Connected components

Linear $O(V + E)$
Component graph is acyclic.

Once DFS visits a component, it discovers all vertices in that component before finishing with $v$.

Largest finishing time is in a source in the component graph.
DFS in a sink, never leaves that component.
edge in component graph

\[ x \rightarrow y \]

\[ f(x) > f(y) \]
Proof Assume not. There is some cut for which the min. wt. edge is not in the MST. (Call the current alleged MST $T$)

$(uv)$ is the min. wt. edge in $T$ crossing the cut $(xy)$ is the min wt edge crossing cut

$w(x,y) < w(u,v)$. Adding $(xy)$ to $T$ creates a cycle which crosses the cut at least twice. Let $(a,b)$ be an edge besides $(x,y)$ that crosses the cut. My tree is $T \cup \{(x,y)\} - (a,b)$

$w(T') = w(T) + w(xy) - w(a,b)$

but $w(xy) < w(a,b)$

$\therefore w(T') < w(T)$ contradicts $T$ being an MST.