Default, Commitment, and Domestic Bank Holdings of Sovereign Debt†

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Abstract

How do the incentives of domestic banks and sovereign governments interact? This paper presents a model of government default and banks that invest in the debt of their own sovereign. In the model, banks demand safe assets to use as collateral, and default affects bank equity. These losses inhibit banks’ ability to attract deposits, leading to lower private credit provision, and lower output. This disincentivizes the sovereign from defaulting. The extent of output losses depends on characteristics of the banking system, including sovereign exposures, equity, and deposits. In turn, bank exposures are affected by default risk. The model is also used to show that policies such as financial repression can improve welfare, but worsen output losses in the event of default.

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1 Introduction

Financial institutions generally hold large amounts of sovereign debt, and, in particular, the debt of their own sovereign. A recent literature has documented this globally (Gennaioli et al., 2014a), and specifically in the context of the European sovereign debt crisis (Broner et al., 2014; Brutti and Saure, 2014). In turn, a growing literature has explored the implications of these holdings for sovereign default, noting that the ability of foreigners to sell their bonds to domestic investors makes selective defaults difficult (Broner et al., 2010), emphasizing that with large bank holdings, sovereign default leads to contractions in credit and output losses (Gennaioli et al., 2014b; Perez, 2015), and arguing that financial repression may explain some of these holdings, and that such policies may be motivated by generating a commitment to not default (Becker and Ivashina, 2014; Chari et al., 2015).

This paper provides a three-period model to further explore how domestic bank holdings of sovereign debt interact with sovereign default and financial repression. In particular, I provide a joint answer to the following three questions: How do domestic banks affect the default incentives of the sovereign? Second, how does sovereign risk affect domestic demand for government debt? Third, how do these holdings affect government policy towards banks, and in particular, financial repression?

The model has two central ingredients and a series of predictions that follow from them. First, banks demand government debt because it serves as collateral in interbank markets, which is valuable for liquidity management, modeled as banks being matched with investment projects that can exceed what they collect as deposits. This use of government debt is emphasized by Bolton and Jeanne (2011) and Gorton and Ordoñez (2013). It implies that banks’ demand for government bonds is downward-sloping, and depends on the extent of liquidity needs as well as on the expected return on the bond. Banks with access to investment opportunities choose to invest in government debt in order to collateralize interbank lending. Without unfettered access
to alternative safe assets, higher sovereign risk may increase the demand for sovereign debt because banks require government assets to pledge as collateral.

Second, banks are a crucial source of credit to firms. However, banks are constrained in their lending by the amount of equity they hold, since financial frictions in the form of an agency problem on bankers that can divert funds puts a limit on their ability to attract deposits. When there is a default on government bonds, bank equity falls, which reduces the ability of banks to attract deposits. This in turn limits the bank funding available to the real economy and leads to declines in output. These endogenous output losses provide a commitment technology to sovereigns, and default costs increase with domestic banks’ holdings of government debt. This also provides a microfounded explanation for the secondary market theory of Broner et al. (2010), demonstrating one way in which bonds are more valuable in the hands of domestic agents.

As in any canonical model of sovereign default, the probability of default is increasing in the amount of foreign borrowing and there is a Laffer curse on the total amount of resources borrowed from abroad. However, the output losses from default in this model are endogenous. Because the output losses are endogenous, the model can generate positive holdings of risky debt and default in equilibrium. Moreover, the model makes the surprising prediction that default is more likely when banks are well-capitalized. In the model, there can be deposit flight both in anticipation of and as a consequence of default, and higher levels of initial deposits are shown to be theoretically associated with lower default risk.

Like in Gennaioli et al. (2014b) and Perez (2015), sovereign defaults are followed by large contractions in private credit, and default has more severe effects but is less likely when banks hold more public debt. Unlike these authors, banks demand public debt to use as collateral, instead of as a store of liquidity. Relative to Perez (2015), this paper assumes that banks with investment opportunities have a motive to hold government bonds, and that default disrupts deposits. In contrast to Gennaioli et al. (2014b), lending to public and
private projects is contemporaneous, the return on both is uncertain, and the probability of default is determined endogenously as a function of the realized level of private productivity.

In addition, banks choose individually how many bonds to hold to maximize profits, but the government chooses to default to maximize social welfare. Because domestic banks take bond prices and default probabilities as given, yet their aggregate holdings of bonds affects the incentives of the policymaker to default, there is a pecuniary externality. Financial repression can increase welfare, since the competitive equilibrium leads to under-holding of government bonds. However, financial repression can seldom prevent default entirely, and in the event of default leads banks and households to be strictly worse off.

The rest of the paper is structured as follows. The remainder of this section reviews related literature. In Section 2 I present the model and define equilibrium. The results of the model are described in Section 3. Section 4 explores the externality that arises from domestic holdings of sovereign debt, and discusses the optimality of financial repression. In Section 5 I develop two extensions of the model, which incorporate a second safe asset, and foreign capital inflows to banks. The final section concludes.

1.1 Related literature

This paper relates to research on sovereign default, financial frictions, and bank holdings of government debt.

There is a considerable literature in economics focused on sovereign default. Canonical sovereign default models (Eaton and Gersovitz, 1981; Arellano, 2008) consider exclusion from financial markets and exogenously assumed output losses to discipline borrowers. That sovereign crises are often coincident with banking crises is shown by Reinhart and Rogoff (2008). Gennaioli et al. 1, 2

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1For surveys, see Eaton and Fernandez (1995) and Aguiar and Amador (2014).
2A number of authors note that this link has been particularly important in the evolving European crises, for example, Angeloni and Wolff (2012), Popov and van Horen (2013), and
(2014b) model banks that lend to the government as a store of liquidity, and can then make fewer private investments in the event of nonrepayment. They also provide empirical evidence that banks’ sovereign exposures are associated with post-default declines in credit. The current paper instead models a contemporaneous choice between public and private investment, and assumes public debt is necessary to collateralize interbank lending. In addition, in this paper the productivity of private investment is subject to uncertainty, and its realization affects bank and household income, which leads default to be a function of the realization of productivity as well as other parameters in the model. This characterization allows further scrutiny of the interaction between government and bank incentives.

A number of papers provide quantitative assessments of the link between government debt, domestic banks, and the real economy. Sosa Padilla (2015) provides a quantitative closed-economy model in which banks lend to the government and firms, and firms require external financing to pay workers up front. Other authors focus on identifying the balance sheet channel in addition to other effects: Bocola (2015) incorporates a risk channel by which banks lend less to a worsening economy, with exogenous default, and Perez (2015) considers the reduced value of public debt as liquidity, with endogenous default. In contrast, the main mechanism by which bank losses occur in this paper are via deposits, which shrink in anticipation and as a consequence of default.

In addition to the costs of exclusion and bank losses, well-functioning secondary markets can help support risky sovereign debt. Broner et al. (2010) argue that if domestic bondholders are relatively more capable of extracting value from a sovereign following a default, or if the sovereign is less likely to default on them, then in anticipation of a default foreigners sell their debts to domestic agents via secondary markets. As a result, it is difficult for governments to default selectively on foreigners. I show that secondary market theory has an additional underlying driver, in the commitment value of domes-

Brunnermeier et al. (2011).
tic creditors. In this model, domestic creditors affect the repayment incentives of debt in a way that better supports the value of debt, which produces a similar outcome as the political economy explanation that domestic agents are more likely to receive compensation from the government after a default.

Although I do not model secondary markets explicitly, I assume that the sovereign is unable to default selectively on foreign lenders. Sturzenegger and Zettelmeyer (2008) document empirically that following a sovereign default, both domestic and foreign creditors suffer losses. Erce (2012) explores historical patterns of selective default, and finds that it is more common for governments to default on domestic creditors when the banks are not weak, a key prediction of my model.

In addition, I draw from a large literature on the impact of credit disruptions on the real economy. Much research has shown how financial frictions can worsen an economic slump (Bernanke and Gertler, 1989; Bernanke et al., 1999). I adopt financial frictions following the form of Gertler and Kiyotaki (2010), and expand the role of government to allow for sovereign default. This provides a simple microfoundation for the credit disruption that occurs when a sovereign defaults on domestic banks.

There is also a large body of research that considers what motivates bank portfolios of sovereign debt. In advanced economies, many financial intermediaries use sovereign assets for risk and liquidity management purposes. Given that government debt often has a nearly “risk-free” status, banks use government debt as collateral for interbank loans or repos (Bolton and Jeanne, 2011). Based on this idea, I model banks requiring government debt for use as collateral in interbank markets. Sovereign debt is also used to access to public liquidity: banks can often access cheap lending from central banks collateralized by government debt and other highly rated securities.

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3 An additional reference for Gertler and Kiyotaki (2010) are teaching notes by Christiano and Zha (2010) and Groth (2011), who demonstrate a two-period version of the model.

4 In developing countries, bank holdings of government bonds may also be driven by a lack of alternatives due to the relatively underdeveloped nature of financial systems (Woodford, 1990; Holmstrom and Tirole, 1998; Kumhof and Tanner, 2008).
As for why banks demand debt of their own sovereign specifically, a number of theories complement the explanations above. Standard theories of portfolio allocation and home bias have some portion of each bank’s portfolio held domestically. Financial repression consists of governments forcing banks to hold sovereign debt or using “moral suasion,” to compel them to do so, and has been a prominent explanation both historically and in present times (Reinhart et al., 2003; Reinhart, 2012; Becker and Ivashina, 2014). In addition, governments may provide incentives for banks to hold domestic sovereign bonds. For example, in Europe, EU government debt receives a zero percent risk weight on the balance sheets of European banks, and is exempt from limits on large exposures.

Finally, this paper relates to a growing body of empirical research on the shifting patterns of sovereign debt ownership in current-day Europe. The height of the European debt crisis coincided with the beginning of a large-scale “repatriation” of sovereign debt holdings, particularly in Greece, Ireland, Italy, Portugal, and Spain (GIIPS). The percentage of debt held by domestic creditors in these countries increased since the mid-2000s, and rose dramatically around the height of the crisis. A number of explanations for this have been put forth: secondary market theory (Broner et al., 2014; Brutti and Saure, 2014), financial repression (Becker and Ivashina, 2014), ECB policy (Crosignani et al., 2015), and reaching for yield (Acharya and Steffen, 2015). This model provides a framework for thinking about risky sovereign debt that is owed both domestically and abroad, and how domestic holdings and default incentives interact.

2 Model

The model combines a canonical model of sovereign default with Gertler and Kiyotaki (2010) style financial frictions, to characterize the interaction between foreign lenders, government default and domestic banks. This is introduced using a three period setup, where the government can borrow only in $t = 0$ debt
that is due in $t = 1$. By using this framework, I abstract from the potential for exclusion from foreign financial markets, to characterize more sharply the role of domestic banks and endogenous output losses.

2.1 Setup

The timing of the model is as follows:

- In $t = 0$, the government borrows from foreign lenders and domestic banks to invest in a public project of fixed size $g$. Households begin with an initial endowment that they can consume or save at banks. Banks lend to the government, to other banks, and to firms. Foreign investors lend to the government. The realization of underlying productivity $A_1$ is uncertain at $t = 0$, for some $A_1 \in [\underline{A}, \overline{A}]$.

- In $t = 1$, the productivity level $A_1$ of the economy is realized, and firms produce. Given $A_1$, the government decides whether to repay its debt or default. Default leads to financial frictions in the form of an incentive constraint on domestic banks. Households consume and save, and bankers lend to firms.

- In $t = 2$, firms produce, and banks shut down and return their net worth to households, who consume.

The detailed optimization problem of each group of agents is described below.

2.2 Households

Households are assumed to be risk-neutral savers who care equally about all three periods, and choose consumption to maximize:

$$c_0 + \mathbb{E}[c_1 + c_2], \quad (1)$$
subject to the budget constraints:

\[ c_0 + d_0 = y_0, \]
\[ c_1 + d_1 = R_1 d_0 - b^{tot} \cdot \mathbb{1}_{\{D=0\}}, \tag{2} \]
\[ c_2 = R_2 d_1 + n_2. \tag{3} \]

Households receive an initial endowment \( y_0 \), earn gross interest rates of \( R_1 \) and \( R_2 \) on intertemporal deposits \( d_1 \) and \( d_2 \), and are assumed to receive a transfer of the equity remaining in banks \( n_2 \) at \( t = 2 \). If the government does not default, denoted by \( D = 0 \), then households pay taxes in order to repay the country’s total debt \( b^{tot} \).

### 2.3 Government

The government’s optimal program satisfies:

\[ V_0 = \max_{b^{tot}} \left[ c_0 + \mathbb{E} \left[ \max_{D \in \{0,1\}} \{V_1, V_1^d\} \right] \right], \tag{4} \]

where \( D \) denotes the government’s default decision, and the government is assumed to choose foreign borrowing \( b^* \) to meet a fixed exogenous level of public spending:

\[ g = q b^{tot}, \]

where \( q \) is the price. Of the total debt issued by the government, some portion \( b \) will be held by domestic banks, and the remainder \( b^* \) is assumed to be held by foreign lenders:

\[ b^{tot} = b + b^*. \]
The continuation values in period 1 are given by:

\[ V_1(b, b^*, A_1) = c_1 + c_2, \]
\[ V_1^d(b, b^*, A_1) = c_1^d + c_2^d, \]

where \( c_t \) and \( c_t^d \) denote consumption conditional on no default and default, respectively, and \( A_1 \in [A, \overline{A}] \) is the stochastic realization of private productivity in \( t = 1 \).

The repayment set is defined as the set of productivity realizations for which repayment is optimal, given \( b \) and \( b^* \):

\[ R(b, b^*) = \{ A_1 \in A : V_1 \geq V_1^d \}, \]

and the default set is defined as the set of realizations for which default is optimal:

\[ \Delta(b, b^*) = \{ A_1 \in A : V_1 < V_1^d \}. \]

Default probabilities and default sets are related in the following way:

\[ p(b, b^*) = \int_{\Delta(b, b^*)} f(A) dA, \quad (5) \]

where \( f(\cdot) \) is a probability density function for the realizations of \( A_1 \).

### 2.4 Foreign lenders

Foreign lenders are assumed to be risk neutral and to lend at an interest rate that compensates them for default risk, maximizing profits in each period. Their opportunity cost is assumed to be the gross risk-free rate, \( R \), yielding profits:

\[ \phi = qb^* - \frac{1 - p(b, b^*)}{R} b^*. \]
In what follows, it is assumed that the government issues enough debt that both domestic and foreign lenders hold bonds, and the price of government debt is accordingly:

\[ q = \frac{1 - p(b, b^*)}{R}, \]  

where \( q \) takes values on a bounded interval \( q \in [0, \frac{1}{R}] \).

\section*{2.5 Banks}

Banks begin with initial equity \( n_0 \) and accept deposits \( d_0 \) from households. The main mechanisms of the model arise from two financial frictions: interbank lending that must be collateralized, and an agency problem in deposit markets. I describe each of these in turn.

\subsection*{2.5.1 Collateralized interbank lending}

Following \cite{Bolton and Jeanne (2011)}, I assume that banks have heterogeneous access to investment projects, but demand deposits homogeneously.

Specifically, at \( t = 0 \), a fraction \( \omega \) of banks get access to the production technology, while \( 1 - \omega \) do not. Banks with projects demand domestic government debt in order to be able to borrow on interbank markets. The size of interbank loans is limited by the value of collateral. Eligible securities are in this case only government bonds, which are assumed to be collateralized with some haircut \( \lambda \geq 1 \).

Banks holding government debt of \( qb \) can borrow up to a limit:

\[ i \leq \lambda qb. \]

Banks are assumed to act competitively in the market for sovereign debt, i.e.,

\footnote{In general, \( \lambda \geq 1 \), because for lower values of the haircut domestic banks are able to borrow less than one dollar on interbank markets for each dollar they lend to the government. If \( \lambda < 1 \), banks are better off not lending to the sovereign at all.}
taking the price $q$ as given. Government debt is first assumed to be the only asset that can be used to collateralize interbank loans, an assumption that is relaxed in Section 5.

Equity and deposits form the liabilities side of banks’ balance sheets, and can be invested in government bonds $qb$ or lent to firms $k_1$. I also allow banks to invest excess funds in a risk-free asset that pays $R$, and denote excess funds by $x$. The balance sheets of banks with and without investment projects are shown in Figure 1.

<table>
<thead>
<tr>
<th>Banks with projects</th>
<th>Without projects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>$k_1$</td>
<td>$\omega d_0$</td>
</tr>
<tr>
<td>$qb$</td>
<td>$\omega n_0$</td>
</tr>
</tbody>
</table>

Figure 1: Heterogeneous bank balance sheets

The rate paid on interbank lending is assumed to equal the overall return earned on private investment and government debt. This makes depositors indifferent between banks with and without investment projects, and allows banks to be treated as homogenous, in aggregate.

At $t = 0$, banks without investment projects have $(1 - \omega)(n_0 + d_0)$ of liabilities that can only be productively invested in interbank markets, government bonds, or the risk-free asset. This places a constraint on the supply of interbank funds:

$$i \leq (1 - \omega)(n_0 + d_0).$$

In equilibrium, banks with projects invest in government debt; banks without projects make collateralized loans to banks with projects. For a given $b$, the
amounts of interbank lending and remaining excess funds are given by:

\[ i = \min \left\{ \frac{b}{bd}, 1 \right\} (1 - \omega)(n_0 + d_0), \]
\[ x = \max \left\{ 1 - \frac{b}{bd}, 0 \right\} (1 - \omega)(n_0 + d_0). \]  

(7)

For a given \( q \), banks with projects demand a sufficient level of government debt to borrow all the funds at banks without investment projects, assuming that the resources held by productive banks is larger than \((1 - \omega)(n_0 + d_0)/\lambda\). This leads to domestic demand for government debt of:

\[ bd = \frac{(1 - \omega)(n_0 + d_0)}{\lambda q}, \]

(8)

which allows all funds in the economy to be intermediated. If the government issues less debt than the amount in equation (8), the lack of collateral hinders interbank lending and leads to efficiency losses. In what follows I focus on cases in which \( b_{tot} > bd \).

In the three-period model, sovereign debt and interbank markets are only open once, from \( t = 0 \) to \( t = 1 \). At \( t = 1 \), all banks are assumed to have equal access to investment projects.

If \( b \geq bd \), the balance sheet constraints of the aggregate banking system are:

\[ k_1 + qb = n_0 + d_0, \]  
\[ k_2 = n_1 + d_1. \]  

(9)  

(10)

Banks choose government debt holdings, lending to firms, and deposits to maximize expected equity in \( t = 2 \):

\[ \mathbb{E}[n_2], \]

(11)
where equity evolves according to:

\[ n_1 = A_1 k_1 + b \cdot I_{\{D=0\}} - R_1 d_0, \]  
(12)

\[ n_2 = A_2 k_2 - R_2 d_1. \]  
(13)

For those banks who have access to the production technology, investment capital yields constant returns to scale, where \( A_1 \) is stochastic, but \( A_2 \) is fixed. If the government defaults, banks suffer a loss to equity relative to the no default case.

So that households are not insulated from the productivity realization in the economy, I assume that the interest rate on deposits in period 1 is state-contingent, i.e. the interest rate on deposits is given by:

\[ R_1 = A_1. \]

While many countries insure deposits, this characterization of the return on deposits is helpful in that it exposes both banks and depositors to the realization of productivity in the economy. If \( R_1 \) were fixed, this would allow banks to become very rich when the realization of productivity is high, but cause them to have outsized losses when the realization of productivity is low. Similarly, if there are no financial frictions in \( t = 1 \):

\[ R_2 = A_2. \]

These assumptions also ensure that banks do not end up with negative equity. Although the possibility of bank failures is not considered explicitly in the model, this would likely worsen the consequences of default.

### 2.5.2 Deposit market frictions

Financial frictions are assumed to arise in interbank markets when bank equity is low, so that depositors are concerned about the health of banks. Following Gertler and Kiyotaki (2010), this is modeled as an agency problem in that
bankers are capable of diverting a fraction $\theta$ of bank assets. This adds an incentive constraint to the banks’ problem: the profits of operations must be greater than the value of assets bankers can divert. In the case that financial frictions arise as a consequence of default, this incentive constraint can be written as:

$$A_2k_2 - R_2d_1 \geq \theta A_2k_2. \quad (14)$$

Using equation (10), the constraint can also be written as a condition on deposits:

$$d_1 \leq \frac{(1-\theta)A_2}{R_2 - (1-\theta)A_2} n_1. \quad (15)$$

This inequality can only hold if $R_2 > (1-\theta)A_2$. Bankers’ demand for funds is thus a well-defined function of $R_2$ in the interval $((1-\theta)A_2, A_2]$, and decreasing in $R_2$. Deposit demand approaches infinity as $R_2 \to (1-\theta)A_2$, and approaches $\frac{1-\theta}{\theta}n_1$ as $R_2 \to A_2$. When $R_2 = A_2$, deposit demand is bounded:

$$0 \leq d_1 \leq \frac{1-\theta}{\theta}n_1.$$

The deposit demand of banks in the case of financial frictions therefore follows:

$$d_1^{\text{demand}} = \begin{cases} \frac{(1-\theta)A_2}{R_2 - (1-\theta)A_2} n_1 & \text{if } R_2 < A_2 \\ \text{Indeterminate in } [0, \frac{1-\theta}{\theta}n_1] & \text{if } R_2 = A_2 \\ 0 & \text{if } R_2 > A_2 \end{cases}.$$  

With risk neutral households that care equally about each period, any interest rate greater than 1 will induce them to save everything, so deposit supply
follows:

\[ d^\text{supply}_1 = \begin{cases} 
R_1d_0 & \text{if } R_2 > 1 \\
\text{Indeterminate in } [0, R_1d_0] & \text{if } R_2 = 1 \\
0 & \text{otherwise}
\end{cases} \]

This can be combined with the bound \( d_1 \leq \frac{1-\theta}{\bar{\theta}} n_1 \) to solve for the bound on bank equity below which financial frictions arise:

\[ n_1 \geq \frac{\theta}{1-\theta} R_1d_0 = \bar{N}. \quad (16) \]

Combining deposit demand and deposit supply results in equilibrium in the deposit market, which pins down both \( d_1 \) and \( R_2 \). For any shock that causes bank equity to decrease below the threshold in (16), banks continue to operate but are subject to an additional incentive constraint given by equation (14). For equation (14) to hold, either the return on deposits must fall, or the amount of deposits must fall, or both. In this way, a government default causes stress in the banking system, and generates an endogenous cost of default.

In the event of default, households have more wealth by the amount of domestic plus foreign debts, as in equation (2). Because the households are wealthier, for deposits in the economy to fall, rather than rise, it must be that the savings that are deposited under financial frictions in equation (15) are less than deposits if there is no default: \( \frac{(1-\theta)A_2}{R_2-(1-\theta)A_2} n_1^d < R_1d_0 - b^{tot} \). If the equilibrium in deposit markets is such that the level of deposits is less than the frictionless level, it can only be that the deposit demand condition intersects deposit supply on the vertical line where \( R_2 = 1 \). This is shown graphically in Figure 2.
2.6 Equilibrium

Equilibrium in this economy consists of the government maximizing welfare, households maximizing expected utility, banks and foreign investors maximizing expected profits, and market clearing for deposits, interbank lending, and sovereign debt.

**Definition 1.** Equilibrium in this economy is defined as a set of policy functions for consumption \( \{c_0, c_1, c_2\} \), deposits \( \{d_0, d_1\} \), lending \( \{k_1, k_2\} \), government asset holdings \( \{b, b^*\} \), repayment sets \( R(b, b^*) \), default sets \( \Delta(b, b^*) \) and bond prices \( q(b, b^*) \) such that:

1. Taking as given government policies and bond prices, consumption and deposit supply plans \( \{c_0, c_1, c_2\} \) and \( \{d_0, d_1\} \) maximize households’ expected utility subject to their budget constraints.

2. Taking as given government policies and bond prices, banks choose deposits and lending to the government and firms to maximize expected equity \( \mathbb{E}[n_2] \), subject to balance sheet constraints, the evolution of net worth, and an incentive constraint that occasionally binds; and, interbank markets clear.

Figure 2: For deposits to fall after default, assuming \( n_1^d < \tilde{N} \)
3. Deposit markets clear, meaning that deposit demand and supply from the banks’ and households’ programs determine the level of deposits and the deposit rate.

4. Taking as given bond prices, the government’s asset holdings $b$ and $b^*$, repayment sets $R(b, b^*)$ and default sets $\Delta(b, b^*)$ satisfy the government’s optimization problem.

5. Bond prices $q(b, b^*)$ reflect the government’s default probabilities and are consistent with foreign lenders’ expected zero profits.

The model can be solved backwards, by first determining the solutions to the problem at $t = 1$. Since there is no uncertainty after $A_1$ is realized, it is possible to compare $V_1$ and $V_1^d$ for given levels of $b$, $b^*$, and realizations of $A_1$ to determine when default will be optimal. These solutions can be then be considered in $t = 0$ along with an assumed probability distribution for the possible realizations of $A_1$, to determine the optimal $b$ and $b^*$.

3 Results

This section characterizes the equilibrium outcomes in the model. First, domestic debt is shown to be a commitment device. Second, the implications of domestic banks’ characteristics for sovereign default are explored: default is more likely on well-capitalized banks, and default risk decreases with the level of bank deposits. Next, because debt has a function that is tied to its value, domestic demand for sovereign debt increases with default risk. Two features of canonical sovereign default model remain true in this setup: default incentives increase in the level of foreign lending, and default occurs in equilibrium. Finally, the model is used to show that financial frictions and deposit flight in anticipation of default are likely to increase the costs of default, while also making default more likely.
Three basic properties of the model are first discussed, before turning to the main results. For the model to generate interesting predictions, one additional restriction is necessary. Banks are assumed to have only a limited amount of equity, which can be stated as a condition on model parameters.

**Assumption 1.** *Initial bank equity is assumed to be lower than the threshold:*

\[ n_0 < d_0 \left[ \frac{1}{A_2(1-\theta)} \frac{\lambda}{\lambda + \omega - 1} - 1 \right]. \]

Assumption 1 guarantees that default occurs when productivity is low. However, in the absence of domestic bond holdings, there is no cost to default. In equilibrium, domestic holdings of sovereign debt must be large enough that a default results in financial frictions in order for there to be any cost associated with non-repayment.

**Lemma 1.** *Domestic bond holdings must be (i) strictly positive and (ii) sufficiently high so that a default results in financial frictions, in order for foreign lending to be supportable in equilibrium.*

**Proof.** *All proofs are included in the Appendix.*

In this three period setup, there is no threat of exclusion from financial markets to discipline government borrowers. If the government has no borrowings from domestic banks, then for any positive value of foreign lending default is certain. Therefore, no lending can be supported in equilibrium. Similarly, for a level of borrowings too small to cause distortions in the domestic banking sector, there is no cost to defaulting to encourage the government to repay. However, to the extent that domestic holdings of sovereign debt can disrupt the banking sector and become a source of output losses in the event of default, foreign lending can be supported in equilibrium.

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\(^6\)The condition in Assumption 1 is derived in the proof of Lemma 3.
If domestic banks are the only holders of sovereign debt, then default is trivial, because it generates output losses as a result of financial frictions, without providing any benefits from retaining resources that would otherwise be transferred abroad. This is demonstrated in the following Lemma, which considers the case where foreigners hold no debt, i.e. $b^* = 0$.

**Lemma 2.** If $b > 0$ and $b^* = 0$, then $\Delta(b, b^*) = \emptyset$, i.e., when all sovereign debt is held by domestic banks, default is never optimal.

The government default decision involves a tradeoff between the benefit of lower lump-sum taxes, and the costs of default on the banking system. A default exclusively on domestic bondholders is never optimal, since this comes with costs and has no associated benefits. Default merely causes a redistribution of wealth from banks to households. Households then cannot save as much at impaired banks, and investment and output fall. Since taxation is lump sum and bank equity is returned to households in $t = 2$, this makes households worse off.

Models of non-contingent sovereign debt typically find that defaults occur when output is low. Given Assumption 1, this is also true in this model, and there is some threshold value of $A_1$ below which default is optimal.

**Lemma 3.** If for a level of borrowing $(b, b^*)$ default is optimal for some $A_1$, then default is also optimal for all $A'_1 < A_1$. Therefore, there exists a threshold level of productivity $\tilde{A} \in [A, \bar{A}]$ below which it is optimal for the government to default.

It follows from Lemma 3 that for each level of domestic and foreign debt, the set of productivity realizations that trigger default is an interval. This simplifies solving the model, because the default probability in equation (5) can be rewritten as:

$$p(b, b^*) = F(\tilde{A}),$$

20
Figure 3: Threshold level of $A_1$

where $F(\tilde{A}) = \int_{\tilde{A}}^{A} f(A) dA$ is the cumulative distribution function of the possible realizations of productivity $A_1$. By extension, agents’ expectations can be written as the sum of two integrals, taken over the default and no default regions of $[A, \overline{A}]$.

3.1 Commitment

Taken together, Lemmas 1-3 demonstrate the importance of domestic bond holders in contributing to the commitment of a sovereign in repaying its debts, through the endogenous costs of default. This result is stated formally in Proposition 1.

**Proposition 1.** For $b < b^*$, $\Delta(b', b^*) \subseteq \Delta(b, b^*)$, i.e. the default set is decreasing in the amount of domestically held sovereign debt.

In the model, the government can borrow from abroad because defaulting leads to financial frictions that limit the ability of banks to channel savings to productive investment. Greater domestic holdings of sovereign debt lead to more severe effects on the banking system in the event of default, which lowers default risk. This is the only effect which serves to discourage default
in this model. In reality, other disciplining devices play a role as well, such as exclusion, reputation, secondary markets, etc.

From the perspective of the government issuer, domestic holders are distinct because their bond holdings come with additional associated losses if the sovereign defaults. This helps to support the value of sovereign debt, by raising the price $q$, or equivalently lowering the default probability $p$. In this sense, a bond in the hands of domestic investors is more valuable due the effect that endogenous output losses have on default incentives.

Importantly, this property of domestic bond holders provides a microfoundation for the assumption in secondary market theory that domestic shareholders place relatively more value on government debt in times of sovereign stress. This can be framed in terms of political economy explanations, such as that domestic creditors are more likely to be able to extract value from the government following a default, perhaps because domestic bond holders are more concentrated and have more sway in domestic political processes. However, the same conclusion – that there is more value to debt in the hands of domestic creditors – emerges from the framework of this model, driven by commitment. Domestic creditors support the value of sovereign debt precisely because defaulting on them is costly.

### 3.2 Bank health

An extension of the idea of domestic banks as a commitment device is how default incentives depend on characteristics of the banking system. A well-capitalized banking system is more easily able to weather a sovereign default. In contrast, fragile banks are more costly for the sovereign to default on. This is the intuition of Proposition 2.

**Proposition 2.** Holding constant the level of foreign debt, $\Delta(b, b^*)$ is increasing in $n_0$, i.e. default is more likely if banks have more equity.

In the event of default, financial frictions limit bank deposits to some multiple
of bank equity. When initial bank equity is low, a default leads to a tighter constraint on deposits, which leads to a higher cost of default. In comparison, a well-capitalized banking system is less constrained in the event of default, leading default to be less costly, and therefore more probable.

Proposition 2 is consistent with the empirical evidence of Erce (2012), who finds default on domestic banks to be less likely when the banking sector is weak. This lends an additional argument to why governments may not want to recapitalize banks during a crisis. Aside from standard moral hazard arguments, Acharya et al. (2014) explore the pressures bank bailouts place on sovereign creditworthiness. Crosignani (2015) argues that governments may avoid recapitalizing banks because they want banks to act as lenders of last resort during crises. If in addition to these factors a stronger banking system increases sovereign risk because it makes sovereign default less costly, then this could contribute to the incentives to not provide bank bailouts when sovereign risk is high.

In contrast, the probability of default is decreasing in the level of household deposits.

**Proposition 3.** \( \Delta(b, b^*) \text{ is decreasing in } d_0, \text{ i.e. default is less likely when banks have more deposits.} \)

This result arises from the distortion to deposit markets in the case of a default. If the sovereign defaults, deposits are limited to a multiple of bank equity. The more deposits households have in excess of this, the greater the distortion and the corresponding disincentive to default.

### 3.3 Domestic demand for government debt

Domestic demand for sovereign debt is determined by the banks’ optimization problem, and features a tradeoff between the use of government bonds as collateral in interbank markets, and the risks of non-repayment. Banks demand government debt in order to intermediate all the assets in the financial
Domestic demand for government debt depends on a number of factors. First, domestic demand for government debt increases with the size of the banking system, which constitutes both deposits and equity. Second, it is increasing in $1 - \omega$, the extent of assets to be borrowed from nonproductive banks. Third, domestic demand for government debt decreases in the “haircut” applied to collateral $\lambda$: the larger amount of interbank loans that can be collateralized by one unit of sovereign debt, the lower is domestic demand.

Fourth, if bond prices fall, more domestic debt is required to provide adequate collateral for the banking system. Following equation 8, this relationship is shown in Figure 4. Since the price of government bonds $q$ is decreasing in the probability of default $p$, domestic demand for sovereign debt may increase with sovereign risk.

\textbf{Proposition 4.} Domestic demand for government debt $b$ is weakly increasing in sovereign risk $p$.

\footnote{This assumes the resources held by productive banks exceed the value of government bonds required to intermediate the remaining assets in the banking system. The alternate case is less interesting: if productive banks’ resources are less than the value of government bonds required to collateralize the non-productive banks’ resources, then the productive banks invest their entire net worth in government debt.}
This dynamic derives from the idea that increased sovereign risk lowers the value of a “safe” asset or collateral. As the value of collateral falls, banks require more of it in order to borrow the same amount on interbank markets. The expected return on government debt is always $R$, so risk-neutral banks prefer government bonds over an alternative safe asset that pays a return $R$ but cannot be used as collateral.

Realistically, other safe assets can likely also be used as collateral, so that banks can substitute away from risky sovereign debt. I discuss this bias and the possibility of alternative collateralizable safe assets in Section 5.

In addition, this model provides a simple lens through which to consider the observed heterogeneity in bank exposures to domestic sovereign debt. As European countries experienced increases in the risk profile of their debt, there was a concurrent repatriation of sovereign ownership from foreign to domestic investors. Brutti and Saure (2014) document the recent European repatriation of sovereign debt in greater detail, and argue that the evidence is consistent with secondary market theory, rather than standard theories of portfolio allocation or home bias. Some of the increases in exposure may be driven by banks being forced to hold additional debt, as argued by Becker and Ivashina (2014). However, to the extent that domestic banks require sovereign bonds to collateralize interbank lending, rising sovereign risk can also lead to increases in domestic demand due to effects on the value of collateral.

Another way to characterize demand for $b$ is to consider domestic holdings as a share of the total face value of debt raised by the government. As shown in Figure 3, for low levels of total debt, domestic banks demand the full share. Once $b = b^d$, this demand levels off, and there is a region over which each marginal bond issued by the government is purchased by foreigners. This can be described as a “safe borrowing region,” in that $q = 1/R$ for some range of total issuance $b^{tot} \in (\ast, \ast\ast)$. Once $b^* \text{ reaches a certain level, debt becomes risky, and as } b^{tot}\text{ increases, } p \text{ increases, which increases domestic demand for sovereign debt } b.$
3.4 Foreign lending and bond prices

With respect to foreign debt, default is increasingly likely for higher levels of borrowing from foreigners. This is consistent with canonical models of sovereign default.

**Proposition 5.** For \( b^* < b^{**} \), \( \Delta(b, b^*) \subseteq \Delta(b, b^{**}) \), i.e. the default set is increasing in the level of borrowing from abroad.

Proposition 5 implies that there exists some maximum level of foreign debt that is supportable in equilibrium. In other words, foreign debt is bounded. This is equivalent to a no-ponzi condition. If \( A_1 \) is bounded, and the default set is increasing in foreign debt, then there is some level of foreign debt that results in default for sure, and is thus unsupportable in equilibrium. However, since default incentives depend on domestic bond holdings, this bound depends on them, as well.

In the model, the government needs to raise a fixed, exogenous \( g \). However, the total resources borrowed by the government is the product of the face value of debt and its price \( q \). The total resources borrowed from abroad follow an endogenous Laffer curve. This is shown along with the bond price in Figure 6 assuming a normal distribution for the potential realization of productivity.
For low levels of foreign borrowing from abroad, \( b^* \in (0, \tilde{b}^*) \), sovereign debt is safe and \( q = 1/R \), so the bond price is defined by the risk free rate. For levels of foreign borrowing \( b^* \in (\tilde{b}^*, \bar{b}^*) \), borrowing is risky. The government never optimally chooses \( b^* > \tilde{b}^* \), because the same amount of total resources can be obtained by choosing a lower level of debt. The shape of the Laffer curve matches Arellano (2008), but in this case the costs which support risky debt are endogenous costs that arise via the banking system.

![Figure 6: Bond price and total resources borrowed from abroad](image)

The total resources borrowed from domestic bondholders is a function of domestic banks’ characteristics. From equation (8) and Proposition 4, \( qb = (1 - \omega)(n_0 + d_0)/\lambda \). As a result, the total resources borrowed from domestic banks does not change with \( q \), but the face value of domestic debts increases mechanically as \( q \) declines.

In the model, the exogenous level of \( g \) and the characteristics of the banking
system then pin down the optimal $b$, which jointly determines the specific amount of resources that need to be borrowed from abroad and the bond price. Since the level of government spending $g$ must be met, whatever the government does not get willingly from domestic banks, it must seek from abroad. An illustrative example of this is shown in Figure 7.

To understand the effect of domestically held debt on both the potential and realized level of borrowing, consider an exogenous change in the parameters that describe the banking system such that more sovereign debt is naturally demanded by domestic agents. This could be a decrease in $\omega$ or $\lambda$, or an increase in $n_0$ or $d_0$. In this case, the additional domestic debt $b' > b$ lowers the probability of default for any given level of $b^*$, so $q$ rises. The effect that this shift has on the total resources borrowed is shown in Figure 8. An increase in $b$ shifts the endogenous Laffer curve up and out, leading to a larger range of possible government spending, and shifts the risky borrowing region to the right.

Figure 7: Total resources borrowed
3.5 Default in equilibrium

Domestic debt discourages, but does not prevent default. This is a direct implication of Lemma 3. Whether default may occur in equilibrium depends in particular on the fiscal needs of the government in $t = 0$, and the characteristics of the banking system.

**Proposition 6.** Default can occur in equilibrium, in spite of large domestic holdings of sovereign debt.

Whether default can occur in equilibrium depends on the parameters of the model, and in particular the level of government spending $g$. If the government were to choose $g$ optimally, in this setting there is no incentive to borrow from
abroad, because government spending does not enter the households utility function and resources borrowed from abroad are not invested productively. With an exogenously determined $g$, which could for example capture fiscal obligations that are difficult to alter in the short run, it may be necessary to borrow from abroad to such an extent that default is a possibility. In this sense, domestic bank holdings of sovereign debt can be thought of as an imperfect commitment device, since despite large domestic holdings, sovereign debt may still be risky.

3.6 Financial frictions in anticipation of default

The above results focus on the implications of default on banks, assuming that banks are not initially subject to financial frictions. However, it is also possible that default risk has effects on the banking system in anticipation of default.

If bank equity is low enough in $t = 0$, then households choose to limit the deposits they channel to the banking system. This can be thought of as a form of deposit flight. Importantly, this reduces the funds available to the banking system to invest, which lowers potential output. This highlights an important limitation of Proposition 2. While weaker banks are in general more costly to default on, if low levels of bank equity act as a constraint on deposits, undercapitalized banks can be associated with higher default risk due to deposit flight.

**Proposition 7.** Deposit flight that occurs due to financial frictions in $t = 0$ makes default more likely.

If banks are sufficiently poorly capitalized that financial frictions arise in $t = 0$, then households save less in the banking system and consume more in $t = 0$. This negatively affects investment and output. Since households have less wealth in $t = 1$, there is less at stake to be affected by a default. This highlights one way in which sovereign and financial crises are interconnected, as sovereign risk may lead to problems in the financial system that reinforce the default risk of the sovereign.
An additional effect of deposit flight is to reduce the size of the banking system \((n_0 + d_0)\), which causes the level of debt demanded domestically to fall. This also contributes to heightened default risk, in that a greater portion of fiscal needs may need to be raised from foreign lenders. Alternately, it may increase the desirability of financial repression, as discussed in the next section.

4 Externalities and financial repression

In setting domestic demand for government debt, banks do not internalize the effect they have on default incentives. In other words, they take \(q\) as given, but \(q\) depends on \(b\). Taking \(q\) and \(\tilde{A}\) as given leads the banks’ optimization to be a problem of taking expectations over intervals:

\[
\max_b E[n_2] = \int_{\tilde{A}}^{\bar{A}} n_2^d f(A)dA + \int_{\tilde{A}}^{\bar{A}} n_2 f(A)dA.
\]

In contrast, if banks were to fully internalize the effect of their debt holdings on bond prices and the default incentives of the sovereign, then the banks’ optimization problem would also consider the effect of their choice of \(b\) on \(\tilde{A}\), \(p\), \(q\), and \(b^*\). This affects the banks’ expectations of equity in the cases of default and no default, as well as the threshold \(\tilde{A}\). In this case, solving the banks’ problem requires first integrating and then taking a first order condition with respect to \(b\).

In this section, the model is altered slightly to accommodate a specific form of government intervention in domestic sovereign debt markets, financial repression. Assume now that the government chooses \(b\) for the banks, which changes the government’s optimization problem in equation (4) to a maximization over \(b\) and \(b^*\), rather than over just \(b^{tot}\), which can be written in terms of intervals:

\[
V_0 = \max_{b,b^*} \left[ c_0 + \int_{\tilde{A}(b)}^{\bar{A}(b)} V_1^d f(A)dA + \int_{\tilde{A}(b)}^{\bar{A}(b)} V_1 f(A)dA \right].
\]
Because domestic banks on their own do not internalize the effect their holdings have on default risk, the government may have an incentive to choose a higher level of domestic bond holdings than the banks would choose on their own. This increases the potential cost of default, which increases the prices at which the government issues debt. At the expense of crowding out private investment, the government can lower its tax bill in $t = 1$.

In the case where the government increases domestic holdings of sovereign debt from $b$ to $b'$, these effects can be understood in the following way. The reduction in external debt $b^*$ to $b^*$ comprises two effects: a mechanical reduction in $b^*$ which occurs because the total fiscal need $g$ is fixed, and an effect via improvement in the price:

$$\frac{db^*}{db} = - \left[ 1 + \frac{g}{q^2} \right] < 0.$$

From Proposition 1, an increase in domestic bond holdings decreases default risk, $dq/db > 0$, and thus $db^*/db < 0$.

However, any increase in $b$ beyond the amount required to intermediate all the assets in the banking system leads to crowding out:

$$\frac{dk_1}{db} = - \left[ \frac{dq}{db} b + q \right] < 0.$$

Since $q \in [0, \frac{1}{\tilde{R}}]$ and $dq/db > 0$, this is unambiguously negative, as well. Whether financial repression has the potential to improve welfare depends on whether the reduction in external debt outweighs the cost of crowding out.

**Proposition 8.** When debt is risky, financial repression crowds out private investment, but can improve welfare. A necessary condition under which financial repression can improve welfare is:

$$\frac{dV_1}{db} \bigg|_{b = \frac{(1-w)(q_0 + q_0)}{\lambda q}} > 0.$$
The idea behind Proposition 8 is shown in Figure 9 for an increase in domestic bond holdings from \( b \) to \( b' \). It is possible that despite crowding out private investment, the decrease in the face value of foreign debt that this achieves will increase welfare if the realization of productivity is such that default does not occur. Financial repression also lowers the threshold \( \tilde{A} \) below which default occurs, which leads to welfare improvements for productivity realizations between \( \tilde{A}' \) and \( \tilde{A} \).

![Figure 9: Financial repression can increase welfare](image)

However, for realizations of productivity below the new threshold, financial repression leads to an unambiguous decrease in welfare. This perhaps relates to historical default episodes such as Argentina in 2001 or Russia in 1998. In these latter two default episodes, domestic banks seem to have piled up on domestic debt, only to then suffer even greater losses when the respective defaults took place [Basu, 2010]. In some sense, financial repression can be seen as taking a gamble that does not necessarily prevent default, and which makes outcomes all the more worse if default occurs.

In addition, the improvements in welfare that result from financial repression are decreasing in the realization of \( A_1 \). This can be shown by the negative second derivative:

\[
\frac{d^2V_1}{dbdA_1} = A_2 \left[ -\frac{dq}{db} - q \right] < 0.
\]

Although there may then be some range of productivity realizations over which
financial repression improves welfare, for high realizations of productivity the losses due to crowding out increase relative to the gains of reduced external debts.

The overall lesson here is that there are limits to the extent to which financial repression can be relied upon to avoid a potential default.

This section relates to the findings of Chari et al. (2015), who argue that when the government cannot commit to repay debts, financial repression may be optimal. In their model, regulation motivates bank holdings of government bonds. While the basic intuition of domestic bank holdings of sovereign debt as commitment is the same, in this setting banks have an added motive to hold government debt as collateral. As such, financial repression must be understood as inducing demand above and beyond what banks would choose to invest in on their own.

With risk-averse households and centralized borrowing (where the government transfers its borrowings to households, instead of spending it on $g$ directly), financial repression can have further benefits in terms of facilitating the ability of households to smooth consumption. In this case, by increasing domestically held sovereign debt, the government can borrow more from abroad, benefiting its citizens directly. A related case is explored in Section 5, where foreign lenders also channel capital inflows to banks.

5 Extensions

Two concerns with the model are that domestic sovereign debt is assumed to be the only eligible type of collateral, and that foreign capital is borrowed exclusively by the government, while in reality banks and firms also receive foreign inflows. In this section, I first discuss the implications of allowing a second safe asset to be used as collateral, and then argue that allowing banks to borrow from abroad in addition to the government reinforces the main mechanisms described in Sections 3 and 4.
5.1 Second safe asset

An easy objection to the role of sovereign debt as collateral in motivating demand for domestic sovereign debt is the presence of alternative safe assets, for example, U.S. Treasuries. If banks are able to costlessly use alternative safe assets as collateral in interbank transactions, then there is little motive to invest in risky domestic sovereign debt. When other assets can also be used as collateral, they could in theory confer all of the same benefits as domestic sovereign debt, and have none of the associated risks.

Under the assumption that a second safe asset is available and completely safe, and if domestic sovereign debt is risky, domestic banks in this model would strictly prefer to use the safe asset in lieu of the domestic one. In this situation, some form of regulation, moral suasion, or financial repression would be necessary to have non-zero domestic holdings of sovereign debt. As discussed in Section 4, there is some scope to undertake these policies while improving overall welfare. Where second safe assets are perfect substitutes for domestic sovereign debt, financial repression is necessary to enable the government to raise its fiscal needs $g$. However, in the presence of regulations that require domestic intermediaries to hold domestic sovereign debt, or that provide incentives for domestic government bondholdings over foreign alternatives, banks face a tradeoff between these factors and the risks of non-repayment.

It may also be that banks can more easily invest in domestic sovereign debt due to issues of proximity, exchange risk, jurisdiction, etc. Diversification of sovereign debt within a bank’s portfolio may require infrastructure in which the bank must invest, raising the barriers to doing so. This would lessen the need for explicit financial repression.

More generally, it is worthwhile to consider to what extent alternative safe assets exist. Some have noted the apparent shortage of safe assets in the global economy (Caballero et al., 2008; Caballero and Farhi, 2015), as well as the consequences of low public debt issuance for the creation of private “safe” securities (Krishnamurthy and Vissing-Jorgensen, 2012, 2013). Issues such as
these complicate the ability of banks to diversify sovereign debt holdings. To the extent that banks and investors demand “safe” assets that are in limited global supply, a security with government backing may be an acceptable alternative. Such preferences are also consistent with “risk-shifting” by banks that maximize their profits, conditional on the absence of a government default.

5.2 Capital inflows to banks

In many countries, a substantial portion of capital inflows are intermediated by banks rather than flowing directly to the government. Allowing banks to borrow from abroad generates some additional implications in the framework of this model.

Assume that foreign lenders are now willing to lend to domestic banks in wholesale markets, and that these loans go into specific foreign-owned investment projects. Consider foreign lending to be similar to domestic interbank lending, in that it requires collateral in the form of government debt and occurs at some (foreign) haircut. At the outset, providing an additional role for collateral would likely increase the domestic demand for sovereign assets (depending of course on the presence of alternative collateralizable securities).

If foreign inflows to banks pay a return that is fixed in advance, a low realization of productivity can lead to even lower equity in the banking sector. This would lead either to financial frictions that are more severe, or in some cases, solvency problems for the banking sector, which may then default on its obligations. One could consider whether bank defaults lead the government to step in and guarantee private sector debts, as occurred in Ireland. If so, a private sector default could precipitate a sovereign default, linking financial and sovereign crises.

Suppose that a sovereign default leads the private sector to also default. This is often effectively the case following a sovereign default due to policies implemented to prevent capital from flowing out of the country, such as deposit freezes, capital controls, and/or exchange controls. In this case, sovereign risk
is tied to private risk, and so when sovereign risk rises it has the additional negative effect of limiting inflows into the banking system. This provides an additional benefit for policies that reduce sovereign risk, such as financial repression, because they facilitate the intermediation of private capital flows to productive investment.

A related policy introduced in the recent European crisis was the provision of long-term refinancing operations (LTROs). Specifically, these refinancing operations offered euros to banks in exchange for a wide array of collateral, including peripheral government debt. This was aimed to prevent liquidity shortages at European banks, but is shown by [Crosignani et al. (2015)] to have contributed to increased bank sovereign exposures. By providing an additional role for collateral, LTROs increased banks’ holdings of domestic sovereign debt.

The banking sector is critical to both facilitating productive investment and also mitigating sovereign risk, and when banks also borrow from abroad, there is more at stake in a sovereign default. LTROs can be thought of as providing not only liquidity, but also as an additional driver of domestic exposure to sovereign debt, and thus reduced default risk. However, this may also worsen outcomes in the event of default, since domestic banks are more exposed.

6 Conclusion

In this paper, I present a model of domestic banks and sovereign default that illustrates the role of domestic banks in supporting risky sovereign debt, through endogenous output losses that result from sovereign default. This provides a specific microfoundation for a political economy argument made in secondary market theory, and a new lens through which to consider the recent trends in ownership of government debt in the European periphery. In addition, the model predicts that default incentives are tied to the health of the banking system and the level of borrowing from abroad, and demonstrates that there are cases where policies such as financial repression can be welfare improving. Importantly, while domestic holdings of sovereign debt can be understood
as strengthening commitment, sovereign governments may default in spite of large domestic exposures to their debt.

In a number of ways, this model is highly stylized and should be taken with a full understanding of its limitations. The treatment of the substitutability of domestic and foreign safe assets is simplistic, default is assumed to be non-selective, and bank recapitalizations are ruled out. I assume banks do not default as a result of a sovereign default. In a sense, the results may then be a lower bound on the endogenous cost of sovereign default, assuming that outright bank failures are more disruptive than financial frictions.

It would be useful to further explore the quantitative importance of the mechanisms described in this paper. For example, it would be good to know how much of an effect incremental increases in domestic bond holdings have on default incentives, conditional on the existing level of exposures. It would also be interesting to compare how the costs of financial frictions compare to disruptions to interbank lending, or to multi-period exclusion from international financial markets. I leave these questions to future work.
References


Angeloni, C. and Wolff, G. B. 2012. Are banks affected by their holdings of government debt?


Appendix: Proofs

Proof of Lemma 1

Proof. The lemma is proved by contradiction. First consider property (i). Set \( b = 0 \) and consider some \( b^* > 0 \). If a default does not occur, there are no financial frictions, i.e. \( n_1 \geq \tilde{N} \). In the event of default, the banking sector loses nothing, since \( b = 0 \). As such, \( n_1^d = n_1 \). In either case, there are no financial frictions, and banks pay an equilibrium interest rate which is greater than 1. Risk neutral households consume \( c_1 = 0 \), irrespective of \( D \). Consumption in \( t = 2 \) is given by equation (3), the sum of savings and net worth of the banking sector, \( c_2 = R_2 d_1 + n_2 \). Substituting equation (13) gives:

\[
c_2 = A_2 k_2.
\]

Further substituting equations (10), (12), and (2) with \( c_1 = 0 \) gives:

\[
c_2 = A_2 [A_1 k_1 - b^*],
\]

\[
c_2^d = A_2 [A_1 k_1].
\]

(A1)

Therefore \( c_2 < c_2^d \) for all \( b^* > 0 \), and thus \( V_1^d > V_1 \). Default is certain, so no lending will occur in equilibrium, contradicting \( b^* > 0 \).

For property (ii), note that \( c_2 \) and \( c_2^d \) are unchanged for \( b > 0 \), so long as the equity of the banking sector does not fall below \( \tilde{N} \), so that there are no financial frictions. As before, \( c_2 < c_2^d \) for all \( b^* > 0 \), and thus \( V_1^d > V_1 \). This contradicts \( b^* > 0 \). \( \square \)

Proof of Lemma 2

Proof. For \( n_1^d < \tilde{N} \), default causes a distortion, and taxation is lump sum, so the result follows directly. For \( n_1^d > \tilde{N} \), the Lemma can be proved by contradiction. Assume that for some \( n_1^d > \tilde{N} \), default is optimal, i.e. \( V_1^d > V_1 \). Although bank equity falls by \( b \) relative to the no default case, households
get additional income which they would otherwise have paid in taxes. Since there is no distortion, households face the same $R_2$ in both cases. As a result, households deposit excess funds not paid in taxes to the banking sector, and $V_1^d = V_1$, a contradiction.

**Proof of Lemma 3.**

*Proof.* With $b$ satisfying Lemma 1 and $b^* > 0$, it is possible to solve for the threshold value of $A_1$ below which default is optimal by setting $V_1^d = V_1$.

In the no default case, as in equation (A1):

$$V_1 = A_2 [A_1 k_1 - b^*].$$

In the case of default and financial frictions, deposits in the banking system are limited by equation (15) with $R_2 = 1$. Consumption in $t = 1$ is the balance of household wealth that could not be deposited in the banking system, $c_1^d = R_1 d_0 - d_1^d$. Consumption in $t = 2$ is given by the sum of deposits (which earn $R_2^d = 1$) and net worth of the banking system, $c_2^d = d_1^d + n_2^d$. The value of default is thus:

$$V_1^d = R_1 d_0 + n_2^d.$$

Substituting equations (13) and (10) gives:

$$V_1^d = R_1 d_0 + (A_2 - 1) d_1^d + A_2 n_1^d.$$

Substituting equation (15) and $R_2 = 1$ gives:

$$V_1^d = R_1 d_0 + \frac{\theta A_2}{1 - (1 - \theta) A_2} n_1^d.$$
Substituting equation (12) and rearranging gives:

\[
V_1^d = \frac{\theta A_2 A_1 k_1}{1 - (1 - \theta) A_2} + \left[ \frac{1 - A_2}{1 - (1 - \theta) A_2} \right] R_1 d_0. \tag{A2}
\]

Then, setting \( V_1^d = V_1 \), it is straightforward to solve for the realization of productivity at which they are equal, assuming that \( R_1 = A_1 \):

\[
\frac{\theta A_2 A_1 k_1}{1 - (1 - \theta) A_2} + \left[ \frac{1 - A_2}{1 - (1 - \theta) A_2} \right] R_1 d_0 = A_2 [A_1 k_1 - b^*],
\]

which can be solved for \( \tilde{A} \):

\[
\tilde{A} = \max \left\{ b^* \left[ \frac{1 - (1 - \theta) A_2}{(A_2 - 1)(\frac{d_0}{A_2} - k_1 (1 - \theta))} \right], A \right\}. \tag{A3}
\]

For default to occur when the realization of productivity is low, the slope of \( V_1 \) with respect to \( A_1 \) must be steeper than the slope of \( V_1^d \). Taking derivatives of equations (A1) and (A2) gives:

\[
\frac{dV_1}{dA_1} = A_2 k_1,
\]

\[
\frac{dV_1^d}{dA_1} = \frac{\theta A_2 k_1}{1 - (1 - \theta) A_2} + \left[ \frac{1 - A_2}{1 - (1 - \theta) A_2} \right] d_0.
\]

For the slope of \( V_1 \) to be steeper than the slope of \( V_1^d \), it must be that:

\[
A_2 k_1 > \frac{\theta A_2 k_1}{1 - (1 - \theta) A_2} + \left[ \frac{1 - A_2}{1 - (1 - \theta) A_2} \right] d_0.
\]

Rearranging gives:

\[
k_1 < d_0 \frac{1}{A_2 (1 - \theta)}.
\]

If \( b \) follows equation (8), then \( k_1 = \frac{\lambda + \omega - 1}{\lambda} (n_0 + d_0) \), and the condition can be
written on $n_0$, $d_0$, $A_2$, $\theta$, $\lambda$, and $\omega$:

$$n_0 < d_0 \left[ \frac{1}{A_2(1-\theta)} \frac{\lambda}{\lambda + \omega - 1} - 1 \right].$$

For parameter values under which this condition holds, when default occurs in equilibrium it will occur for low realizations of output, i.e. below the threshold $\tilde{A}$.

Proof of Proposition 1.

**Proof.** This follows from Lemma 3. Although equation (A3) does not explicitly include $b$, note from equation (9) that increasing $b$ beyond the banks’ optimal choice (equation (8)) leads to crowding out of private investment, i.e. $\frac{d_k}{db} < 0$. Since $\tilde{A}$ is increasing in $k_1$, it follows that the threshold decreases for increasing levels of $b$.

There is a secondary effect of increased $b$ on $b^*$ because the level of fiscal spending is fixed, wherein increasing $b$ decreases $b^*$, since $b^* = g/q - b$. To the extent that $q$ rises because of the increase in $b$, $b^*$ is smaller still. This contributes to the decline in $\tilde{A}$. □

Proof of Proposition 2.

**Proof.** This also follows from Lemma 3. Since $\tilde{A}$ is increasing in $k_1$, and $k_1$ increases with $n_0$ in equation (9), higher $n_0$ leads to a higher threshold $\tilde{A}$. □

Proof of Proposition 3.

**Proof.** This also follows from Lemma 3. $\tilde{A}$ is decreasing in $d_0$, so higher $d_0$ leads to a lower threshold $\tilde{A}$. To the extent that higher deposits lead to higher levels of domestic debt demanded $b$, this also decreases the extent of fiscal needs which need to be financed abroad, due to both quantity and price effects, as in Proposition 1. □
Proof of Proposition 4.

Proof. To show that domestic demand for sovereign debt is increasing in \( p \), there are two cases to consider. Provided that equation (8) describes the optimal domestic demand for sovereign debt, this follows directly, because the derivative of \( b^d \) with respect to \( p \) is positive. The second case is one in which banks demand more as part of a strategy of reaching for yield.

To show that equation (8) indeed pins down the domestic demand for sovereign debt, we proceed in two steps: first, that it is never optimal to demand less than \( b^d \), and then that if it is optimal to demand more, then demand remains increasing in \( p \).

The first step is proved by contradiction. Assume that \( 0 < b < b^d \) and \( b^* > 0 \), so that \( x > 0 \), i.e. not all of the funds of the banking sector can be invested in productive projects. In this case the \( t = 0 \) balance sheet constraint of the aggregate banking system is:

\[
k_1 + qb + x = n_0 + d_0.
\]

Substituting equations (7) and (8) gives:

\[
k_1 = \omega(n_0 + d_0) + (\lambda - 1)qb,
\]

and the evolution of bank equity in \( t = 1 \) is:

\[
n_1 = A_1 k_1 + b \cdot \mathbb{1}_{\{D=0\}} + Rx - R_1 d_0.
\]

Although the safe asset returns \( R \), the risk-adjusted return on government debt is also \( R \). However, government debt provides the additional benefit from facilitating interbank transactions, and so increasing \( b \) up to \( b^d \) increases \( k_1 \), as shown in equation (A4).

Since the default threshold is solved for in equation (A2), the banks’ optimiza-
tion problem can be written as:

$$\max E[n_2] = \int_{\tilde{A}} n_2^d f(A) dA + \int_{\tilde{A}} n_2 f(A) dA.$$  \hspace{1cm} (A5)

Bank equity is given by equation (13):

$$n_2 = A_2 k_2 - R_2 d_1.$$  \hspace{1cm} (A6)

In the no default case, substituting equations (10) and \( R_2 = A_2 \) gives:

$$n_2 = A_2 n_1.$$  \hspace{1cm} (A6)

In the event of default, substituting equations (10), (15) and \( R_2 = 1 \) gives:

$$n_2^d = \frac{\theta A_2}{1 - (1 - \theta) A_2} n_1^d.$$  \hspace{1cm} (A7)

In either case, bank equity in \( t = 2 \) increases monotonically with \( n_1 \). Therefore, it cannot be optimal to have \( x > 0 \) if \( 0 < b < b^d \).

To show that it is also not optimal to demand \( b > b^d \), note that in this case, demand for government debt would crowd out private investment, i.e. \( k_1 \) is decreasing in \( b \) if \( b > b^d \):

$$k_1 = n_0 + d_0 - qb.$$  \hspace{1cm} (A8)

Taking equation (A5) and substituting equations (A6), (A7), \( R_1 = A_1 \), and (12) gives:

$$\max E[n_2] = \int_{\tilde{A}} \frac{\theta A_2}{1 - (1 - \theta) A_2} A[k_1 - d_0] f(A) dA + \int_{\tilde{A}} A_2 [A(k_1 - d_0) + b] f(A) dA.$$  \hspace{1cm} (A9)

Substituting equation (A8) and taking derivatives with respect to \( b \) yields the
first order condition for the bank problem:

\[
\int_\tilde{A}^{A} \frac{\theta A_2}{1 - (1 - \theta)A_2} [-Aq] f(A)dA + \int_{\tilde{A}}^{A} A_2 [-Aq + 1] f(A)dA. \tag{A10}
\]

If the sum of these two terms is negative, then demand for sovereign debt is at the optimal level, i.e. equation (8). The first term is unambiguously negative. However, the second term may be positive or negative. If it is positive and larger than the first term, then domestic demand for sovereign debt increases not only because of the need for collateral, but also because banks are reaching for yield. In other words, the expected return on bonds is higher than the expected losses from crowding out.

From equation (6), the second term of equation (A10) equals:

\[
1 - Aq = \frac{R - A(1 - p)}{R},
\]

which is also increasing in \(p\).

Proof of Proposition 5.

Proof. This follows directly from Lemma 3. Equation (A3) is increasing in \(b^*\), thus the higher foreign debts are, the higher the threshold below which default results.

Proof of Proposition 6.

Proof. This follows directly from Lemma 3.

Proof of Proposition 7.

Proof. If financial frictions bind in \(t = 0\), then bank deposits in \(t = 0\) are limited to a multiple of bank equity:

\[
d_0 \leq \frac{(1 - \theta)A_1 \left(\frac{\lambda + \omega - 1}{\lambda}\right)}{R_1 - (1 - \theta)A_1 \left(\frac{\lambda + \omega - 1}{\lambda}\right)n_0}
\]
This limits deposits to an increasing function of \( n_0 \). From Proposition 3, \( \Delta(b, b^*) \) is decreasing in \( d_0 \). Conversely, if \( d_0 \) falls due to a constraint \( t = 0 \), default risk increases.

Proof of Proposition 8.

Proof. Formally, for financial repression to improve welfare requires that \( dV_1/db > 0 \). This condition can be written:

\[
\frac{dV_1}{db} = A_2 \left[ A_1 \frac{dk_1}{db} - \frac{db^*}{db} \right], \\
= A_2 \left[ A_1 \left( -\frac{dq}{db} b - q \right) + \frac{g}{q^2} \frac{dq}{db} + 1 \right], \\
= A_2 \left[ \frac{dq}{db} \left( \frac{g}{q^2} - A_1 \right) + 1 - qA_1 \right] > 0.
\]

If debt is not risky, i.e. \( q = 1 \), this is unambiguously negative, i.e. financial repression will only decrease welfare. However, for \( q < 1 \), there are cases where \( dV_1/db > 0 \).