= all comparators are with pivot.

\[ T(n) = T(x) + T(n-x-1) + O(n) \]

when the split puts \( x \) els. on left side.

if \( x = \frac{n}{2} \) \[ T(n) = 2T(\frac{n}{2}) + O(n) \Rightarrow O(n \log n) \]

\( x = 1 \) \[ T(n) = T(1) + T(n-2) + O(n) \]
\[ = T(n-2) + O(n) \]
\[ = n + (n-2) + (n-1) + \ldots + 1 = O(n^2) \]
What happens in avg. case?

If \( x = \frac{n}{10} \)

\[
T(n) = T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + O(n)
\]

= \( O(n \log n) \). (need proof)

How often is the split \( \frac{1}{10}, \frac{9}{10} \) or better?

\[
\begin{array}{c|c|c}
10\% & n & 70\% \\
10\% & n & 70\%
\end{array}
\]

80\% of time
Formally,

\[ T(n) = \text{expected running time of q.s.} \]

\[ T(n) = \sum \Pr(\text{ith smallest is pivot} \mid \text{ith smallest is pivot}) \cdot \left( \frac{1}{n} (T(i-1) + T(n-i) + O(1)) \right) \]

One can show by induction that \( T(n) = O(n \log n) \).
Different analysis:

- Count executions of line 6
  - counting comparisons
- all comps. w/ pivot
- each pair of elts. is compared at most once

rename data \(Z_1, \ldots, Z_n\) in sorted order.

\[Z_{ij} = \{Z_{i1}, Z_{i2}, \ldots, Z_j\}\]
\[ X_{ij} = I\{Z_i \text{ is compared with } Z_j\} \]

\[ X = \text{total \# comparisons} \]

\[ X = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} X_{ij} \]

\[ E[X] = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E[X_{ij}] \]

\[ E[X] = \sum_{i=1}^{n} \sum_{j=i+1}^{n} P_i \left(Z_i \text{ is compared with } Z_j\right) \]
\( z_i \) is compared to \( z_j \) iff

either \( z_i \) or \( z_j \) is chosen as pivot before any other element in \( Z_{ij} \).

\[
\Pr(z_i \text{ comp. to } z_j) = \frac{2}{j+1}
\]

\[
\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{2}{j-i+1} \quad (k = j-i+1)
\]

\[
\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{2}{j-i+1} \leq \sum_{i=1}^{n-1} \sum_{k=2}^{n+1} \frac{2}{k} \leq \sum_{i=1}^{n-1} \sum_{k=2}^{n+1} \frac{2}{k} \leq 2(n+1) = O(n \log n).
\]