Sorting restricted ranges of numbers

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- Assume each of the $n$ input elements is an integer in the range $1 \ldots k$. 
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Idea For each $A[i]$ compute the number of elements less than or equal to $A[i]$, and use that to compute position.

• Array $A[1 \ldots n]$ – holds input
• Array $C[1 \ldots k]$ – $C[j]$ holds number of elements of $A$ less than or equal to $j$

Example:

<table>
<thead>
<tr>
<th>index</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>2</td>
<td>9</td>
<td>1</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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- Assume each of the \( n \) input elements is an integer in the range \( 1 \ldots k \).

**Idea** For each \( A[i] \) compute the number of elements less than or equal to \( A[i] \), and use that to compute position.

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<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>2</td>
<td>9</td>
<td>1</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C )</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

**Questions**

- How do we compute \( C \)
- We need to be careful dealing with duplicates (stability)
Counting Sort

\textit{Counting − Sort}(A, B, k)

1 \textbf{for} \ i \leftarrow 0 \ \textbf{to} \ k
2 \textbf{do} \ C[i] \leftarrow 0
3 \textbf{for} \ j \leftarrow 1 \ \textbf{to} \ \text{length}[A]
4 \textbf{do} \ C[A[j]] \leftarrow C[A[j]] + 1
5 \triangleright \ C[i] \ now \ contains \ the \ number \ of \ elements \ equal \ to \ i.
6 \textbf{for} \ i \leftarrow 1 \ \textbf{to} \ k
7 \textbf{do} \ C[i] \leftarrow C[i] + C[i − 1]
8 \triangleright \ C[i] \ now \ contains \ the \ number \ of \ elements \ less \ than \ or \ equal \ to \ i.
9 \textbf{for} \ j \leftarrow \text{length}[A] \ \textbf{downto} \ 1
10 \textbf{do} \ B[C[A[j]]] \leftarrow A[j]
11 \quad C[A[j]] \leftarrow C[A[j]] − 1
Analysis

- Running Time $O(n + k)$
- No Comparisons
- Doesn’t work on all data
- Good when $k$ is small
- Examples?

Question: Is Counting Sort appropriate for alphabetizing the Columbia directory?
Radix Sort

Radix – Sort\((A, d)\)

1. for \(i \leftarrow 1\) to \(d\)
2. do use a stable sort to sort array \(A\) on digit \(i\)

Example

<table>
<thead>
<tr>
<th></th>
<th>STABLE SORT</th>
<th></th>
<th>STABLE SORT</th>
<th></th>
<th>STABLE SORT</th>
</tr>
</thead>
<tbody>
<tr>
<td>379</td>
<td>STABLE SORT</td>
<td>912</td>
<td>STABLE SORT</td>
<td>802</td>
<td>STABLE SORT</td>
</tr>
<tr>
<td>912</td>
<td>(\Rightarrow)</td>
<td>802</td>
<td>(\Rightarrow)</td>
<td>803</td>
<td>(\Rightarrow)</td>
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<td>258</td>
<td>823</td>
<td>804</td>
<td>259</td>
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<td>269</td>
<td>803</td>
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