The Informational Effect of Monetary Policy and the Case for Policy Commitment *

(Job Market Paper)

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Abstract

I study how the informational effect of monetary policy leads to gains from commitment. Monetary policy has an informational effect when the private sector has imperfect information about the underlying economy and extracts information about unobserved shocks from the central bank's interest rate decisions. With serially uncorrelated shocks, I show that the optimal monetary policy rule responds more aggressively to natural-rate shocks and less aggressively to cost-push shocks, relative to the central bank's optimizing response under discretion. The optimal policy rule improves ex-ante welfare by reducing the information revealed on cost push-shocks, which consequently reduces the stabilization bias caused by actual cost-push shocks under perfect information. In addition, I study how external information and serial correlation in shocks affect the size of gains from commitment. A calibrated dynamic model shows that, with relatively precise external information, committing to the optimal rule improves ex-ante welfare by 54 percent compared with the equilibrium outcome under optimizing discretionary policy.

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1 Introduction

It has become widely accepted that the effect that monetary policy has on the economy depends on the beliefs held by the private sector. Past literature has demonstrated that there are gains from commitment when beliefs in the private sector can be optimally controlled by the central bank when it commits to policy rules.[1] While the importance of expectations is well established, previous literature has only studied the case in which commitment changes the expectations regarding the policy itself. In this paper, I study how gains from commitment can come from the informational effect of monetary policy. Monetary policy has an informational effect when the central bank has better information than the private sector about the state of the economy. Consequently, the private sector can extract information about the underlying shocks from changes in the interest rate. I demonstrate that the central bank can change how beliefs about different shocks are formed in the private sector by committing to a state-contingent policy rule, which leads to welfare gains from commitment.

The informational effect of monetary policy builds on the assumption of informational frictions in the private sector. Previous literature has studied both the case in which the central bank is better informed about relevant economic fundamentals than the private sector and the case in which the central bank has less precise information than the private sector does. With few exceptions, the majority of these papers assume that the expectations formed in the private sector about the underlying state of the economy are independent of monetary policy decisions. However, recent empirical papers demonstrate that changes in the interest rate also affect the beliefs in the private sector about economic fundamentals.[2] In this paper, I study how the optimal conduct of monetary policy should be affected by the central bank's awareness of the information that will be revealed by its policy decisions. In particular, I examine whether using a policy rule can achieve welfare gains relative to discretionary policy.

I build a New Keynesian model with Calvo price rigidity and information frictions in the private sector. There are two types of shocks: natural-rate shocks and cost-push shocks. Due to imperfect information, the equilibrium output gap and inflation depend on both the actual shocks and the beliefs about the shocks, as well as the interest rate decisions by the central bank.

The central bank is assumed to have perfect information about both types of shocks. It sets the interest rate conditional on the actual shocks to minimize its loss function given by the weighted sum of squared inflation and the output gap. Private agents with rational expectations correctly

[1] See Kydland and Prescott (1977), Barro and Gordon (1983), Clarida, Gali and Gertler (2000), Woodford (1999), Eggertsson et al. (2003), among others. A more comprehensive review on gains from commitment is provided in the literature review section.

[2] See Campbell et al. (2012) and Nakamura and Steinsson (2013) as examples of empirical studies on the informational effect of monetary policy.

understand the reaction function of the interest rate. Therefore, they regard the interest rate as a public signal which simultaneously provides information about the two shocks. In this situation, the interest rate has two effects on the equilibrium in the private sector: the traditionally studied direct effect on the cost of borrowing for consumers and the informational effect on the beliefs in the private sector.

A central bank can either be discretionary or commit to an interest rate rule. The discretionary central bank sets an optimizing interest-rate at any given state of the economy and takes the informational effect of its interest rate decisions to be exogenous. In comparison, a central bank with commitment can change the beliefs in the private sector by announcing and committing to a state-contingent rule. When choosing the ex-ante policy rule, the central bank with commitment internalizes the informational effect of its interest rate decisions and balances between the direct effect and the informational effect of the interest rate.

To study how the optimal rule differs from the equilibrium interest rate decision of the discretionary central bank, I start with the simple case in which shocks have no serial correlations. Private agents are rational. They correctly understand how interest rates react to both shocks but have imperfect information about the shocks. Private agents form beliefs through a Bayesian updating process, whereby they regard the interest rate set by the central bank as a signal to extract information about the two shocks. When the interest rate reacts positively to both shocks, it becomes one signal that jointly provides information to the two shocks. When the private sector forms expectation about one shock, the prior distribution of the other shock becomes the source of noise in the signal. I demonstrate that beliefs formed through a Bayesian updating process are more sensitive to the shock to which the interest rate responds more aggressively or has a higher ex-ante dispersion.

The informational effect of the interest rate applies differently to the equilibrium output gap and inflation. Under the assumptions that shocks have no serial correlation and the interest rate only responds to shocks in the current period, the expectations about future equilibrium variables are at their steady-state levels. Consequently, although private agents are forward-looking, expectations about future equilibrium do not affect their decisions in the current period. I assume that the consumer is able to observe current price levels, but that each individual firm does not observe the aggregate price level. Consequently, the output gap is free from the expectations. However, the inflation depends on the beliefs in the private sector, as optimal pricing decisions are strategic complements, where the resetting price of each firm also depends on the firm's expectation about the aggregate price level. Thus, the interest rate changes the output gap only through the direct effect, but affects inflation through both the direct effect and the informational effect. When the central bank reacts to expansionary shocks[3] by increasing the interest rate, the informational effect

[3] I use the term "expansionary shocks" to refer to the shocks that cause positive output gap or inflation without the

dampens the direct effect of the increase in the interest rate, as the private sector updates its beliefs about the expansionary shocks.

To compare the optimizing discretionary policy with the optimal policy rule, I first examine how the informational effect of the interest rate changes the Phillips curve. The Phillips curve is the constraint that a central bank faces, which captures the co-movement of the output gap and the inflation as a result of changes in interest rates. After a marginal increase in the interest rate, the direct effect on a household's cost of borrowing decreases both the output gap and inflation, which results in a positively sloped Phillips curve under perfect information. However, under imperfect information, as the informational effect dampens the direct effect on inflation, the Phillips curve becomes flatter than that under perfect information. In addition, under perfect information, a cost-push shock induces a positive intercept for the Phillips curve, as a cost-push shock increases inflation only without changing the natural output level. This positive intercept of the Phillips curve leads to stabilization bias, which is the conflict between the closing the output gap and minimizing inflation. Under perfect information, a central bank increases the interest rate to partially offset the effect of the cost-push shock on inflation, which results in a positive inflation and a negative output gap. However, with imperfect information, as the private sector simultaneously updates beliefs about both the cost-push shock and the natural-rate shock from a positive change in the interest rate, the intercept caused by the cost-push shock is reduced. For the same reason, although a natural-rate shock does not result in a positive intercept under perfect information, it does so under imperfect information.

In the case of a discretionary central bank, I solve for the Markov perfect equilibrium between the central bank and the private sector. The private sector forms beliefs and makes optimal consumption and pricing decisions while expecting the central bank to play the equilibrium optimizing interest rate at any state of the economy. The central bank optimizes the interest rate to minimize the deviations of inflation and the output gap from their targets, taking as given the informational effect of its interest rate decision. A discretionary central bank does not internalize the change in the informational effect when making interest rate decisions.

The change in the Phillips curve under imperfect information leads to a change in the optimizing discretionary monetary policy in equilibrium. Although the natural-rate shock can be completely offset by discretionary monetary policy under perfect information, this "divine coincidence" cannot be achieved in the presence of informational frictions. This is because even if the actual shock is a natural-rate shock, the private sector still assigns a positive possibility to the event that the interest rate is reacting to a cost-push shock. Consequently, optimizing discretionary policy is "leaning against the wind" after both shocks, seeking a negative correlation between output

response of interest rates. That is, positive natural-rate shocks (negative current TFP shocks) and positive cost-push shocks.

gap and inflation. I show that the optimizing discretionary interest rate reacts more to natural-rate shocks and less to cost-push shocks than what is optimal under perfect information.

Second, I demonstrate the gains from committing to the optimal policy rule. To isolate the informational gains, I focus on the case in which the interest rate only responds to current shocks. This removes the traditionally studied gains from commitment to a delayed response, which comes from the change in expected future equilibrium. I show that even without the traditionally studied effect on the expected future equilibrium, the optimal policy rule still improves ex-ante welfare compared with the equilibrium under discretion, as the policy rule optimally controls the information revealed about the unobserved shocks.

In the case of commitment, the central bank controls the informational effect by announcing and committing to a state-contingent interest rate rule. I demonstrate that relative to the optimizing discretionary interest rate, the optimal interest rate rule responds more aggressively to natural-rate shocks and less aggressively to cost-push shocks. When the private sector believes that the interest rate is less sensitive to the cost-push shocks, beliefs about the cost-push shocks are less sensitive to changes in the interest rate. Consequently, both the slope and the intercept of the Phillips curve are endogenously determined by the policy rule. The optimal policy rule improves ex-ante welfare, because it reduces the marginal increase in the expected cost-push shock after a marginal increase in the interest rate, which consequently reduces the stabilization bias caused by an actual cost-push shock under perfect information.

The informational effect of monetary policy results in a novel time-inconsistency problem. Different from the traditional time inconsistency, in which the incentives to deviate apply across time periods, the time inconsistency problem in my model applies across states. Once the central bank has committed to a policy rule, it has fixed the informational effect of the interest rate, and thus the Phillips curve. Ex-post, the central bank has an incentive to deviate from its committed rule, assuming that such a change in the interest rate response will not change the Phillips curve. Suppose that there is a positive natural-rate shock; then, prior to the realization of the shock, the central bank commits to react more aggressively, relative to the optimizing response under discretion, to reduce the informational effect on the expected cost-push shock. This policy rule reduces the intercept of the Phillips curve. Once the Phillips curve is fixed, the central bank wants to reduce the increase in the interest rate, assuming that such a one-time deviation from the rule will not change the Phillips curve after the natural-rate shock.

I extend the analysis in two ways. First, I incorporate external signals, which models any other source of information obtained by the private agents, including direct communication by the central bank. The external signals are distributed independently around the actual shocks. Without the informational effect of the interest rate, increasing the precision of the signal about one shock only makes the expected shock closer to the actual shock ex-ante. However, in the presence of an informational effect of the interest rate, the effects of external signals are not independent. Increasing the precision of the external signal about one shock also makes the interest rate a more precise signal about the other shock. Consequently, this interaction effect yields different welfare implications for central bank communication than argued by the conventional wisdom. Providing more precise information about the efficient shock (natural-rate shock) through central bank communication may reduce welfare if the private sector also simultaneously has more precise information about the inefficient shock (cost-push shock) from the interest rate.

Second, I extend the analysis to serially correlated shocks to study the dynamic informational effect of the interest rate. In this case, the dynamic informational effect of the current interest rate comes from the persistent belief-formation process in the private sector. The private agents forms beliefs in the current period by optimally combining current signals and past beliefs. Consequently, the current interest rate has a lagged effect on future equilibrium through its effect on current beliefs. When the central bank considers the dynamic effect of its interest rate decisions, the objective function of a discretionary central bank includes deviations of the output gap and inflation in both current and future periods. The optimal discretionary policy can be characterized as "dynamically leaning against the wind": it is willing to tolerate a positive sum of current inflation and the current output gap if the sum of inflation and the output gap in the future is expected to be negative.

To quantify the gains from commitment, I calibrate the full version of my model, including external signals, serially correlated shocks, and policy implementation errors. In my calibrated model, I adopt parameter values from previous macroeconomics studies, except for the precision of external information. Varying the precision of external information critically changes the size of the gains from commitment. In the extreme case in which external information is infinitely imprecise, the gains from commitment are negligible. However, when external signals are as precise as actual shocks, the optimal policy rule can improve welfare by 54 percent relative to the equilibrium under optimizing discretionary policy.

1.1 Related Literature

My paper connects three strands of literature: (i) the comparison of monetary policy under discretion versus commitment, (ii) optimal monetary policy under information frictions, and (iii) the informational effect of monetary policy.

(A) Discretionary Monetary Policy versus Monetary Policy Rule

There is a long history of studying the gains from monetary policy commitment. The original treatments can be found in Kydland and Prescott (1977) and Barro and Gordon (1983), who discuss the classical inflationary bias that results from a discretionary central bank having an objective function that contains a positive output gap target. A large literature has developed various methods to overcome the inflationary bias under discretion, including central bank reputation (Barro (1986) and Cukierman and Meltzer (1986) ect). and different central bank preferences (Rogoff (1985), Lohmann (1992) and Svensson (1995) etc).

Another mechanism that leads to gains from commitment is when a discretionary central bank faces stabilization bias. This occurs when there is a trade-off between closing the output gap and minimizing inflation in the current period. By committing to a delayed interest rate response, the central bank is able to decrease current inflation without sacrificing the current output gap; instead it does so through the decrease in expected future inflation. Clarida, Gali and Gertler (2000) study how an ad-hoc cost-push shock introduces a conflict between inflation stabilization and output gap stabilization and describe the optimal commitment to a future interest rate path. Woodford (1999) studies how an interest rate smoothing objective helps the central bank to commit to a history-dependent policy, to steer private sector expectations about future policy rates. Eggertsson et al. (2003) show that optimal commitment to delayed response can mitigate the distortions created by the zero lower bound on the interest rate.

(B) Optimal Monetary Policy with Informatioaln Frictions

My paper builds on the studies of optimal monetary policy under imperfect information. This field is revived by Woodford (2001), which shows how higher order beliefs lead to a persistent effect of monetary policy, under the assumption of imperfect information which was initially introduced in Phelps (1970) and Lucas (1972).

The majority of papers that study optimal monetary policy under informational frictions assume that beliefs in the private sector are formed independently from monetary policy decisions. Under this assumption, a central bank makes policy decisions every period, taking as given the exogenous beliefs in the private sector. Ball, Mankiw and Reis (2005) assume that information is rigid in the private sector and characterize optimal policy as an elastic price standard. Adam (2007) assumes an endogenous learning process in the private sector and demonstrates that the target of the optimal monetary policy changes from output gap stabilization to price stabilization when information becomes more precise. Angeletos and La'O (2011) solve the Ramsey problem for optimal monetary policy and show that the flexible-price equilibrium is no longer the first-best when information frictions affect real variables.

There are also papers that discuss the gains from policy commitment under imperfect information. Svensson and Woodford (2003) and Svensson and Woodford (2004) assume that the central bank has imperfect information and show that the optimal policy under commitment displays considerable inertia, relative to the discretionary policy, due to the persistence in the learning process. Lorenzoni (2010) and Paciello and Wiederholt (2013) explore the idea that the central bank is able to change the learning process in the private sector if it is able to commit to completely offset inefficient shocks.

Recent papers have begun to investigate the situation in which the private sector extracts information about the underlying economy from monetary policy decisions. Baeriswyl and Cornand (2010) note that because monetary policy cannot fully neutralize markup shocks, the central bank alters its policy response to reduce the information revealed about the cost push shock through monetary policy. Berkelmans (2011) demonstrates that with multiple shocks, tightening policy may initially increase inflation. The paper most related to the present work is Tang (2013), which shows that when the private sector has rational expectations, the stabilization bias is reduced when monetary policy has an information effect.

To the best of my knowledge, the only paper that discusses the time inconsistency problem resulting from the informational effect of monetary policy is Stein and Sunderam (2016). The authors use a reduced-form model in which the central bank balances between implementing the optimal target rate and minimizing the information revealed about this target. In their paper, private agents are assumed not to have rational expectations about the central bank's behaviors. The discretionary central bank always has incentives to deviate from the target interest rate, to reveal less information about its target. In my paper, I assume that private agents have rational expectations about how the central bank would react under both discretionary policy and a policy rule. Relative to the perfect information case, both optimizing discretionary policy and the optimal policy rule exhibit an inertial response to cost-push shocks, but the degree of inertia is higher under commitment.

(C) Empirical Evidence on the Informational Effect of Monetary Policy

My study is also motivated by the empirical evidence on the informational effect of monetary policy. Romer and Romer (2000) and Romer and Romer (2004) are the first contributions to provide empirical evidence on information asymmetry between the Federal Reserve and the private sector. They show that inflation forecasts by private agents respond to changes in the policy-rate after FOMC announcements. Faust, Swanson and Wright (2004) further confirm that the private sector revises its forecasts in response to monetary policy surprises. In more recent papers, Campbell et al. (2012) show that unemployment forecasts decrease and CPI inflation forecasts increase after a positive innovation to future federal funds rates. Nakamura and Steinsson (2013) identify the informational effect of the federal funds rate suing high-frequency data. In addition, Melosi (2016) captures this empirical pattern using a DSGE model with dispersed information. Garcia-Schmidt (2015) uses Brazilian Survey data to show that inflation forecasts in the private sector increase in the short run after an unexpected tightening policy.

The remainder of the paper is organized as follows. Section 2 characterizes the optimization decisions by the representative household in the private sector, and expresses aggregate output gap and inflation as functions of beliefs. Section 3 analyzes optimizing discretionary policy and gains from commitment to policy rule in the baseline case where shocks are not serially correlated. Section 4 and section 5 discuss two factors that affect the size of gains from commitment: external information and serial correlation in shocks. To quantitatively assess the gains from commitment, I calibrate the full version of my model with serially correlated shocks, external signals and policy implementation error in section 6. Section 7 concludes the paper.

2 Private Sector

In this section, I incorporate informational frictions to an otherwise standard New Keynesian model with Calvo-type price rigidity. Fluctuations are driven by two types of shocks: a technology shock (expressed in terms of the "Wicksellian natural rate" in the output gap) and a wage markup shock (expressed in terms of a cost push shock in inflation). I assume that the central bank has perfect information about the two shocks, whereas the private sector cannot directly observe the shocks. The private sector has rational expectations about the central bank's behavior. In particular, the private sector correctly understands how the central bank will respond to both shocks and infers information about the shocks from observing the interest rate decision. This section describes the equilibrium level of the aggregate output gap and inflation as functions of beliefs in the private sector.

2.1 Information Frictions

Following Phelps (1970), Woodford (2001), and Angeletos and La'O (2010), I model an "island economy", in which the informational friction is the result of geographical isolation. There is a continuum of islands, indexed by j, and a representative household. The household consists of a consumer and a continuum of workers. At the beginning of each period, the household sends one worker to each island, j. There is a continuum of monopolistic firms, each located on one island and indexed by the island. Each firm demands labor in the local labor market in the island and produces a differentiated intermediate good, j. Information is symmetric within an island, as each firm is able to observe its firm-specific shocks. Information is asymmetric across islands, as firms are unable to observe shocks or decisions made by other firms. Consequently, the resetting price of each firm depends on the firm's expectation of the aggregate price level, which makes aggregate inflation a function of beliefs in the private sector. The consumer of the representative household makes inter-temporal consumption decisions. He is able to observe the current prices of all intermediate goods, but unable to directly observe shocks. Consequently, the inter-temporal consumption decisions are also subject to informational frictions.

2.2 Private Sector Optimization Problem

2.2.1 Household

The preferences of the representative household are defined over the aggregate consumption good, C_t, and the labor supplied to each firm, $N_t(j)$, as

$$E_t^H \Sigma_{t=0}^{\infty} \beta^t \left\{ U(C_t) - \int V(N_t(j)) dj \right\}, \tag{1}$$

where E_t^H denotes the household's subjective expectations conditional on its information set, ω_H. The aggregate good C_t consists a continuum of intermediate goods:

$$C_t = \left(\int_0^1 C_t(j)^{1-\frac{1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where $C_t(j)$ is the consumption of intermediate good j in period t.

The economy is cashless. The household maximizes expected utility subject to the inter-temporal budget constraint:

$$\int P_t(j)C_t(j)dj + B_{t+1} \leq \int W_t(j)N_t(j)dj + (1+i_t)B_t + \Pi_t, \tag{3}$$

where B_t is a risk-free bond with nominal interest i_t, which is determined by the central bank. Π_t is the lump-sum component of household income, which includes tax payments and profits from all firms. $W_t(j)$ and $N_t(j)$ are the labor wage and labor supply for firm j, respectively.

The household's optimization problem can be solved in two stages. First, conditional on the level of aggregate consumption, the household allocates intermediate goods consumption to minimize the cost of expenditure conditional on the level of aggregate good consumption. The allocation of intermediate good consumption that minimizes expenditure yields

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} C_t, \tag{4}$$

where $P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$.

In the second stage, given the aggregate price level, P_t, the household chooses its aggregate consumption, C_t, labor supply to all firms, $N_t(j)\ \forall j$, and savings in the risk-free bond, B_{t+1}. I assume that the utility of aggregate good consumption and the utility of labor supply take the following forms: $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$, and $V(N_{jt}) = \frac{N_{jt}^{1+\varphi}}{1-\varphi}$, where σ is the inverse of the inter-temporal elasticity of substitution and the parameter φ is the inverse of the Frisch elasticity of labor supply.

The inter-temporal consumption decision leads to the following Euler equation:

$$C_t^{-\sigma} = \beta(1+i_t)E_t^H\left(C_{t+1}^{-\sigma}\frac{P_t}{P_{t+1}}\right). \tag{5}$$

Equation (5) shows that consumption decisions are forward-looking. Current demand depends the relative cost of consumption today versus consumption tomorrow.

The intra-temporal labor supply decision sets the marginal rate of substitution between leisure and consumption equal to the real wage:

$$\frac{N_t^{\varphi}(j)}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \tag{6}$$

2.2.2 Firms

Firms make two decisions to maximize expected profits: the intra-period cost minimization and the optimal pricing decisions. As the cost minimization problem only involves information within the island and information is symmetric within islands, the intra-period cost minimization problem is free from any informational frictions. The optimal pricing decision, by contrast, is affected by both the Calvo price rigidity and the informational frictions. In each period, a measure $1-\theta$ of firms get the Calvo lottery to reset their prices. Other firms charge their previous prices. A firm j that resets its price in period t chooses $P_t^*(j)$ to maximize its own expectation of the sum of all discounted profits while $P_t^*(j)$ remains effective. The profit optimization problem can be written as follows:

$$max_{P_t^*(j)}\Sigma_{k=0}^{\infty}\theta^k E_t^j\left\{Q_{t,t+k}\left[P_t^*(j)Y_{t+k}(j) - U_{t+k}^w(j)W_{t+k}(j)N_t(j)\right]\right\}, \tag{7}$$

where E_t^j denotes firm j's expectation conditional on its information set, ω_j. $Q_{t,t+k}$ is the stochastic discount factor given by: $Q_{t,t+k} = \beta^k\frac{U'(C_{t+k})}{U'(C_t)}\frac{P_t}{P_{t+k}}$. $U_{t+k}^w(j)$ denotes the wage markup for firm j.

Firms face two constraints. The first is the demand for their products, which results from the household's optimal allocation among intermediate goods. The second constraint is the production technology. Following the tradition of New Keynesian literature, I assume that labor is the only input and each firm produces according to a constant return to scale technology,

$$Y_t(j) = A_t(j)L_t(j), \tag{8}$$

where $A_t(j)$ denotes the technology of firm j.

There are two sources of uncertainty that affect the pricing decisions of each firm: technology shocks and wage markup shocks. I assume that both shocks have an aggregate component and an idiosyncratic component. The idiosyncratic components are drawn independently in every period, and are distributed log-normally around their aggregate components.

$$log(A_t(j)) \equiv a_t(j) = a_t + s_t^a(j), \qquad s_t^a(j) \sim N(0,\ \sigma_{sa}^2)$$
$$log(U_t^w(j)) \equiv u_t^w(j) = u_t^w + s_t^u(j), \qquad s_t^u(j) \sim N(0,\ \sigma_{su}^2)$$

I assume that the aggregate components of both shocks follow AR(1) processes:

$$a_t = \phi^a a_{t-1} + v_t^a, \qquad v_t^a \sim N(0,\ \sigma_{va}^2)$$
$$u_t^w = \phi^u u_{t-1}^w + v_t^{uw}, \qquad v_t^{uw} \sim N(0,\ \sigma_{vuw}^2)$$

The first order condition for labor input implies that the nominal marginal cost of production is $U_t(j)W_t(j)/A_t(j)$. Substituting the marginal cost of production into the optimal pricing decision results in

$$P_t^*(j) = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t^j \Sigma(\beta\theta)^k u'(C_{t+k}) P_{t+k}^{\varepsilon} Y_{t+k} \frac{u_{t+k}(j) w_{t+k(j)}}{A_{t+k}(j)}}{E_t^j \Sigma(\beta\theta)^k u'(C_{t+k}) P_{t+k}^{\varepsilon-1} Y_{t+k}}. \tag{9}$$

Equation (9) implies that individual resetting prices are forward-looking and strategic complements. The optimal resetting price of firm j increases with the expectation of a higher firm-specific marginal cost of production and a higher aggregate price level in both the current and all future periods.

2.3 Aggregation and Equilibrium in the Private Sector

Equilibrium variables in the private sector are solved in log deviations from steady state values (i.e., $x_t \equiv ln(X_t/X)$), and denoted by lower-case letters. (See Appendix A for details.)

The Output Gap

Following the New Keynesian tradition, I express output in terms of the output gap, $\hat{y}_t$, which is defined as the difference between y_t and the natural level of output, y_t^n. The natural level of output is defined as the output level under flexible prices and perfect information. In this situation, y_t^n becomes a linear function of a_t $y_t^n = \frac{\varphi+\sigma}{1+\varphi} a_t$, and follows an AR(1) process, $y_t^n = \phi y_{t-1}^n + v_t$, where $\phi = \phi^a$, and $\sigma_v = \frac{\varphi+\sigma}{1+\varphi}\sigma_{va}$.

The output gap is derived as follows:

$$\hat{y}_t \equiv y_t - y_t^n = E_t^H \hat{y}_{t+1} - \frac{1}{\sigma}\left[i_t - \left(\frac{1}{1-\phi} r_t^n - \frac{\phi}{1-\phi} E_t^H r_t^n\right) - E_t^H \pi_{t+1}\right], \tag{10}$$

where $E_t^H \hat{y}_{t+1} = E_t^H y_{t+1} - E_t^H y_{t+1}^n = E_t^H y_{t+1} - \phi E_t^H y_t^n$. r_t^n denotes the natural rate of interest, which is the equilibrium real interest rate that equates output to its natural level under perfect information and flexible prices. It is calculated as $r_t^n \equiv \sigma\left(E_t y_{t+1} - y_t^n\right) = \sigma(\phi-1)y_t^n$.

If information is perfect, $E_t^H r_t^n = r_t^n$, and expectations about future equilibrium are objective i. e., $E_t^H \hat{y}_{t+1} = E_t \hat{y}_{t+1}$ and $E_t^H \pi_{t+1} = E_t \pi_{t+1}$. Substituting them into the above equation results in the IS curve under perfect information:

$$\hat{y}_t = E_t^s \hat{y}_t - \frac{1}{\sigma}\left[i_t - r_t^n - E_t \pi_{t+1}\right] \tag{11}$$

The difference between equation (10) with equation (11) illustrates how the output gap under imperfect information differs from that under perfect information. Specifically, under perfect information, a positive natural-rate shock increases the output gap by $\frac{1}{\sigma} r_t^n$. The positive output gap is caused by the price rigidity, as the adjustments in prices are insufficient, so that the reduction in the equilibrium output is smaller than the reduction in the natural output. In comparison, this output gap is enlarged under imperfect information. Absent an interest rate response, the private agents do not update their beliefs about the natural rate. Substituting $E_t^s r_t^n = 0$ into equation (10) shows that the output gap becomes $\frac{1}{1-\phi}\frac{1}{\sigma} r_t^n$. Intuitively, as the household does not know about the change in the natural output level in the next period, the household does not reduce current consumption, which is equivalent to a larger positive output gap.

Inflation

According to the assumption of Calvo-type price rigidity, the current aggregate price level is the composite of the aggregate price in the previous period and the average resetting prices:

$$p_t = \theta p_{t-1} + (1-\theta)\int p_t^*(j)dj. \tag{12}$$

The integral of resetting prices potentially leads to the higher order beliefs problem. As equation (9) shows, $p_t^*(j)$ includes firm j's expectation about the aggregate price level P_t, and, thus, includes other firms' expectations. This leads to the infinite regress problem, in which each firm uses its firm-specific shock as a private signal, and guesses the private signals observed by other firms. As the focus of my study is on aggregate variables instead of on the distribution of prices across firms, I abstract from this higher order beliefs problem by modeling homogeneous subjective beliefs.[4] This means that when all private agents, including both firms and the household, form expectations about the aggregate variables, all agents use only public signals. Therefore, the information sets are the same across all agents. I denote the homogeneous subjective beliefs in the

[4]There are many papers that address how higher order beliefs lead to monetary policy to have more persistent effects, for example Woodford (2001) and Angeletos and La'O (2009). For the solution method to the infinite regress problem, see Huo and Takayama (2015), Melosi (2016) and Nimark (2017).

private sector as E_t^s.[5] Mathematically, I assume that the idiosyncratic components of firm-specific shocks have infinite variance. In this case, private signals are completely uninformative, so that firms do not use their private signals about firm-specific shocks to form beliefs about aggregate variables. [6]

The aggregation of individual resetting prices leads to the New Keynesian Phillips curve under subjective beliefs: (see Appendix A for the detailed derivation.)

$$\pi_t = \beta\theta E_t^s \pi_{t+1} + (1-\theta)E_t^s \pi_t + \kappa\theta\hat{y}_t + u_t, \tag{13}$$

where $\kappa = \frac{(1-\beta\theta)(1-\theta)(\varphi+\sigma)}{\theta}$, and u_t denotes the cost push shock, which is related to the wage markup shock as $u_t = (1-\theta)(1-\beta\theta)u_t^w$.

If information is perfect, expected inflation is the same as actual inflation, i.e., $E_t^s \pi_t = \pi_t$, and expectations about future equilibrium are objective i.e., $E_t^s \pi_{t+1} = E_t \pi_{t+1}$. Substituting them into equation (13) results in the Phillips curve under perfect information:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\hat{y}_t + \frac{1}{\theta}u_t \tag{14}$$

The difference between equation (13) and equation (14) shows how the inflation under imperfect information differs from that under perfect information. Under perfect information, a positive cost-push shock increases inflation by $\frac{1}{\theta}u_t$. As this cost-push shock does not increase the output gap, the central bank faces a conflict between stabilizing inflation and closing the output gap. If it increases the interest rate to dampen inflation, it also creates a negative output gap. When information is imperfect, only a fraction θ of the actual cost-push shock is observed by individual firms, as firms only observe their firm-specific shocks. Absent an interest rate response, firms do not update beliefs regarding the aggregate cost-push shock, meaning that the resetting prices change by less than under perfect information. Therefore, imperfect information reduces the stabilization bias under perfect information.

[5] Note that subjective expectations in this paper refer to the rational expectations formed as a result of imperfect information about the state variables.

[6] Another way to generate homogeneous beliefs is to assume that firms have the same technology and face the same wage markup but do not observe them when setting prices. This assumption, however, implies that aggregate inflation consists of only the firms' expectations, and does not consist of actual shocks. Consequently, there will be no trade-off between inflation and the output gap due to the lack of actual cost-push shocks, which makes the optimal monetary policy becomes less interesting.

3 Monetary Policy with Serially Uncorrelated Shocks

I start by comparing discretionary monetary policy and policy rule commitment in a simple scenario, in which underlying shocks have no serial correlation. In this case, although private agents are forward-looking, the expectations of future equilibrium variables do not matter for current choices, as future equilibrium variables are expected to be at their steady state levels. In addition, I shut down the gains from committing to a delayed response, as I impose the restriction that the central bank can only respond to current states. These two assumptions allow me to focus on the within-period gains from commitment through the informational effect of monetary policy.

3.1 Equilibrium under an Arbitrary Interest Rate Policy

This section studies how interest rates affect the output gap and inflation through both the direct effect on the borrowing cost and the informational effect on beliefs. In addition, it illustrates how the informational effect on beliefs about different shocks are determined by the interest rate reaction function.

First, since shocks have no correlation, substituting $\phi = 0$ and $E_t^s \hat{y}_{t+1} = E_t^s \pi_{t+1} = 0$ in the IS function and the Phillips curve results in:[7]

$$\hat{y}_t = -\frac{1}{\sigma}\left(i_t - r_t^n\right) \tag{15}$$

$$\pi_t = (1-\theta)E_t^s \pi_t + \kappa\theta\hat{y}_t + u_t \tag{16}$$

As shown in the IS equation, the output gap is free from subjective beliefs and thus the informational effect of the interest rate does not play a role in determining the output gap. This is because future equilibrium variables are expected to be at steady state levels and the current aggregate price level is observed by the consumer.

In contrast, inflation is affected by subjective beliefs, as individual firms do not observe the aggregate price level when setting optimal prices. Consequently, to express actual inflation in terms of shocks, further substitute the expected aggregate inflation by $E_t^s \pi_t = \kappa E_t^s \hat{y}_t + \frac{1}{\theta} E_t^s u_t$. The expected output gap is different from the actual output gap, as the private sector has imperfect knowledge of the actual r_t^n. Specifically, $E_t^s \hat{y}_t = \hat{y}_t - \frac{1}{\sigma} r_t^n + \frac{1}{\sigma} E_t^s r_t^n$. As a result, inflation can be expressed in terms of the output gap, the actual shocks and the expected shocks as follows:

$$\pi_t = \kappa\hat{y}_t + (1-\theta)\frac{\kappa}{\sigma}\left(E_t^s r_t^n - r_t^n\right) + \frac{1-\theta}{\theta} E_t^s u_t + u_t. \tag{17}$$

[7] Following the conventional New Keynesian literature, the long-run distortion has been eliminated via Pigouvian tax as an employment subsidy, so that the steady state levels of the output gap and inflation are all zero.

The interest rate has two effects on equilibrium in the private sector. The first one is the direct effect, which is the conventionally studied effect on the borrowing cost for the household. The direct effect of a marginal increase in the interest rate reduces current consumption, as it increases the relative cost of current consumption versus future consumption. In addition, the direct effect of an increase in the interest rate also reduces the aggregate price level, as each firm reduces its resetting price when facing a lower demand. The direct effect of the interest rate on the output gap and inflation are as follows:

$$\frac{\partial \hat{y}_t}{\partial i_t}|_{direct} = -\frac{1}{\sigma}, \qquad \frac{\partial \pi_t}{\partial i_t}|_{direct} = \frac{\partial \pi_t}{\partial \hat{y}_t}\frac{\partial \hat{y}_t}{\partial i_t} = -\frac{\kappa}{\sigma}.$$

The informational effect captures how the interest rate changes the beliefs in the private sector about the two underlying shocks, $E_t^s r_t^n$ and $E_t^s u_t$. As the output gap is not affected by the subjective beliefs, it is free from the informational effect of the interest rate. The marginal informational effect of the interest rate on inflation is the combination of the marginal change in the expected cost-push shock and in the expected natural-rate shock. The marginal informational effect of the interest rate on output gap and inflation is

$$\frac{\partial \hat{y}_t}{\partial i_t}|_{informational} = 0, \qquad \frac{\partial \pi_t}{\partial i_t}|_{informational} = \frac{\partial \pi_t}{\partial E_t^s r_t^n}\frac{\partial E_t^s r_t^n}{\partial i_t} + \frac{\partial \pi_t}{\partial E_t^s u_t}\frac{\partial E_t^s u_t}{\partial i_t}.$$

where the partial derivatives of inflation on the expected natural-rate and the expected cost-push shock are defined in equation (17) as: $\frac{\partial E_t^s r_t^n}{\partial i_t} = (1-\theta)\frac{\kappa}{\sigma}$, $\frac{\partial E_t^s u_t}{\partial i_t} = \frac{1-\theta}{\theta}$.

State and Signals

To study the informational effect of the interest rate, one first needs to specify the (unobserved) state variables and the signals about the state variables. As shown in the IS curve and the Phillips curve, only the aggregate part of the shocks matter in determining the output gap and inflation. In addition, technology shocks and wage markup shocks can be written in terms of natural-rate shocks and cost-push shocks, r_t^n and u_t, respectively.

$$r_t^n = \phi r_{t-1}^n + v_t,$$
$$u_t = \phi_u u_{t-1} + v_t^u,$$

where the natural-rate shock and the cost-push shock are mapped from the technology shock and the wage markup shock as $r_t^n = \frac{\varphi+\sigma}{1+\varphi}\sigma(\phi-1)a_t$, and $u_t = (1-\theta)(1-\beta\theta)u_t^w$.

Denote the auto-coefficients of the natural-rate shock and the cost-push shock as ϕ and ϕ_u. By construction, they are the same as the auto-coefficients of the aggregate technology process and the wage markup process. In this section, I assume that the two shocks are serially uncorrelated.

($\phi = \phi^u = 0$) Denote the standard deviation of the natural-rate shock and the cost-push shock as σ_r and σ_u. By construction, $\sigma_r = \frac{\varphi+\sigma}{1+\varphi}\sigma(\phi-1)\sigma_{va}$, and $\sigma_u = (1-\theta)(1-\beta\theta)\sigma_{vuw}$

I assume that private agents have rational expectations regarding the interest rate response function. Under an arbitrary linear interest rate function which responds linearly to the two aggregate shocks, i.e., $i_t = F_r r_t^n + F_u u_t$, the interest rate becomes *one signal* that simultaneously provides information about *two shocks*.

If there is only one shock to which the interest rate responds linearly, the private sector will be able to perfectly infer the actual shock. In this case, the economy becomes is identical to the perfect information case. [8] In the case with two shocks, when private agents regard the interest rate as a signal about one shock, the prior distribution of the other shock becomes the source of noise in this signal.

Belief Formation

Agents in the private sector are Bayesian, and form best linear forecasts by optimally weighting their prior beliefs (shocks have zero ex-ante mean) and the current signal (the interest rate). Let K_r and K_u denote the optimal weights on the two states after observing interest rate changes, which are determined through the optimal filtering process. Beliefs formed about the two states obtained through the Kalman Filtering process are

$$\begin{bmatrix} E_t^s r_t^n \\ E_t^s u_t \end{bmatrix} = \begin{bmatrix} 1-K_r \\ 1-K_u \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} K_r \\ K_u \end{bmatrix} \hat{i}_t = \begin{bmatrix} K_r F_r & K_r F_u \\ K_u F_r & K_u F_u \end{bmatrix} \begin{bmatrix} r_t \\ u_t \end{bmatrix}, \tag{18}$$

where

$$K_r F_r = \frac{F_r^2 \sigma_r^2}{F_r^2 \sigma_r^2 + F_u^2 \sigma_u^2},$$
$$K_u F_u = \frac{F_u^2 \sigma_u^2}{F_r^2 \sigma_r^2 + F_u^2 \sigma_u^2}.$$

Equation (18) shows that in the solution of the Kalman filtering process with an arbitrary interest rate reaction function, the sensitivity of beliefs to the actual shock is the product of the sensitivity of beliefs to the interest rate (K_r or K_u) and the sensitivity of the interest rate to the actual shocks, (F_r or F_u). The following lemma provides an interpretation of equation (18).

Lemma 1: *Beliefs are more sensitive to the shock (1) to which the interest rate responds more aggressively, and (2) that has higher ex-ante dispersion.*

Lemma 1 describes, for a given ex-ante dispersion of the shocks, how the precision of the

[8] Another way to maintain imperfect information while having only one state variable is to include an implementation error in the interest rate, meaning the interest rate becomes a noisy signal. In Section 6 where I quantitative assess the gains from commitment, I also incorporate implementation error.

interest rate as a signal is determined by the interest rate response function of the two shocks. Private agents in the private sector do not know whether a changes in interest rate responds to the natural rate shock or to the cost push shock. They believe that the interest rate is more likely to respond to the shock to which it is more sensitive. For example, if the interest rate barely responds to cost-push shocks, then after observing a change in the interest rate, agents in the private sector infer that the change in the interest rate is less likely to be a response to a cost-push shock. Otherwise, provided that F_u is very small, the change in the interest rate has to come from a large cost-push shock, which is less likely to realize given the prior distribution of the cost-push shock. However, for any given interest rate reaction function, agents in the private sector update more toward the shock that has higher ex-ante dispersion, as the ex-ante mean of the shock has a smaller weight in belief-formation process.

Notice the difference between the sensitivity of beliefs to actual shocks and the sensitivity of beliefs to the interest rate. I illustrate the difference in the following figure.

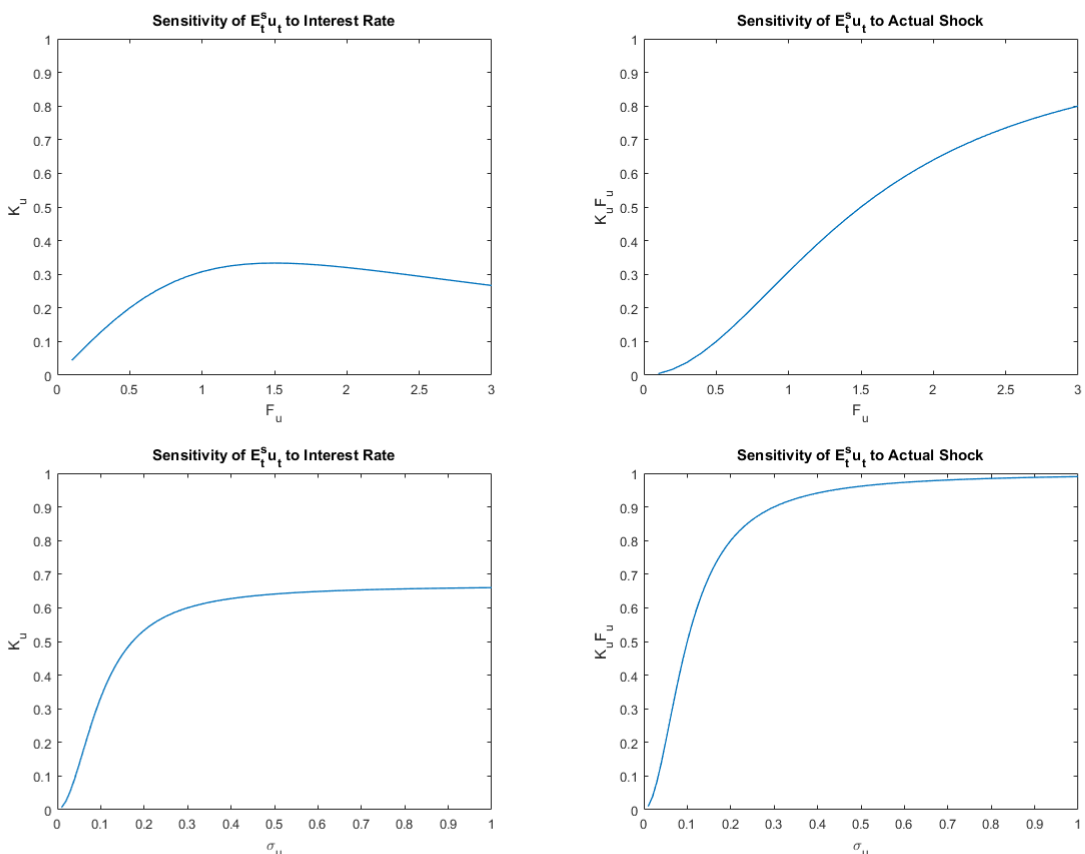

Figure 1: The Informational Effect of Interest Rate (Lemma 1)

In the first row, F_r is fixed at 1.5, and $\sigma_r = \sigma_u = 0.1$. When F_u increases from 0.1 to 3, $K_u F_u$ (right figure) increases monotonically. However, as shown in the left figure, K_u increases first, but then decreases at larger value of F_u. In the second row, I hold $F_r = F_u = 2$, and $\sigma_r = 0.1$. Increasing σ_u monotonically increases both the sensitivity of beliefs to interest rate and the sensitivity of beliefs (left figure) to the actual shock (right figure).

In this figure, I first hold $\sigma_r = \sigma_u = 0.1$, and illustrate the change in the sensitivity of the expected cost-push shock to the interest rate (K_u) and the sensitivity of the expected cost-push shock to the actual shock ($K_r F_r$) while holding F_r fixed at 1.5. Lemma 1 suggests that for a given F_r, the sensitivity of the expected cost-push shock to the actual cost-push shock, $\frac{\partial E_t^s u_t}{\partial u_t}(K_u F_u)$, increases as F_u increases, but it is not necessarily the case for the sensitivity of expected cost push shock to interest rates, $\frac{\partial E_t^s u_t}{\partial i_t}(K_u)$.

When F_u begins to increase from a small value, both the sensitivity of beliefs to interest rate and the sensitivity of beliefs to the actual shock increases. However, as F_u becomes larger, the change in the informational effect is dominated by the interest rate becoming more sensitive to shocks rather than beliefs being more sensitive to interest rate changes. As shown in the left figure in the first row, the sensitivity of $E_t^s u_t$ to the change in i_t decreases at higher level of F_u. Next, in the second row, I fix $F_r = F_u = 2$, and $\sigma_r = 0.1$, and analyze changes in σ_u from 0.01 to 1. Both the sensitivity of beliefs to interest rate and to the actual shock increases.

3.2 Discretionary Monetary Policy

In the previous section, I analyzed the informational effect for a given interest rate rule. Here, I analyze the equilibrium between the private sector and the central bank in which the central bank optimizes in a discretionary way. Specifically, the central bank sets the interest rate to maximize its objective at any given state, taking as given the informational effect of the interest rate. The private sector has rational expectations, in the sense that it perfectly understands the best response function of the interest rate, and extracts information about the current states through the optimal filtering process. Simultaneously, the household chooses consumption and firms optimally set prices.

The optimizing interest rate is an endogenous decision by the central bank, whose objective function consists of equilibrium variables in the private sector. The equilibrium variables in the private sector depend on the beliefs in the private sector, which in turn depend on the equilibrium interest rate reaction function. This introduces circularity into the belief-formation problem. The solution of this problem is discussed by Svensson and Woodford (2003). Following their method, I study the optimizing interest rate in equilibrium by first conjecturing an interest rate reaction function, with which private agents form beliefs. Next, I show how the constraint of the discretionary central bank is affected by the informational effect of the interest rate in equilibrium, and then solve for the optimizing interest rate decision under this constraint. Finally, I solve for a Markov perfect equilibrium such that the response of the interest rate is consistent with the previously conjectured interest rate reaction function.

3.2.1 The Phillips Curve

I begin the analysis of the discretionary monetary policy by discussing the constraint faced by the central bank, which is the Phillips curve. The Phillips curve captures the trade-off between output gap stabilization and inflation stabilization, as the interest rate changes both the output gap and the inflation. With perfect information, the slope of the Phillips curve is exogenous to the interest rate decision. Moreover, with perfect information, the Phillips curve crosses the origin of the $(\hat{y}_t,\ \pi_t)$ plane after a natural-rate shock and has a positive intercept after a cost-push shock.

However, with imperfect information, the Phillips curve depends not only on the realization of actual shocks, but also on the expectations about the shocks. As the expectations about the two shocks are determined by the reaction function of the interest rate, I first guess a linear reaction function of the interest rate: $i_t = F_r r_t^n + F_u u_t$. Then, I substitute the expected shocks under this interest rate reaction function, to solve for the Phillips curve with the informational effect of the interest rate:

$$\pi_t = \kappa\hat{y}_t + \left[(1-\theta)\frac{\kappa}{\sigma}K_r + \frac{1-\theta}{\theta}K_u\right] i_t - (1-\theta)\frac{\kappa}{\sigma}r_t^n + u_t. \tag{19}$$

To express the trade-off between output gap stabilization and inflation stabilization, I substitute interest rate by its relation with output gap from the IS equation, $i_t = -\sigma\hat{y}_t + r_t^n$. This results in the Phillips curve with the informational effect of the interest rate:

$$\pi_t = \left\{\kappa - \sigma\left[(1-\theta)\frac{\kappa}{\sigma}K_r + \frac{1-\theta}{\theta}K_u\right]\right\}\hat{y}_t + \left\{(1-\theta)\frac{\kappa}{\sigma}(K_r-1) + \frac{1-\theta}{\theta}K_u\right\} r_t^n + u, \tag{20}$$

where K_r and K_u are determined through the optimal filtering process in equation (18).

For a given interest rate function that responds positively to the two shocks, I plot the Phillips curve under imperfect information, in comparison with the Phillips curve under perfect information in Figure 2.[9]

The following lemma summarizes the differences between the Phillips curve under perfect information and the Phillips curve under imperfect information.

Lemma 2: *For a given interest rate function that reacts to both shocks in a linear way, the informational effect of the interest rate changes the Phillips curve in three aspects, relative to the Phillips curve under perfect information:*

1. The slope of the Phillips curve is flatter than that under perfect information.

2. The intercept after a cost push shock is reduced.

3. There is non-zero intercept after a natural rate shock.

Proof: see Appendix

[9] see Section 6 or parameter values

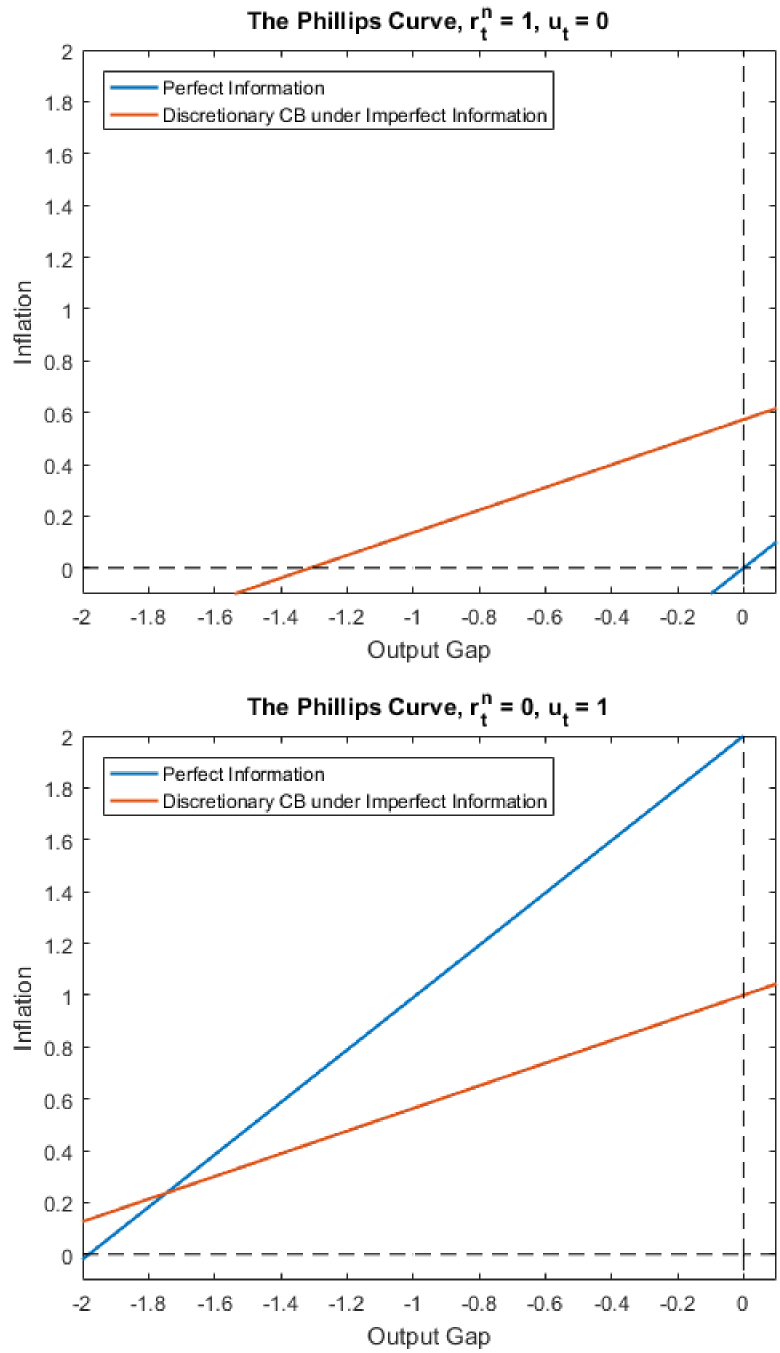

Figure 2: The Phillips Curve with Discretionary Monetary Policy

In the above figures, I plot the Phillips curve under discretionary policy while fixing the interest rate reaction function to be $F_r = 1$ and $F_u = 1$. The prior distribution of the two shocks are set equal, $\sigma_r = \sigma_u = 0.1$.

The intuition follows:

For (1), the slope captures the co-movement between the output gap and inflation due to changes in the interest rate. The informational effect of the interest rate on inflation reduces the co-movement between the output gap and inflation. After a tightening monetary policy, the direct effect of the interest rate reduces the output gap, as the higher nominal interest rate increases the real cost of borrowing. Under perfect information, the direct effect on inflation is given by κ, but under imperfect information, this direct effect is dampened by the informational effect. When observing a higher interest rate, private agents assign a positive possibility to the event that the interest rate is reacting to a positive cost-push shock. This update in the expected cost-push shock leads to an increase in expected inflation. This update of beliefs reduces the direct tightening effect of the interest rate on inflation, which reduces the co-movement between the output gap and inflation.

For (2), the intercept caused by an actual cost-push shock is reduced because information on the actual cost-push shocks is only partially revealed through the interest rate. Note that although both the actual cost-push shock and the expected cost-push shock induce an increase in inflation, the expected cost-push shock does not cause an intercept. This is because when the interest rate does not change, meaning the output gap stays at zero after in absence of natural rate shock, the private agents do not update expected cost-push shock. In fact, the effect of an expected cost push shock is captured in the slope, rather than the intercept, of the Phillips curve.

For (3), after a positive natural-rate shock, the intercept of the Phillips curve represents the equilibrium output gap and inflation when when the interest rate tracks the natural rate one-to-one, $i_t = r_t^n$. Due to the informational effect of the interest rate, the change in the interest rate makes the private agents simultaneously update beliefs about both shocks. Therefore, inflation changes, with the sign depending on the expected cost-push shock and the difference between the expected and the actual natural-rate shock. First, the expected cost-push shocks increase inflation, because each firm believes the aggregate price level increases, when other firms all have higher wage markups. Second, as the private agents underestimate the realization of the natural-rate shock, each firm expects aggregate demand to be less than the actual level. Consequently, the negative difference between the expected and the actual natural-rate shock decreases inflation. The relative size of the two effects determines the sign of the intercept.

3.2.2 Optimal Discretionary Monetary Policy

When shocks are serially uncorrelated and the interest rate does not respond to lagged variables, the current interest rate does not affect the future output gap or inflation. Thus, when choosing the current interest rate, although the central bank is forward-looking, it only considers the effect on current inflation and the output gap when making current interest rate decision. The optimization problem for the discretionary central bank is given by:

$$min_{i_t} L(t) = \begin{bmatrix} \pi_t & \hat{y}_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \pi_t \\ \hat{y}_t \end{bmatrix} + indept.\, terms \tag{21}$$

subject to

$$\hat{y}_t = -\frac{1}{\sigma}(i_t - r_t^n) \tag{22}$$

$$\pi_t = \kappa \hat{y}_t + (1-\theta)\frac{\kappa}{\sigma}(E_t^s r_t^n - r_t^n) + \frac{1-\theta}{\theta} u_t \tag{23}$$

$$E_t^s r_t^n = K_r i_t \tag{24}$$

$$E_t^s u_t = K_u i_t \tag{25}$$

where ω is a constant that results from the second-order approximation of the household's utility.[10]

Definition: *A Markov perfect equilibrium between a discretionary central bank and the private sector with rational expectations can be described in aggregate terms in the following way:*

(i) Inflation and the output gap result from the household's optimal consumption choices and firms' optimal price-setting behavior, which are shown in equations (10) and (13).

(ii) Beliefs in the private sector about the realization of shocks are formed through the Kalman Filtering process as shown in equation (18);

(iii) The interest rate is set by the central bank's constrained optimization problem as specified in (21).

To solve for the equilibrium interest rate, we first need to conjecture an interest rate reaction function, $i_t = F_r^0 r_t^n + F_u^0 u_t$, which determines the Phillips curve. Then, the central bank chooses the interest rate to maximize its objective function under the constraint of the Phillips curve. The equilibrium interest rate under rational expectations is found as the fixed point between the conjectured interest rate function and the optimizing interest rate solution. I analyze the characteristics of the optimizing discretionary interest rate in the rest of this section.[11]

The first-order condition with respect to i_t from equation (21) is given by

$$\pi_t = -\left(\frac{\partial \pi_t}{\partial i_t^*}\right)^{-1} \frac{\partial \hat{y}_t}{\partial i_t^*} \omega \hat{y}_t \equiv = -R\hat{y}_t. \tag{26}$$

Lemma 3: *When shocks are serially uncorrelated, discretionary monetary policy seeks a neg-*

[10] see Woodford (2011) for general derivation of the second-order approximation of the household's utility under perfect information, and Adam (2007) for the application to imperfect information. Appendix D.2 shows the derivation that applies to the specific assumptions in this paper.

[11] A detailed derivation for solving for the equilibrium optimizing interest rate is provided in Appendix B.

ative correlation between the current output gap and inflation after both natural-rate shocks and cost-push shocks. The absolute value of the correlation coefficient is greater than that under full information.

The intuition of this result is as follows: As the intercept is generally not zero after both shocks (Lemma 2), the interest rate increases after positive realizations of both shocks. As the optimizing central bank chooses the tangent point between its indifference curve $L(\hat{y}_t, \pi_t)$ and the Phillips curve, the equilibrium $(\hat{y}_t^*, \hat{\pi}_t^*)$ vector is orthogonal to the Phillips curve. As the Phillips curve has a smaller slope under the informational effect, the resulting vector of $(\pi_t^*, \hat{y}_t^*)$ becomes steeper.

More explicitly, under full information, the absolute value of the correlation between output gap and inflation is

$$R_{perfect\ info} = \left(\frac{\partial \pi_t}{\partial i_t^*}\right)^{-1} \frac{\partial \hat{y}_t}{\partial i_t^*} \omega = \left(-\frac{\kappa}{\sigma}\right)^{-1} \left(-\frac{1}{\sigma}\right) \omega. \tag{27}$$

With information frictions, the marginal effect of the interest rate on inflation is dampened by the informational effect, and thus

$$R_{imperfect\ info} = \left(\frac{\partial \pi_t}{\partial i_t^*}\right)^{-1} \frac{\partial \hat{y}_t}{\partial i_t^*} \omega = \left(-\frac{\kappa}{\sigma} + (1-\theta)\frac{\kappa}{\sigma} K_r + \frac{1-\theta}{\theta} K_u\right)^{-1} \left(-\frac{1}{\sigma}\right) \omega. \tag{28}$$

Under usual parameter values, the interest rate responds positively to both shocks, i.e., $F_r > 0$, $F_u > 0$. Therefore, the Kalman gains are positive, $K_r > 0$, $K_u > 0$, which results in $R_{imperfect\ info} > R_{perfect\ info}$.

We now turn to finding the equilibrium interest rate that achieves the target characterized in Lemma 3.

First, recall that the equilibrium interest rate tracks one-to-one with the change in natural rate, as doing so completely closes the output gap and stabilizes inflation. The optimal response to cost-push shock is "leaning against the wind", which results in $\pi_t = -\frac{\omega}{\kappa} \hat{y}_t$.

Denote the equilibrium interest rate under discretionary central bank and perfect information as $i_t = F_r^p r_t^n + F_u^p u_t$, where

$$F_r^p = 1, \qquad F_u^p = \left(\kappa + \frac{\omega}{\kappa}\right)^{-1} \frac{\sigma}{\theta}.$$

Denote the equilibrium interest rate of discretionary monetary policy under imperfect information as $i_t = F_r^d r_t^n + F_u^d u_t$. The following assumptions help me compare the equilibrium discretionary interest rate under imperfect information and under perfect information.

Assumption 1: $(1-\theta)\frac{\kappa}{\sigma}(K_r(F_r^p,\ F_u^p) - 1) + \frac{1-\theta}{\theta} K_u(F_r^p,\ F_u^p) > 1$, where K_r and K_u denote the Kalman gains from updating beliefs about the expected natural-rate and cost-push shock as specified in equation (18).

Assumption 2: $R^p > \bar{R}$, where R^p represents the absolute value of the correlation between the output gap and inflation, which is given by equation (23).[12]

Lemma 4: *Under Assumptions 1 and 2, the equilibrium discretionary interest rate reacts more aggressively to natural-rate shocks and less aggressively to cost-push shocks under imperfect information, relative to its equilibrium response under perfect information, i.e.,* $F_r^d > F_r^p$ *and* $F_u^d < \frac{1}{\theta} F_u^p$

Proof: see the Appendix

Assumption 1 guarantees that under imperfect information, when the central bank implements $\left(F_r^p,\ F_u^p\right)$, the Phillips curve has a positive intercept after a natural-rate shock. In this situation, as suggested by Lemma 3, the discretionary central bank should increase the interest rate to achieve a negative output gap, which is equivalent to an F_r^d that is greater than F_r^p.

Two factors drive the change in the optimal response to cost-push shocks under imperfect information, as the informational effect changes both the slope and the intercept of the Phillips curve. First, as suggested by Lemma 2, after a cost-push shock, the intercept decreases from $\frac{1}{\theta} u_t$ under perfect information to u_t under imperfect information. Holing the slope constant, this reduction proportionally reduces the decrease in the equilibrium output gap and the equilibrium response of the interest rate. Second, holding the intercept fixed, Assumption 2 dictates that the change in the slope also results in an increase in the equilibrium output gap. Therefore, the two factors result in a smaller response of the interest rate to a cost-push shock, $F_u^d < \frac{1}{\theta} F_u^p$.

3.3 Monetary Policy Rule

In contrast to the problem for a discretionary central bank, which takes as given how beliefs will be formed in the private sector, a committed central bank is able to control beliefs by announcing a monetary policy rule prior to the realization of shocks. After the realization of shocks, the central bank perfectly observes the shocks and implements the interest rate implied by the rule. Private agents observe the interest rate, form beliefs about the realized shocks and simultaneously choose consumption and pricing decisions.

With forward-looking agents, expectations about future equilibrium matter for current consumption and pricing decisions. Consequently, even with serially uncorrelated shocks, a committed central bank may choose a policy rule that responds to past shocks, meaning that the expectations about the direct effect of future interest rates also change the current equilibrium, which potentially leads to gains from commitment. The gains from committing to a delayed response still apply under imperfect information. However, to focus on the within-period gains from the informational effect, I study a state-contingent policy rule that only responds to current shocks.

[12] See the Appendix for specific expression for $\bar{R}$

As the expectation about the interest rate response function determines the beliefs formed about the two shocks, by announcing a state-contingent policy rule, a central bank also chooses a direct mapping from actual shocks to the beliefs about those shocks. In this section, I show that the Phillips curve becomes endogenous to the choice of the policy rule. In addition, I show that there is always a profitable deviation from the optimal policy rule, when the private sector does not update beliefs about the deviation.

3.3.1 Phillips Curve

For a committed central bank, the Phillips curve is no longer an exogenous trade-off between the output gap and inflation. A committed central bank internalizes the fact that its policy-rule decisions will change the marginal informational effect of the interest rate and, consequently, changes the trade-off between inflation and the output gap.

Specifically, the Phillips curve applying to a central bank with commitment that describes the available trade-off between the output gap and inflation is given by

$$\pi_t = \left\{ \kappa - \sigma \left[(1-\theta)\frac{\kappa}{\sigma} K_r^* + \frac{1-\theta}{\theta} K_u^* \right] \right\} \hat{y}_t + \left\{ (1-\theta)\frac{\kappa}{\sigma}(K_r^* - 1) + \frac{1-\theta}{\theta} K_u^* \right\} r_t^n + u, \tag{29}$$

where K_r^* and K_u^* are no longer constant, but endogenously determined by the choice of policy rule, i.e., $K_r^* = K(F_r, F_u)$, $K_u^* = K(F_r, F_u)$ as specified in equation (18).

I plot the Phillips curve for a committed central bank in Figure 3. In the first figure, the blue line illustrates the Phillips curve under perfect information, after a natural-rate shock such that $r_t^n = 1$ and $u_t = 0$. It crosses the origin and has a positive slope of κ. The red curve represents the Phillips curve. Tracing the Phillips curve from a positive output gap to a negative output gap corresponds to an increase in the positive response of the interest rate to the natural-rate shock, which is equivalent to an increasing F_r. Importantly, the effect of the increasing F_r to inflation also depends on the value of F_u, as (F_r, F_u) jointly determines (K_r, K_u). Therefore, I fix $F_u = 1$ to illustrate the effect of the change in F_r. When F_r increases, its marginal effect on the output gap is constant, $-\frac{1}{\sigma}$, but its marginal effect on inflation changes, because the marginal informational effect of the interest rate changes as $(K_r,\ K_u)$ changes with respect to F_r.

In this figure, this change is illustrated by a steeper slope as the output gap decreases. Intuitively, as the private agents expect the interest rate to respond more aggressively to the natural-rate shock, they assign a lower probability to the event that a cost-push shock will be realized. Therefore, the informational effect of the tightening monetary policy leads to a smaller increase in expected inflation, which results in a steeper slope.

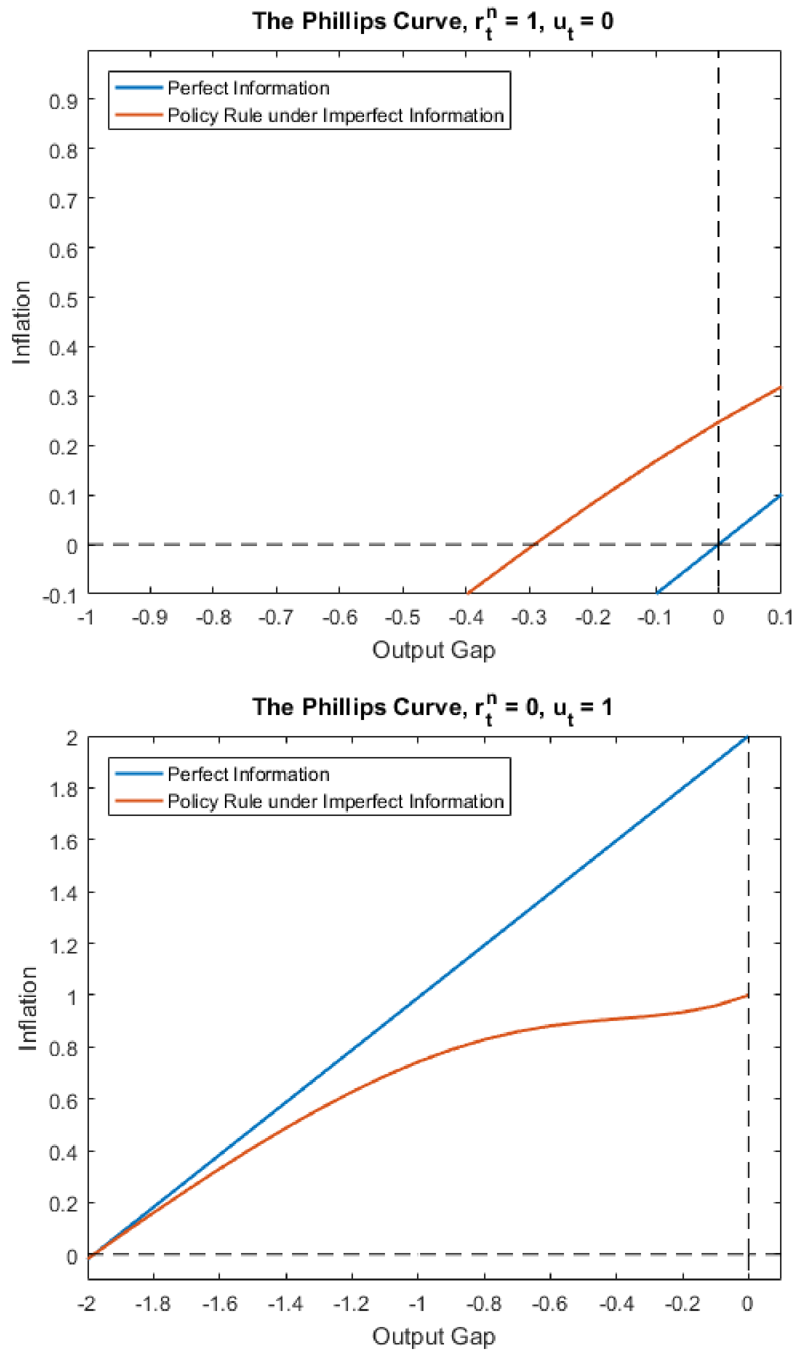

Figure 3: The Phillips Curve with Monetary Policy Rule

In the above figures, I plot the Phillips curve under policy rule. The first figure shows the Phillips curve after a natural-rate shock, where I fixed $F_u = 1$ and vary F_r. The second figure shows the Phillips curve after a cost-push shock, I fixed $F_r = 1$ and vary F_u. Prior distribution of shocks are set equal to each other, such that $\sigma_r = \sigma_u = 0.1$.

In the second figure, the blue line is the Phillips curve under perfect information after a cost-push shock, such that $r_t^n = 0$ and $u_t = 1$. The red curve is the Phillips curve under imperfect information, where I vary the value of F_u while fixing $F_r = 1$. In this figure, the change in the marginal informational effect is more significant than that the first figure. Tracing the Phillips curve from left to right, it represents a decreasing response of the interest rate to a cost-push shock, which is equivalent to a decrease in F_u. As Lemma 1 suggests, a smaller F_u increases K_r, and has a non-monotonic effect on K_u. The combined effect depends on the value of F_u, together with other parameters. When F_u is very small, an increase in its value decreases the output gap, but barely decreases inflation. This is because at this level of F_u, an increase in its value increases both K_r and K_u, (see Figure 1, top row). Therefore, the informational effect almost completely offsets the direct effect, which increases the borrowing cost.

3.3.2 Optimal Policy Rule

The optimal simple rule is found by choosing the interest rate feedback rule $i_t = f\left(r_t^n,\ u_t,\ \pi_t,\ \hat{y}_t\right)$ prior to the realization of shocks, which becomes $i_t = F_r r_t^n + F_u u_t$ in equilibrium. The optimal simple rule is found by choosing F_r and F_u to minimize the central bank's ex-ante loss over the state space:

$$min_{F_r,\ F_u} \int\int \pi_t^2(r_t,\ u_t) + \omega \hat{y}_t^2(r_t,\ u_t) dr_t^n du_t, \tag{30}$$

subject to

$$\hat{y}_t = -\frac{1}{\sigma}\left[(F_r - 1) r_t^n + F_u u_t\right], \tag{31}$$

$$\begin{aligned} \pi_t = & \left\{ -\frac{\kappa}{\sigma}(F_r - 1) + (1-\theta)\frac{\kappa}{\sigma}(K_r F_r - 1) + \frac{1-\theta}{\theta} K_u F_r \right\} r_t^n \\ & + \left\{ -\frac{\kappa}{\sigma} F_u + (1-\theta)\frac{\kappa}{\sigma} K_r F_u + \frac{1-\theta}{\theta} K_u F_u + 1 \right\} u_t \end{aligned} \tag{32}$$

$$E_t^s r_t^n = K_r F_r r_t^n + K_r F_u u_t, \tag{33}$$

$$E_t^s u_t = K_u F_u r_t^n + K_u F_u u_t. \tag{34}$$

Comparing this problem with the problem for a discretionary central bank (equation 21), we find that the available set of combinations of $(\hat{y}_t,\ \pi_t)$ is expanded due to the additional degree of freedom, i.e. instead of choosing interest rate, the central bank with commitment chooses F_r and F_u. By committing to a state-contingent rule that is different from $i_t = F_r^d r_t^n + F_u^d u_t$, the central bank chooses a direct mapping from the actual shocks to the expected shocks. In comparison, even if a discretionary central bank changes its response of interest rate, the private sector still expects that it will follow its equilibrium response, which is described by $\left(F_r^d, F_u^d\right)$, meaning that the informational effect of the interest rate cannot be changed when the central bank does not have credible commitment. In other words, (K_r, K_r) are endogenous choice variables only when the central bank has credible commitment. Otherwise, the central bank regards (K_r, K_r) as exogenous to its interest rate decisions.

To illustrate how the control of the informational effect of the interest rate changes the available trade-off between the output gap and inflation, consider the case in which $r_t = 1$ and $u_t = 0$. Suppose that the central bank commits to being completely inelastic to the cost-push shock, i.e. $F_u = 0$; then the private agents assign probability 1 to the event that the natural-rate is realized when observing a change in the interest rate, i.e., $E_t^s r_t^n = 1$ and $E_t^s u_t = 0$. In this case, information on both shocks is perfectly revealed through the interest rate, and the economy is identical to the perfect informational economy in which the divine coincidence holds. In comparison, without credible commitment, private agents do not believe that if a cost-push shock were to be realized, the central bank would not respond with a positive interest rate. Consequently, the private agents still assign positive probability to the event that a cost-push shock is realized even if the increase in the interest rate is completely due to the realization of the natural-rate shock. The updates in the expected cost-push shock break down the divine coincidence.

However, although such a policy rule is desirable after a natural rate shock, it is not desirable in all states, as not responding to a realized cost push shock is clearly not optimal. This is because when an actual cost-push shock induces positive inflation, if the interest rate does not increase, it results in positive inflation and zero output gap, contradicting Lemma 3. The optimal monetary policy balances between the optimal informational effect and the optimal direct effect across all states.

We now turn to comparing the optimal policy rule with the equilibrium interest rate under discretionary central bank using the first-order conditions. The key factor in this analysis is that the effect of the interest rate after one shock also depends on how it would react to the other shock, because the informational effect, $(K_r,\ K_u)$ is jointly determined by the response of the interest rate to both shocks, $(F_r,\ F_u)$. (F_r, F_u) are jointly determined by the first-order condition on F_r after the r_t^n shock, and the first-order condition on F_u after the u_t shock.

The first-order condition on F_r after the r_t^n shock is

$$\underbrace{-\frac{\kappa}{\sigma}}_{direct} + \underbrace{\Omega_r K_r + \Omega_u K_u}_{informational} + \underbrace{\Omega_r \frac{\partial K_r}{\partial F_r} F_r + \Omega_u \frac{\partial K_u}{\partial F_r} F_r}_{change\ of\ informational} = \frac{\omega}{\sigma} \frac{\hat{y}_t}{\pi_t}, \tag{35}$$

where $\Omega_r = (1-\theta)\frac{\kappa}{\sigma}$, and $\Omega_u = \frac{1-\theta}{\theta} K_u$.

Similarly, the first-order condition on F_u after the u_t shock is

$$\underbrace{-\frac{\kappa}{\sigma}}_{direct} + \underbrace{\Omega_r K_r + \Omega_u K_u}_{informational} + \underbrace{\Omega_r \frac{\partial K_r}{\partial F_r} F_r + \Omega_u \frac{\partial K_u}{\partial F_r} F_r}_{change\ of\ informational} = \frac{\omega}{\sigma} \frac{\hat{y}_t}{\pi_t} \tag{36}$$

In Figure 4, I draw the optimal F_r^* at varying values of F_u which is the solution to equation (35), and the optimal F_u^* for varying values of F_r, which is the solution to equation (36). The point where two lines cross is (F_r^*, F_u^*).

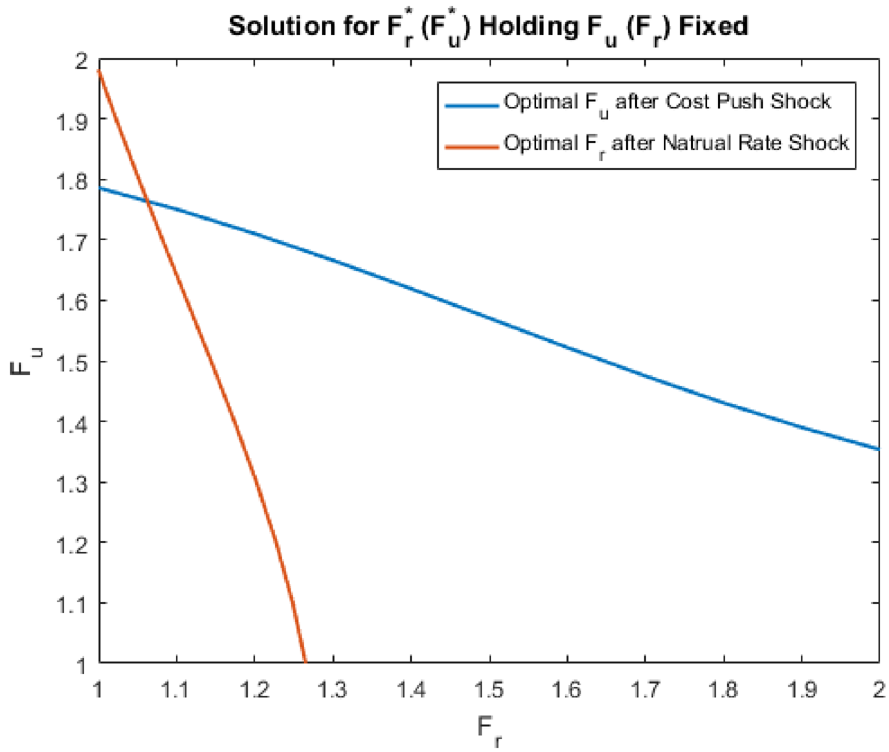

Figure 4: Solution for the Optimal Commitment Rule

In this figure, each point on the red line represents solution of $F_r^*(F_u)$ that satisfies the first order condition on F_r as specified in equation (35), and each point on the blue line represents solution of $F_u^*(F_r)$ that satisfies the first order condition on F_u as specified in equation (36). The point where two line cross defines (F_r^*, F_u^*).

To illustrate the difference between the equilibrium interest rate under the discretionary optimizing policy and under the optimal policy rule, I write the first-order condition on the interest rate for a discretionary central bank in terms of F_r after an r_t^n shock, and F_u after a u_t shock.

$$-\frac{\kappa}{\sigma} + \Omega_r K_r + \Omega_u K_u = \frac{\omega}{\sigma}\frac{\hat{y}_t}{\pi_t} \tag{37}$$

$$-\frac{\kappa}{\sigma} + \Omega_r K_r + \Omega_u K_u = \frac{\omega}{\sigma}\frac{\hat{y}_t}{\pi_t} \tag{38}$$

where $\Omega_r = (1-\theta)\frac{\kappa}{\sigma}$, and $\Omega_u = \frac{1-\theta}{\theta}K_u$.

Comparing the first-order conditions under commitment, the discretionary central bank regards the informational effect of the interest rate as exogenous to its interest rate decisions. Specifically, it does not internalize the change in the Kalman gain with respect to a change in the interest rate.

This is because $(K_r,\ K_u)$ are determined by the private sector's expectations about the interest rate reaction function. Then, if the private agents believe that the central bank will optimize at any given state, they believe the interest rate will follow $i_t = F_r^d r_t^n + F_u^d u_t$.

As suggested by Lemma 1, both the sensitivity of the interest rate to shocks, and the prior distribution of shocks matter for the informational effect of the interest rate. Next, I posit assumptions on the prior distribution of shocks, which help me compare the optimal policy rule with the equilibrium interest rate response under discretionary central bank.

Assumption 3. $\sigma_{r^n} = \sigma_u$.

Proposition 1: *Under Assumptions 1, 2, and 3, i) the optimal policy rule responds more aggressively to a natural-rate shock than the equilibrium response of the interest rate under discretionary policy, for a given response to cost-push shocks, and ii) the optimal policy rule responds less aggressively to a cost-push shock than the equilibrium response of the interest rate under discretionary policy, for a given response to natural-rate shocks.*

Proof: see the Appendix.

Under the above assumptions, increasing the response to natural-rate shocks (F_r) or decreasing the response to cost-push shocks (F_u) from the equilibrium response under discretion $\left(F_r^d,\ F_u^d\right)$, decreases the informational effect of the interest rate. Specifically, it means that (1) $\Omega_r \frac{\partial K_r}{\partial F_r} + \Omega_u \frac{\partial K_u}{\partial F_r} < 0$, and (2) $\Omega_r \frac{\partial K_r}{\partial F_u} + \Omega_u \frac{\partial K_u}{\partial F_u} > 0$,

As the central bank with commitment internalizes the effect of the interest rate decisions on the Phillips curve, it wants to reduce the marginal informational effect of interest rate, making interest rate more "effective" in offsetting the shocks. Assumption (1) guarantees that a higher value of F_r decreases the marginal informational effect of interest rate after natural-rate shocks, and Assumption (2) guarantees that a lower value of F_u decreases the marginal informational effect of the interest rate after cost-push shocks.

The gains from commitment come from (a) the increase in the slope of the Phillips curve after both shocks and (b) the decrease in the intercept after natural-rate shocks. As shown in the Phillips curve expressed in equation (29), (a) and (b) are equivalent, and thus have same implication for the value of F_r^c and F_u^c. Intuitively, more precise information on natural-rate shocks and less precise information on cost-push shocks reduces the conflict between the direct effect and the informational effect of the interest rate.

Another way to investigate the comparison between the optimal policy rule and the equilibrium interest rate under discretionary policy is that as a central bank with commitment internalizes the informational effect of the interest rate, it balances between the optimal informational effect and the optimal direct effect on the borrowing cost.

The optimal informational effect is such that the central bank reveals perfect information about the natural-rate shock, and completely withholds information about the cost-push shock. This is because the natural-rate shock is efficient, as it changes the natural level of output together with the price level. Therefore, the natural-rate shock does not cause a conflict between output gap stabilization and inflation stabilization under perfect information. In comparison, the cost-push shock only changes the price level without chancing the natural level of output. Thus, it is inefficient, and leads to a conflict between output gap stabilization and inflation stabilization. [13] The optimal informational effect of the interest rate can be achieved by either setting $F_r \to \infty$ or setting $F_u = 0$. Balancing the optimal informational effect and the optimal direct effect results in the interest rate being more sensitive to the natural-rate shocks and less sensitive to the cost-push shocks, i.e. $F_r^c > F_r^d$ and $F_u^c < F_u^d$.

3.4 Time Inconsistency

In this section, I analyze the time inconsistency problem which refers to the situation in which the central bank has an incentive to deviate from its previously committed policy rule. The conventional wisdom on the time inconsistency problem applies across time periods. For example, as discussed in Eggertsson et al. (2003), when the current interest rate hits the zero lower bound, the central bank can encourage current consumption by committing to a lower interest rate in future periods such that $E_t\pi_{t+1} > 0$, when consumption decisions are forward-looking. However, the central bank will face a time inconsistency problem at $t+1$, because $\pi_{t+1} > 0$ is sub-optimal. In my baseline model, I have shut down this conventional channel of commitment, so that the time inconsistency across time periods does not apply. Instead, I present a novel time inconsistency problem that applies across states. Specifically, the central bank wants to implement a different interest rate conditional on the realization of shocks, rather than according to its previously announced policy rule.

The intuition for the time inconsistency problem is that a discretionary central bank does not take into account the change of the informational effect of interest rates when deviating from the policy rule. Mathematically, if a central bank has convinced the private sector that it will implement $i_t = F_r^c r_t^n + F_u^c u_t$, the sensitivity of expected shocks to changes in the interest rate is fixed. At this point, the central bank wants to re-optimize its interest rate decisions. The incentives for deviation are summarized in the following proposition.

Proposition 2: *After the central bank has committed to a policy rule, there is always a profitable deviation after either natural-rate shocks or cost-push shocks, as long as the deviation is unexpected and thus the informational effect of the interest rate remains unchanged.*

Proof: see the Appendix.

[13] Existing literature has discussed how information on efficient shocks is beneficial. See Morris and Shin (2002), Angeletos and Pavan (2007), Angeletos, Iovino and La'O (2016) as examples.

The intuition for Proposition 2 is the following. Prior to the realization of shocks, the optimal policy rule has committed to respond more aggressively to natural-rate shocks (Proposition 1), as it optimally weighs between decreasing the combined informational effect of the interest rate and the direct effect on the borrowing cost. If the central bank decides to implement an one-time deviation which is not expected by the private sector, it is able to keep the informational effect fixed and considers only its direct effect. By doing so, the central bank takes the informational advantage such that the private sector believes the shock is more likely to be a natural-rate shock than a cost-push shock, without actually sacrificing a lower output gap when a natural-rate shock is realized. However, if the private sector anticipates such deviation, the private sector will update the sensitivity of its beliefs to changes in the interest rate, leaving no profitable deviation available for the central bank.

I illustrate the incentives for deviation after a natural-rate shock in the following graph.

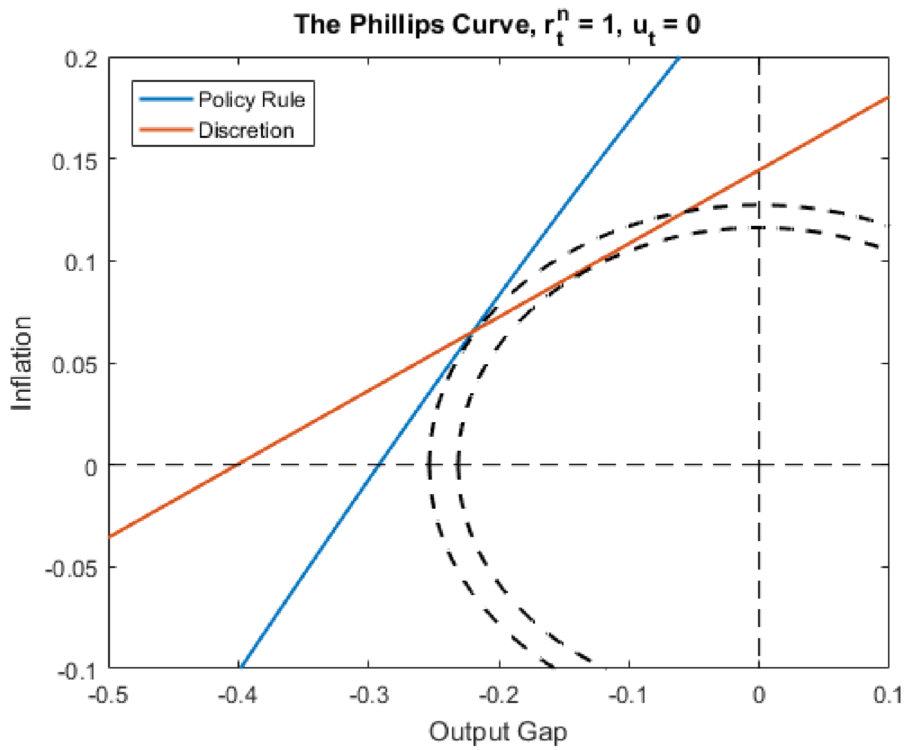

Figure 5: Time Inconsistency Problem

The blue line is the Phillips curve under policy rule, and the red line is the Phillips curve for one-time deviation, assuming K_r and K_u are fixed. The dotted line is the indifference curve of the central bank.

The dotted ellipse is the indifference curve for the central bank whose objective function consists the weighed sum of squared inflation and the squared output gap. As explained above, the Phillips curve under commitment policy is endogenous to the choice of policy rule. To find $F_r^*(F_u)$, the optimal response to a natural-rate shock under commitment, we first need to specify the available trade-off between the output gap and inflation. The blue line represents the available trade-off by varying the value of F_r while holding fixed $F_u = 1$. F_r^* is chosen as the tangent point between the endogenous Phillips curve and the indifference curve.

At this point, if a central bank deviates from its commitment, but such deviation is not anticipated by the private sector, then the central bank faces a different Phillips curve. This is because the informational effect of the interest rate is fixed by the commitment, so that changing F_r will not change the slope of the Phillips curve. Mathematically, it means that the marginal informational effect of the interest rate, $\Omega_r K_r(F_r^c, F_u^c) + \Omega_u K_u(F_r^c, F_u^c)$, will not change as F_r changes, as the private sector expects the central bank to implement $(F_r^c,\ F_u^c)$.

Specifically, after a natural-rate shock, when the central bank deviates to a smaller interest rate response than it had committed to ($F_r < F_r^c$), the Phillips curve has a lower π_t at any level of $\hat{y}_t$. This change in the Phillips curve suggests that by deviating to a smaller interest rate response to a natural-rate shock, the central bank achieves a one-time welfare improvement.

4 External Information

In a more realistic setting, the interest rate is not the only signal that the private sector receives about the aggregate state of the economy. In this section, I discuss central bank direct communication, which is an example of the external information in addition to the informational effect of the interest rate. Unlike the informational effect through the interest rate, which is restricted by the signal dimension, central bank direct communication is not bounded by the signal dimension.

Central bank direct communication can be modeled by providing additional signals to the actual shocks independently , and controlling for the precision of these external signals. The previous literature has discussed the value of central bank communication, and the general consensus is that more precise information about the efficient shocks (natural-rate shocks in this setting) is welfare improving and more precise information about the inefficient shocks (cost-push shocks in this setting) is detrimental.[14] In my model, however, the value of central bank communication interacts with the informational effect through policy rate.

4.1 Interaction between the Informational Effect of Monetary Policy and Central Bank Direct Communication

Denote the external signals sent through the central bank communications as m_t^r and m_t^u, which are distributed log normally around the actual shocks, r_t^n and u_t. I assume that the interest rate does not

[14] see Kramer et al. (2008) for survey of literature on central bank communication.

react to the external signals. However, the existence of external signals changes the informational effect of the interest rate, which consequently changes both the equilibrium interest rate under discretionary central bank and the optimal policy rule.

Signals

The signals consist of both the interest rate and external signals sent through the central bank direct communication, which are summarized as follows:

$$\begin{bmatrix} \hat{i}_t \\ m_t^r \\ m_t^u \end{bmatrix} = \begin{bmatrix} F_1 & F_3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^r \\ \varepsilon_t^u \end{bmatrix} \tag{39}$$

Beliefs

The private sector updates beliefs using both the interest rate and the external signals:

$$\begin{bmatrix} E_t^s r_t^n \\ E_t^s u_t \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \end{bmatrix} \begin{bmatrix} i_t \\ m_t^r \\ m_t^u \end{bmatrix} \tag{40}$$

where the Kalman gains in the K matrix are determined through the optimal filtering process.

The informational effect of the interest rate interact with central bank communication. When the central bank communicates more precisely about one shock, the private sector assigns a greater weight on the information from direct communication versus the information from the interest rate. At the same time, the interest rate becomes a more precise signal of the other shock. For example, suppose that the central bank precisely communicates about the r_t^n shock; then after a positive cost-push shock, the private agents know that $r_t^n = 0$ through the direct communication by the central bank. In addition, the private agents also observe that the interest rate responds positively, so that they infer precisely that the increase in the interest rate is due to the positive realization of a cost-push shock.

However, the existence of the informational effect of policy rates also changes the effect of central bank communication. K_{13} and K_{22} measure how much information is "falsely" updated to beliefs via external signals. Without an informational effect transmitted through the policy rate, K_{13} and K_{22} would be equal to zero, as signals are distributed independently, and the signal of one shock does not provide information about the other shock. However, as the interest rate is one signal about the two shocks, the interaction with the informational effect of the interest rate makes the central bank unable to separately convey information. Specifically, both K_{13} and K_{22} are negative. Intuitively, suppose that the interest rate does no change and that external signals on natural rate goes up; in this case, the private sector would then back out a negative change in the cost-push shock.

K_{12} and K_{23} measure how much information is "correctly" updated through external signals. In Figure 6, I plot how the sensitivity of beliefs to each signal changes when the interest rate reacts more aggressively to a natural-rate shock, while holding its response to a cost-push shock fixed (varying F_r from 0.1 to 2 while fixing $F_u = 1$).

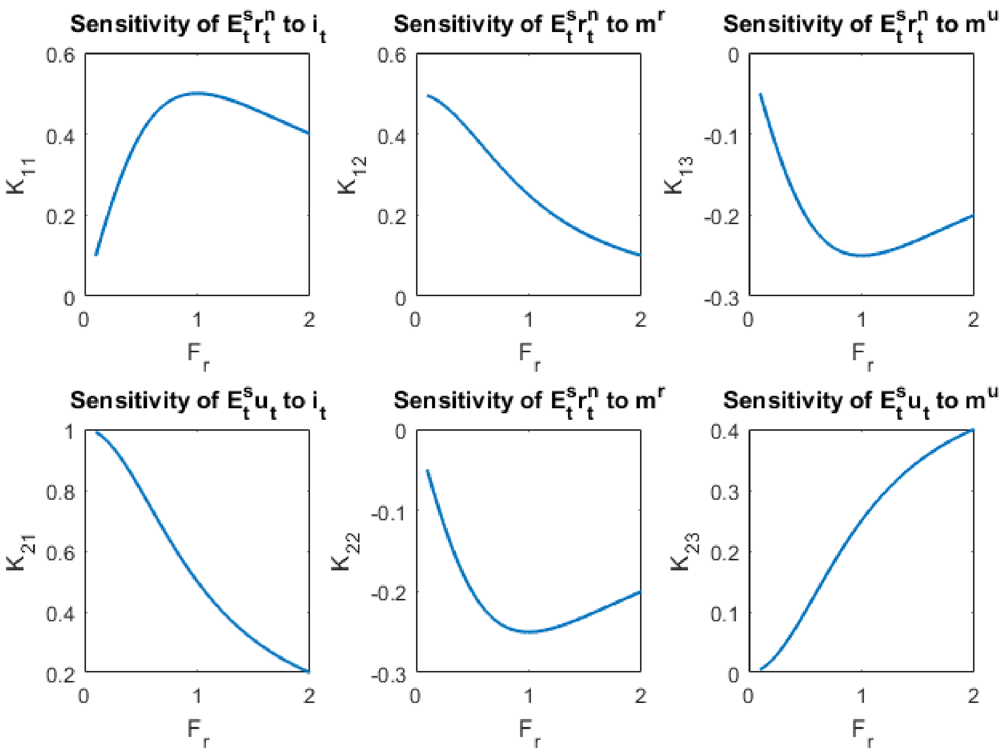

Figure 6: Sensitivity of Beliefs to Different Signals

In all 6 figures, I set $F_u = 1$, and vary F_r from 0.1 to 2. Ex-ante dispersion of shocks are set to be equal with each other, such that $\sigma_r = \sigma_u = 0.1$. The top row is the sensitivity of beliefs about natural-rate shocks with respect to all three signals, the interest rate, the external information about the natural-rate shock, and the external information about the cost-push shock. The second row is the sensitivity of beliefs about cost-push shocks with respect to all three signals, the interest rate, the external information about the natural-rate shock, and the external information about the cost-push shock.

As illustrated in the first row, when the sensitivity of $E_t^s r_t^n$ to the change in the interest rate is not monotonic, as suggested by Lemma 1. The second figure in the first row shows the weight on m_t^r decreases as the interest rate becomes a more precise signal about r_t^n. The second row shows that increasing the sensitivity of interest rate to the natural-rate shock also decreases the sensitivity of $E_t^s u_t$ to interest rate changes, and increases the sensitivity of $E_t^s u_t$ to central bank communication about the cost-push shock. Intuitively, since the private agents optimally weight these three signals, when the interest rate reacts more aggressively to the natural-rate shock, it becomes a more precise signal than m_t^r. At the same time, it becomes a less precise signal than m_t^u. For this reason, K_{12} decreases and K_{23} increases.

4.2 Value of (External) Information

To assess the value of external information through the direct communication from the central bank, we first need to study how the optimal response of the interest rate changes under discretion and with commitment, as the central bank takes into account the interaction between the informational effect of the interest rate and the direct communication. The Phillips curve with all signals can be obtained as follows:

$$\begin{aligned}\pi_t = & \left\{\kappa - \sigma\left[(1-\theta)\frac{\kappa}{\sigma}K_{11} + \frac{1-\theta}{\theta}K_{21}\right]\right\}\hat{y}_t \\ & + \left\{(1-\theta)\frac{\kappa}{\sigma}(K_{11}+K_{12}-1) + \frac{1-\theta}{\theta}(K_{21}+K_{22})\right\}r_t^n + \left\{(1-\theta)\frac{\kappa}{\sigma}K_{13} + \frac{1-\theta}{\theta}K_{23}+1\right\}u_t\end{aligned} \tag{41}$$

The existence of the informational effect of the interest rate complicates the welfare effect of central bank communication. Without the information effect of interest rate, welfare is maximized when the central bank provides perfectly precise signal about the efficient shock (the natural-rate shock), and completely uninformative signal about the inefficient shock (the cost-push shock). However, with the information effect of the interest rate, if agents in the private agents have precise information about one shock, they are able to infer precise information about the other shock from the interest rate.

In Figure 1, I illustrate the welfare implications of direct communication of the central bank. In the first row of Figure 7, I show the contour plot at varying levels of precision of central bank communication under optimizing discretionary policy (left) and under optimal policy rule (right). It shows that when communication about the cost-push shock becomes more imprecise, which is modeled by a lower σ_{eu}, the ex-ante loss increases. This is consistent with the conventional wisdom that more precise information about the inefficient shock is welfare reducing. However, when the precision of central bank communication about natural-rate shocks increases, which is modeled by a smaller σ_{er}, the ex-ante loss also increases. This contradicts the conventional wisdom. [15] In summary, when the interest rate is able to provide sufficiently precise information about the efficient shock, additional direct communication about either shocks reduces ex-ante welfare.

[15] For the conventional wisdom on the value of information, see Morris and Shin (2002), Angeletos and Pavan (2007), Angeletos, Iovino and La'O (2016), for examples

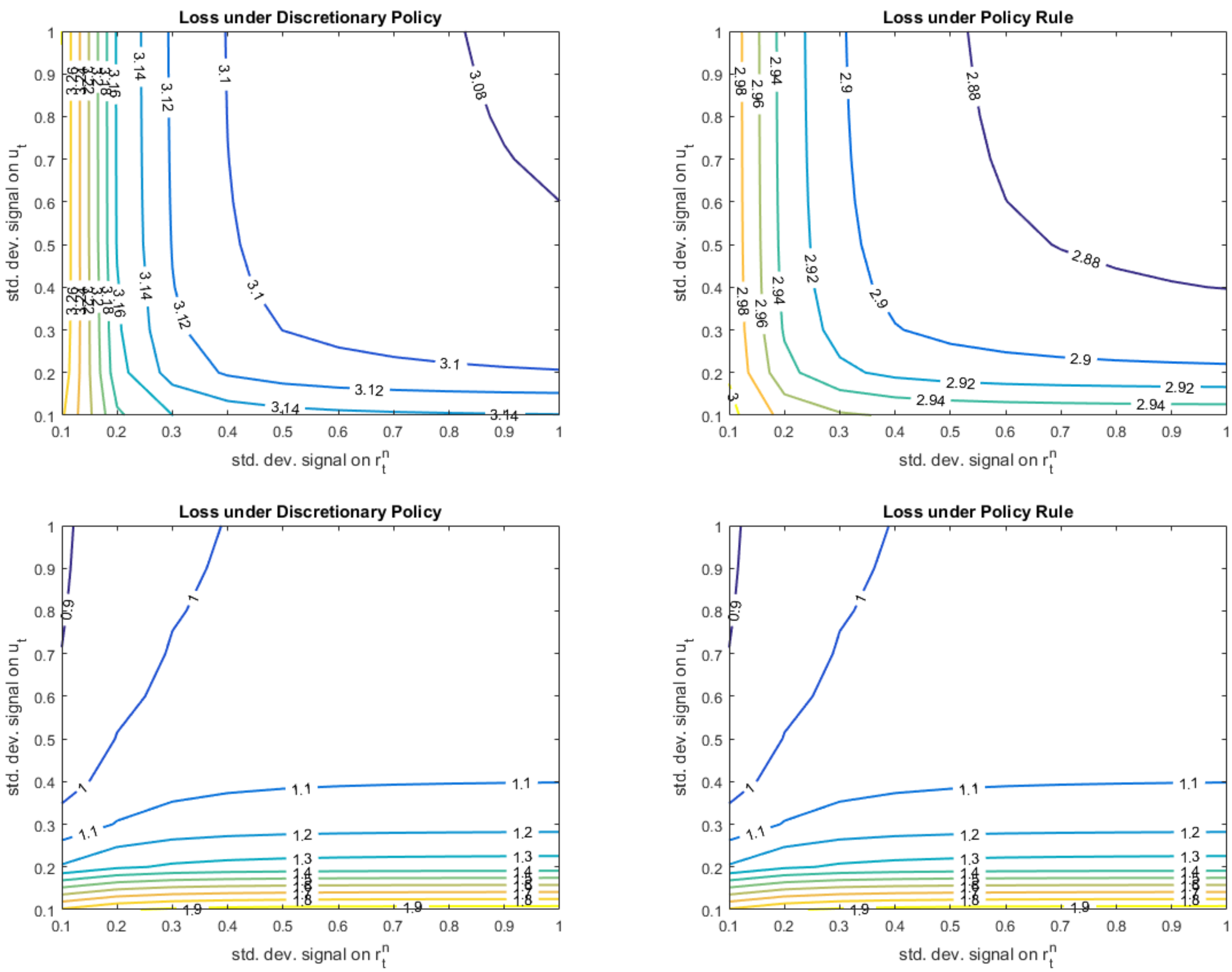

Figure 7: The Value of (External) Information

Next, I add an implementation error to the interest rate function, such that $i_t = F_r r_t^n + F_u u_t + e_t$. The interest rate becomes a noisier signal of both shocks when the variance of the implementation error increases. I show the contour plot at varying levels of precision of central bank communication, assuming the implementation error of the interest rate has a standard deviation of 0.5. Since interest rate becomes a relatively imprecise signal now, the value of (external) information becomes the same as the conventional wisdom. The loss is minimized when information on natural-rate shock is most precise and information on the cost-push shock is least precise.

5 Dynamic Informational Effect

I extend the analysis to the dynamic informational effect of the interest rate by introducing serially correlated shocks. Since the consumption and pricing decisions are both forward-looking, the expectations about the future states matter for current output gap and inflation. When shocks have serial correlation, current interest rates also affect expectations about future shocks, which leads to the dynamic informational effect of interest rates.

5.1 States, Beliefs and Equilibrium in Private Sector

To analyze the direct informational effect of interest rates, I first study the dynamic learning process in the private sector.

State

Natural-rate shocks and cost push-shocks follow an AR(1) process:

$$\begin{bmatrix} r_t^n \\ u_t \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix} \begin{bmatrix} r_{t-1}^n \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_t^r \\ v_t^u \end{bmatrix}. \tag{42}$$

Signals

The information set of the private sector includes the values of all parameters and the entire history of interest rates upon t. I first conjecture and then show that the equilibrium interest rate becomes a function of the state variables in period t, which includes both the actual shocks at time t and beliefs in period $t-t$.

$$i_t = F_1 r_t^n + F_2 E_{t-1}^s r_{t-1}^n + F_3 u_t + F_4 E_{t-1}^s u_{t-1}. \tag{43}$$

The inertial components in the equilibrium interest rate comes from the persistent belief updating process. As the private sector optimally weights the signals in the current period and the beliefs in the last period to form current expectations, the current output gap and inflation become functions of past beliefs. Therefore, when a discretionary central bank sets the current interest rate to minimize deviations of the current output gap and inflation, the interest rate in equilibrium also reacts to beliefs in the past period.

As the private agents have perfect memory of their beliefs in the past, they are able to distinguish the fraction of the interest rate that reacts to current shocks from the fraction of the interest rate that reacts to past beliefs. Let $\hat{i}_t$ denote the fraction of i_t that reacts to current shocks, which follows:

$$\hat{i}_t \equiv i_t - F_3 E_{t-1}^s r_{t-1}^n - F_4 E_{t-1}^s u_{t-1} = F_1 r_t^n + F_3 u_t. \tag{44}$$

Belief Formation

The private sector forms expectations about current states through the Kalman filtering process. Denote the hidden state variables as

$$z_t = \Phi z_{t-1} + v_t \tag{45}$$

where $z_t = [r_t^n,\ u_t]'$, $\Phi = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix}$, and $v_t = [v_t^r,\ v_t^u]'$ with white noise of variance Q.

Denote the observable signal as

$$s_t = Dz_t \tag{46}$$

where $s_t = i_t$, and $D = [F_1, F_3]'$.

The Kalman filtering process makes beliefs about the current state variables be the optimal combination of beliefs in the last period and signals in the current period:

$$E_t^s z_t = \Phi E_{t-1}^s z_{t-1}^n + K\left(s_t - D\Phi E_{t-1}^s z_{t-1}\right) \tag{47}$$

where the optimal weight, K, is determined by Ricatti iteration

$$K = PD'(DPD')^{-1}, \tag{48}$$

$$P = \Phi\left(P - PD'(DPD')^{-1}DP\right)\Phi + Q. \tag{49}$$

Solution in the Private Sector under Arbitrary Policy Coefficients

The equilibrium in the private sector is described by the system of equations summarizing private sector optimization decisions in aggregate variables (equations 10 and 12), shock evolution (equation 32), the interest rate reaction function (equation 33), and belief updating process characterized in equation (37).

Since the equilibrium involves forward-looking variables, I solve for it by the undetermined coefficients method. I first conjecture that $\hat{y}_t$ and π_t are linear functions of the state variables in period t, that is, $\left[r_t^n,\ u_t,\ E_{t-1}^s r_{t-1}^n,\ E_{t-1}^s u_{t-1}\right]$

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \\ \gamma_5 & \gamma_6 & \gamma_7 & \gamma_8 \end{bmatrix} \begin{bmatrix} r_t^n \\ E_{t-1}^s r_{t-1}^n \\ u_t \\ E_{t-1}^s u_{t-1} \end{bmatrix} \tag{50}$$

This conjecture allows for the expression of expected future equilibrium variables in terms of the beliefs about current shocks, $E^s_t r^n_t$ and $E^s_t u_t$:

$$\begin{bmatrix} E_t \hat{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \gamma_1 \phi + \gamma_2 & \gamma_3 \phi^u + \gamma_4 \\ \gamma_5 \phi + \gamma_6 & \gamma_7 \phi^u + \gamma_8 \end{bmatrix} \begin{bmatrix} E^s_t r^n_t \\ E^s_t u_t \end{bmatrix} \tag{51}$$

Substituting these into the the IS and the Phillips curve results in expressions of $\hat{y}_t$ and π_t as functions of the actual shocks $[r^n_t,\ u_t]$ and beliefs $[E^s_t r^n_t,\ E^s_t u_t]$. Applying the belief-updating process yields the expressions as functions that consist only of predetermined states. (See Appendix B for the detailed derivation.)

5.2 Discretionary Monetary Policy

A discretionary central bank minimizes the expected output gap and inflation deviations in all periods. The central bank's optimization problem can be written as follows:

$$E_t L(t) = E_t[\pi_t^2 + \omega \hat{y}_t^2] + \beta E_t(L(t+1)) \tag{52}$$

where the output gap follows equation (10), inflation follows equation (12), the actual shocks evolve following equation (35), and beliefs are formed using Kalman filtering process specified in equations (37 - 39).

E_t denotes the *objective* expectation. The information set of the central bank at t includes the entire history of natural-rate and cost-push shocks upon t and the beliefs formed in the private sector upon $t-1$, i.e.,

$$I_t = \left\{ r^n_T,\ E^s_{T-1} r^n_{T-1}\ u_T,\ E^s_{T-1} u_{T-1} \quad \forall T = 0...t \right\}$$

$E_t(L(t+1))$ includes the deviations of equilibrium inflation and the output gap in all future periods:

$$\begin{aligned} E_t\left(L(t_1)\right) &= \Sigma^{\infty}_{j=1} \beta^j E_t \left\{ \begin{bmatrix} \pi_{t+1} & \hat{y}_{t+j} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \pi_{t+j} \\ \hat{y}_{t+j} \end{bmatrix} \right\} \\ &= \Sigma^{\infty}_{j=1} \beta^j \left\{ \begin{bmatrix} E_t \pi_{t+1} & E_t \hat{y}_{t+j} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} E_t \pi_{t+j} \\ E_t \hat{y}_{t+j} \end{bmatrix} + indept.\ terms \right\} \end{aligned} \tag{53}$$

When there is serial correlation in shocks, the interest rate has a dynamic informational effect due to the persistent learning process in the private sector. Consequently, this dynamic informational effect changes the objective function of a discretionary central bank. [16]

[16] As long as there are shocks that the central bank is unable to completely offset, optimal policy can be described

Lemma 5 *With dynamic informational effect, the optimizing discretionary monetary policy is dynamically "leaning against the wind" as it targets a negative correlation between current and future deviations of the output gap and inflation.*

This can be shown as the first-order condition of the central bank's objective function:

$$\left\{\frac{\partial E_t\pi_t}{\partial i_t^*}E_t\pi_t+\omega\frac{\partial E_t\hat{y}_t}{\partial i_t^*}E_t\hat{y}_t\right\}=-\frac{1}{2}\Sigma_{j=1}^{\infty}\beta^j\left\{\frac{\partial E_t\pi_{t+j}}{\partial i_t^*}E_t\pi_{t+j}+\omega\frac{\partial E_t\hat{y}_{t+j}}{\partial i_t^*}E_t\hat{y}_{t+j}\right\} \tag{54}$$

To see that the right-hand side is non-zero, we need to first specify how future equilibrium is affected by current beliefs, and how the current interest rate affects current beliefs. Denote the predetermined state variables at t as: $z_t=\left[r_t^n,\ E_{t-1}^s r_{t-1}^n,\ u_t, E_{t-1}^s u_{t-1}\right]$. Due to the projected linear relationship, the objective expectation of the inflation and the output gap in j periods ahead becomes:

$$\begin{bmatrix}E_t\pi_{t+j}\\ E_t\hat{y}_{t+j}\end{bmatrix}=\begin{bmatrix}\gamma_1 & \gamma_2 & \gamma_3 & \gamma_4\\ \gamma_5 & \gamma_6 & \gamma_7 & \gamma_8\end{bmatrix}E_t z_{t+j}\equiv\Gamma E_t z_{t+j} \tag{55}$$

As long as shocks cannot be completely offset by the interest rate, Γ is non-zero.

The evolution of $E_t z_{t+j}$ includes the auto-correlated actual shocks, and the dynamic process of belief formation. The belief formation yields:

$$E_t^s r_t^n=K_{11}F_1 r_t^n+\phi\left(1-K_{11}F_1\right)E_{t-1}^s r_{t-1}^n+K_{11}F_3 u_t-\phi^u K_{11}F_3 E_{t-1}^s u_{t-1} \tag{56}$$

$$E_t^s u_t=K_{21}F_1 r_t^n-\phi K_{21}F_1 E_{t-1}^s r_{t-1}^n+K_{21}F_3 u_t+\phi^u\left(1-K_{21}F_3\right)E_{t-1}^s u_{t-1} \tag{57}$$

Thus, the evolution of $E_t z_{t+j}$ can be summarized as

$$\begin{bmatrix}E_t r_{t+j}^n\\ E_t E_{t+j-1}^s r_{t+j-1}^n\\ E_t u_{t+j}\\ E_t E_{t+j-1}^s u_{t+j-1}\end{bmatrix}=\begin{bmatrix}\phi & 0 & 0 & 0\\ K_{11}F_1 & \phi-K_{11}F_1\phi & K_{11}F_3 & -K_{11}F_3\phi^u\\ 0 & 0 & \phi^u & 0\\ K_{21}F_1 & -K_{21}F_1\phi & K_{21}F_3 & \phi^u-K_{21}F_3\phi^u\end{bmatrix}\begin{bmatrix}E_t r_{t+j-1}^n\\ E_t E_{t+j-2}^s r_{t+j-2}^n\\ E_t u_{t+j-1}\\ E_t E_{t+j-2}^s u_{t+j-2}\end{bmatrix}\equiv\Lambda E_t z_{t+j-1} \tag{58}$$

Combine equations (45) and (48) to express the future equilibrium in terms of current beliefs as follows:

$$\begin{bmatrix}E_t\pi_{t+j}\\ E_t\hat{y}_{t+j}\end{bmatrix}=\Gamma\Lambda^{j-1}E_t z_t \tag{59}$$

Substituting this expression into the central bank's objective function transforms the objective function into a weighted sum of current inflation, the current output gap and the persistent state

as "leaning against the wind" - seeking a contemporary negative correlation between the output gap and inflation. For discussion about the conventional within-period "leaning against" policy that is caused by informational frictions, see Adam (2005), Angeletos and La'O (2013), and Tang (2015), among others.

variables which include the current actual shocks and current beliefs. The first-order condition on i_t^* results in:

$$\left\{\frac{\partial E_t\pi_t}{\partial i_t^*}E_t\pi_t+\omega\frac{\partial E_t\hat{y}_t}{\partial i_t^*}E_t\hat{y}_t\right\}+\frac{1}{2}\Sigma_{j=1}^{\infty}\beta^j\Delta(j-1)=0 \tag{60}$$

where Δ captures how the current interest rate affects future deviations through its informational effect on $[E_t^s r_t^n,\ E_t^s u_t]'$. (See the Appendix for the derivations.)

Proposition 3: *With serially correlated shocks, the interest rate in the current period affects future the equilibrium through the dynamic informational effect. The consideration of the dynamic informational effect makes the equilibrium interest rate target beliefs in addition to targeting the current inflation and the output gap.*

The consideration of the dynamic informational effect consists of two parts. The first part is captured by the effect on current equilibrium, because both consumption and pricing decisions are forward-looking. The second part is due to the persistence in the learning process, which is in addition to stabilizing the current economy. This additional beliefs-targeting does not exist with serially uncorrelated shocks.

The effects of discretionary policy on future variables are different from the effects on the actual future variables, as the private sector cannot distinguish the actual shocks from the beliefs.

To see this, use the output gap as an example. First, express the future output gap as the actual shocks, the expected shocks and the interest rate.

$$\hat{y}_{t+1}=\Xi(1)E_{t+1}^s r_{t+1}^n+\Xi(2)E_t^s u_{t+1}+\frac{1}{\sigma}\frac{1}{1-\phi}r_{t+1}^n-\frac{1}{\sigma}i_{t+1} \tag{61}$$

Next, express the expected shocks as the beliefs formed with weights assigned on both past beliefs and signals in this period. [17]

$$E_{t+1}^s r_{t+1}^n=\Lambda_1 E_t^s r_t^n+\Lambda_2 E_t^s u_t+K_{11}i_{t+1}$$
$$E_{t+1}^s u_{t+1}=\Lambda_3 E_t^s r_t^n+\Lambda_4 E_t^s u_t+K_{21}i_{t+1}$$

The marginal effect of the interest rate on $\hat{y}_{t+1}$ can then be expressed as the the combination of the informational effects on $E_t^s r_t^n$ and $E_t^s u_t$.

$$\frac{\partial\hat{y}_{t+1}}{\partial i_t}=\Xi(1)\left(\Lambda_1\frac{\partial E_t^s r_t^n}{\partial i_t}+\Lambda_2\frac{\partial E_t^s u_t}{\partial i_t}\right)+\Xi(2)\left(\Lambda_1\frac{\partial E_t^s r_t^n}{\partial i_t}+\Lambda_2\frac{\partial E_t^s u_t}{\partial i_t}\right) \tag{62}$$

However, the effect of interest rate on the expected future output gap has an additional term, as the private sector is not able to separate the beliefs from the actual r_{t+1}. The effect of discretionary

[17] See Appendix for expressions of Ξ and Λ

policy on the expected output gap is

$$\frac{\partial E_t^s \hat{y}_{t+1}}{\partial i_t} = \left(\Xi(1) + \frac{1}{\sigma}\frac{1}{1-\phi}\phi\right)\left(\Lambda_1 \frac{\partial E_t^s r_t^n}{\partial i_t} + \Lambda_2 \frac{\partial E_t^s u_t}{\partial i_t}\right) + \Xi(2)\left(\Lambda_1 \frac{\partial E_t^s r_t^n}{\partial i_t} + \Lambda_2 \frac{\partial E_t^s u_t}{\partial i_t}\right) \quad (63)$$

In addition, the effect of discretionary policy on future variables should be distinguished from the effect of committing to a future interest rate, as the former consists of the informational effect on the current beliefs, and the latter consists only of the direct effect on future borrowing costs. Both of the effects are able to influence the current equilibrium when private agents are forward-looking. The marginal effect of an increase in the future interest rate on $\hat{y}_{t+1}$ is:

$$\frac{\partial \hat{y}_{t+1}}{\partial i_{t+1}} = -\frac{1}{\sigma}. \quad (64)$$

To solve for the equilibrium interest rate under discretion, I first propose that the interest rate follows a linear function, $i_t = F_1 r_t^n + F_2 E_{t-1}^s r_{t-1}^n + F_3 u_t + F_4 E_{t-1} u_{t-1}$, with which the private sector updates beliefs on $E_t^s r_t^n$ and $E_t^s u_t$. The central bank then chooses the interest rate to minimize the loss function specified in equation (45). If the optimizing interest rate is different from the proposed one, the private sector then changes its beliefs about the interest rate reaction function. The optimal interest rate is found as the fixed-point solution in this iteration process. Details of this solution method are provided in Appendix B.

The persistence in underlying shocks strengthens the informational effect of interest rate, because it increases the effect of expected future deviations on current consumption and pricing decisions. If the serial correlation is high enough, it may cause optimal discretionary interest rate to fail to exist. The intuition is the following. Suppose that the private sector believes the best response of central bank is to increase the interest rate to the two shocks. If cost push shock is realized to be positive, which makes the inflation positively deviate from steady-state, the nominal effect of the interest rate decreases inflation and the informational effect of the interest rate increases inflation. If the informational effect dominates the direct effect, the inflation increases even further. As a result, a discretionary central bank wants to choose a negative interest rate, which contradicts the beliefs in the private sector that the best response of interest rate is to react positively to the two shocks.

5.3 Monetary Policy Rule

The objective function for the committed central bank is the same as the discretionary central bank. I require that the committed central bank can only commit to a rule which responds linearly to current state variables. Notice that as past beliefs determine current beliefs, they also become current state variables. In equilibrium, the optimal rule follows same functional form as the discretionary interest rate, i.e., $i_t = F_1 r_t^n + F_2 E_{t-1}^s r_{t-1}^n + F_3 u_t + F_4 E_{t-1}^s u_{t-1}$. The coefficients of the optimal rule, $[F_1,\ F_2,\ F_3,\ F_4]$ are selected to minimize the ex-ante loss from the steady state. [18]

$$min_{F_1,\ F_2,\ F_3,\ F_4} E_t L(t) = \int\int \left(\pi_t^2 + \omega \hat{y}_t^2 + \beta E_t L(t+1)\right) dr_t^n du_t \tag{65}$$

where output gap follows equation (10), inflation follows equation (12), actual shocks evolve as equation (34), and beliefs are formed using Kalman filtering process as specified in equation (39 - 41).

In contrast to the serially uncorrelated case, in which I completely shut down the gains from commitment through delayed response, I allow for such gains in the dynamic case. Potentially, the policy rule can react to current cost-push shocks by a lesser extent and commits to a large response to past beliefs than a discretionary interest rate does. In doing so, not only does interest rate reveal less information about the cost-push shock, it also decreases expected inflation. The gains from committing to a delayed response strengthen the gains from the informational effect.

6 Quantitative Assessment

The goal of this section is to quantify the size of the gains from commitment using a dynamic model with varying degrees of information precision. I begin with the case where there are no external signals. In this case, the information precision depends on the prior distribution of the actual shocks. I then consider the case where there are external signals, and in addition, I allow for an implementation error in the interest rate. By varying the precision of signals, I quantitatively assess how the gains from commitment depend on the precision of external information.

6.1 No External Information

In the baseline model, I assume that there are no external signals, and calibrate the model parameters in line with the convention in the macroeconomics literature. As noted in Section 2, I set $\varphi = 1$ and $\sigma = 1$, assuming a unitary Frisch elasticity of labor supply and log utility of consumption. I use $\beta = 0.99$, which implies a steady state real return on financial assets of four percent. For price rigidity, I calibrate θ, the price stickiness parameter, to be 0.5, which is indicated by the average price duration from macro and micro empirical evidences. [19] For the parameter that governs the elasticity of substitution between intermediate goods, I set $\varepsilon = 4$, which implies a steady state price markup of one-third of revenue.

[18] In steady state, $E_{t-1}^s r_{t-1}^n = 0$, and $E_{t-1}^s u_{t-1} = 0$.

[19] Sources:Bils and Klenow (2004), Galı and Gertler (1999), Nakamura and Steinsson (2010)

For the evolution of underlying shocks, I set the auto-correlation of natural-rate shocks to be 0.9, with a standard deviation of 3 percent, as measured by Laubach and Williams (2003). There is less consensus in the persistence and volatility of cost-push shocks, as they stems from a various sources. I set the auto-correlation for cost-push shocks to be 0.3 to avoid informational effect of interest rate being so strong that kills the equilibrium of an optimizing discretionary interest rate. I set the standard deviation of cost-push shocks to be the same as that of natural-rate shocks. In addition, I set the standard deviation of policy implementation error to be the same as the standard deviation of natural rate shock. I assume that there are no external signals apart from the interest rate. A summary of parameter values in the baseline calibration is provided in the Appendix.

I numerically solve for both the equilibrium interest rate under discretion and the optimal policy rule under the baseline dynamic model, which yields the following results:

$$\begin{aligned} i_{discretionary} &= 1.3887 r_t^n - 0.3498 E_{t-1}^s r_{t-1}^n + 0.1852 u_t + 0.3370 E_{t-1}^s u_{t-1} \\ i_{rule} &= 1.3727 r_t^n - 0.3374 E_{t-1}^s r_{t-1} + 0.1830 u_t + 0.3332 E_{t-1}^s u_{t-1} \end{aligned}$$

Regarding the equilibrium interest rate under discretion, the novelties of the dynamic case are the value of F_2 and F_4. They capture how the interest rate optimally responds to the beliefs in the past period. As beliefs are persistent, reacting to past beliefs leads to inertia in the interest rate. As analyzed in the belief-formation process, this response has no informational effect, as the private sector is able to subtract the part of $F_2 E_{t-1}^s r_{t-1}^n + F_4 E_{t-1}^s u_{t-1}$ to obtain signals in current period. Specifically, a negative F_2 means central bank counteracts the excess response to the natural-rate shock in the first period. A positive F_4 means that the central bank makes up for the deficient response to the cost-push shock in the first period. The intuition can be found in the output gap equation and inflation equation, which show that an expected natural rate decreases output gap and expected cost push shock increases inflation. As past beliefs contribute positively to current beliefs, the interest rate optimally responds to past beliefs to offset their contribution to current deviations.

Compared with the optimal response of a discretionary central bank, the response coefficients in the optimal policy rule do not differ substantially. This suggests that the welfare gains from commitment is not significant. I calculate the ex-ante welfare loss as the loss function of the central bank, which is 0.0325 for the discretionary case and 0.0324 for the commitment case. In Figure 8, I plot the impulse response after a natural-rate shock, a cost-push shock and a policy error, which also show that the difference in equilibrium under discretionary policy and under policy rule is negligible when the interest rate is the only source of information.

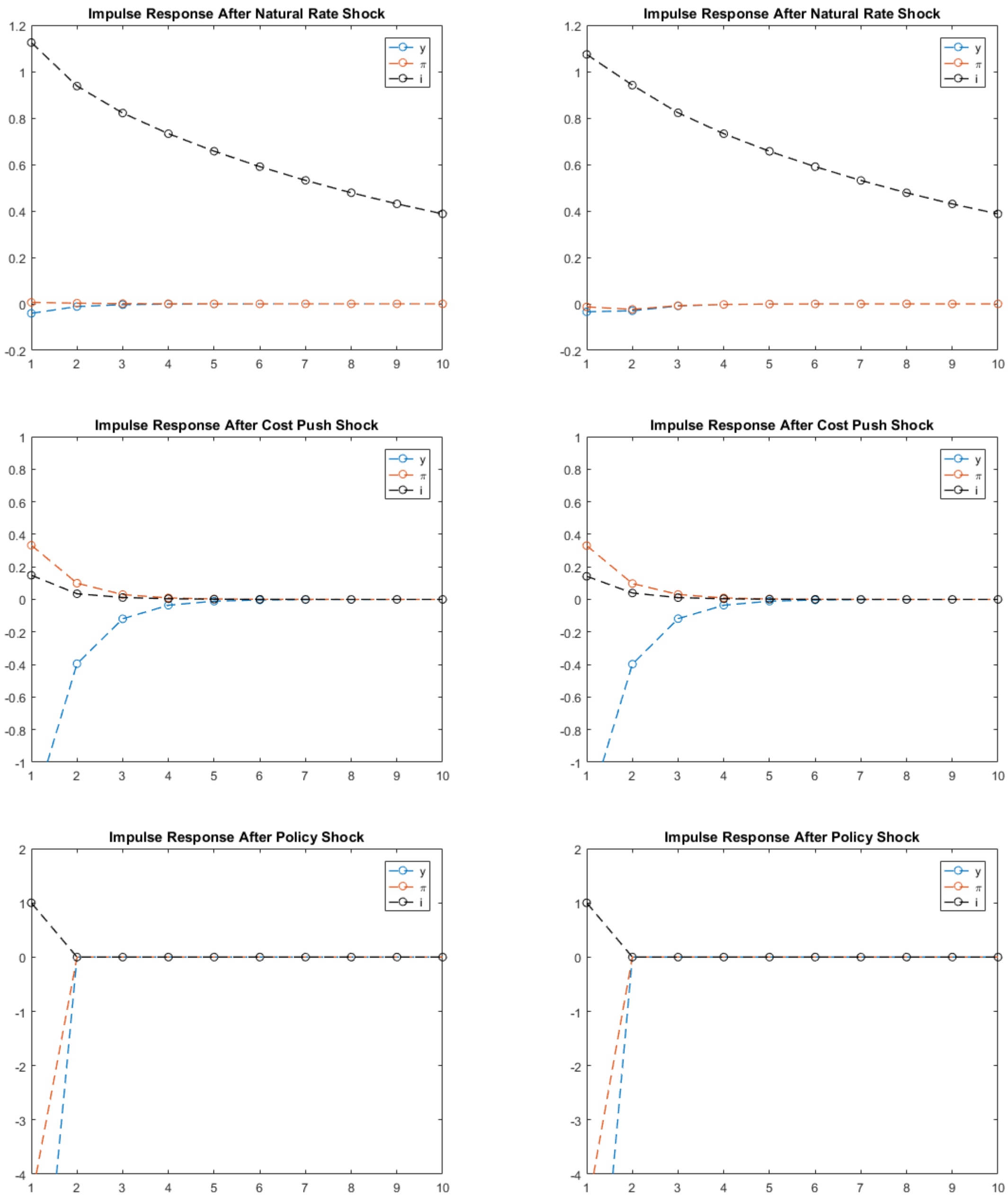

Figure 8: Impulse Response under Discretionary Policy (Left) and Optimal Policy Rule (Right)

6.2 Varying Precision of External Information

As analyzed in Section 4, the precision of external signals crucially determines the gains from commitment, as it affects the size of the informational effect of the interest rate. However, the previous literature does not provide consensus on the degree of information frictions in the private sector. Instead of calibrating the precision of external signals, I investigate how the size of the gains varies with the precision of external signals. I first set the variance of the external signals to be same as the variance of the ex-ante dispersion of the actual shocks. In addition, I also allow for an implementation error in the interest rate.

Figure 9 compares the impulse response function after a natural rate shock, a cost push shock and a policy rate shock under discretionary policy and under policy rule.

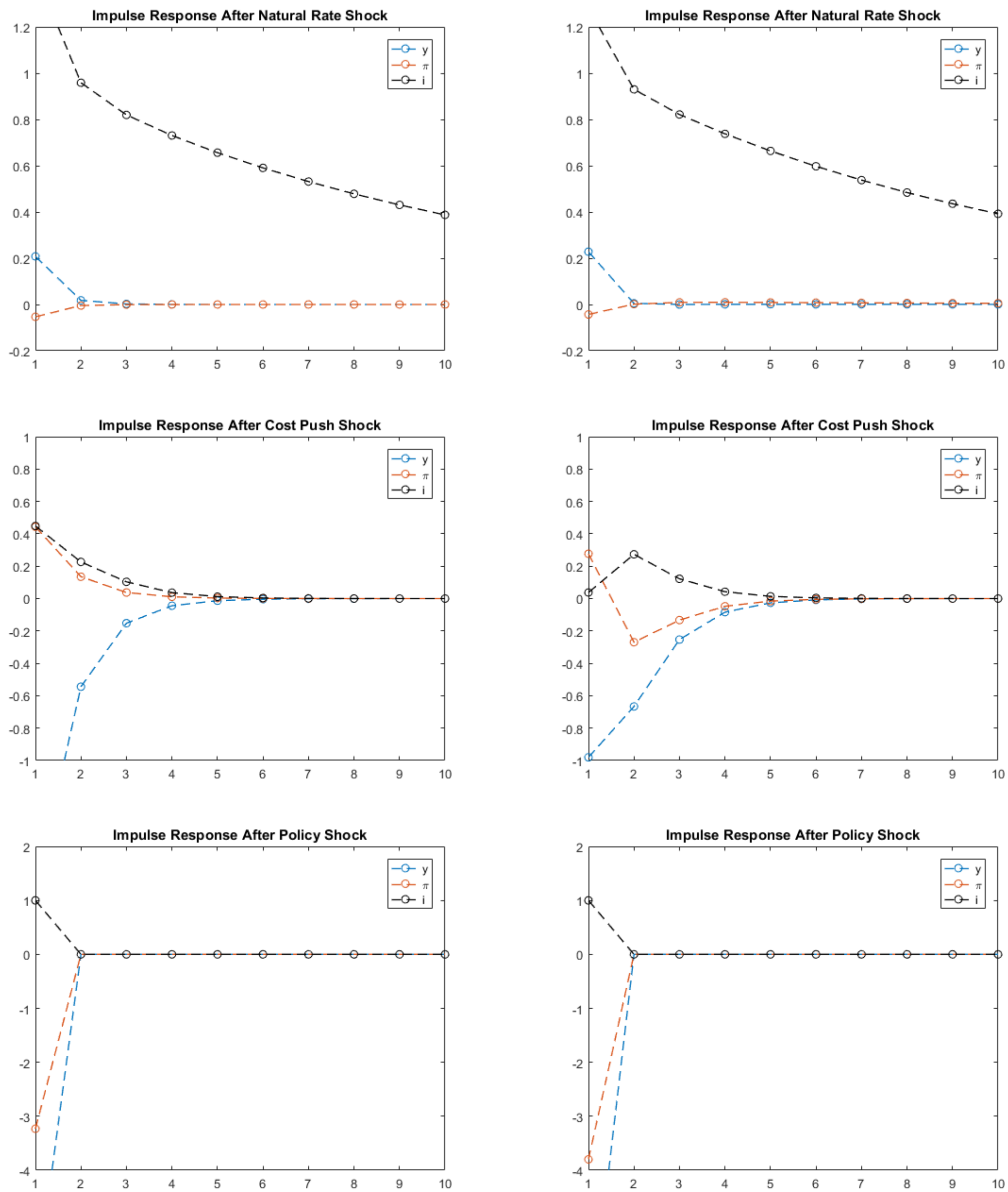

Figure 9: Impulse Response under Discretionary Policy (Left) and Optimal Policy Rule (Right), with No External Information

In Figure 9, the interest rate is numerically solved as follows:

$$i_{discretionary} = 1.4402 r_t^n - 0.3962 E_{t-1}^s r_{t-1}^n + 0.4485 u_t + 0.2578 E_{t-1} u_{t-1},$$
$$i_{rule} = 1.2818 r_t^n - 0.2430 E_{t-1}^s r_{t-1} + 0.0373 u_t + 0.5184 E_{t-1}^s u_{t-1}.$$

The most significant difference is that the interest rate under commitment responds much less to the cost-push shock, as $F_3^{comm} = 0.0373$ and in comparison, $F_e^{disc} = 0.4485$. At the same time, the conflict between output gap stabilization and inflation stabilization in the period where cost push shock occurs is largely reduced. There are two reasons why optimal policy rule is able to largely reduce stabilization bias. First, as interest rate commits to react less to cost push in current period, it reduces information revealed on cost push shock, which decreases the sensitivity of beliefs to interest rate. Second, as interest rate commits to react more in future period, this commitment of tightening policy further reduces expected inflation and expected output gap in future periods.

In the rest of this section, I show how the size of gains depends on the precision of external signals. First of all, holding the standard deviation of m_t^r fixed at 0.1, I vary the precision of external signal on natural rate shock from 0.01 to 0.1. [20] In the following table I report the welfare gains measured by the ex-ante loss function of the central bank [21] standard deviation of inflation and standard deviation of output gap for both types of central bank. When signal on natural rate changes from relatively imprecise $\sigma_{\varepsilon r} = 0.1$ to very precise $\sigma_{\varepsilon r} = 0.01$, the welfare gains measured by the ex-ante loss function of the central bank increases from 64 percent to 70.4 percent.

	Discretionary			Rule		
	$\sigma_{\varepsilon r} = 0.01$	$\sigma_{\varepsilon r} = 0.05$	$\sigma_{\varepsilon r} = 0.1$	$\sigma_{\varepsilon r} = 0.01$	$\sigma_{\varepsilon r} = 0.05$	$\sigma_{\varepsilon r} = 0.1$
Ex-ante Loss	5.13	3.78	3.76	1.84	1.89	1.10
Inflation	1.45	1.11	1.1	1.2	1.12	1.10
Output Gap	4.63	4.42	4.46	2.35	2.31	2.33

Table 1: Gains from Commitment at Varying Levels of Precision of External Information on r_t^n
The ex-ante loss is calculated as the objective function of the central bank defined in equation (57) $\times 10^2$. The numbers for inflation and output gap are noted in percentage points.

In the following table, I report the size of gains from commitment when holding the standard deviation of m_t^u fixed at 0.1, and varying the standard deviation of external signal on cost-push shock from 0.01 to 0.1. When signal on natural rate changes from relatively imprecise $\sigma_{\varepsilon r} = 0.1$ to very precise $\sigma_{\varepsilon r} = 0.01$, the welfare gains measured by the ex-ante loss function of the central

[20] standard deviation of interest rate implementation error is fixed to be 0.01 in all calibration exercises.

[21] This is defined as the objective function for commitment central bank in equation (57), which is the weighted sum of squared deviations from steady state.

bank increases from 28 percent to 49 percent.

	Discretionary			Rule		
	$\sigma_{\varepsilon u}=0.01$	$\sigma_{\varepsilon u}=0.05$	$\sigma_{\varepsilon u}=0.1$	$\sigma_{\varepsilon u}=0.01$	$\sigma_{\varepsilon u}=0.05$	$\sigma_{\varepsilon u}=0.1$
Ex-ante Loss	6.28	3.76	3.76	4.53	1.90	1.89
Inflation	1.39	1.12	1.1	1.12	1.14	1.10
Output Gap	5.99	4.56	4.46	5.22	2.39	2.33

Table 2: Gains from Commitment at Varying Levels of Precision of External Information on u_t
The ex-ante loss is calculated as the objective function of the central bank defined in equation (57) $\times 10^2$. The numbers for inflation and output gap are noted in percentage points.

7 Conclusion

In this paper, I studied an economy in which the private sector has imperfect information about the underlying shocks and the central bank has perfect information when making interest rate decisions. Consequently, the interest rate decisions have an informational effect, as the private sector regards the equilibrium interest rate as a signal about the unobserved shocks, and extracts information from the interest rate. I showed that with serially correlated shocks and relatively precise external information, the size of gains from commitment is quantitatively important.

To theoretically study the gains from commitment, I built a New Keynesian model with both nominal frictions and information frictions, and studied the optimal response of interest rates to natural-rate shocks and cost-push shocks. I started with the simple scenario in which both shocks are serially uncorrelated, which allowed me to isolate the informational gains from commitment. Using an arbitrary interest rate function that responds positively to both shocks, I showed that beliefs in the private sector are more sensitive to the shock to which the interest rate reacts more aggressively or that has higher ex-ante dispersion.

I began the analysis of monetary policy by characterizing how the informational effect of interest rates changes the Phillips curve, which is the constraint faced by the central bank. As the Phillips curve measures the co-movement of the output gap and inflation that results from changes in the interest rate, it is affected by the informational effect of the interest rate, and is further determined by how private agents expect the interest rate to respond to different shocks.

A discretionary central bank sets interest rates to optimize its objective function at any state of the economy, taking as given the informational effect of its interest rate decision. In comparison, under commitment, a central bank can change the informational effect of interest rates by committing to a different response function than its optimizing response under discretion. Consequently, the informational effect of interest rates makes the Phillips curve endogenous to the central bank's decision about the optimal policy rule.

The responses of the optimal policy rule to the two shocks are jointly determined. Assuming that the natural-rate shock and the cost-push shock have the same ex-ante dispersion, I show that the optimal policy rule responds more aggressively to the natural-rate shock and less aggressively to the cost-push shock, relative to the equilibrium optimizing interest rate set by discretionary central bank. By doing so, it achieves an informational advantage as it withholds information about the cost-push shock, which consequently reduces the stabilization bias caused by the actual cost-push shocks under perfect information.

I extend the analysis by studying the interaction between the informational effect of the interest rate and external signals. Central bank direct communication can also be modeled as providing more precise external signals independent of the informational effect of the interest rates. I presented the situation in which providing more precise information about the efficient shocks might reduce welfare. In this case, communicating about the natural-rate shock also makes the interest-rate a more precise signal about the cost-push shock.

Finally, I quantified the size of the gains from commitment by adopting conventionally used parameter values while varying the precision of external signals. I found that when external signals are extremely imprecise, the size of gains from commitment is negligible. However, more precise external information about both shocks increases the size of gains from commitment. Specifically, when the precision of external signals is equal to the prior distribution of actual shocks, committing to the optimal policy rule improves ex-ante welfare by 54 percent relative to the equilibrium under the optimizing discretionary policy.

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Appendices

A Log-Linearization and Aggregation

From the household first order conditions, we first do log-linear approximation to the Euler equation in (A.6) by

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) \tag{A.1}$$

The log-linear approximation to the labor supply of equation (A.7) is $\varphi n_t(j) + \sigma y_t = w_t(j)$ where w_t denotes the log approximated real wage, $log(W_t/P_t)$. Recall that resource constraint implies that $c_t^j = y_t^j \; \forall j$, which further implies $c_t = y_t$. We can then write the labor supply as follows:

$$\varphi n_t(j) + \sigma y_t = w_t(j) \tag{A.2}$$

Next, we want to relate individual firm's real marginal cost of production to aggregate output. To to this, first integrate equation (A.13):

$$\int w_t(j) = \varphi \int n_t(j) dj + \sigma y_t \tag{A.3}$$

Then, substitute the log-linear approximation of the individual good demand, i.e., $y_t(j) - y_t = -\varepsilon(p_t(j) - p_t)$, which results in:

$$\int n_t(j) dj = y_t + \int (-\varepsilon)(p_t(j) - p_t) - \int a_t(j) = y_t - a_t \tag{A.4}$$

Substitute this into $\int w_t(j)$, and then deduct a_t from both sides:

$$\int w_t(j) - a_t(j) = (\phi + \sigma) y_t - (1 + \varphi) a_t \tag{A.5}$$

Define natural level of output as the equilibrium output level without price rigidity and under perfect information, which makes y_t^n as a linear function of aggregate technology. Then, write the above equation in terms of output gap:

$$\int w_t(j) - a_t(j) = (\phi + \sigma)(y_t - y_t^n) \tag{A.6}$$

We know move on to the firm's side. Taking log-linear approximation of individual firm's optimal resetting prices:

$$p_t^*(j) = (1 - \beta\theta) E_t^j \left\{ \Sigma(\beta\theta)^k \left[p_{t+k} + u_{t+k}(j) + w_{t+k}(j) - a_{t+k}(j) \right] \right\} \tag{A.7}$$

The Calvo assumption implies that the aggregate price index is an average of the price charged by the fraction of $1-\theta$ of firms which reset their prices at t, and the fraction of θ of firms whose prices remain as the last period prices. Thus, the log-linear approximation of the aggregate price in period t becomes:

$$p_t = \theta p_{t-1} + (1-\theta)\int p_t^*(j)dj \tag{A.8}$$

Subtract p_{t-1} from both sides to express in terms of inflation:

$$\pi_t = (1-\theta)\left(\int p_t^*(j) - p_{t-1}\right) \tag{A.9}$$

As explained in Section 2.3, assume homogeneous, subjective believes in order to abstract from the higher order beliefs problem in aggregating prices. This assumption allows me to write individual resetting prices as:

$$p_t^*(j) = (1-\beta\theta)\left(E_t^s p_t + u_t(j) + w_t(j) - a_t(j)\right) + (1-\beta\theta)\Sigma_{k=1}^{\infty}(\beta\theta)^k E_t^s\left(p_{t+k} + u_{t+k}(j) + w_{t+k} - a_{t+k}\right) \tag{A.10}$$

Integrate over j:

$$\int p_t^*(j)dj = (1-\beta\theta)\left(E_t^s p_t + u_t + w_t - a_t\right) + (1-\beta\theta)\Sigma_{k=1}^{\infty}(\beta\theta)^k E_t^s\left(p_{t+k} + w_{t+k} - a_{t+k}\right) \tag{A.11}$$

To write in difference equation, first calculate:

$$\beta\theta\int E_t^s p_{t+1}^*(j)dj = (1-\beta\theta)\Sigma_{k=1}^{\infty} E_t^s\left(p_{t+k} + u_{t+k} + w_{t+k} - a_{t+k}\right) = \beta\theta E_t^s p_{t+k}^* \tag{A.12}$$

The second equation holds due to homogeneous beliefs.

Subtract equation (A. 23) from equation (A. 22):

$$\int p_t^*(j)dj - \beta\theta E_t^s p_{t+1} = (1-\beta\theta)E_t^s p_t + (1-\beta\theta)u_t + (1-\beta\theta)(\varphi+\sigma)\hat{y} \tag{A.13}$$

$$\int p_t^*(j)dj - p_{t-1} = \beta\theta\left(E_t^s p_{t+1}^* - E_t^s p_t\right) + E_t^s p_t - p_{t-1} + (1-\beta\theta)u_t + (1-\beta\theta)(\varphi+\sigma)\hat{y}_t$$

$$\pi_t = \beta\theta E_t^s \pi_{t+1} + (1-\theta)E_t^s \pi_t + (1-\theta)(1-\beta\theta)u_t + (1-\beta\theta)(1-\theta)(\varphi+\sigma)\hat{y}_t$$

In the last equation, I assume that aggregate price is observable after one period, i.e., $p_{t-1} = E_t^s p_{t-1}$

Write inflation as:

$$\pi_t = \beta\theta E_t^s \pi_{t+1} + (1-\theta)E_t^s \pi_t + \kappa\theta\hat{y}_t + u_t \tag{A.14}$$

where $\kappa = \frac{(1-\beta\theta)(1-\theta)(\varphi+\sigma)}{\theta}$, and $u_t = (1-\theta)(1-\beta\theta)u_t$

B Solution to the Markov Perfect Equilibrium under Discretionary Monetary Policy

In this section, I first solve the model with serially uncorrelated shocks and then solve the model with serially correlated shocks. For both cases, I solve for the fixed point where the beliefs by people in the private sector on the best response of interest rate at any state match the optimizing discretionary interest rate. This means that in equilibrium people have rational expectation.

B.1 Equilibrium Optimizing Discretionary Policy with Serially Uncorrelated Shocks

The solution takes the following steps:

1. I conjecture that interest rate reacts linear to both shocks, i.e., $i_t = F_r^0 r_t^n + F_u^0 u_t$.
2. With this interest rate, I solve for the beliefs formed about natural-rate shock and cost-push shock in the private sector as functions of interest rate.
3. With beliefs formed in private sector, $E_t^s r_t^n$ and $E_t^s u_t$, the actual shocks, r_t^n and u_t, I solve for $\hat{y}_t$ and π_t as a function of i_t.
4. Solve for i_t that minimizes the loss function, $L_t = \pi_t^2 + \omega \hat{y}_t$, and express interest rate as actual shocks, $i_t = F_r r_t^n + F_u u_t$.
5. Iterate the process until convergence.

Specifically, in step 1, $i_t = F_r^0 r_t^n + F_u^0 u_t$.

In step 2, beliefs about underlying shocks follow:

$$E_t^s r_t^n = K_r i_t \tag{B.1}$$

$$E_t^s u_t = K_u i_t \tag{B.2}$$

where $K_r F_r^0 = \frac{F_r^{02}\sigma_r^2}{F_r^{02}\sigma_r^2 + F_u^{02}\sigma_u^2}$, and $K_u F_u^0 = \frac{F_u^{02}\sigma_u^2}{F_r^{02}\sigma_r^2 + F_u^{02}\sigma_u^2}$.

In step 3, write out the expression of output gap and inflation as function of interest rate:

$$\hat{y}_t = -\frac{1}{\sigma}\left(i_t - r_t^n\right) \tag{B.3}$$

$$\pi_t = \kappa \hat{y}_t + (1-\theta)\frac{\kappa}{\sigma}\left(E_t^s r_t^n(i_t) - r_t^n\right) + \frac{1-\theta}{\theta} E_t^s u_t(i_t) + u_t \tag{B.4}$$

In step 4, I first write out the first order condition of interest rate:

$$\pi_t \frac{\partial \pi_t}{\partial i_t} + \omega \hat{y}_t \frac{\partial \hat{y}_t}{\partial i_t} = 0 \tag{B.5}$$

Substitute π_t and $\hat{y}_t$ by equation (B.3) and (B.4):

$$\left\{(1-\theta)\frac{\kappa}{\sigma}(E_t^s r_t^n - r_t^n) + \frac{1-\theta}{\theta}E_t^s u_t + u_t\right\}\frac{\partial \pi_t}{\partial i_t} + \left(\omega\frac{\partial \hat{y}_t}{\partial i_t} + \kappa\frac{\partial \pi_t}{\partial i_t}\right)\left\{-\frac{1}{\sigma}(i_t - r_t^n)\right\} = 0 \quad \text{(B.6)}$$

Substituting $E_t^s r_t^n$ and $E_t^s u_t$ as i_t leads to:

$$\lambda_1 r_t^n + \lambda_2 u_t + \lambda_3 i_t = 0 \tag{B.7}$$

where $\frac{\partial \hat{y}_t}{\partial i_t} = -\frac{1}{\sigma}$, and $\frac{\partial \pi_t}{\partial i_t} = -\frac{\kappa}{\sigma} + (1-\theta)\frac{\kappa}{\sigma}K_r + \frac{1-\theta}{\theta}K_u$, and

$$\begin{aligned}
\lambda_1 &= \left\{\left(\kappa\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\frac{1}{\sigma} - \frac{\partial \pi_t}{\partial i_t}(1-\theta)\frac{\kappa}{\sigma}\right\} \\
\lambda_2 &= \frac{\partial \pi_t}{\partial i_t} \\
\lambda_3 &= \frac{\partial \pi_t}{\partial i_t}(1-\theta)\frac{\kappa}{\sigma}K_{11} + \frac{\partial \pi_t}{\partial i_t}\frac{1-\theta}{\theta}K_{21} - \left(\kappa\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\frac{1}{\sigma}
\end{aligned}$$

Rearranging the above equation to get:

$$i_t = F_1 r_t^n + F_3 u_t \tag{B.8}$$

where $F_1 = -\frac{\lambda_1}{\lambda_3}$, and $F_3 = -\frac{\lambda_2}{\lambda_3}$.

In step 5, I iterate the above process until $F_r = F_r^0$ and $F_u = F_u^0$.

B.2 Equilibrium Optimizing Discretionary Policy with Serially Correlated Shocks

In this section, I solve for the general version of the dynamic information case where I have serially correlated shocks, external signals which captures central bank direct communication, and implementation error.

The solution method is similar to the case with serially uncorrelated shocks, as solving for optimizing interest rate in equilibrium involves conjecture of interest rate response function. In addition to this conjecture, solving equilibrium variables in the private sector also requires additional step of undetermined coefficient to deal with the subjective expectation of future equilibrium variables.

1. I conjecture that interest rate reacts linear to both current shocks and past beliefs, i.e., $i_t = F_1^0 r_t^n + F_2^0 E_{t-1}^s r_{t-1}^n + F_3^0 u_t + F_4^0 E_{t-1}^s u_{t-1}$.

2. With this interest rate, I solve for the beliefs formed about natural-rate shock and cost-push shock in the private sector as functions of current signals (interest rate and central bank

communication) plus past beliefs.

3. (Undetermined Coefficient) I conjecture that output gap and inflation are linear functions of current state variables which include actual shocks and past beliefs. As a result, I am able to express the forward-looking output gap and inflation as functions of current actual shocks and current beliefs.

4. With beliefs formed in private sector, $E_t^s r_t^n$ and $E_t^s u_t$, the actual shocks, r_t^n and u_t, I solve for $\hat{y}_t$ and π_t as a function of i_t.

5. Solve for i_t that minimizes the loss function, $L_t = \pi_t^2 + \omega \hat{y}_t$, and express interest rate as actual shocks, $i_t = F_r r_t^n + F_u u_t$.

6. Iterate the process until convergence.

Specifically, in step 1, I conjecture that $i_t = F_1 r_t^n + F_r E_{t-1}^s r_{t-1}^n + F_3 u_t + F_4 E_{t-1}^s u_{t-1}$.

In Step 2, to solve the beliefs formed in the pirvate sector, I first specify the evolution of actual shocks:

State:

$$\begin{bmatrix} r_t^n \\ u_t \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u u_t \end{bmatrix} + \begin{bmatrix} v_t \\ v_t^u \end{bmatrix} \tag{B.9}$$

which I denote as $z_t = \Phi z_{t-1} + v_t$, where $\Phi = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix}$ and $v_t = [v_t, v_t^u]$ with the white noise of variance Q.

Signals

As people in private sector have perfect memory of beliefs they have in the past, they are able to back out the part of interest rate that reacts to current shocks, which I denote as

$$\hat{i}_t \equiv i_t - F_3 E_{t-1}^s r_{t-1}^n - F_4 E_{t-1}^s u_{t-1} \tag{B.10}$$

All signals are summarized as.

$$\begin{bmatrix} \hat{i}_t \\ m_t^r \\ m_t^u \end{bmatrix} = \begin{bmatrix} F_1 & F_3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_t \\ \varepsilon_t^r \\ \varepsilon_t^u \end{bmatrix} \tag{B.11}$$

which I denote as $s_t = D z_t + R_t$

Beliefs

People in private sector are Bayesian, and update beliefs through the Kalman Filtering process, in which they optimally weigh between all current signals and past beliefs by their variances. The beliefs follow:

$$\begin{bmatrix} E_t^s r_t^n \\ E_t^s u_t \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix} \begin{bmatrix} E_{t-1}^s r_{t-1}^n \\ E_{t-1}^s u_{t-1} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \end{bmatrix} \left(\begin{bmatrix} \hat{i}_t \\ m_t^r \\ m_t^u \end{bmatrix} - \begin{bmatrix} F_1 & F_3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix} \begin{bmatrix} E_{t-1}^s r_{t-1}^n \\ E_{t-1}^s u_{t-1} \end{bmatrix} \right) \tag{B.12}$$

Write out the expression for $\hat{i}_t$ and collect terms:

$$\begin{aligned} E_t^s r_t^n &= (K_{11}F_1 + K_{12})\, r_t^n + \phi\,(1 - K_{11}F_1 - K_{12})\, E_{t-1}^s r_{t-1}^n \\ &\quad + (K_{11}F_3 + K_{13})\, u_t + \phi^u\,(-K_{11}F_3 - K_{13})\, E_{t-1}^s u_{t-1} + K_{12}\varepsilon_t^r + K_{13}\varepsilon_t^u + K_{11} e_t \end{aligned} \tag{B.13}$$

$$\begin{aligned} E_t^s u_t &= (K_{21}F_1 + K_{22})\, r_t^n + \phi\,(-K_{21}F_1 - K_{22})\, E_{t-1}^s r_{t-1}^n \\ &\quad + (K_{21}F_3 + K_{23})\, u_t + \phi^u\,(1 - K_{21}F_3 - K_{23})\, E_{t-1}^s u_{t-1} + K_{22}\varepsilon_t^r + K_{23}\varepsilon_t^u + K_{21} e_t \end{aligned} \tag{B.14}$$

Denote the above equations as $E_t^s r_t^n = \Psi(1) r_t^n + \Psi(2) E_{t-1}^s r_{t-1}^n + \Psi(3) u_t + \Psi(4) E_{t-1}^s u_{t-1} + \Psi(5)\varepsilon_t^r + \Psi(6)\varepsilon_t^u + \Psi(7) e_t$, and $E_t^s u_t = \Psi(8) r_t^n + \Psi(9) E_{t-1}^s r_{t-1}^n + \Psi(10) u_t + \Psi(11) E_{t-1}^s u_{t-1} + \Psi(12)\varepsilon_t^r + \Psi(13)\varepsilon_t^u + \Psi(14) e_t$. I will use this notation in solving equilibrium in the private sector by the method of undetermined coefficients.

In step 3, the first write out the the forward-looking output gap and inflation as:

$$\hat{y}_t = E_t^s \hat{y}_{t+1} - \frac{1}{\sigma}\left[i_t - \left(\frac{1}{1-\phi} r_t^n - \frac{\phi}{1-\phi} E_t^s r_t^n \right) - E_t^s \pi_{t+1} \right] \tag{B.15}$$

$$\pi_t = \beta\theta E_t^s \pi_{t+1} + (1-\theta) E_t^s \pi_t + \kappa\theta \hat{y}_t + u_t \tag{B.16}$$

Following the method of undetermined coefficients, I first need to conjecture that equilibrium variables are linear functions to current state variables, which include current actual shocks $(r_t^n,\ u_t)$, past beliefs, $\left(E_{t-1}^s r_{t-1}^n,\ E_{t-1}^s u_{t-1}\right)$, and noise in current signals, $(\varepsilon_t^r,\ \varepsilon_t^u,\ e_t)$.

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \\ \gamma_8 & \gamma_9 & \gamma_{10} & \gamma_{11} \end{bmatrix} \begin{bmatrix} r_t^n \\ E_{t-1}^s r_{t-1}^n \\ u_t \\ E_{t-1}^s u_{t-1} \end{bmatrix} + \begin{bmatrix} \gamma_5 & \gamma_6 & \gamma_7 \\ \gamma_{12} & \gamma_{13} & \gamma_{14} \end{bmatrix} \begin{bmatrix} \varepsilon_t^r \\ \varepsilon_t^u \\ e_t \end{bmatrix} \tag{B.17}$$

Next, substitute this conjecture into the forward-looking variables, $E_t \hat{y}_{t+1}$ and $E_t^s \pi_{t+1}$. Notice that noise of all signals are temporary, which are expected to be zero in future period.

$$\begin{bmatrix} E_t^s \hat{y}_{t+1} \\ E_t^s \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \gamma_1 \phi + \gamma_2 & \gamma_3 \phi^u + \gamma_4 \\ \gamma_8 \phi + \gamma_9 & \gamma_{10} \phi^u + \gamma_{11} \end{bmatrix} \begin{bmatrix} E_t^s r_t^n \\ E_t^s u_t \end{bmatrix} \tag{B.18}$$

First substitute this into the output gap expression:

$$\hat{y}_t = \left[(\gamma_1 \phi + \gamma_2) + \frac{1}{\sigma}(\gamma_8 \phi + \gamma_9) - \frac{1}{\sigma} \frac{\phi}{1-\phi} \right] E_t^s r_t^n + \left[\gamma_3 \phi^u + \gamma_4 + \frac{1}{\sigma}(\gamma_{10} \phi^u + \gamma_{11}) \right] E_t^s u_t - \frac{1}{\sigma} i_t + \frac{1}{\sigma} \frac{1}{1-\phi} r_t^n \tag{B.19}$$

Next work on π_t, as the actual inflation also includes the expected current inflation, and expected current inflation includes expected current output gap, I first need to calculate:

$$E_t^s \hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma} \left[i_t - E_t^s r_t^n - E_t^s \pi_{t+1} \right] \tag{B.20}$$

$$E_t^s \pi_t = \beta E_t^s \pi_{t+1} + \kappa \left\{ E_t^s \hat{y}_{t+1} - \frac{1}{\sigma} \left[i_t - E_t^s r_t^n - E_t^s \pi_{t+1} \right] \right\} + \frac{1}{\theta} E_t^s u_t \tag{B.21}$$

Substitute $E_t \pi_t$ into π_t:

$$\begin{aligned}
\pi_t &= \beta \theta E_t^s \pi_{t+1} + (1-\theta) \left\{ \beta E_t^s \pi_{t+1} + \kappa E_t^s \hat{y}_t + \frac{1}{\theta} E_t^s u_t \right\} + \kappa \theta \hat{y}_t + u_t \qquad \text{(B.22)} \\
&= \beta E_t^s \pi_{t+1} + (1-\theta) \kappa \left\{ (\gamma_1 \phi + \gamma_2) E_t^s r_t^n + (\gamma_3 \phi^u + \gamma_4) E_t^s u_t \right\} - (1-\theta) \frac{\kappa}{\sigma} i_t + (1-\theta) \frac{\kappa}{\sigma} E_t^s r_t^n \\
&+ (1-\theta) \frac{\kappa}{\sigma} \left\{ (\gamma_8 \phi + \gamma_9) E_t^s r_t^n + (\gamma_{10} \phi^u + \gamma_{11}) E_t^s u_t \right\} + \frac{1-\theta}{\theta} E_t^s u_t + \kappa \theta \hat{y}_t + u_t \\
&= \left\{ (1-\theta) \kappa (\gamma_1 \phi + \gamma_2) + (1-\theta) \frac{\kappa}{\sigma} + \left(\beta + (1-\theta) \frac{\kappa}{\sigma} \right) (\gamma_8 \phi + \gamma_9) \right\} E_t^s r_t^n \\
&+ \left\{ (1-\theta) \kappa (\gamma_3 \phi^u + \gamma_4) + \frac{1-\theta}{\theta} + \left(\beta + (1-\theta) \frac{\kappa}{\sigma} \right) (\gamma_{10} \phi^u + \gamma_{11}) \right\} E_t^s u_t - (1-\theta) \frac{\kappa}{\sigma} i_t + \kappa \theta \hat{y}_t + u_t
\end{aligned}$$

The values of γ can be solved in the following matrix:

$$\gamma = M\gamma + c$$

where

$$\gamma = [\gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_4,\ \gamma_5,\ \gamma_6,\ \gamma_7,\ \gamma_8]'$$

$$M =$$

$$\left[\begin{array}{ccccccccccccc}
\phi\Psi_1 & \Psi_1 & \phi^u\Psi_8 & \Psi_8 & 0 & 0 & 0 & \frac{1}{\sigma}\phi\Psi_1 & \frac{1}{\sigma}\Psi_1 & \frac{1}{\sigma}\phi^u\Psi_8 & \frac{1}{\sigma}\Psi_8 & 0 & 0 \\
\phi\Psi_2 & \Psi_2 & \phi^u\Psi_9 & \Psi_9 & 0 & 0 & 0 & \frac{1}{\sigma}\phi\Psi_2 & \frac{1}{\sigma}\Psi_2 & \frac{1}{\sigma}\phi^u\Psi_9 & \frac{1}{\sigma}\Psi_9 & 0 & 0 \\
\phi\Psi_3 & \Psi_3 & \phi^u\Psi_{10} & \Psi_{10} & 0 & 0 & 0 & \frac{1}{\sigma}\phi\Psi_3 & \frac{1}{\sigma}\Psi_3 & \frac{1}{\sigma}\phi^u\Psi_{10} & \frac{1}{\sigma}\Psi_{10} & 0 & 0 \\
\phi\Psi_4 & \Psi_4 & \phi^u\Psi_{11} & \Psi_{11} & 0 & 0 & 0 & \frac{1}{\sigma}\phi\Psi_4 & \frac{1}{\sigma}\Psi_4 & \frac{1}{\sigma}\phi^u\Psi_{11} & \frac{1}{\sigma}\Psi_{11} & 0 & 0 \\
\phi\Psi_5 & \Psi_5 & \phi^u\Psi_{12} & \Psi_{12} & 0 & 0 & 0 & \frac{1}{\sigma}\phi\Psi_5 & \frac{1}{\sigma}\Psi_5 & \frac{1}{\sigma}\phi^u\Psi_{12} & \frac{1}{\sigma}\Psi_{12} & 0 & 0 \\
\phi\Psi_6 & \Psi_6 & \phi^u\Psi_{13} & \Psi_{13} & 0 & 0 & 0 & \frac{1}{\sigma}\phi\Psi_6 & \frac{1}{\sigma}\Psi_6 & \frac{1}{\sigma}\phi^u\Psi_{13} & \frac{1}{\sigma}\Psi_{13} & 0 & 0 \\
\phi\Psi_7 & \Psi_7 & \phi^u\Psi_{14} & \Psi_{14} & 0 & 0 & 0 & \frac{1}{\sigma}\phi\Psi_7 & \frac{1}{\sigma}\Psi_7 & \frac{1}{\sigma}\phi^u\Psi_{14} & \frac{1}{\sigma}\Psi_{14} & 0 & 0 \\
(1-\theta)\kappa\phi\Psi_1+\kappa\theta & (1-\theta)\kappa\Psi_1 & (1-\theta)\kappa\phi^u\Psi_8 & (1-\theta)\kappa\Psi_8 & 0 & 0 & 0 & \left((1-\theta)\frac{\kappa}{\sigma}\phi+\beta\phi\right)\Psi_1 & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_1 & \left((1-\theta)\frac{\kappa}{\sigma}\phi^u+\beta\phi^u\right)\Psi_8 & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_8 & 0 & 0 \\
(1-\theta)\kappa\phi\Psi_2 & (1-\theta)\kappa\Psi_2+\kappa\theta & (1-\theta)\kappa\phi^u\Psi_8 & (1-\theta)\kappa\Psi_8 & 0 & 0 & 0 & \left((1-\theta)\frac{\kappa}{\sigma}\phi+\beta\phi\right)\Psi_2 & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_2 & \left((1-\theta)\frac{\kappa}{\sigma}\phi^u+\beta\phi^u\right)\Psi_9 & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_9 & 0 & 0 \\
(1-\theta)\kappa\phi\Psi_3 & (1-\theta)\kappa\Psi_3 & (1-\theta)\kappa\phi^u\Psi_{10}+\kappa\theta & (1-\theta)\kappa\Psi_{10} & 0 & 0 & 0 & \left((1-\theta)\frac{\kappa}{\sigma}\phi+\beta\phi\right)\Psi_3 & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_3 & \left((1-\theta)\frac{\kappa}{\sigma}\phi^u+\beta\phi^u\right)\Psi_{10} & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_{10} & 0 & 0 \\
(1-\theta)\kappa\phi\Psi_4 & (1-\theta)\kappa\Psi_4 & (1-\theta)\kappa\phi^u\Psi_{11} & (1-\theta)\kappa\Psi_{11}+\kappa\theta & 0 & 0 & 0 & \left((1-\theta)\frac{\kappa}{\sigma}\phi+\beta\phi\right)\Psi_4 & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_4 & \left((1-\theta)\frac{\kappa}{\sigma}\phi^u+\beta\phi^u\right)\Psi_{11} & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_{11} & 0 & 0 \\
(1-\theta)\kappa\phi\Psi_5 & (1-\theta)\kappa\Psi_5 & (1-\theta)\kappa\phi^u\Psi_{12} & (1-\theta)\kappa\Psi_{12} & +\kappa\theta & 0 & 0 & \left((1-\theta)\frac{\kappa}{\sigma}\phi+\beta\phi\right)\Psi_5 & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_5 & \left((1-\theta)\frac{\kappa}{\sigma}\phi^u+\beta\phi^u\right)\Psi_{12} & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_{12} & 0 & 0 \\
(1-\theta)\kappa\phi\Psi_6 & (1-\theta)\kappa\Psi_6 & (1-\theta)\kappa\phi^u\Psi_{13} & (1-\theta)\kappa\Psi_{13} & 0 & +\kappa\theta & 0 & \left((1-\theta)\frac{\kappa}{\sigma}\phi+\beta\phi\right)\Psi_6 & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_6 & \left((1-\theta)\frac{\kappa}{\sigma}\phi^u+\beta\phi^u\right)\Psi_{13} & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_{13} & 0 & 0 \\
(1-\theta)\kappa\phi\Psi_7 & (1-\theta)\kappa\Psi_7 & (1-\theta)\kappa\phi^u\Psi_{14} & (1-\theta)\kappa\Psi_{14} & 0 & 0 & +\kappa\theta & \left((1-\theta)\frac{\kappa}{\sigma}\phi+\beta\phi\right)\Psi_7 & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_7 & \left((1-\theta)\frac{\kappa}{\sigma}\phi^u+\beta\phi^u\right)\Psi_{14} & \left((1-\theta)\frac{\kappa}{\sigma}+\beta\right)\Psi_{14} & 0 & 0
\end{array}\right.$$

$$c = \begin{bmatrix}
-\frac{1}{\sigma}\frac{\phi}{1-\phi}\Psi_1 - \frac{1}{\sigma}F_1 + \frac{1}{\sigma}\frac{1}{1-\phi} \\
-\frac{1}{\sigma}\frac{\phi}{1-\phi}\Psi_2 - \frac{1}{\sigma}F_2 \\
-\frac{1}{\sigma}\frac{\phi}{1-\phi}\Psi_3 - \frac{1}{\sigma}F_3 \\
-\frac{1}{\sigma}\frac{\phi}{1-\phi}\Psi_4 - \frac{1}{\sigma}F_4 \\
-\frac{1}{\sigma}\frac{\phi}{1-\phi}\Psi_5 \\
-\frac{1}{\sigma}\frac{\phi}{1-\phi}\Psi_6 \\
-\frac{1}{\sigma}\frac{\phi}{1-\phi}\Psi_7 - \frac{1}{\sigma} \\
(1-\theta)\frac{\kappa}{\sigma}\Psi_1 + \frac{1-\theta}{\theta}\Psi_8 - (1-\theta)\frac{\kappa}{\sigma}F_1 \\
(1-\theta)\frac{\kappa}{\sigma}\Psi_2 + \frac{1-\theta}{\theta}\Psi_9 - (1-\theta)\frac{\kappa}{\sigma}F_2 \\
(1-\theta)\frac{\kappa}{\sigma}\Psi_3 + \frac{1-\theta}{\theta}\Psi_{10} - (1-\theta)\frac{\kappa}{\sigma}F_3 + 1 \\
(1-\theta)\frac{\kappa}{\sigma}\Psi_4 + \frac{1-\theta}{\theta}\Psi_{11} - (1-\theta)\frac{\kappa}{\sigma}F_4 \\
(1-\theta)\frac{\kappa}{\sigma}\Psi_5 + \frac{1-\theta}{\theta}\Psi_{12} \\
(1-\theta)\frac{\kappa}{\sigma}\Psi_6 + \frac{1-\theta}{\theta}\Psi_{13} \\
(1-\theta)\frac{\kappa}{\sigma}\Psi_7 + \frac{1-\theta}{\theta}\Psi_{14} - (1-\theta)\frac{\kappa}{\sigma}
\end{bmatrix}$$

γ can be uniquely pinned down by the above linear system.

In step 5, in order to solve for the optimizing interest rate, I first need to specify central bank's objective function.

Central Bank Objective Function

As current interest rate has persistent effect through the dynamic learning process, central bank also considers how current interest rate affect future equilibrium. Consequently, the loss function includes output gap and inflation of current and all future periods.

$$E_t L(t) = [\pi_t^2 + \omega \hat{y}_t^2] + \beta E_t(L(t+1)) \tag{B.23}$$

where the $E_t(L(t+1))$ is:

$$\Sigma_{j=1}^{\infty} \beta^j E_t \left\{ \begin{bmatrix} \pi_{t+1} & \hat{y}_{t+j} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \pi_{t+j} \\ \hat{y}_{t+j} \end{bmatrix} \right\} = \Sigma_{j=1}^{\infty} \beta^j \left\{ \begin{bmatrix} E_t \pi_{t+1} & E_t \hat{y}_{t+j} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} E_t \pi_{t+j} \\ E_t \hat{y}_{t+j} \end{bmatrix} + indept.\ terms \right\} \tag{B.24}$$

The central banks expectation is *objective*, denoted by E_t, in the sense that it observes all past shocks, and expects all future shocks to be zero. The information set of central bank at period t is:

$$I_t = \{r_T^n,\ u_T,\ \forall T = 0...t\}$$

Let $z_t = \left[r_t^n,\ E_{t-1}^s r_{t-1}^n, u_t,\ E_{t-1}^s u_{t-1}\right]'$ denote the persistent state variables. So the central bank's objective expectation of future period output gap and inflation becomes a linear function of $E_t z_{t+j}$:

$$\begin{bmatrix} E_t \pi_{t+j} \\ E_t \hat{y}_{t+j} \end{bmatrix} = \begin{bmatrix} \gamma_8 & \gamma_9 & \gamma_{10} & \gamma_{11} \\ \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \end{bmatrix} E_t z_{t+j} \equiv \Gamma E_t z_{t+j} \tag{B.25}$$

$E_t z_{t+j}$ follows:

$$\begin{bmatrix} E_t r_{t+j}^n \\ E_t E_{t+j-1}^s r_{t+j-1}^n \\ E_t u_{t+j} \\ E_t E_{t+j-1}^s u_{t+j-1} \end{bmatrix} = \begin{bmatrix} \phi & 0 & 0 & 0 \\ K_{11}F_1 + K_{12} & \phi(1 - K_{11}F_1 - K_{12}) & K_{11}F_3 + K_{13} & -\phi^u(K_{11}F_3 + K_{13}) \\ 0 & 0 & \phi^u & 0 \\ K_{21}F_1 + K_{22} & -\phi(K_{21}F_1 + K_{22}) & K_{21}F_3 + K_{23} & \phi^u(1 - K_{21}F_3 - K_{23} \end{bmatrix} \begin{bmatrix} E_t r_{t+j-1}^n \\ E_t E_{t+j-2}^s r_{t+j-2}^n \\ E_t u_{t+j-1} \\ E_t E_{t+j-2}^s u_{t+j-2} \end{bmatrix} \tag{B.26}$$

$$\begin{bmatrix} E_t \pi_{t+j} \\ E_t \hat{y}_{t+j} \end{bmatrix} = \Gamma \Lambda^{j-1} E_t z_{t+1} \tag{B.27}$$

Substitute into the $E_t(L(t+1))$:

$$\Sigma \beta^j E_t z_{t+1}' (\Lambda^{j-1})' \Gamma' \Omega \Gamma \Lambda^{j-1} E_t z_{t+1} \equiv \Sigma \beta^j E_t z_{t+1}' \Theta_{j-1} E_t z_{t+1} \tag{B.28}$$

Take the first order condition on i_t^* of $E_t L(t+1)$:

$$\left\{\frac{\partial E_t\pi_t}{\partial i_t^*}E_t\pi_t+\omega\frac{\partial E_t\hat{y}_t}{\partial i_t^*}E_t\hat{y}_t\right\}+\frac{1}{2}\Sigma_{j=1}^{\infty}\beta^j\Delta(j-1)=0 \tag{B.29}$$

where

$$\begin{aligned}
\Delta_{j-1} &= (\Theta_{j-1}^{21}+\Theta_{j-1}^{12})\phi r_t^n\frac{\partial E_t^s r_t^n}{\partial i_t}+(\Theta_{j-1}^{32}+\Theta_{j-1}^{23})\phi^u u_t\frac{\partial E_t^s r_t^n}{\partial i_t}+(\Theta_{j-1}^{42}+\Theta_{j-1}^{24})E_t^s u_t\frac{\partial E_t^s r_t^n}{\partial i_t}+\Theta_{j-1}^{22}\cdot 2E_t^s r_t^n\frac{\partial E_t^s r_t^n}{\partial i_t}\\
&\quad+(\theta_{j-1}^{41}+\Theta_{j-1}^{14})\phi r_t^n\frac{\partial E_t^s u_t}{\partial i_t}+(\Theta_{j-1}^{43}+\Theta_{j-1}^{34})\phi^u u_t\frac{\partial E_t^s u_t}{\partial i_t}+(\Theta_{j-1}^{42}+\Theta_{j-1}^{24})E_t^s r_t^n\frac{\partial E_t^s u_t}{\partial i_t}+\Theta_{j-1}^{44}\cdot 2E_t^s u_t\frac{\partial E_t^s u_t}{\partial i_t}\\
&\quad\equiv \Delta_{j-1}(1)r_t^n+\Delta_{j-1}(2)u_t+\Delta_{j-1}(3)E_t^s u_t+\Delta_{j-1}(4)E_t^s r_t^n\\
&\quad+\Delta_{j-1}(5)r_t^n+\Delta_{j-1}(6)u_t+\Delta_{j-1}(7)E_t^s r_t^n+\Delta_{j-1}(8)E_t^s u_t
\end{aligned}$$

To solve for the first order condition on interest rate, first write equilibrium variables in terms of i_t:

Beliefs:

$$\begin{aligned}
E_t^s r_t^n &= (\phi(1-K_{11}F_1-K_{12})-K_{11}F_2)E_{t-1}^s r_{t-1}^n-(K_{11}F_4+\phi^u(K_{11}F_3+K_{13}))E_{t-1}^s u_{t-1} && \text{(B.30)}\\
&\quad+K_{12}r_t^n+K_{13}u_t+K_{11}i_t\\
E_t^s u_t &= (\phi^u(1-K_{21}F_3-K_{23})-K_{21}F_4)E_{t-1}^s u_{t-1}-(\phi(K_{21}F_1+K_{22})+K_{21}F_2)E_{t-1}^s r_{t-1}^n && \text{(B.31)}\\
&\quad+K_{22}r_t^n+K_{23}u_t+K_{21}i_t && \text{(B.32)}
\end{aligned}$$

Output gap:

$$\hat{y}_t=\Xi(1)E_t^s r_t^n+\Xi(2)E_t^s u_t-\frac{1}{\sigma}i_t+\frac{1}{\sigma}\frac{1}{1-\phi}r_t^n \tag{B.33}$$

Inflation:

$$\pi_t=\kappa\theta\hat{y}_t+\Xi(3)E_t^s r_t^n+\Xi(4)E_t^s u_t-(1-\theta)\frac{\kappa}{\sigma}i_t+u_t \tag{B.34}$$

Substitute the above endogenous variables into the first order condition on i_t:

$$\lambda_1 E_t^s r_t^n+\lambda_2 E_t^s u_t+\lambda_3 r_t^n+\lambda_4 u_t+\lambda_5 i_t=0 \tag{B.35}$$

where

$$\lambda_1 = \left(\kappa\theta\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\Xi(1) + \frac{\partial \pi_t}{\partial i_t}\Xi(3) + \frac{1}{2}\Sigma\beta^j\left(\Delta_{j-1}(4) + \Delta(7)\right) \tag{B.36}$$

$$\lambda_2 = \left(\kappa\theta\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\Xi(2) + \frac{\partial \pi_t}{\partial i_t}\Xi(4) + \frac{1}{2}\Sigma\beta^j\left(\Delta_{j-1}(3) + \Delta(8)\right) \tag{B.37}$$

$$\lambda_3 = \left(\kappa\theta\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\frac{1}{\sigma}\frac{1}{1-\phi} + \frac{1}{2}\Sigma\beta^j\left(\Delta_{j-1}(1) + \Delta(5)\right) \tag{B.38}$$

$$\lambda_4 = \frac{\partial \pi_t}{\partial i_t} + \frac{1}{2}\Sigma\beta^j\left(\Delta(2) + \Delta(6)\right) \tag{B.39}$$

$$\lambda_5 = \left(\kappa\theta\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\left(-\frac{1}{\sigma}\right) + \frac{\partial \pi_t}{\partial i_t}\left(-(1-\theta)\frac{\kappa}{\theta}\frac{1}{\sigma}\right) \tag{B.40}$$

and partial derivatives are derived as:

$$\frac{\partial E_t^s r_t^n}{\partial i_t} = K_{11} \tag{B.41}$$

$$\frac{\partial E_t^s u_t}{\partial i_t} = K_{21} \tag{B.42}$$

$$\frac{\partial \hat{y}_t}{\partial i_t} = \Xi(1)\frac{\partial E_t^s r_t^n}{\partial i_t} + \Xi(2)\frac{\partial E_t^s u_t}{\partial i_t} - \frac{1}{\sigma} \tag{B.43}$$

$$\frac{\partial \pi_t}{\partial i_t} = \kappa\theta\frac{\partial \hat{y}_t}{\partial i_t} + \Xi(3)\frac{\partial E_t^s r_t^n}{\partial i_t} + \Xi(4)\frac{\partial E_t^s u_t}{\partial i_t} - (1-\theta)\frac{\kappa}{\sigma} \tag{B.44}$$

Further substitute $E_t^s r_t^n$ and $E_t^s u_t$:

$$\begin{aligned}
0 = {} & \lambda_1\left\{(\phi(1-K_{11}F_1-K_{12}) - K_{11}F_2)E_{t-1}^s r_{t-1}^n - (K_{11}F_4 + \phi^u(K_{11}F_3+K_{13}))E_{t-1}^s u_{t-1} + K_{12}r_t^n + K_{13}u_t + K_{11}i_t\right\} \\
& + \lambda_2\left\{(\phi^u(1-K_{21}F_3-K_{23}) - K_{21}F_4)E_{t-1}^s u_{t-1} - (\phi(K_{21}F_1+K_{22}) + K_{21}F_2)E_{t-1}^s r_{t-1}^n + K_{22}r_t^n + K_{23}u_t + K_{21}i_t\right\} \\
& + \lambda_3 r_t^n + \lambda_4 u_t + \lambda_5 i_t
\end{aligned}$$

The above equation solves the optimal nominal interest rate. Comparing with the guessed form yields the solution of $[F_1,\ F_2,\ F_3,\ F_4]$

$$F_1 = -\frac{\lambda_1 K_{12} + \lambda_2 K_{22} + \lambda_3}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \tag{B.45}$$

$$F_2 = -\frac{\lambda_1 \left(\phi(1 - K_{11}F_1 - K_{12}) - K_{12}F_2\right) - \lambda_2 \left(\phi(K_{21}F_1 + K_{22}) + K_{21}F_2\right)}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \tag{B.46}$$

$$F_3 = -\frac{\lambda_1 K_{13} + \lambda_2 K_{23} + \lambda_4}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \tag{B.47}$$

$$F_4 = -\frac{-\lambda_1 \left(K_{11}F_4 + \phi^u(K_{11}F_3 + K_{13})\right) + \lambda_2 \left(\phi^u(1 - K_{21}F_3 - K_{23}) - K_{21}F_4\right)}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \tag{B.48}$$

I iterate the process until the conjectured interest rate function matches the above solution.

C Proofs

C.1 Proof of Lemma 2

The first (change of slope) and second point (intercept after cost push shock) are obvious. The following shows the proof for the their point (the sign of intercept after natural rate shock). The sign depends on the combination of parameter values and expectation on interest rate reaction function. Specifically, from Kalman Filtering, we know that $K_{11}F_1 + K_{21}F_3 = 1$, which indicates $0 < \frac{\partial E_t^s r_t^n}{\partial r_t^n} < 1$ and $0 < \frac{\partial E_t^s u_t}{\partial u} < 1$ as long as both F_1 and F_3 are non-zero. However, $K_{11} = \frac{\partial E_t^s r_t^n}{\partial i_t}$ may be greater or less than one, depending on whether F_1 is greater or smaller than 1. Consequently, the sign of the impact of r_t^n on the intercept of Phillips Curve under imperfect information, $\left\{(1-\theta)\frac{\kappa}{\sigma}(K_{11}-1) + \frac{1-\theta}{\theta}K_{21}\right\}$ is ambiguous.

C.2 Second Order Approximation of Household's Utility Function

Follow Woodford (2003), Gali (2010), Walsh (2010) to prove that maximizing the utility of household is equivalent, up to second order approximation, to

$$W = -\frac{1}{2}E_0\Sigma\beta^t\left((\varepsilon^{-1}+\varphi)\varepsilon^2 var_j(p_t(j)) + (\sigma+\varphi)\hat{y}_t^2\right) \tag{C.1}$$

The next step is to prove the relationship between $var_j(p_t(j))$ with $var(\pi_t)$. Denote $\Delta_t = var_j[logp_{jt}]$. Since $var_j\bar{P}_{t-1} = 0$, we have

$$\begin{aligned}\Delta_t &= var_j[logp_{jt} - \bar{P}_{t-1}] \\ &= E_j[logp_{jt} - \bar{P}_{t-1}]^2 - [E_j logp_{jt} - \bar{P}_{t-1}]^2 \\ &= E_j[logp_{jt-1} - \bar{P}_{t-1}]^2 + (1-\theta)(\int p_{tj}^* - \bar{P}_{t-1})^2 - (\bar{P}_t - \bar{P}_{t-1})^2\end{aligned} \tag{C.2}$$

As noted in Appendix A, $\bar{P}_t = (1-\theta)\int p_{tj}^* + \theta\bar{P}_{t-1}$, we have $(1-\theta)\int logp_{tj}^* + \theta\bar{p}_{t-1}$, which implies that $(1-\theta)\int logp_{tj}^* - (1-\theta)p_{t-1} = \bar{p} - \bar{p}_{t-1}$. So, we have:

$$\int logp_{tj}^* = \left(\frac{1}{1-\theta}\right)(\bar{p}_t - \bar{p}_{t-1}) \tag{C.3}$$

Substitute this into (D.2) and get $\Delta_t = \theta\omega_{t-1} + \left(\frac{\theta}{1-\theta}\right)(\bar{p} - \bar{p}_{t-1})^2$. Applying the definition of inflation results in:

$$E_t\Sigma\beta^t\Delta_t = \frac{\theta}{(1-\theta)(1-\theta\beta)}E_t\Sigma\beta^t\pi_t^2 + t.i.p. \tag{C.4}$$

C.3 Proof of Lemma 4

To prove the optimal response for a discretionary rate, express out the inflation expression in Lemma 3, $\pi_t = -R\hat{y}_t$ as follows:

$$\kappa\hat{y}_t + \Omega_r\left(K_r - 1\right)r_t^n + \Omega_u K_u + u_t = -R\hat{y}_t \tag{C.5}$$

Then, substitute $\hat{y}_t$ and π_t by interest rate, which results in:

$$\left[(R+\kappa)\frac{1}{\sigma} - \Omega_r\right] r_t^n + u_t = \left[(R+\kappa)\frac{1}{\sigma} - \left(\Omega_r K_r + \Omega_u K_u\right)\right] i_t \tag{C.6}$$

To prove that $F_r > 1$ is equivalent to prove $\Omega)rK_r + \Omega_u K_u > \Omega_r$ which is just Assumption 1.

To prove that $F_u < \theta F_u^p$ is equivalent to prove:

$$\begin{aligned}(R^p+\kappa)\frac{1}{\sigma} &< (R+\kappa)\frac{1}{\sigma} - \left[\Omega_r K_r + \Omega_u K_u\right]\\ \Leftrightarrow \Omega_r K_r + \Omega_u K_u &< \frac{1}{\sigma}\left(R - R^p\right)\end{aligned}$$

As $(\hat{y}_t, \pi_t)$ is chosen optimally by the central bank, we know that $R \cdot slope = \omega$ where ω is in the CB's objective function:

$$\pi_t^2 + \omega\hat{y}_t^2$$

So,

$$\frac{1}{\sigma}\left(\frac{\omega}{R^p} - \frac{\omega}{R}\right) = (1-\theta)\frac{\kappa}{\sigma}K_r + \frac{1-\theta}{\theta}K_u \tag{C.7}$$

Importantly, the RHS is just the difference between R_p and R, as it measures the sensitivity inflation to beliefs.

Substitute this into the above equation leads to:

$$\begin{aligned}\frac{\omega}{R^p} - \frac{\omega}{R} &< R - R^p\\ \Leftrightarrow \omega &< R^p \cdot R\end{aligned}$$

As Lemma 3 suggests that $R_{>}R^p$, the sufficient condition for the above inequality to hold is that $R > \bar{R} \equiv \sqrt{\omega}$, where ω is defined as the second order approximation of household utility.

C.4 Proof of Proposition 1

The proof of Proposition 1 take two steps. First I show that evaluated at the equilibrium optimizing discretionary interest rate, the partial derivative of combined Kalman gains weighted by their effects on inflation is negative with respect to F_r and is positive with respect to F_u.

Step 1: Under the assumption that $\sigma_r = \sigma_u$, I prove the following result.

Result 1. $\Omega_r \frac{\partial K_r}{\partial F_r} + \Omega_u \frac{\partial K_u}{\partial F_r} < 0$

Result 2. $\Omega_r \frac{\partial K_r}{\partial F_u} + \Omega_u \frac{\partial K_u}{\partial F_u} > 0$

The partial derivatives of Kalman gains on F:

$$\frac{\partial K_r}{\partial F_r} = D^{-1}\left(F_u^2\sigma_r^2\sigma_u^2 - F_r^2\sigma_r^2\sigma_r^2\right)$$
$$\frac{\partial K_u}{\partial F_u} = D^{-1}\left(F_r^2\sigma_r^2\sigma_u^2 - F_u^2\sigma_u^2\sigma_u^2\right)$$
$$\frac{\partial K_u}{\partial F_r} = D^{-1}\left(2F_rF_u\sigma_r^2\sigma_u^2\right)$$
$$\frac{\partial K_r}{\partial F_u} = D^{-1}\left(2F_rF_u\sigma_r^2\sigma_u^2\right)$$

where $D = F_r^2\sigma_r^2 + F_u^2\sigma_u^2$.

To show Result 1, notice that $\frac{\partial K_u}{\partial F_r} < 0$, so it becomes sufficient to show $\frac{\partial K_r}{\partial F_r}$. This inequality holds when $F_u < F_r$ under discretion. From Lemma 4, we know that $F_r > F_r^p = 1$, and $F_u < \theta F_u^p$. In addition, we can derive the optimizing discretionary interest rate under perfect information to be: $F_u^p = \frac{\kappa^2}{1+\kappa^2}\frac{\sigma}{\theta}$, which leads to $F_u < 1$. Under $\sigma_r =$ and $F_r > F_u$, we have $\frac{\partial K_r}{\partial F_r} < 0$.

To show Result 2 is involves more steps. First, we know that $\frac{\partial K_r}{\partial F_u}$,0, and under parameter specifications, $0 < \Omega_r < \Omega_u$. Thus, the sufficient condition for Result 2 to be hold is $\Omega_u \frac{\partial K_r}{\partial F_u} + \Omega_u \frac{\partial K_u}{\partial F_u} > 0$, which is equivalent to prove that $\frac{\partial K_r}{\partial F_u} + \frac{\partial K_u}{\partial F_u} > 0$.

From the partial derivatives of K to F, and with the assumption $\sigma_r = \sigma_u$, we have

$$\frac{\partial K_r}{\partial F_u} + \frac{\partial K_u}{\partial F_u} = D^{-1}\left(F_r^2\sigma_r^2\sigma_u^2 - F_u^2\sigma_u^2\sigma_u^2 - 2F_rF_u\sigma_r^2\sigma_u^2\right) \tag{C.8}$$

Holding F_r fixed, the quadratic function $f(F_u) = F_r^2\sigma_r^2\sigma_u^2 - F_u^2\sigma_u^2\sigma_u^2 - 2F_rF_u\sigma_r^2\sigma_u^2$ reaches maximum at negative value of F_u. The range of F_u is $[0, F_r]$ (Lemma 4). So, I only need to show $f(0) > 0$ and $f(F_r) < 0$.

$$f(0) = F_r^2\sigma_r^2\sigma_u^2 > 0 \tag{C.9}$$
$$f(F_r) = -2F_rF_r\sigma_r^2\sigma_u^2 < 0 \tag{C.10}$$

This completes the prove of Result 2.

Step 2: I prove that Result 1 and Result 2 makes Proposition 2 to hold.

Part 1, $F_r^c > F_r^d$

Suppose on the contrary that $F_r^c = F_r^d$. Then, express out the first order condition on F_r after

r_t^n shock for both discretionary central bank and central bank with commitment, which is $\pi_t \frac{\partial \pi_t}{\partial F_r} + \omega \hat{y}_t \frac{\partial \hat{y}_t}{\partial F_r} = 0$. As $\Omega_u > 0$ and $D > 0$

$$-\frac{\kappa}{\sigma} + \Omega_r K_r + \Omega_r \frac{\partial K_r}{\partial F_r} F_r + \Omega_u K_u + \Omega_u \frac{\partial K_u}{\partial F_r} F_r |_{comm} = \frac{\omega}{\sigma} \frac{\hat{y}_t}{\pi_t} |_{comm} \tag{C.11}$$

$$-\frac{\kappa}{\sigma} + \Omega_r K_r + \Omega_u K_u |_{disc} = \frac{\omega}{\sigma} \frac{\hat{y}_t}{\pi_t} |_{disc} \tag{C.12}$$

Under Assumption 1, The LHS of E.1 is smaller than the LHS of E.2, which makes $\frac{\hat{y}_t}{\pi_t}$ to be smaller for commitment central bank. Substitute $\hat{y}_t = -\frac{1}{\sigma} (F_r - 1) r_t^n$, we have

$$\frac{F_r^c}{\pi_t^c} > \frac{F_r^d}{\pi_t^d} \tag{C.13}$$

The necessary condition for this inequality to hold is 1: $F_r^c > F_r^d$, or $\pi_t^c < \pi_t^d$.

Next, write out the expression for $\pi_t^c < \pi_t^d$:

$$-\frac{1}{\sigma} (F_r - 1) + \Omega_r K_r F_r + \Omega_u K_u F_r |_{comm} < -\frac{1}{\sigma} (F_r - 1) + \Omega_r K_r F_r + \Omega_u K_u F_r |_{disc} \tag{C.14}$$

The necessary condition for this inequality to hold is: either $F_r^c > F_r^d$, or $\Omega_r K_r F_r + \Omega_u K_u F_r |_{comm} < \Omega_r K_r F_r + \Omega_u K_u F_r |_{disc}$. Under Assumption, this implies $F_r^c > F_r^d$. In conclusion, we have $F_r^c > F_r^d$.

Part 2, $F_u^c < F_u^d$

Suppose on the contrary that $F_u^c = F_u^d$. Then, express out the first order condition on F_u after u_t shock for both discretionary central bank and central bank with commitment, which is $\pi_t \frac{\partial \pi_t}{\partial F_u} + \omega \hat{y}_t \frac{\partial \hat{y}_t}{\partial F_u} = 0$

$$-\frac{\kappa}{\sigma} + \Omega_r K_r + \Omega_r \frac{\partial K_r}{\partial F_u} F_u + \Omega_u K_u + \Omega_u \frac{\partial K_u}{\partial F_u} F_u |_{comm} = \frac{\omega}{\sigma} \frac{\hat{y}_t}{\pi_t} |_{comm} \tag{C.15}$$

$$-\frac{\kappa}{\sigma} + \Omega_r K_r + \Omega_u K_u |_{disc} = \frac{\omega}{\sigma} \frac{\hat{y}_t}{\pi_t} |_{disc} \tag{C.16}$$

Under Assumption 2, The LHS of E.5 is smaller than the LHS of E.6, which makes $\frac{\hat{y}_t}{\pi_t}$ to be bigger for commitment central bank. Substitute $\hat{y}_t = -\frac{1}{\sigma} F_u u_t$, we have

$$\frac{F_u^c}{\pi_t^c} < \frac{F_u^d}{\pi_t^d} \tag{C.17}$$

The necessary condition for this inequality to hold is 1: $F_u^c < F_u^d$, or $\pi_t^c > \pi_t^d$.

Next, write out the expression for $\pi_t^c < \pi_t^d$ after u_t shock:

$$-\frac{1}{\sigma}F_u + \Omega_r K_r F_u + \Omega_u K_u F_u|_{comm} < -\frac{1}{\sigma}F_u + \Omega_r K_r F_u + \Omega_u K_u F_u|_{disc} \tag{C.18}$$

The necessary condition for this inequality to hold is: either $F_u^c < F_u^d$, or $\Omega_r K_r F_u + \Omega_u K_u F_u|_{comm} < \Omega_r K_r F_u + \Omega_u K_u F_u|_{disc}$. Under Assumption 2, this implies $F_u^c < F_u^d$. In conclusion, we have $F_u^c > F_u^d$.

C.5 Proof of Proposition 2

As analyzed in Section 3.3.2, the optimal polity rule satisfies:

$$-\frac{\kappa}{\sigma} + \Omega_r\left(K_r + \frac{\partial K_r}{\partial F_r}F_r\right) + \Omega_u\left(K_u + \frac{\partial K_u}{\partial F_u}F_r\right) = \frac{\omega}{\sigma}\frac{\hat{y}_t}{\pi_t} \tag{C.19}$$

$$-\frac{\kappa}{\sigma} + \Omega_r\left(K_r + \frac{\partial K_r}{\partial F_r}F_r\right) + \Omega_u\left(K_u + \frac{\partial K_u}{\partial F_u}F_r\right) = \frac{\omega}{\sigma}\frac{\hat{y}_t}{\pi_t} \tag{C.20}$$

After K_r and K_u are determined, the optimal one-time discretionary interest rate aims to minimize the ex-post loss:

$$\pi_t^2 + \omega \hat{y}_t^2 \tag{C.21}$$

subjected to the constraint that $\pi_t = \{\kappa - \sigma(\Omega_r K_r + \Omega_u K_u)\}\hat{y}_t + (\Omega_r(K_r - 1) + \Omega_u K_u) r_t + u$. The first order condition of interest rate satisfies:

$$-\frac{\kappa}{\sigma} + \Omega_r K_r + \Omega_u K_u = \frac{\omega}{\sigma}\frac{\hat{y}_t}{\pi_t} \tag{C.22}$$

As long as $F_r \neq 0$ and $F_u \neq 0$, equation (C.16) is different from equation (C.13) and (C.14), which implies that one-time discretionary interest rate is different from the response of interest rate under optimal policy rule. Since the one-time discretionary interest rate is derived from the ex-post loss minimization problem, it implies that there is always a profitable deviation.