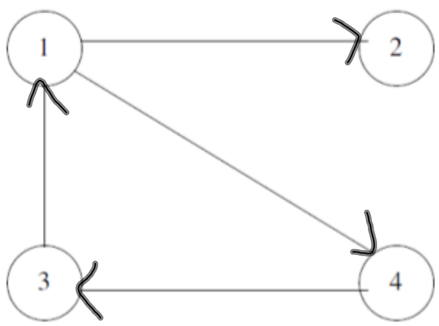
node = object
edge = relation between objects

Oct 27-4:09 PM

graph.pdf (application/pdf Object) - Mozilla Firefox

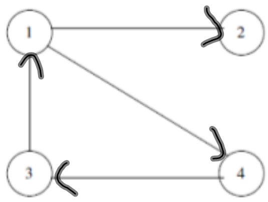
## Graphs

- Graph $G = (V, E)$ has vertices (nodes) V and edges (arcs) E.
- Graph can be directed or undirected
- Graph can represent any situation with objects and pairwise relationships.




Oct 27-4:15 PM

# Representations




$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

directed

Adjacency Matrix

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 1 |
| 2 | 1 | 0 | 0 | 0 |
| 3 | 1 | 0 | 0 | 1 |
| 4 | 1 | 0 | 1 | 0 |



Oct 27-4:18 PM

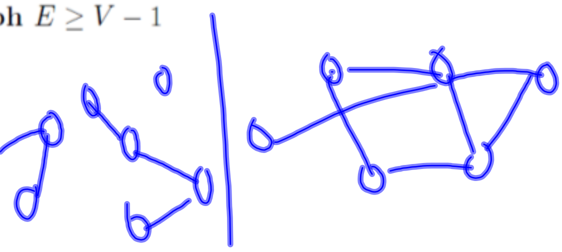


# Comparison

| | Space | Query Time | All neighbors time |
|---|---|---|---|
| Matrix | $O(V^2)$ | $O(1)$ | $O(V)$ |
| List | $O(E)$ | $O(\text{degree})$ | $O(\text{degree})$ |

- For a simple graph (no double edges) $E \leq V^2 = O(V^2)$
- For a connected graph $E \geq V - 1$
- For a tree $E = V - 1$

$\Omega(V) = E = O(V^2)$

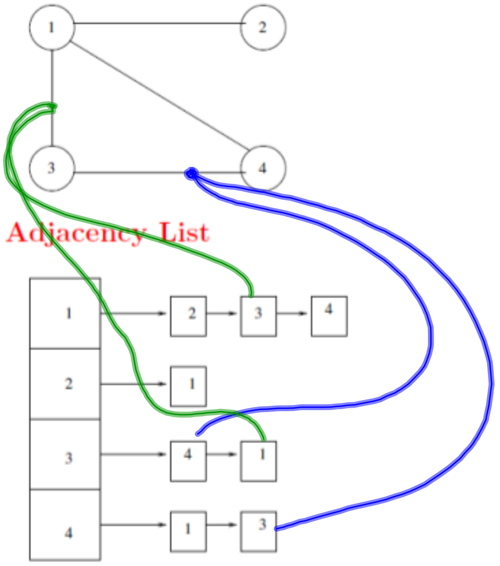




Oct 27-4:26 PM

graph.pdf (application/pdf Object) - Mozilla Firefox

http://www.columbia.edu/~cs2035/courses/csor4231.F09/graph.pdf




total size

un. $V + 2E$

dir. $V + E$

$O(E)$

Oct 27-4:28 PM

## Which rep?

Depends on operations,

$E, V$

$E \sim V$ sparse (adj list.)

$E \sim V^2$ dense (adj matrix)

sparse graph: $10^6$ nodes

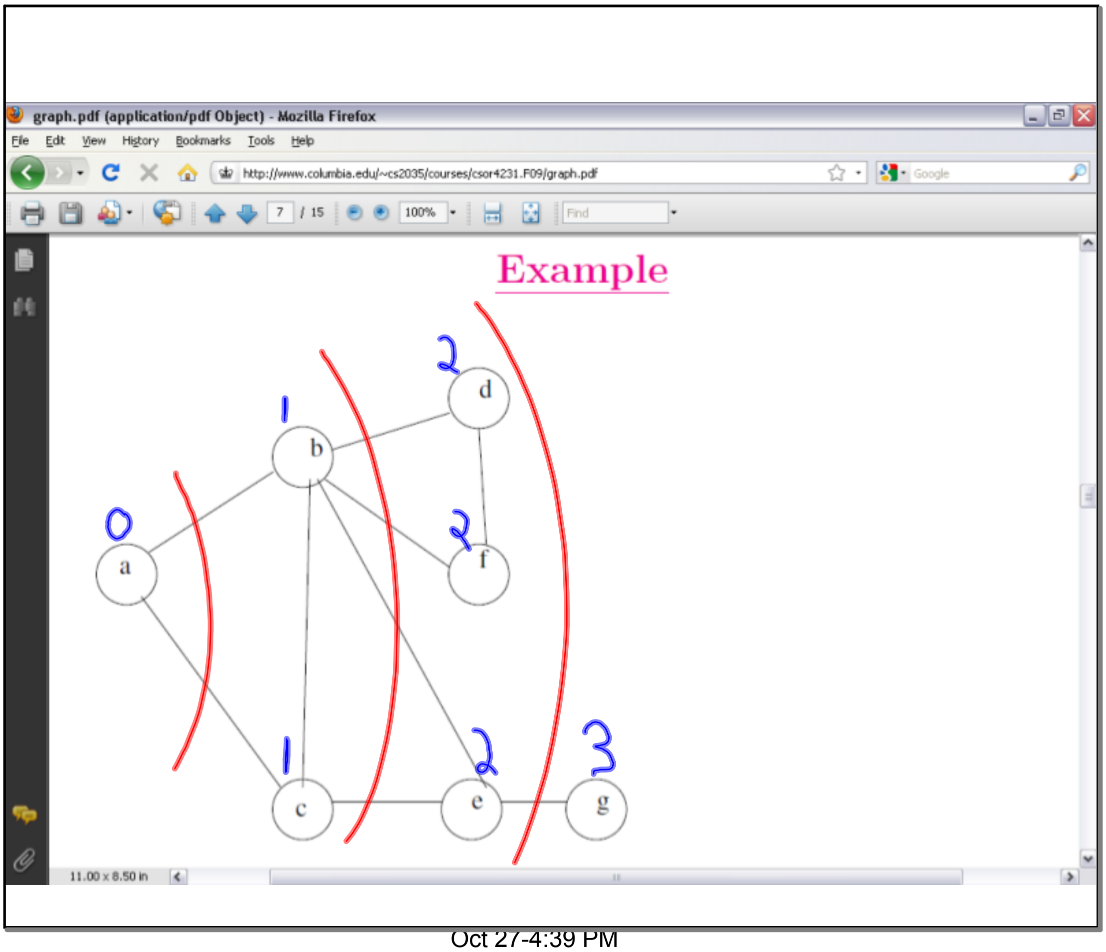
$3 \times 10^6$ edges

| | storage | access | all neighbors |
|---|---|---|---|
| AM | $10^{12}$ | 1 | $10^6$ |
| AL. | $4 \times 10^6$ | $\sim 3$ | $\sim 3$ |

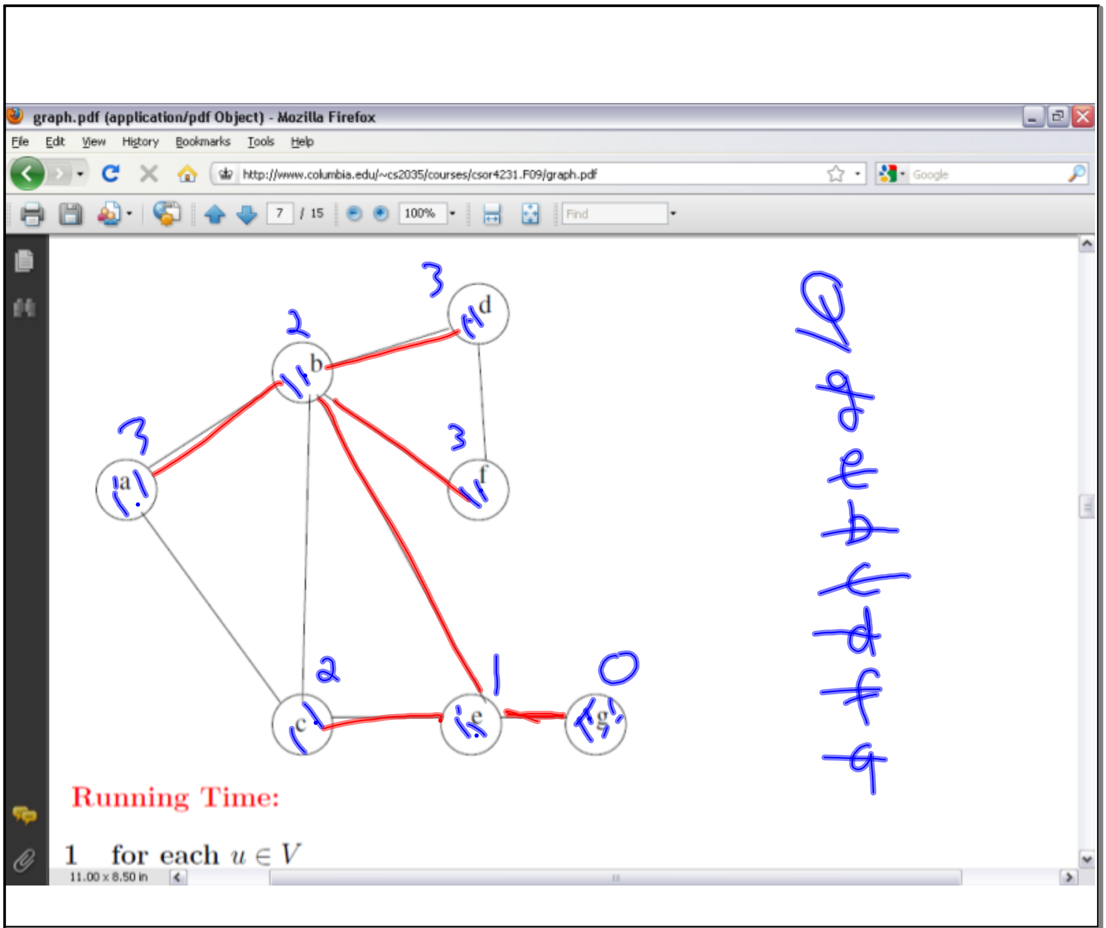
$10^6$

$10^6$

Oct 27-4:31 PM




Oct 27-4:39 PM




Oct 27-4:44 PM

graph.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.columbia.edu/~cs2035/courses/csor4231.F09/graph.pdf

6 / 15 100% Find

$BFS(G, s)$

for each vertex $u \in V[G] - \{s\}$
    do $color[u] \leftarrow$ WHITE
        $d[u] \leftarrow \infty$
        $\pi[u] \leftarrow$ NIL

V

$color[s] \leftarrow$ GRAY
$d[s] \leftarrow 0$
$\pi[s] \leftarrow$ NIL
$Q \leftarrow \emptyset$
ENQUEUE$(Q, s)$

1

while $Q \neq \emptyset$
    do $u \leftarrow$ DEQUEUE$(Q)$
        for each $v \in Adj[u]$
            do if $color[v] =$ WHITE
                then $color[v] \leftarrow$ GRAY
                    $d[v] \leftarrow d[u] + 1$
                    $\pi[v] \leftarrow u$

11.00 x 8.50 in

Oct 27-4:45 PM

graph.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.columbia.edu/~cs2035/courses/csor4231.F09/graph.pdf

6 / 15 100% Find

for each vertex $u \in V[G] - \{s\}$
    do $color[u] \leftarrow$ WHITE
        $d[u] \leftarrow \infty$
        $\pi[u] \leftarrow$ NIL

V

$color[s] \leftarrow$ GRAY
$d[s] \leftarrow 0$
$\pi[s] \leftarrow$ NIL
$Q \leftarrow \emptyset$
ENQUEUE$(Q, s)$

1

$V^2$

while $Q \neq \emptyset$
    do $u \leftarrow$ DEQUEUE$(Q)$
        for each $v \in Adj[u]$
            do if $color[v] =$ WHITE
                then $color[v] \leftarrow$ GRAY
                    $d[v] \leftarrow d[u] + 1$
                    $\pi[v] \leftarrow u$
                    ENQUEUE$(Q, v)$
        $color[u] \leftarrow$ BLACK

1

at most 2E times

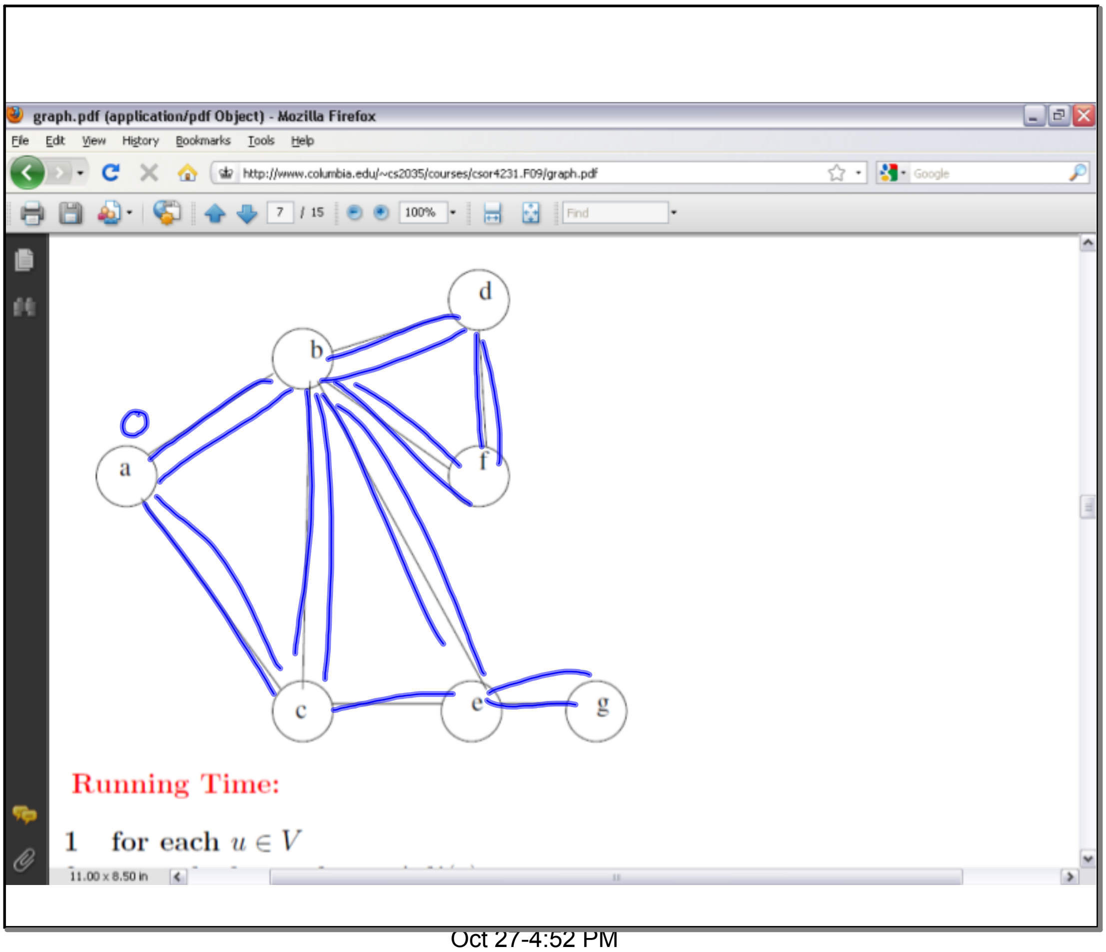
for all vertices v

for all neighbors of v

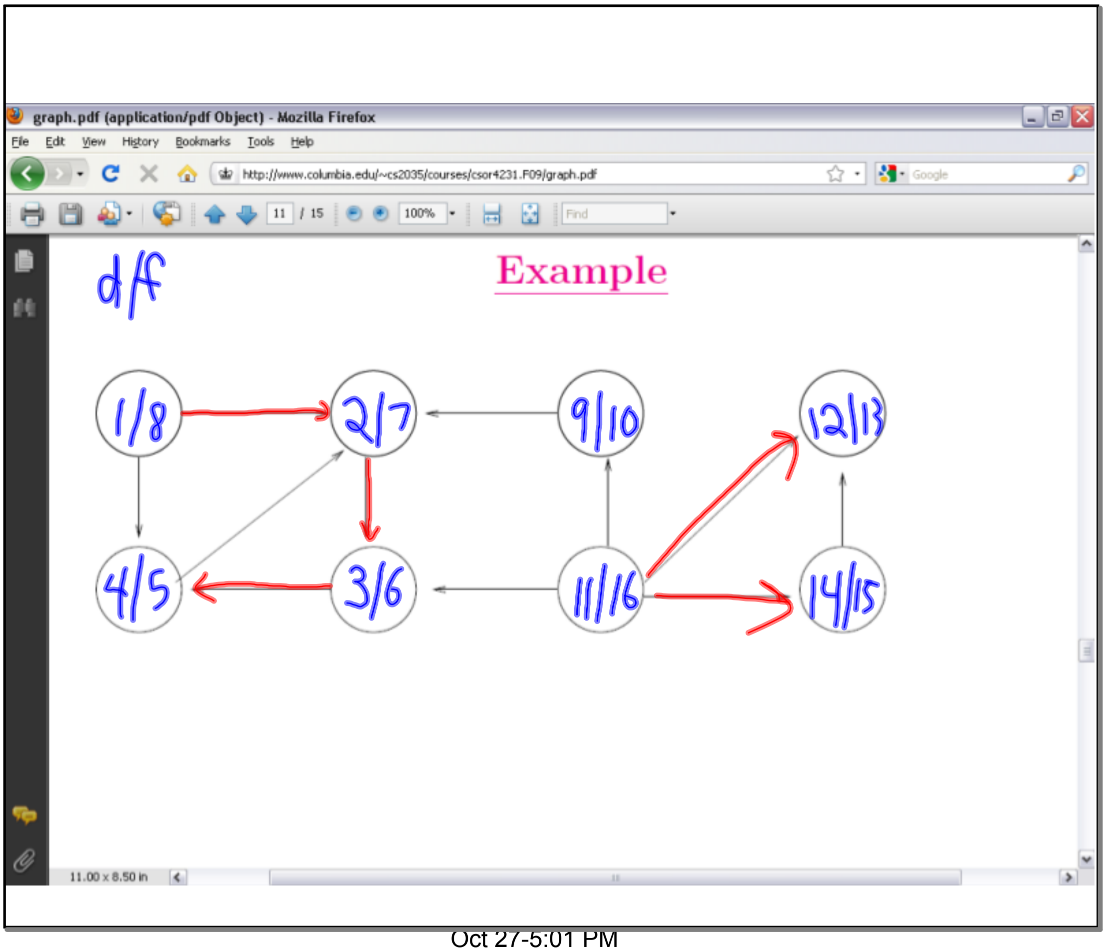
O(1)

11.00 x 8.50 in

Oct 27-4:50 PM




Oct 27-4:52 PM




Oct 27-5:01 PM

graph.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.columbia.edu/~cs2035/courses/csor4231.F09/graph.pdf

10 / 15 100% Find

*DFS*$(G)$

```
for each vertex u ∈ V[G]
      do color[u] ← WHITE
         π[u] ← NIL
time ← 0
for each vertex u ∈ V[G]
      do if color[u] = WHITE
            then DFS-VISIT(u)
```

each vertex is DFS-visited exactly once.

**DFS-Visit**$(u)$

```
color[u] ← GRAY                  ▷ White vertex u has just been discovered.
time ← time + 1
d[u] ← time
for each v ∈ Adj[u]              ▷ Explore edge (u, v).
      do if color[v] = WHITE
            then π[v] ← u
                 DFS-VISIT(v)
```

- within a call, you explore adjacent edges.

$O(E) \Leftarrow$ running time.

for all v

for all neighbor $O(1)$

11.00 x 8.50 in

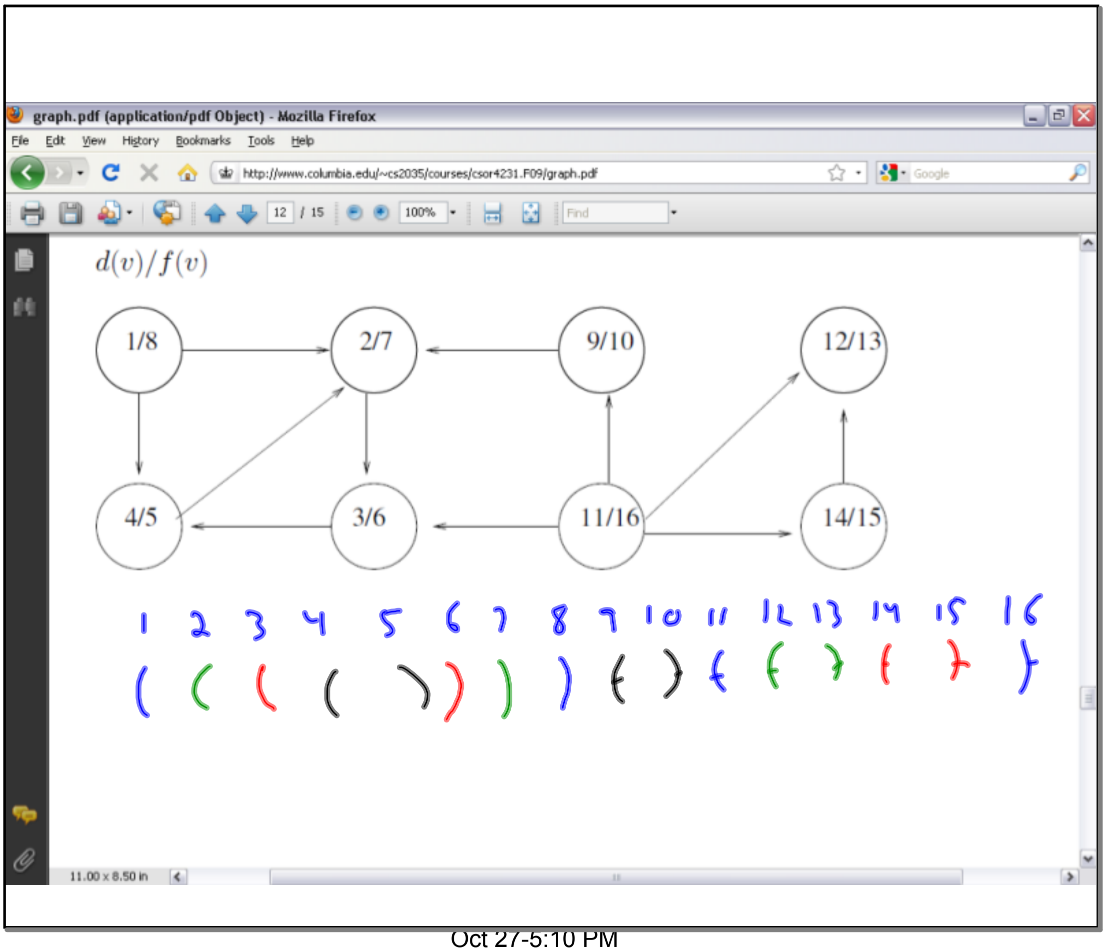
Oct 27-5:04 PM

Well-formed expression

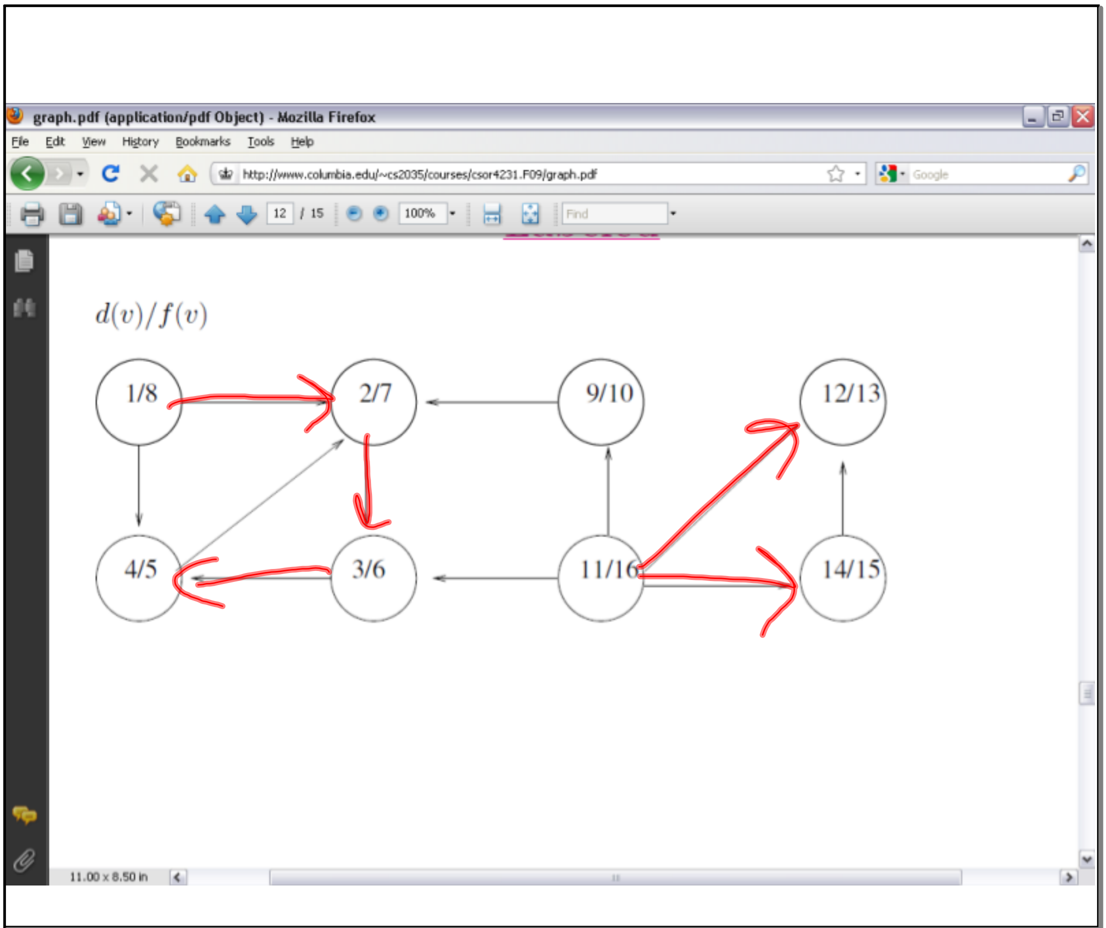
$(x+[y-3])$ ( [ ] )

$\cancel{(x+[y-3)]}$ ( ) [ ]

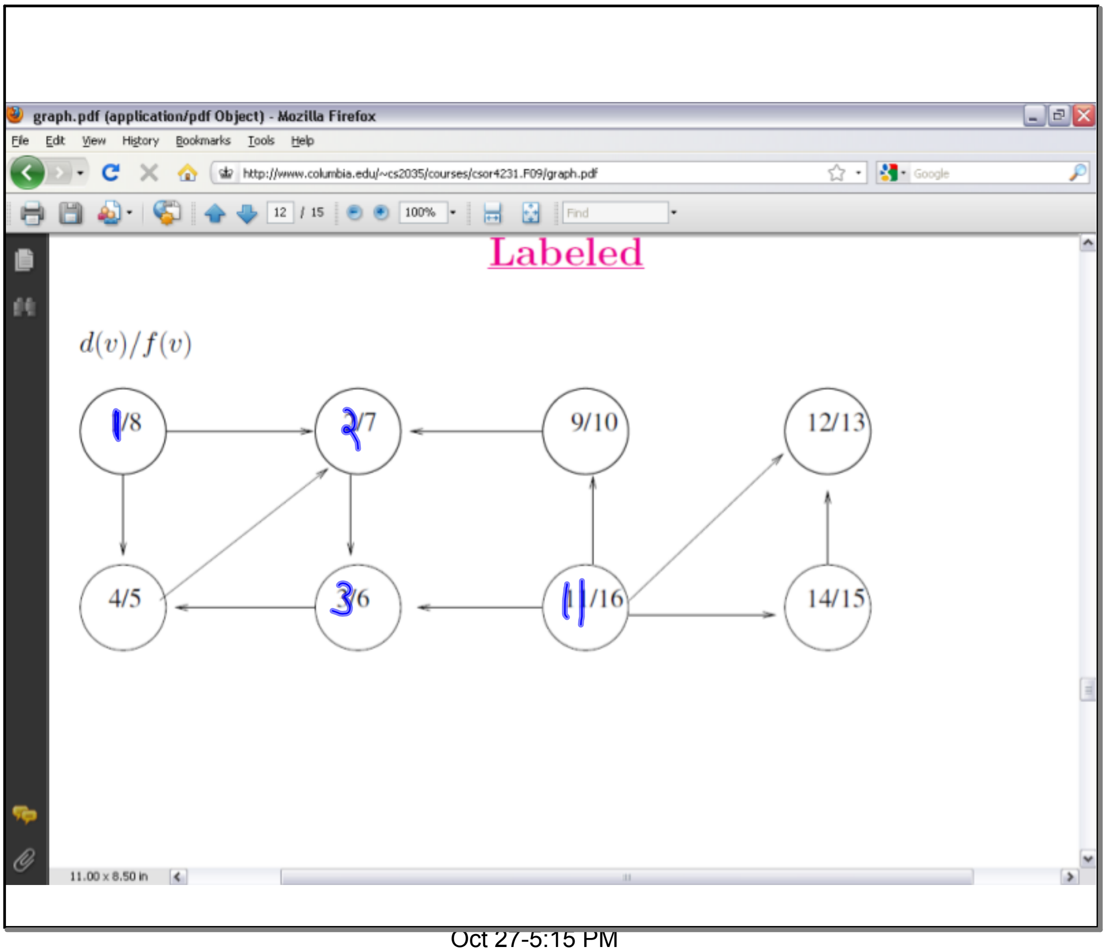
Oct 27-5:05 PM




Oct 27-5:10 PM



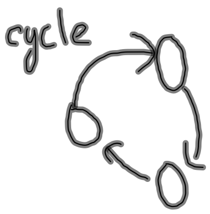

Oct 27-5:12 PM

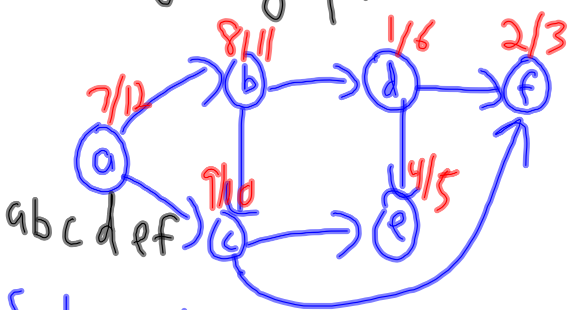



Oct 27-5:15 PM

# Topological Sorting

DAG - directed acyclic graph




Topological Sort: ordering of vertices s.t. if $(v,w) \in E$, then v is before w in the ordering.

abdcfe ~~eacbdf~~

Oct 27-5:17 PM

## Topological Sort Algorithm (G)

1. Do DFS (G)
2. Output nodes in order of decreasing finishing times.

$O(E)$

Oct 27-5:22 PM