Lecture Note 14: Unit Commitmment

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1 Introduction

So far, we have talked about the economic dispatch problem as if we solve it once, using a simple LP for finding the optimal generation and demand allocations. However, this is not how the ISOs actually decide on how to allocate. Instead, as mentioned briefly, there are several stages of allocation at various points in time. A key issue that we mentioned last time is that many types of power-generating plants require long startup and shutdown times (on the order of hours to a day). This is one reason to consider day-ahead (DA) markets, where we commit some plants to producing energy on the following day. Beyond startup and shutdown times, another attractive property of DA markets is that they reduce uncertainty for the parties that settle on generation and load taking in the DA market. This may, for example, simplify staffing scheduling.

2 Unit Commitment

In this section we study how to handle binary operational decisions. For example, a nuclear or coal power plant must decide ahead of time whether to commit to turning the plant on or not. If they do commit, they usually have some minimum power output level (in addition to an upper bound), and if they do not, then they cannot generate any power. This binary decision problem obviously causes some problems for our market-based mechanism from the last lecture note: we used strong duality to get locational marginal prices for each node in the network. But with binary variables, we will not have strong duality! This section will discuss a few potential remedies to this problem, though none of them are perfect.

For simplicity, let us consider a single-node problem, where demand is fixed at p_d. Then we get the following market clearing problem with non-convexity due to binary decisions, which is a mixed-integer linear program (MILP):

$$
\begin{aligned}
\min_{p,z} \quad & \sum_{g \in \Psi^G} c_g p_g + C_g z_g && && (1)\\
\text{s.t.} \quad & \sum_{g \in \Psi^G} p_g \geq p_d && && (2)\\
& p_g \leq z_g \overline{p}_g, && \forall g \in \Psi^G && (3)\\
& p_g \geq z_g \underline{p}_g, && \forall g \in \Psi^G && (4)\\
& z_g \in \{0,1\} && \forall g \in \Psi^G && (5)
\end{aligned}
$$

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Now suppose we solve this problem, and get a set of optimal binary variables z^*. Then it turns out that we can in fact construct prices using these binary variables. The idea is to introduce a continuous version of the MILP, where we constrain each continuous variable z_g to take on exactly the value z_g^*, and then we will use the Lagrange multiplier on that constraint to price the non-convexity. This yields the following LP, which we call EDLP:

$$\min_{p,z} \sum_{g \in \Psi^G} c_g p_g + C_g z_g \tag{6}$$

$$\text{s.t.} \sum_{g \in \Psi^G} p_g \geq p_d \tag{7}$$

$$p_g \leq z_g \overline{p}_g, \qquad \forall g \in \Psi^G \tag{8}$$

$$p_g \geq z_g \underline{p}_g, \qquad \forall g \in \Psi^G \tag{9}$$

$$z_g = z_g^* \qquad \forall g \in \Psi^G \tag{10}$$

Now consider an optimal solution x^*, z^*, and let λ^* be the corresponding Lagrange multiplier on Eq. (20) and μ_g^* be the Lagrange multiplier for Eq. (10) for each g. We will pay λ^* for generating electricity, and for each generator g such that $z_g^* = 1$, we pay them μ_g^* for turning on.

This turns out to yield a market equilibrium, as we will now show. Consider a generator g; they wish to solve the following problem:

$$\max_{p_g, z_g} \sum_{g \in \Psi^G} (\lambda^* - c_g) p_g + (\mu_g - C_g) z_g \tag{11}$$

$$\text{s.t. } p_g \leq z_g \overline{p}_g \tag{12}$$

$$p_g \geq z_g \underline{p}_g \tag{13}$$

$$z_g \in \{0, 1\} \tag{14}$$

One way to solve this problem is to make z_g continuous, and hope that an integral solution happens to pop out. That yields the following program

$$\max_{p_g, z_g} \sum_{g \in \Psi^G} (\lambda^* - c_g) p_g + (\mu_g - C_g) z_g \tag{15}$$

$$\text{s.t. } p_g \leq z_g \overline{p}_g \tag{16}$$

$$p_g \geq z_g \underline{p}_g \tag{17}$$

$$z_g \in \mathbb{R} \tag{18}$$

Clearly an optimal solution to this problem upper bounds the optimal solution to the integral version. But now it is easy to see that if we form the Lagrangian of EDLP:

$$\min_{p,z} \sum_{g \in \Psi^G} c_g p_g + C_g z_g + \lambda^* \left(p_d - \sum_{g \in \Psi^G} p_g \right) + \sum_{g \in \Psi^G} \mu_g^* \left(z_g^* - z_g \right) \tag{19}$$

$$\text{s.t.} \tag{20}$$

$$p_g \leq z_g \overline{p}_g, \qquad \forall g \in \Psi^G \tag{21}$$

$$p_g \geq z_g \underline{p}_g, \qquad \forall g \in \Psi^G, \tag{22}$$

then we get a problem which includes exactly the same constraints on p_g, z_g, and has the same coefficients in the objective. But then by strong duality we know that $p_g = p_g^*, z_g = z_g^*$ is an optimal solution to this problem, which shows that it must be an optimal solution to the LP for generator i.

While the above approach was described in the context of unit commitment, it works much more broadly. If a generator has multiple binary decision then we can simply add one per constraint per decision, and we will then get a price for each of their binary decisions.

One drawback of this pricing approach is that it tends to produce highly volatile prices, which can be both negative and positive. This can lead to prices that can seem very unfair (and materialize suddenly through minor changes to the pricing problem). A second concern is that we may no longer have budget balance, meaning that the ISO could potentially fall short on money due to the unit commitment prices.

3 Uplift Payments

In practice, ISOs often use what are called *uplift payments*. Uplift payments are an asymmetric variant of the previous pricing approach. The ISO will compute only locational marginal prices. Then, for generators with discrete decisions such as unit commitment, if the LMPs do not support their assigned decisions and power output, the ISO will pay the difference. Note that this can make the generator better or worse off depending on context. For example, μ_g being negative is ignored which helps the generator, but when μ_g is positive the uplift payment could be smaller than μ_g still.

4 Convex Hull Pricing

An alternative pricing approach is that of *convex hull pricing* (CH pricing). CH pricing is very easy to set up. We simply Lagrangify the demand constraint, and solve the resulting minimization problem over electricity prices. Formally, we solve

$$\min_{\lambda} q(\lambda),$$

where $q(\lambda)$ is defined as

$$q(\lambda) := \begin{array}{lll} \min\limits_{p,z} & \sum\limits_{g \in \Psi^G} c_g p_g + C_g z_g + \lambda\big(p_d - \sum\limits_{g \in \Psi^G} p_g\big) & \\ \text{s.t.} & p_g \le z_g \overline{p}_g, & \forall g \in \Psi^G \\ & p_g \ge z_g \underline{p}_g, & \forall g \in \Psi^G \\ & z_g \in \{0,1\} & \forall g \in \Psi^G \end{array}$$

From an optimization perspective this approach has some attractive properties, especially the fact that given a fixed λ, solving $q(\lambda)$ decomposes into simple per-generator optimization problems. On the other hand, since we do not have strong duality, this approach does not necessarily give us a feasible solution. In practice, the resulting CH prices λ^* would be extracted, but the allocation would use the original MILP for finding a feasible allocation. This means that in general CH pricing will not be such that generators get allocations that are in their demand set. To fix this issue, ISOs would then provide additional uplift payments.

5 Connecting DA and RT Markets

So far we have discussed RT and DA markets in isolation. In practice, the RT market operates after a number of contracts for consumption and generation have been settled in the DA market. For example, suppose a generator was assigned 100 megawatt (MW) of generation for an RT period, but it turns out that they will only be able to produce 97MW. In that case, the remaining 3MW must be purchased in the RT market. Financially speaking, the generator would then be viewed as having purchased 3MW of power in that RT market. Similarly, a demand that purchased 100MW of power in the DA market but then consumed only 90MW would be viewed as selling 10MW of power in the RT market. In general, we can view the RT market as a balancing operation that corrects any imbalances that occur due to increased or decreased consumption or generation specified in the DA market.

If not for uncertainty, it is easy to convince yourself that the price in the DA and RT markets should be the same. If they were not, then any generator that was assigned to generate in the market with the lower price would simply wish to change their bids such that they end up getting assigned the same generation in the market with the higher price. A similar argument holds for demands.

A key reason why the RT market may nonetheless require balancing is that consumer electricity usage as forecasted in the DA market will differ from the realized usage in the RT market. This causes relatively manageable imbalances in the market, and the ISO needs to correct these imbalances in order to keep the system functioning. A second and more severe imbalance issue that can occur is generator outages. A generator outage can lead to large imbalances that require significant additional generation allocation in the RT market.

Due to these imbalances, and the very short-term nature of the RT market, flexible generation and consumption entities will be rewarded at a higher rate in the RT market when realized demands turns out to be higher than realized generation. On the other hand, expensive generators that are primarily used to cover the case of excess demand in the RT market will not make any money when realized demand is lower than realized generation. Thus, the cost of generation for such plants is often high, which can lead to higher volatility in RT market prices.

6 Historical Notes

The approach for pricing binary decision by using the MIP solution as constraints in the LP was introduced by O'Neill et al. [2]. Convex hull pricing was introduced by Gribik et al. [1].

References

[1] Paul R Gribik, William W Hogan, Susan L Pope, et al. Market-clearing electricity prices and energy uplift. *Cambridge, MA*, pages 1–46, 2007.

[2] Richard P O'Neill, Paul M Sotkiewicz, Benjamin F Hobbs, Michael H Rothkopf, and William R Stewart Jr. Efficient market-clearing prices in markets with nonconvexities. *European journal of operational research*, 164(1):269–285, 2005.