central bank which does not provide forward guidance, the optimal monetary policy shifts from output gap stabilization to price level stabilization, as the precision of private signal increases.*

This results is consistent with the conclusion in Adam (2007). With precise private information, real dichotomy holds and real output are independent with monetary policy. Thus, optimal monetary policy stabilizes price level. On the other hand, with imprecise information, monetary policy has little impact on firms pricing decisions, as firms are reluctant to update beliefs on aggregate shocks and the reaction of monetary policy on aggregate shocks.

3.2 Forward Guidance with Commitment

In this situation, the central bank reveals its imperfect information on aggregate technology, m, through announcing its forward guidance of its best-forecast monetary policy, n. In addition, the central bank commits to implement this forward guidance policy, which gives all firm perfect information about n.

Since the forward guidance of nominal demand is made when the central bank is still subjected to imperfect information, n is based on m. I guess and verify that the optimal monetary policy under forward guidance with commitment is a linear in the central bank's information:

$$n = g\left(m\right) = \gamma_{fc} m$$

. As firms are assumed to have rational expectation, they correctly understand the response function of the central bank. Consequently, upon receiving the information provided by the central bank, the information set of individual firms becomes $\omega_i = \{m,\ a_i\}$, with which each firm forms conditional expectation on aggregate technology as:

$$E_i \bar{a} = \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} m + \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a} a_i \tag{22}$$

Optimal policy becomes the choice of γ_{fc} to maximize the expected objective function conditional on m:

$$max_{\{n\}}\, E\left[f(n)|m\right] = max_{\{\gamma_{fc}\}} E\left[f(\gamma_{fc})|m\right] \tag{23}$$

Equilibrium:

For every realization of $\{\bar{a},\ \upsilon\}$, there exists an unique Nash Equilibrium where $\left(\vec{P},\ \gamma^*_{fc}\right)$ is the best response of firms and the central bank such that:

$$E\left[\pi_i\left(p_i,\ p_{-i},\ \gamma^*_{fc}\right)|\omega_i\right] \geq E\left[\pi_i\left(p'_i,\ p_{-i},\ \gamma^*_{fc}\right)|\omega_i\right] \quad \forall p'_i,\ \forall i \tag{24}$$

$$E\left[f(\gamma^*_{fc},\ \vec{P})|m\right] \geq E\left[f(\gamma_{fc},\ \vec{P})|m\right] \quad \forall \gamma_{fgc} \tag{25}$$

where $\vec{P}$ denotes the vector of all p_i.

The expression of $\left(\vec{P},\ \gamma_{fc}^{*}\right)$ is found to be (see Appendix for derivations)

$$p_i = n - \beta\left[\frac{1-\alpha}{\alpha}\frac{\kappa_m}{\alpha\kappa_s+\kappa_a+\kappa_m}\frac{n}{\gamma_{fgc}^{*}} + \frac{(1-\alpha)\kappa_s}{\kappa_m+\kappa_a+\alpha\kappa_s}a_i\right] - \beta a_i \tag{26}$$

$$\gamma_{fc}^{*} = \left(\sigma_a^2+\sigma_v^2\right)^{-1}\left\{\left[\beta\frac{1-\alpha}{\alpha}\frac{\kappa_m}{\alpha\kappa_s+\kappa_a+\kappa_m} + \beta\left(\frac{(1-\alpha)\kappa_s}{\kappa_m+\kappa_a+\alpha\kappa_s}+1\right)\right]\sigma_a^2 + \left[\beta\frac{1-\alpha}{\alpha}\frac{\kappa_m}{\alpha\kappa_s+\kappa_a+\kappa_m}\right]\sigma_v^2\right\} \tag{27}$$

The aggregate price level and output are:

$$p = \left[\gamma_{fc}^{*} - \beta\frac{1-\alpha}{\alpha}\frac{\kappa_m}{\alpha\kappa_s+\kappa_a+\kappa_m}\right]m - \beta\left[\frac{(1-\alpha)\kappa_s}{\kappa_m+\kappa_a+\alpha\kappa_s}+1\right]\bar{a} \tag{28}$$

$$y = \left[\beta\frac{1-\alpha}{\alpha}\frac{\kappa_m}{\alpha\kappa_s+\kappa_a+\kappa_m}\right]m + \beta\left[\frac{(1-\alpha)\kappa_s}{\kappa_m+\kappa_a+\alpha\kappa_s}+1\right]\bar{a} \tag{29}$$

Discussion:

The first thing to notice is that output in equation (29) is independent with the γ_{fc}. This is because as monetary policy is known by firms make pricing decisions, prices adjust completely to offset the effect of monetary policy on nominal budget of the household, leaving monetary policy unable to affect households real demand. As a result, the central bank does not have trade-off between price level stabilization and output gap stabilization, and optimal monetary policy stabilizes price.

Fixed γ_{fc}

Equation (28) and (29) describe the equilibrium price level and output as functions of signal precision. A positive policy error is defined to be realization of positive υ in the central bank's measurement. After a positive policy shock, efficient output is zero, and output gap positively deviates from zero. In addition, more precise public information amplifies the deviation in output gap, as firms weigh more on the signal from forward guidance when forming beliefs on the aggregate technology. This is the same logic as Morris and Shin (2002) on the trade-off of public signal. Providing public signal to private sectors and committing to a policy action based on imprecise information improves coordination but may moves equilibrium away from efficient level, depending on the nature of the shock. On the other hand, more precise private information moves output closer to the efficient level after aggregate technology shocks. This is because the more that firms are able to distinguish the change in monetary policy is due to policy error or due to reaction on aggregate technology shocks, the more they adjust prices to incorporate the policy shock.

Optimal Monetary Policy

Proposition 2: *Under forward guidance with commitment, the real output is independent of the monetary policy, and optimal monetary policy minimizes price deviation.*

As monetary policy is set in the forward guidance, monetary policy has only nominal effect. Consequently, there is no conflict between price level stabilization and output gap stabilization. Figure 3 plots γ^*_{fc} and the equilibrium of the aggregate price level and the output with the precision of public signal increases from 0 to 5000.

Compared with fixed γ_{fc}, we find that γ^*_{fc} decreases price stabilization after a technology shock. This is achieved by having a smaller γ_{fc} when the precision of public signal is less precise. The intuition is that when precision of public signal is less precise, the central bank are less confident on its measurement and thus reacts less aggressively to its own measurement. On the other hand, both optimal monetary policy and effects on price and output have different dynamics when the precision of private signals increases.

Figure 4 plots γ^*_{fc} with respect to private signal varying from 0 to 5000 below, which shows that optimal monetary policy does not change with varying precision of private signals. We see that output approaches to efficient level with infinite precision of private signals after both technology shock and policy shock. However since optimal monetary policy does not change with respect to precision of private signals, price deviate further away from zero as precision of private signals increases.

4 Forward Guidance Without commitment

In this situation, the central bank announces its information on aggregate technology before firms set prices. It then waits until the last stage to implement an re-optimized monetary policy conditional on the accurate information that becomes available in the last stage. As a result, the optimal monetary policy can react on two actual state variables, the aggregate technology shock, and the error in the public signal which it conditions the forward guidance policy upon. I guess and verify that the optimal monetary policy is a linear function with respect to the two state variables, i.e., $n = \gamma^a_{fnc}\bar{a} + \gamma^\upsilon_{fnc}\upsilon$

When setting optimal prices, all firms need to form expectations on both $\bar{a}$ and n. Firm i's conditional expectation on $\bar{a}$ is same as in the case of forward guidance with commitment, since in both cases firms are able to infer the imperfect information held by the central bank from the forward guidance policy. The conditional expectation on the aggregate technology is formed using both private and public signal:

$$E_i\bar{a} = \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a}m + \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a}a_i \tag{30}$$

To guess n, firms separately expect the two state variables that n reacts to: the aggregate technology and the policy error:

$$E_i n = \gamma^a_{fnc} E_i \bar{a} + \gamma^{\nu}_{fnc} E_i \upsilon \tag{31}$$

From the assumption on public signal, $m = \bar{a} + \upsilon$, firms form expectation on υ as the difference between m and $E_i \bar{a}$:

$$E_i \upsilon = E_i m - E_i \bar{a} = m - E_i \bar{a} \tag{32}$$

Substitute $E_i \bar{a}$ and $E_i \upsilon$ and get

$$E_i n = \left(\gamma^a_{fnc} \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} + \gamma^{\upsilon}_{fnc} \frac{\kappa_s + \kappa_a}{\kappa_m + \kappa_s + \kappa_a} \right) m + \left(\left(\gamma^a_{fnc} - \gamma^{\nu}_{fnc} \right) \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a} \right) a_i \tag{33}$$

In the following passage, I simplify the expression of equation (33) as

$$E_i n = \rho_m m + \rho_a a_i \tag{34}$$

$$\bar{E} n = (\rho_m + \rho_a) \bar{a} + \rho_m \upsilon \tag{35}$$

where $\rho_m = \gamma^a_{fnc} \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} + \gamma^{\upsilon}_{fnc} \frac{\kappa_s + \kappa_a}{\kappa_m + \kappa_s + \kappa_a}$, and $\rho_a = \left(\gamma^a_{fnc} - \gamma^{\nu}_{fnc} \right) \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a}$.

Since $\rho_m + \rho_a \neq \gamma^a_{fnc}$, and $\rho_m \neq \gamma^{\upsilon}_{fnc}$ as long as κ_s does not approaches to infinity, we know that with firms having imperfect information, the monetary policy cannot be perfectly known when firms set prices. Consequently, monetary policy can affect nominal demand by by unanticipated "shock" to household nominal budget, and the "shock" comes from the central bank's reaction to previous error its released public signal.

Equilibrium:

For every realization of $\{a,\ \upsilon\}$, there exists an unique Nash Equilibrium in which $\left(\vec{P},\ \gamma^*_{fnc} \right)$ is the strategy for all firms and the central bank such that

$$E\left[\pi_i \left(p_i,\ p_{-i},\ \gamma^*_{fnc} \right) | \omega_i \right] \geq E\left[\pi_i \left(p'_i,\ p_{-i},\ \gamma^*_{fnc} \right) | \omega_i \right] \quad \forall p'_i,\ \forall i f(\gamma^*_{fnc},$$
$$\vec{P}) \geq f(\gamma'_{fnc},\ \vec{P}) \quad \forall \gamma'_{fnc}$$

where γ^*_{fnc} represents the pair of parameters $\left(\gamma^a_{fnc},\ \gamma^{\upsilon}_{fnc} \right)$.

With derivation in Appendix Section 2.3, the expression of aggregate variables are found to be

$$p = (\phi_m + \phi_a) \bar{a} + \phi_m \upsilon \tag{36}$$

$$y = (\gamma^a_{fnc} - \phi_m - \phi_a) \bar{a} + (\gamma^{\upsilon}_{fnc} - \phi_m) \upsilon \tag{37}$$

where

$$\phi_m = \rho_m + \left[\rho_a(1-\alpha) - \beta\frac{1-\alpha}{\alpha}\right]\frac{\kappa_m}{\alpha\kappa_s + \kappa_a + \kappa_m} \tag{38}$$

$$\phi_a = [\alpha\rho_a - \beta]\frac{(1-\alpha)\kappa_s}{\kappa_m + \kappa_a + \alpha\kappa_s} + \alpha\rho_a - \beta \tag{39}$$

$$\rho_m = \gamma^a_{fnc}\frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} + \gamma^\upsilon_{fnc}\frac{\kappa_s + \kappa_a}{\kappa_m + \kappa_s + \kappa_a} \tag{40}$$

$$\rho_a = \left(\gamma^a_{fnc} - \gamma^v_{fnc}\right)\frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a} \tag{41}$$

Discussion:

$\gamma^a_{fnc} = \gamma^\upsilon_{fnc}$

If the monetary policy is restricted to $\gamma^a_{fnc} = \gamma^\upsilon_{fnc}$, its response to aggregate technology shock and policy error in the same way. In other words, it acts on m, not $\bar{a}$ and υ separately, which makes the policy exactly the same as forward guidance with commitment.

Optimal Monetary Policy

Since monetary policy affects both price level and real output, optimal monetary policy needs to weigh between price stabilization and output gap stabilization.

If the central bank only seeks to minimize price deviation, it chooses $\left\{\gamma^a_{fnc},\ \gamma^\upsilon_{fnc}\right\}$ to make $\phi_m = 0$ and $\phi_a = 0$. We first solve for $\{\rho_m,\ \rho_a\}$ as

$$\rho_m^{price\ stab} = 0, \quad \rho_a^{price\ stab} = \frac{\beta}{\alpha}$$

Then plug the values to equation (40) and (41) to solve for$\left\{\gamma^a_{fnc},\ \gamma^\upsilon_{fnc}\right\}$ as

$$\gamma^a_{fnc} = \frac{\beta}{\alpha}\frac{\kappa_a + \kappa_s}{\kappa_s} \quad \gamma^\upsilon_{fnc} = -\frac{\beta}{\alpha}\frac{\kappa_m}{\kappa_s}$$

The sign of price-stabilization γ^a_{fnc} is positive, and the sign of price-stabilization γ^υ_{fnc} is negative, meaning the monetary policy will counteract its previous policy error in public signal. If the central bank overestimate aggregate technology, it reduces n afterwards, and vice-versa. In addition, the magnitude of this counteraction increases as the precision of public signal increases. This is because as firms weigh more on public signal when it is precise, it has larger effect on price, and thus price stabilization policy needs to have larger counteraction.

If the central bank only seeks to minimize output gap, it chooses $\left\{\gamma^a_{fnc},\ \gamma^\upsilon_{fnc}\right\}$ to make

$$\gamma^a_{cc} - \phi_m - \phi_a = \frac{\beta}{\alpha} \quad \gamma^\upsilon_{cc} - \phi_m = 0 \tag{42}$$

Solve $\{\rho_a,\ \rho_m\}$ and substitute their values in equation (40) and (41) to solve for $\left\{\gamma^a_{fnc},\ \gamma^\upsilon_{fnc}\right\}$, I find only the difference in parameters can be identified as: $\gamma^a_{fnc} - \gamma^\upsilon_{fnc} = \frac{\beta}{\alpha} - \beta$

In addition, the value of output-gap-stabilization $\gamma^a_{fnc} - \gamma^\upsilon_{fnc}$ is the same as the output-gap-stabilization optimal policy in perfect opacity case, independent of the precision of public signal.

Figure 7 plots optimal monetary policy, and Figure 8 plots the equilibrium price and real output under optimal monetary policy after a positive aggregate technology shock and a positive policy shock. I allow the precision of public signal varies from 0 to 5000, while setting the precision of private signals to be 1.

Notice that in the first diagram, I use dashed line to plot the output gap stabilization policy implied γ^υ_{fnc} when γ^{*a}_{fnc} is chosen optimally to maximize central bank objective function. We merely see the difference between γ^{*a}_{fnc} and output gap stabilization policy implied γ^a_{fnc}. In addition, Figure 8 shows that the output is nearly completely stabilized in the whole range of public signal precision. However, price moves further away from zero as the precision of public signal increases under both aggregate technology shock and policy shock. This is because when firms receive a positive m, they cannot distinguish whether it is a realization of positive aggregate technology shock or a positive policy error. However, as they know at equilibrium, the central bank response by increase n to both the two shocks, they increase prices, and the more precise the public signal is, the more they increase prices. In equilibrium, it is also the central bank's best response to increase nominal demand, otherwise the nominal budget of household will shrink after the increase in price level. The spiral effect makes price level deviation to be larger with more precise public signal.

However, this negligible trade-off between output gap stabilization and price stabilization only holds true when private signal are imprecise. In Figure 9 and Figure 10, I set $\sigma_m = 5\sigma_a$, and let the precision of private signal varies from 0 to 5000. Again, dashed line in the first diagram is the output gap stabilization policy implied γ^υ_{fnc} , which is no longer the same one as $\gamma^{\upsilon *}_{fnc}$. The trade-off between output gap stabilization and price stabilization tightens as the precision of private signal increases. In Figure 10, we see the while output approaches to efficient level and price approaches to zero as the precision of signal approaches infinity, they largely deviate from efficient level when private signals are less precise.

Proposition 3: *Under forward guidance without commitment, output achieves efficient level after either technology shocks or policy shocks. The aggregate price level deviates further away from zero as the precision of both private signals and public signals increases.*

5 Policy Trade-off of Commitment under Uncertainty

To compare the gains and losses of forward guidance with commitment, I set the $\kappa_s = 1$, and let κ_m varies to compare the equilibrium aggregate price level and the output gap under forward guidance with and without commitment. Figure 11 shows that after both technology shocks and policy shocks, the output gap under forward guidance without commitment is zero, independent of precision of public signals. This shows that gains from re-optimization where central bank can offset the policy error when making forward guidance announcement. On the other hand, after both shocks, the aggregate price level deviates further away from zero under forward guidance without commitment, in comparison with forward guidance with commitment. This shows the gains from commitment, where central bank reduces the uncertainty in pricing decisions about the monetary policy. The combined effect on ex-ante welfare thus depends on the relative importance of price stabilization and output gap stabilization.

6 Conclusion

In this paper, I compare three strategies of central bank communication. Perfect opacity is defined as the situation where the central bank does not provide forward guidance, so that central bank cannot reveal its information to the private sector. Forward guidance is defined as providing the central bank's forecast on the monetary policy conditional on its imperfect information. I discuss both the case where central bank commits to the forward guidance policy and the case where central bank re-optimizes policy conditional on accurate information which becomes available in later stage.

The important assumption that drives the policy trade-off is that both the central bank and the private sector have imperfect information in the early stage. Under this assumption, the announcement of forward guidance is conditional on the imperfect information of the central bank and has informational effect on the price-setting behaviors. As the private sector is rational, and understands that the forward guidance policy is a linear function with one variable of the central bank's imperfect information, the private sector regards the forward guidance as a public signal about the aggregate technology shock.

As the effect of monetary policy is to control the nominal aggregate demand, central bank has the option to re-optimize monetary policy in the last stage when it has accurate information. However, although re-optimization can completely closes the output gap, it increases fluctuations in the aggregate price level. The reason is that as prices are flexible, firms incorporate their expectations about the monetary policy in their pricing decisions. Consequently, under forward guidance without commitment, all firms need to form expectation about how monetary policy will re-optimize in later stage. The private sector is rational and understands the re-optimized monetary policy will react on the policy error in its announced forward guidance policy. Consequently, firms need to form expectation both about the aggregate technology shock and the policy error. The effect of this additional uncertainty is amplified through the higher order beliefs. In summary, forward guidance with commitment reduces deviations in the aggregate price level and forward guidance without commitment reduces the deviations in aggregate output gap.

References

Adam, Klaus. 2007. "Optimal monetary policy with imperfect common knowledge." *Journal of monetary Economics*, 54(2): 267–301.

Angeletos, George-Marios, and Chen Lian. 2016. "Forward guidance without common knowledge." National Bureau of Economic Research.

Angeletos, George-Marios, and Jennifer La'O. 2010. "Noisy business cycles." *NBER Macroeconomics Annual*, 24(1): 319–378.

Angeletos, George-Marios, and Jennifer La'O. 2011. "Optimal monetary policy with informational frictions." National Bureau of Economic Research.

Baeriswyl, Romain, and Camille Cornand. 2010. "The signaling role of policy actions." *Journal of Monetary Economics*, 57(6): 682–695.

Ball, Laurence, N Gregory Mankiw, and Ricardo Reis. 2005. "Monetary policy for inattentive economies." *Journal of monetary economics*, 52(4): 703–725.

Barro, Robert J, and David B Gordon. 1983. "Rules, discretion and reputation in a model of monetary policy." *Journal of monetary economics*, 12(1): 101–121.

Berkelmans, Leon. 2011. "Imperfect information, multiple shocks, and policy's signaling role." *Journal of Monetary Economics*, 58(4): 373–386.

Campbell, Jeffrey R, Charles L Evans, Jonas DM Fisher, and Alejandro Justiniano. 2012. "Macroeconomic effects of Federal Reserve forward guidance." *Brookings Papers on Economic Activity*, 2012(1): 1–80.

Clarida, Richard, Jordi Gali, and Mark Gertler. 2000. "Monetary policy rules and macroeconomic stability: evidence and some theory." *The Quarterly journal of economics*, 115(1): 147–180.

Del Negro, Marco, Marc P Giannoni, and Christina Patterson. 2012. "The forward guidance puzzle."

Eggertsson, Gauti B, et al. 2003. "Zero bound on interest rates and optimal monetary policy." *Brookings papers on economic activity*, 2003(1): 139–233.

Faust, Jon, Eric T Swanson, and Jonathan H Wright. 2004. "Do Federal Reserve policy surprises reveal superior information about the economy?" *Contributions in Macroeconomics*, 4(1).

Feroli, Michael, David Greenlaw, Peter Hooper, Frederic S Mishkin, and Amir Sufi. 2017. "Language after liftoff: Fed communication away from the zero lower bound." *Research in Economics*.

Huo, Zhen, and Naoki Takayama. 2015. "Higher order beliefs, confidence, and business cycles." University of Minnesota working paper.

Kydland, Finn E, and Edward C Prescott. 1977. "Rules rather than discretion: The inconsistency of optimal plans." *Journal of political economy*, 85(3): 473–491.

Lorenzoni, Guido. 2010. "Optimal monetary policy with uncertain fundamentals and dispersed information." *The Review of Economic Studies*, 77(1): 305–338.

Lucas, Robert E. 1972. "Expectations and the Neutrality of Money." *Journal of economic theory*, 4(2): 103–124.

Melosi, Leonardo. 2016. "Signalling Effects of Monetary Policy." *The Review of Economic Studies*, 84(2): 853–884.

Morris, Stephen, and Hyun Song Shin. 2002. "Social value of public information." *The American Economic Review*, 92(5): 1521–1521.

Nakamura, Emi, and Jón Steinsson. 2013. "High frequency identification of monetary non-neutrality: The information effect." National Bureau of Economic Research.

Nimark, Kristoffer P. 2017. "Dynamic higher order expectations."

Phelps, Edmund S. 1970. "Microeconomic foundations of employment and inflation theory."

Romer, Christina, and David Romer. 2003. "A New Measure of Monetary Shocks: Derivation and Implications." National Bureau of Economic Research, Inc.

Romer, Christina D, and David H Romer. 2000. "Federal Reserve information and the behavior of interest rates." *American Economic Review*, 429–457.

Tang, Jenny. 2013. "Uncertainty and the signaling channel of monetary policy."

Woodford, Michael. 1999. "Optimal monetary policy inertia." *The Manchester School*, 67(s1): 1–35.

Woodford, Michael. 2001. "Imperfect common knowledge and the effects of monetary policy." National Bureau of Economic Research.

Appendices

A Higher Order Belief

From equation (4), $p_i = E_i[p+\alpha y] - \beta a_i$, substitute y by $y = n - p$. To express the approximation of p in log-linear form, I first take log-linear approximation of the aggregate price, $P^{1-\varepsilon} = \int_0^1 P_i^{1-\varepsilon} di$ yields $p = \int_0^1 p_i di$. Then, substitute this expression of p into to optimal price for individual firms to get:

$$p_i = E_i\left[p+\alpha(n-p)\right] - \beta a_i \tag{A.1}$$

$$= (1-\alpha)E_i p + \alpha E_i n - \beta a_i \tag{A.2}$$

$$= (1-\alpha)E_i\left[\int_0^1 E_j\left(p+\alpha(n-p)\right) - \beta a_j\right] + \alpha E_i n - \beta a_i \tag{A.3}$$

I use $\bar{E}[\cdot]$ to be average expectation operator given by

$$\int_0^1 E_j(\cdot)\, dj = \bar{E}(\cdot) \tag{A.4}$$

$$\bar{E}^j[\cdot] = \int E_i \bar{E}^{j-1}[\cdot]\, di = \bar{E}\bar{E}^{j-1} \tag{A.5}$$

Equation (3) can be simplified as

$$p_i = (1-\alpha)^2 E_i \bar{E} p + \alpha(1-\alpha) E_i \bar{E} n + \alpha E_i \bar{E} n - (1-\alpha)\beta E_i \bar{a} - \beta a_i \tag{A.6}$$

Iterate the substitution leads to the optimal individual price with higher order beliefs on aggregate technology shocks:

$$p_i = (1-\alpha)^\infty E_i \bar{E}^\infty p + \alpha \Sigma_{j=0}^\infty (1-\alpha)^j E_i \bar{E}^j n - \beta \Sigma_{j=0}^\infty (1-\alpha)^{j+1} E_i \bar{E}^j \bar{a} - \beta a_i \tag{A.7}$$

B Price-setting and Central Bank Communication

B.1 Perfect Opacity

Firm i needs to form conditional expectation on both money supply and aggregate technology. I guess and verify that money supply follows a linear form: $n = \gamma\alpha$, so that $E_i n = \gamma E_i a$. Firm i sees only its own technology a_i as the private signal, and thus the conditional expectation becomes

$$E_i \bar{a} = \frac{\kappa_s}{\kappa_s + \kappa_a} a_i + \frac{\kappa_a}{\kappa_s + \kappa_a} \mu_a = \frac{\kappa_s}{\kappa_s + \kappa_a} a_i \tag{B.1}$$

Average over i and get

$$\bar{E}\bar{a} = \frac{\kappa_s}{\kappa_s + \kappa_a}\bar{a} \tag{B.2}$$

$$E_i\bar{E}\bar{a} = \frac{\kappa_s}{\kappa_s + \kappa_a} \tag{B.3}$$

$$E_i\bar{a} = \left(\frac{\kappa_s}{\kappa_s + \kappa_a}\right)^2 a_i E_i\bar{E}^j\bar{a} = \left(\frac{\kappa_s}{\kappa_s + \kappa_a}\right)^{j+1} a_i \tag{B.4}$$

Apply equation (B.4) into equation (A.7):

$$p_i = (1-\alpha)^\infty E_i\bar{E}^\infty p + \alpha\gamma_{po}\Sigma_{j=0}^{\infty}(1-\alpha)^j \left(\frac{\kappa_s}{\kappa_s + \kappa_a}\right)^{j+1} a_i - \beta\Sigma_{j=0}^{\infty}(1-\alpha)^{j+1}\left(\frac{\kappa_s}{\kappa_s + \kappa_a}\right)^{j+1} a_i - \beta a_i \tag{B.5}$$

I guess and verify that the higher order expectation on p is less than $\frac{1}{1-\alpha}$ so that $(1-\alpha)^\infty E_i\bar{E}^\infty p \to 0$. Equation (B.5) becomes

$$p_i = \frac{(\alpha\gamma_{po} - \beta)\kappa_s - \beta\kappa_a}{\kappa_a + \alpha\kappa_s} a_i \tag{B.6}$$

Integrate over i and apply $y = n - p = \gamma_{po}\bar{a} - p$ to get the equilibrium aggregate price level and output:

$$p = \frac{(\alpha\gamma_{po} - \beta)\kappa_s - \beta\kappa_a}{\kappa_a + \alpha\kappa_s}\bar{a} \tag{B.7}$$

$$y = \frac{\beta\kappa_s + (\gamma_{po} + \beta)\kappa_a}{\kappa_a + \alpha\kappa_s}\bar{a} \tag{B.8}$$

B.2 Forward Guidance with Commitment

Now, firms use not only its own technology, but also the forward guidance monetary policy as a public signal to form conditional expectation on $\bar{a}$.

$$\begin{aligned} E_i\bar{a} &= \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_\xi} m + \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_\xi} a_i + \frac{\kappa_\xi}{\kappa_s + \kappa_\xi}\mu_a \\ &= \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_\xi} m + \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_\xi} a_i \end{aligned} \tag{B.9}$$

Integrate over all i yields:

$$\bar{E}\bar{a} = \frac{\kappa_m}{K}m + \frac{\kappa_s}{K}\bar{a} \tag{B.10}$$

where I denote $K = \kappa_m + \kappa_s + \kappa_\xi$. It follows that

$$E_i\bar{E}\bar{a} = \frac{\kappa_m}{K}m + \frac{\kappa_s}{K}E_i\bar{a} = \frac{\kappa_m}{K}m + \frac{\kappa_s}{K}\left[\frac{\kappa_m}{K}m + \frac{\kappa_s}{K}a_i\right] = \left[\frac{\kappa_m}{K} + \frac{\kappa_s}{K}\frac{\kappa_m}{K}\right]m + \left(\frac{\kappa_s}{K}\right)^2 a_i \tag{B.11}$$

Successively take average and then apply E_i leads to

$$E_i\bar{E}^{j-1}\bar{a} = \left(\frac{\kappa_m}{K}\right)\Sigma_{k=1}^{k=j}\left(\frac{\kappa_s}{K}\right)^{k-1} m + \left(\frac{\kappa_s}{K}\right)^{j} a_i \tag{B.12}$$

In addition, firms do not need to form expectation on n, since they observe n perfectly. Similarly as in section 2.1, I guess and verify that higher order expectation on p does not explode, and thus equation (A.7) now becomes

$$p_i = \alpha\Sigma_{j=1}^{\infty}(1-\alpha)^{j-1}n - \beta\Sigma_{j=1}^{\infty}(1-\alpha)^{j}E_i\bar{E}^{j-1}\bar{a} - \beta a_i \tag{B.13}$$

$$p_i = \alpha\Sigma_{j=1}^{\infty}(1-\alpha)^{j-1}n - \beta\Sigma_{j=1}^{\infty}(1-\alpha)^{j}\left\{\left(\frac{\kappa_m}{K}\right)\Sigma_{k=1}^{k=j}\left(\frac{\kappa_s}{K}\right)^{j-1} m + \left(\frac{\kappa_s}{K}\right)^{j} a_i\right\} - \beta a_i \tag{B.14}$$

To simplify equation (B.14), first work on $\Sigma_{j=1}^{\infty}(1-\alpha)^{j}\left\{\left(\frac{\kappa_m}{K}\right)\Sigma_{k=1}^{k=j}\left(\frac{\kappa_s}{K}\right)^{j-1}\right\}$ as

$$(1-\alpha)\frac{\kappa_m}{K} + (1-\alpha)^2\frac{\kappa_m}{K} + (1-\alpha)^2\frac{\kappa_m}{K}\frac{\kappa_s}{K} + (1-\alpha)^3\frac{\kappa_m}{K} + (1-\alpha)^3\frac{\kappa_m}{K}\left(\frac{\kappa_s}{K}\right)^2 + \cdots \tag{B.15}$$

Collecting terms gives:

$$(1-\alpha)\frac{\kappa_m}{K}\left\{\Lambda_1\cdot 1 + \Lambda_2\cdot\frac{\kappa_s}{K} + \Lambda_3\cdot\left(\frac{\kappa_s}{K}\right)^2 + \cdots\right\} \tag{B.16}$$

where $\Lambda_1 = \frac{1}{\alpha}$, $\Lambda_2 = \frac{1-\alpha}{\alpha}$, $\Lambda_3 = \frac{(1-\alpha)^2}{\alpha}$. Thus, equation (B.15) becomes

$$\Sigma_{j=1}^{\infty}(1-\alpha)^{j}\left\{\left(\frac{\kappa_m}{K}\right)\Sigma_{k=1}^{k=j}\left(\frac{\kappa_s}{K}\right)^{k-1}\right\} = \frac{1-\alpha}{\alpha}\frac{\kappa_m}{\alpha\kappa_s+\kappa_\xi+\kappa_m} \tag{B.17}$$

Substitute equation (B.17) into equation (B.14), and find

$$p_i = n - \beta\left[\frac{1-\alpha}{\alpha}\frac{\kappa_m}{\alpha\kappa_s+\kappa_\xi+\kappa_m}m + \frac{(1-\alpha)\kappa_s}{\kappa_m+\kappa_\xi+\alpha\kappa_s}a_i\right] - \beta a_i \tag{B.18}$$

$$p = \left[\gamma_{fc} - \beta\frac{1-\alpha}{\alpha}\frac{\kappa_m}{\alpha\kappa_s+\kappa_\xi+\kappa_m}\right] - \beta\left[\frac{(1-\alpha)\kappa_s}{\kappa_m+\kappa_\xi+\alpha\kappa_s}+1\right]\bar{a} \tag{B.19}$$

$$y = \left[\beta\frac{1-\alpha}{\alpha}\frac{\kappa_m}{\alpha\kappa_s+\kappa_\xi+\kappa_m}\right]m + \beta\left[\frac{(1-\alpha)\kappa_s}{\kappa_m+\kappa_\xi+\alpha\kappa_s}+1\right]\bar{a} \tag{B.20}$$

B.3 Forward Guidance without Commitment

p_i is defined as equation (A.7), and firms need to form conditional expectation on both n and $\bar{a}$. The conditional expectation on $\bar{a}$ is the same as in forward guidance with commitment situation. The expectation on n is formed as the difference between the forward guidance policy and the firm's expectation on the aggregate technology shocks as:

$$E_i n = \rho_m m + \rho_a a_i \tag{B.21}$$

where $\rho_m = \gamma^a_{cc}\frac{\kappa_m}{\kappa_m+\kappa_s+\kappa_a} + \gamma^{\upsilon}_{cc}\frac{\kappa_s+\kappa_a}{\kappa_m+\kappa_s+\kappa_a}, \rho_a = (\gamma^a_{cc} - \gamma^v_{cc})\frac{\kappa_s}{\kappa_m+\kappa_s+\kappa_a}$

Integrate over i,

$$\int_0^i E_i n = \bar{E}n = \rho_m m + \rho_a \bar{a} \tag{B.22}$$

$$E_i \bar{E}n = \rho_m m + \rho_a E_i \bar{a} \tag{B.23}$$

$$\bar{E}^2 n = \rho_m m + \rho_a \bar{E}\bar{a} \tag{B.24}$$

Continue this procedure to get:

$$E_i \bar{E}^j n = \rho_m m + \rho_a E_i \bar{E}^{j-1}\bar{a} \tag{B.25}$$

From equation (A.7):

$$p_i = \alpha \left[\Sigma_{j=1}^{\infty}(1-\alpha)^j E_i \bar{E}^j n + E_i n\right] - \beta\Sigma_{j=1}^{\infty}(1-\alpha)^j E_i \bar{E}^{j-1}\bar{a} - \beta a_i \tag{B.26}$$

The second term, $-\beta\Sigma_{j=1}^{\infty}(1-\alpha)^j E_i \bar{E}^{j-1}\bar{a} - \beta a_i$ has already been solved in the case of forward guidance with commitment, so that we only need to work on the first term here. Substitute$E_i\bar{E}^j n$ by the equation (B.25) and get

$$\begin{aligned}\alpha \left[\Sigma_{j=1}^{\infty}(1-\alpha)^j E_i \bar{E}^j n + E_i n\right] &= \alpha\left\{\Sigma_{j=1}^{\infty}(1-\alpha)^j\left[\rho_m m + \rho_a E_i \bar{E}^{j-1}\bar{a}\right] + \rho_m m + \rho_a a_i\right\} \\ &= \alpha\left\{\Sigma_{j=0}^{\infty}(1-\alpha)^j \rho_m m + \rho_a \Sigma_{j=1}^{\infty}(1-\alpha)^j E_i \bar{E}^{j-1}\bar{a} + \rho_a a_i\right\}\end{aligned} \tag{B.27}$$

Substitute the expression for $E_i\bar{E}^{j-1}\bar{a}$ as in the case of forward guidance with commitment,

$$\begin{aligned}E_i \bar{E}^{j-1}\bar{a} &= \alpha\left\{\rho_m m \frac{1}{\alpha} + \rho_a \Sigma_{j=1}^{\infty}(1-\alpha)^j\left[\left(\frac{\kappa_m}{K}\Sigma_{k=1}^{k=j}\right)\left(\frac{\kappa_s}{K}\right)^{k-1} m + \left(\frac{\kappa_s}{K}\right)^j a_i\right] + \rho_a a_i\right\} \\ &= \alpha\left\{\rho_m m \frac{1}{\alpha} + \rho_a\left[\frac{1-\alpha}{\alpha}\frac{\kappa_m}{\alpha\kappa_s + \kappa_a + \kappa_m} m + \frac{(1-\alpha)\kappa_s}{\kappa_m + \kappa_a + \alpha\kappa_s} a_i\right] + \rho_a a_i\right\}\end{aligned} \tag{B.28}$$

Substitute equation (B.28) as the first term in equation (B.26):

$$p_i = m\left\{\rho_m + \left[\rho_a(1-\alpha) - \beta\tfrac{1-\alpha}{\alpha}\right]\tfrac{\kappa_m}{\alpha\kappa_s+\kappa_a+\kappa_m} + \rho_a a_i\right\} + a_i\left\{[\alpha\rho_a - \beta]\tfrac{(1-\alpha)\kappa_s}{\kappa_m+\kappa_a+\alpha\kappa_s} + \rho_a a_i - \beta\right\} \tag{B.29}$$

Aggregate over i to get the aggregate price level. Then, take the difference between the aggregate nominal demand and the price level to get the real output.

$$p = \phi_m m + \phi_a \bar{a} = (\phi_m + \phi_a)\,\bar{a} + \phi_m \upsilon \tag{B.30}$$

$$y = n - p = (\gamma_a^{cc} - \phi_m - \phi_a)\,\bar{a} + (\gamma_\upsilon^{cc} - \phi_m)\,\upsilon \tag{B.31}$$

where

$$\phi_m = \rho_m + \left[\rho_a(1-\alpha) - \beta\frac{1-\alpha}{\alpha}\right]\frac{\kappa_m}{\alpha\kappa_s + \kappa_a + \kappa_m} \tag{B.32}$$

$$\phi_a = [\alpha\rho_a - \beta]\frac{(1-\alpha)\kappa_s}{\kappa_m + \kappa_a + \alpha\kappa_s} + \alpha\rho_a - \beta \tag{B.33}$$

$$\rho_m = \gamma_{fnc}^a \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} + \gamma_{fnc}^\upsilon \frac{\kappa_s + \kappa_a}{\kappa_m + \kappa_s + \kappa_a} \tag{B.34}$$

$$\rho_a = \left(\gamma_{fnc}^a - \gamma_{fnc}^\upsilon\right)\frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a} \tag{B.35}$$

Output gap stabilization

First consider the case where the re-optimized monetary policy considers only output gap stabilization.

$$\phi_m = \rho_m + \left[\rho_a(1-\alpha) - \beta\frac{1-\alpha}{\alpha}\right]\frac{\kappa_m}{\alpha\kappa_s + \kappa_a + \kappa_m} = \gamma_{fnc}^\upsilon \tag{B.36}$$

$$\phi_a = \gamma_{fnc}^a - \gamma_{fnc}^\upsilon - \frac{\beta}{\alpha} \equiv \gamma_{fnc}^d - \frac{\beta}{\alpha} \tag{B.37}$$

Substitute the above equations to the expression of ϕ_m and ϕ_a in equation (B.32), (B.33), and solve for ρ_m and ρ_a as a function of γ_{fnc}^a and γ_{fnc}^υ:

$$\gamma_{fnc}^a - \gamma_{fnc}^\upsilon - \frac{\beta}{\alpha} = (\alpha\rho_a - \beta)\left(\frac{(1-\alpha)\kappa_s}{\kappa_m + \kappa_a + \alpha\kappa_s} + 1\right) \tag{B.38}$$

$$\gamma_{fnc}^\upsilon = \rho_m + \left[\rho_a(1-\alpha) - \beta\frac{1-\alpha}{\alpha}\right]\frac{\kappa_m}{\alpha\kappa_s + \kappa_a + \kappa_m} \tag{B.39}$$

This result leads to the solution of ρ_m as a function of ρ_a:

$$\rho_m = \gamma^{\upsilon}_{fnc} - \left[\rho_a(1-\alpha) - \beta\frac{1-\alpha}{\alpha}\right]\frac{\kappa_m}{\alpha\kappa_s + \kappa_a + \kappa_m} \tag{B.40}$$

Use this result to solve the expression of ρ_a

$$\rho_a = \frac{\left(\gamma^a_{fnc} - \gamma^{\upsilon}_{fnc} - \frac{\beta}{\alpha}\right)\frac{\alpha\kappa_s+\kappa_a+\kappa_m}{K} + \beta}{\alpha} \tag{B.41}$$

Use this result to solve for $\gamma^a_{fnc} - \gamma^{\upsilon}_{fnc}$ by equating it with the definition of ρ_a as specified in equation (B.35):

$$\left(\gamma^a_{fnc} - \gamma^{\upsilon}_{fnc}\right)\frac{\kappa_s}{K} = \frac{\left(\gamma^a_{fnc} - \gamma^{\upsilon}_{fnc} - \frac{\beta}{\alpha}\right)\frac{\alpha\kappa_s+\kappa_a+\kappa_m}{K} + \beta}{\alpha} \tag{B.42}$$

$$\left(\gamma^a_{fnc} - \gamma^{\upsilon}_{fnc}\right) = \frac{\beta}{\alpha}(1-\alpha) \tag{B.43}$$

Plug the expression of $\left(\gamma^a_{fnc} - \gamma^{\upsilon}_{fnc}\right)$ in the above equation to ρ_a in equation (B.41) to solve for ρ_a

$$\rho_a = \frac{\left(\frac{\beta}{\alpha}(1-\alpha) - \frac{\beta}{\alpha}\right)\frac{\alpha\kappa_s+\kappa_a+\kappa_m}{K} + \beta}{\alpha} = \beta\frac{1-\alpha}{\alpha}\frac{\kappa_s}{K} \tag{B.44}$$

Plug this to equation (48) to solve for ρ_m, which we use to equate with the definition of ρ_m in equation (42)

$$\rho_m = \gamma^a_{fnc}\frac{\kappa_m}{K} + \gamma^{\upsilon}_{fnc}\frac{\kappa_a+\kappa_s}{K} = \gamma^{\upsilon}_{fnc} - \left[\beta\frac{1-\alpha}{\alpha}\frac{\kappa_s}{K}(1-\alpha) - \beta\frac{1-\alpha}{\alpha}\right]\frac{\kappa_m}{\alpha\kappa_s + \kappa_a + \kappa_m} \tag{B.45}$$

which results in

$$\gamma^a_{fnc} - \gamma^{\upsilon}_{fnc} = \beta\frac{1-\alpha}{\alpha} \tag{B.46}$$

Optimal γ

In this section, I solve for the optimal monetary policy which minimizes the weighted sum of squared deviations of the output gap and the aggregate price level:

$$\left(\gamma^a_{fnc} - \phi_m - \phi_a\right)^2\sigma_a^2 + \left(\gamma^{\upsilon}_{fnc} - \phi_m\right)^2\sigma_{\upsilon}^2 + \tau\left(\phi_m + \phi_a\right)^2\sigma_a^2 + \phi_m^2\sigma_{\upsilon}^2$$

The first order condition on γ^a_{fnc} results in:

$$\sigma_a^2\left[\left(\gamma^a_{fnc}-\phi_m-\phi_a\right)\frac{d\left(\gamma^a-\phi_m-\phi_a\right)}{d\gamma^a_{fnc}}+\left(\phi_m+\phi_a\right)\frac{d\left(\phi_m+\phi_a\right)}{d\gamma^a_{fnc}}\right] \tag{B.47}$$

$$+\tau\sigma_\upsilon^2\left[\left(\gamma^\upsilon_{fnc}-\phi_m\right)\frac{d\left(\gamma^\upsilon_{fnc}-\phi_m\right)}{d\gamma^a_{fnc}}+\phi_m\frac{d\phi_m}{d\gamma^a_{fnc}}\right]=0$$

Re-arrange the above equation to get:

$$\sigma_a^2\left\{\left(\gamma^a_{fnc}-\phi_m-\phi_a\right)+\left(-\gamma^a_{fnc}+2(\phi_m+\phi_a)\right)\frac{d\left(\phi_m+\phi_a\right)}{d\gamma^a_{fnc}}\right\}+\tau\sigma_\upsilon^2\left\{\left(-\gamma^\upsilon_{fnc}+2\phi_m\right)\frac{d\phi_m}{d\gamma^a_{fnc}}\right\}=0 \tag{B.48}$$

By symmetry, the first order condition on γ^υ takes the form:

$$\sigma_a^2\left\{\left(\gamma^a_{fnc}-\phi_m-\phi_a\right)+\left(-\gamma^a_{fnc}+2(\phi_m+\phi_a)\right)\frac{d\left(\phi_m+\phi_a\right)}{d\gamma^\upsilon_{fnc}}\right\}+\tau\sigma_\upsilon^2\left\{\left(-\gamma^\upsilon_{fnc}+2\phi_m\right)\frac{d\phi_m}{d\gamma^\upsilon_{fnc}}\right\}=0 \tag{B.49}$$

where

$$\frac{d\left(\phi_m+\phi_a\right)}{d\gamma^a_{fnc}}=\frac{\partial\phi_m}{\partial\rho_m}\frac{\partial\rho_a}{\partial\gamma^a_{fnc}}+\frac{\partial\phi_m}{\partial\rho_a}\frac{\partial\rho_a}{\partial\gamma^a_{fnc}}+\frac{\partial\phi_a}{\partial\rho_m}\frac{\partial\rho_m}{\partial\gamma^a_{fnc}}+\frac{\partial\phi_a}{\partial\rho_a}\frac{\partial\rho_a}{\partial\gamma^a_{fnc}} \tag{B.50}$$

$$\frac{d\phi_m}{d\gamma^a_{fnc}}=\frac{\partial\phi_m}{\partial\rho_m}\frac{\partial\rho_a}{\partial\gamma^a_{fnc}}+\frac{\partial\phi_m}{\partial\rho_a}\frac{\partial\rho_a}{\partial\gamma^a_{fnc}} \tag{B.51}$$

The partial derivatives in the above expression are calculated as follows:

$$\frac{\partial \phi_m}{\partial \rho_m} = 1 \tag{B.52}$$

$$\frac{\partial \phi_a}{\partial \rho_m} = 0 \tag{B.53}$$

$$\frac{\partial \phi_m}{\partial \rho_a} = (1-\alpha)\frac{\kappa_m}{\alpha\kappa_s + \kappa_a + \kappa_m} + (1-\alpha)\alpha\frac{\kappa_m}{K} \tag{B.54}$$

$$\frac{\partial \phi_a}{\partial \rho_a} = \frac{\alpha(1-\alpha)\kappa_s}{\kappa_m + \kappa_a + \alpha\kappa_s} + (1-\alpha)\alpha\frac{\kappa_s}{K} \tag{B.55}$$

$$\frac{\partial \rho_m}{\partial \gamma^a} = \frac{\kappa_m}{K} \tag{B.56}$$

$$\frac{\partial \rho_a}{\partial \gamma^a} = \frac{\kappa_s}{K} \tag{B.57}$$

$$\frac{\partial \rho_a}{\partial \gamma^v} = \frac{\kappa_s + \kappa_a}{K} \tag{B.58}$$

$$\frac{\partial \rho_a}{\partial \gamma^v} = -\frac{\kappa_s}{K} \tag{B.59}$$

C Figures

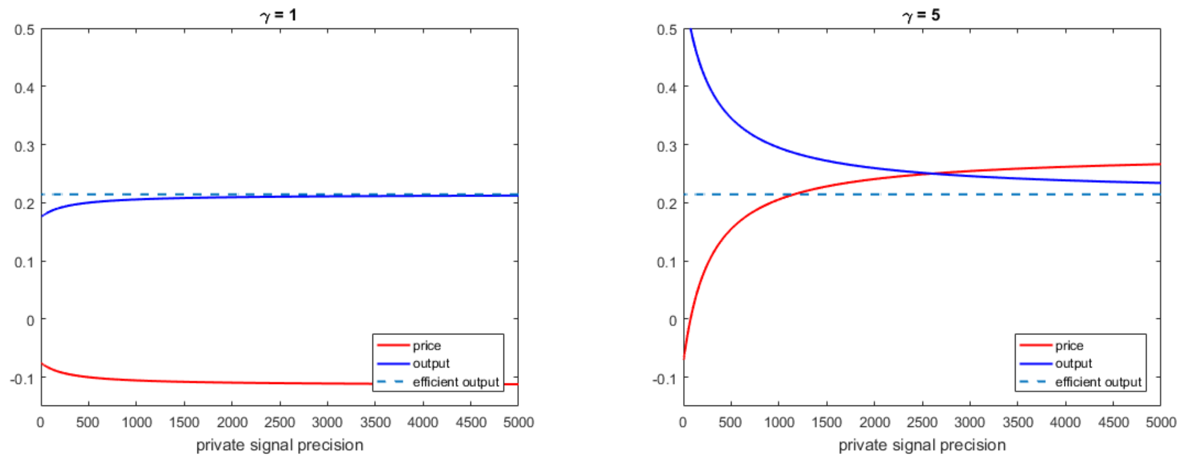

Figure 1: Perfect Opacity, Equilibrium Price and Output Under Fixed γ

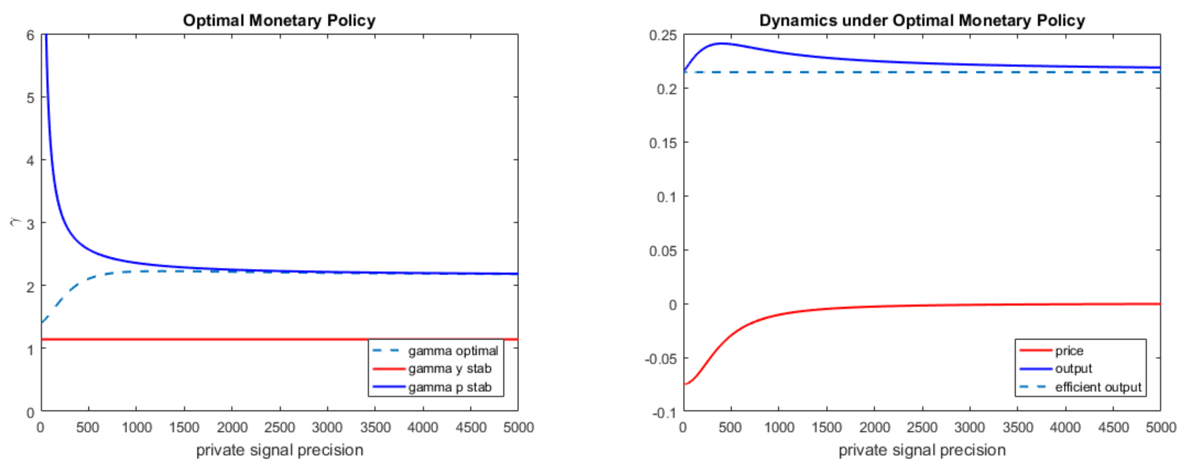

Figure 2: Perfect Opacity, Optimal Monetary Policy and Equilibrium of the Aggregate Price and the Output Gap

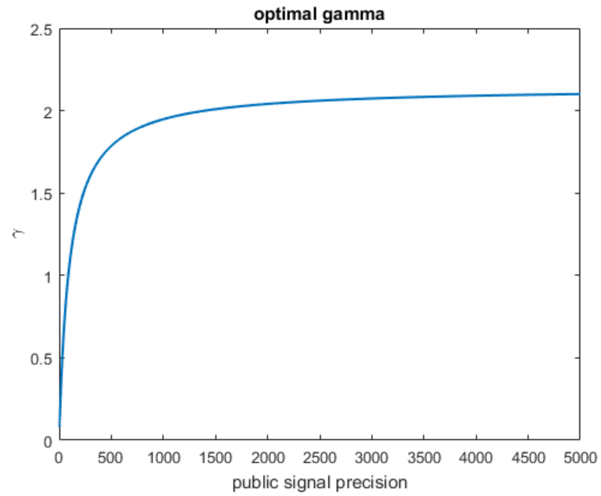

Figure 3: Forward Guidance with Commitment, Optimal Monetary Policy with Varying Precision of Public Signal

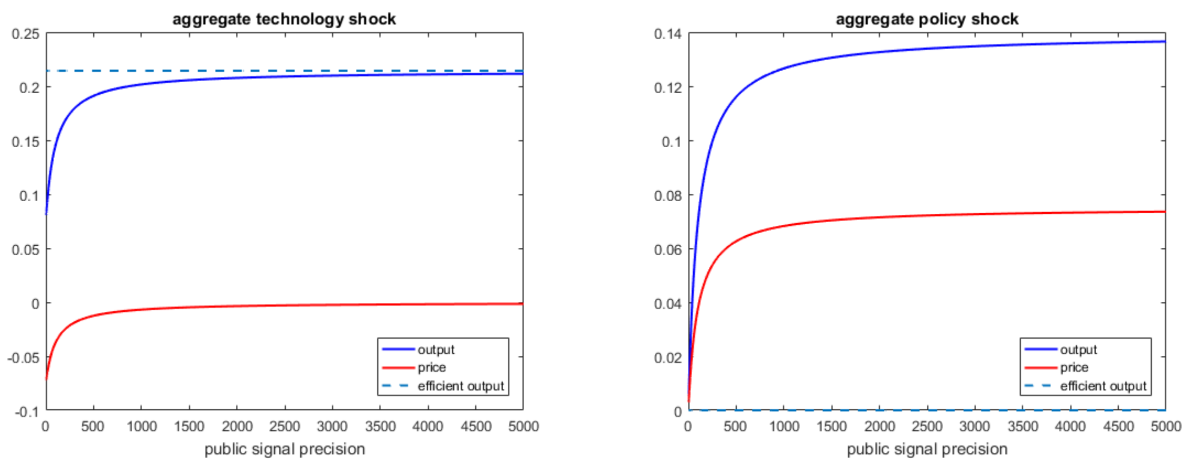

Figure 4: Forward Guidance with Commitment, Equilibrium under Optimal Monetary Policy with Varying Precision of Public Signal

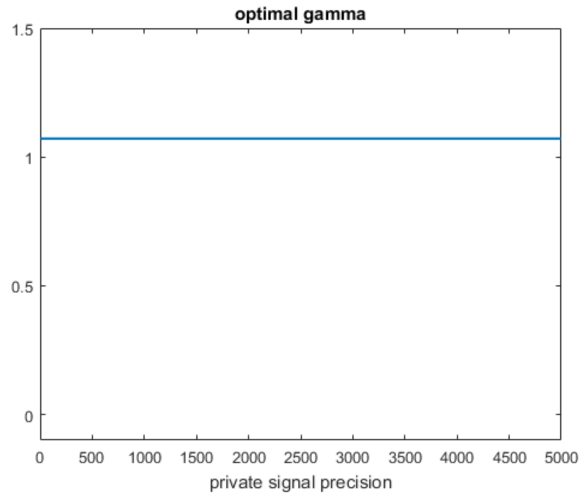

Figure 5: Forward Guidance with Commitment, Optimal Monetary Policy with Varying Precision of Private Signal

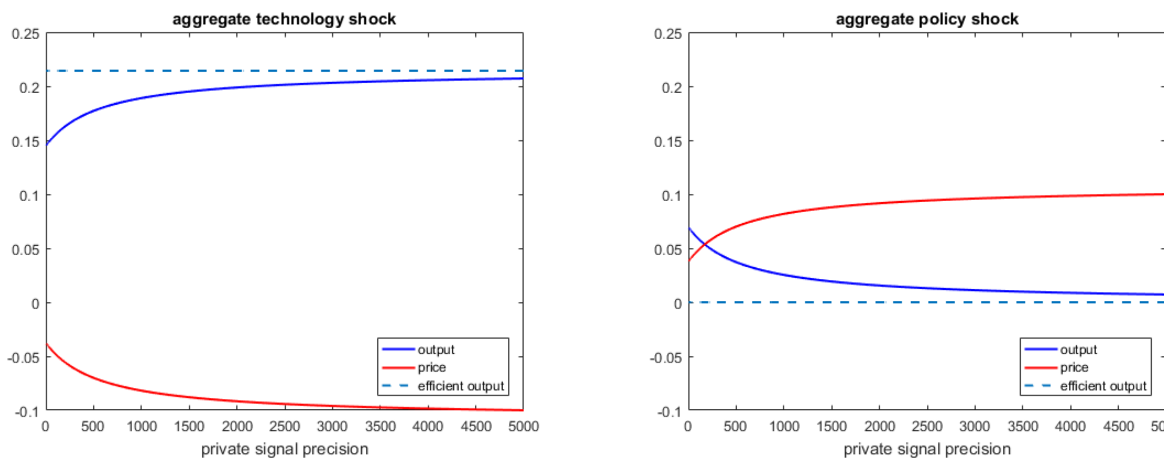

Figure 6: Forward Guidance with Commitment, Equilibrium under Optimal Monetary Policy with Varying Precision of Private Signal

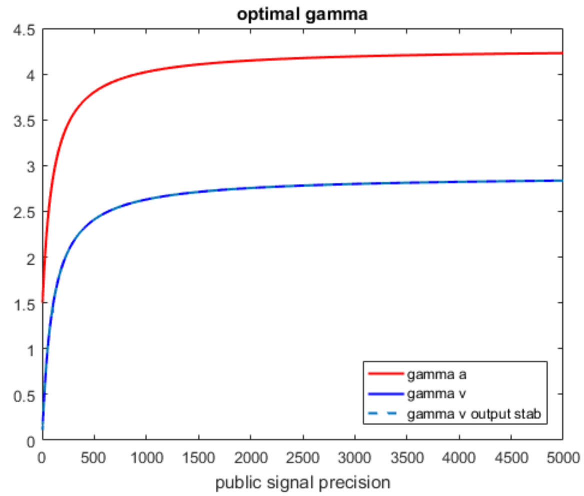

Figure 7: Forward Guidance without Commitment, Optimal Monetary Policy with Varying Precision of Public Signal

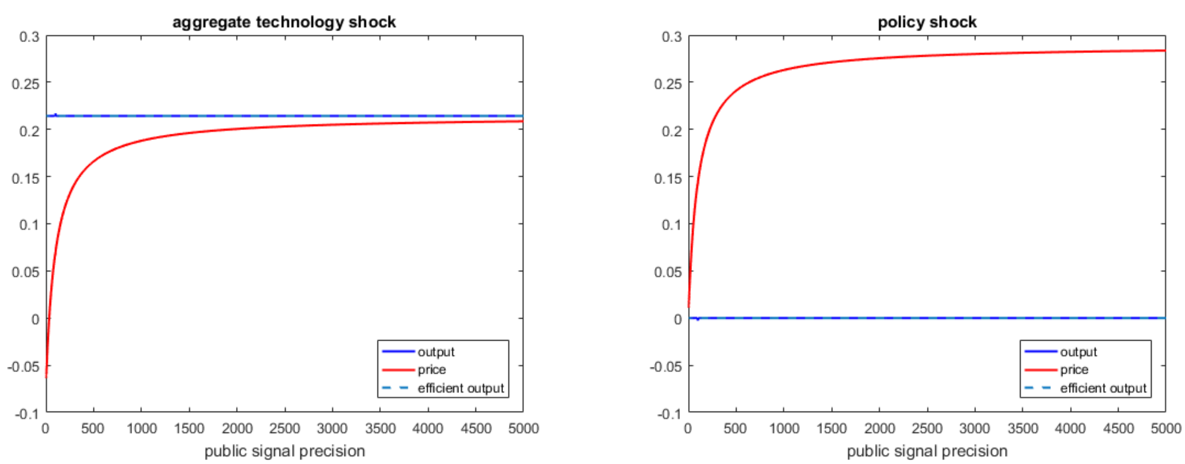

Figure 8: Forward Guidance without Commitment, Equilibrium under Optimal Monetary Policy with Varying Precision of Public Signal

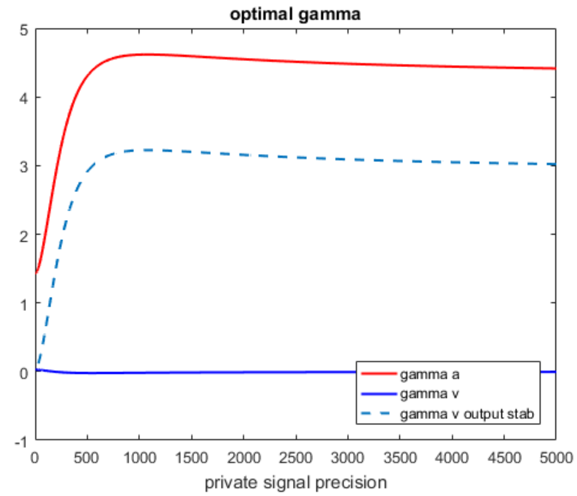

Figure 9: Forward Guidance without Commitment, Optimal Monetary Policy with Varying Precision of Private Signal

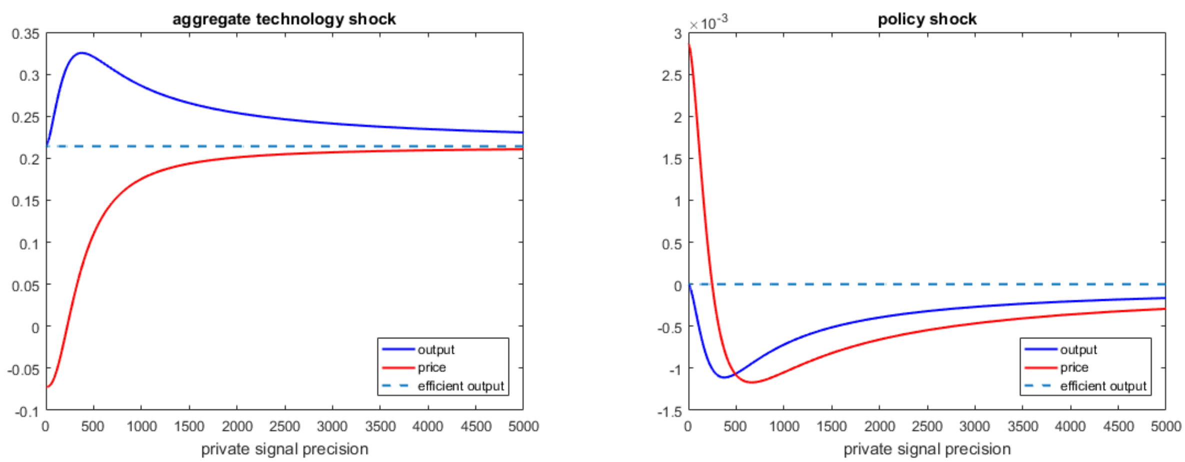

Figure 10: Forward Guidance without Commitment, Equilibrium under Optimal Monetary Policy with Varying Precision of Private Signal

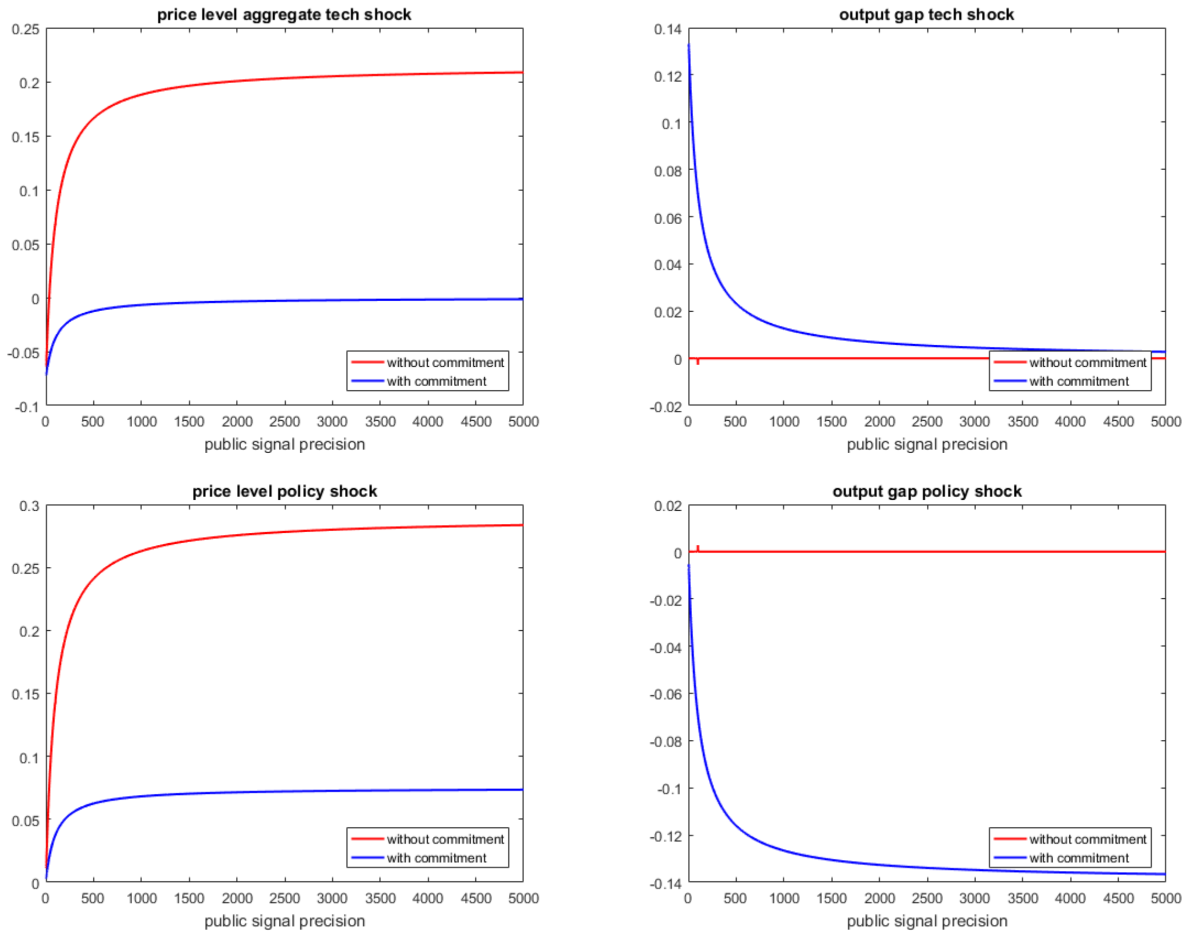

Figure 11: Equilibrium Price Level and Output under Forward Guidance with and without Commitment