Dealing w/ NP-complete Problems

- solve small instances
- solve instances w/ special structure
- Heuristics (find not nec. opt. solutions)
  - greedy
  - simulated annealing, genetic algs., tabu search, GRASP, ...
  - no guarantee on quality of solution
Approximation Algs

Problem $X$, instance $I$, want to minimize

Let $OPT(I)$ be the best possible solution (min)

A $\rho$-approx. alg. for $X$, called $A$,

1) runs in polynomial time

2) return a solution of value $A(I)$ where $A(I) \leq \rho \cdot OPT(I)$.

($\rho \geq 1$, small $\rho$ are better)
If \( p = 1.1 \), 10% relative error.

Need to accomplish this, w/o knowing \( \text{OPT}(I) \).

1) Compute a lower bound \( \text{LB}(I) \) on \( \text{OPT}(I) \), some poly-time computable value (object)
   
   \[ \text{st. } \text{LB}(I) \leq \text{OPT}(I) \]

2) Use \( \text{LB}(I) \) to find a solution \( A(I) \)
   
   \[ \text{w/ } A(I) \leq p \cdot \text{LB}(I) \].

Conclude \( A(I) \leq p \cdot \text{LB}(I) \leq p \cdot \text{OPT}(I) \).
Give a 2-approx alg \((p=2)\)

Matching: A set of edges that share no endpoints (find in polynomial time)

Maximum matching is matching with most edges

This is \(0.5\) lower bound
\[ MM(\mathcal{I}) \leq \text{size of } MM \]
\[ OPT(\mathcal{I}) = \text{size of } OPT \text{. vertex cover} \]

\[ MM(\mathcal{I}) \leq OPT(\mathcal{I}). \]

Alg.
1) Compute a max. matching
2) For each edge in the matching, include both endpoints in the vertex cover. \((VC)\)
The set of vertices we choose is a vertex cover, because if some edge \((u, v)\) has neither \(u\) nor \(v\) in the cover, then \((u, v)\) could be added to the matching, contradicting the fact that we have a maximum matching.

\[ VC(I) = 2MM(I) \leq 2OPT(I) \]

\[ VC(I) \leq 2OPT(I) . \]
2-approx for TSP w/ Δ-inequality

\[ w(a, b) - \text{dist between } a \text{ and } b \]

For any \( a, b, c \):
\[ w(a, b) \leq w(a, c) + w(c, b) \]

w.d. Δ-ineq, you cannot approx. TSP (unless P=NP)
dist = Eucl. dist.

LB = MST

OPT = opt.

TSP tour

MST(I) ≤ OPT(I)

double MST(I) = 2 MST(I)

double MST(I) ≤ 2 OPT(I).

2 approximation
1. Find MST
2. Double each edge
3. Find an Euler tour in doubled edges
4. Shortcut to Euler tour to STSP tour
Matching $\leq \frac{1}{2} \text{OPT}(E)$

$\text{TSW} \leq \text{OPT}(E)$