Thm Top Sort Alg is correct.

i.e. $(x,y) \in E \Rightarrow f(x) > f(y)$.

Pf

$x \rightarrow y$

i) $d(x) < d(y)$

Then at $d(x)$, y is white

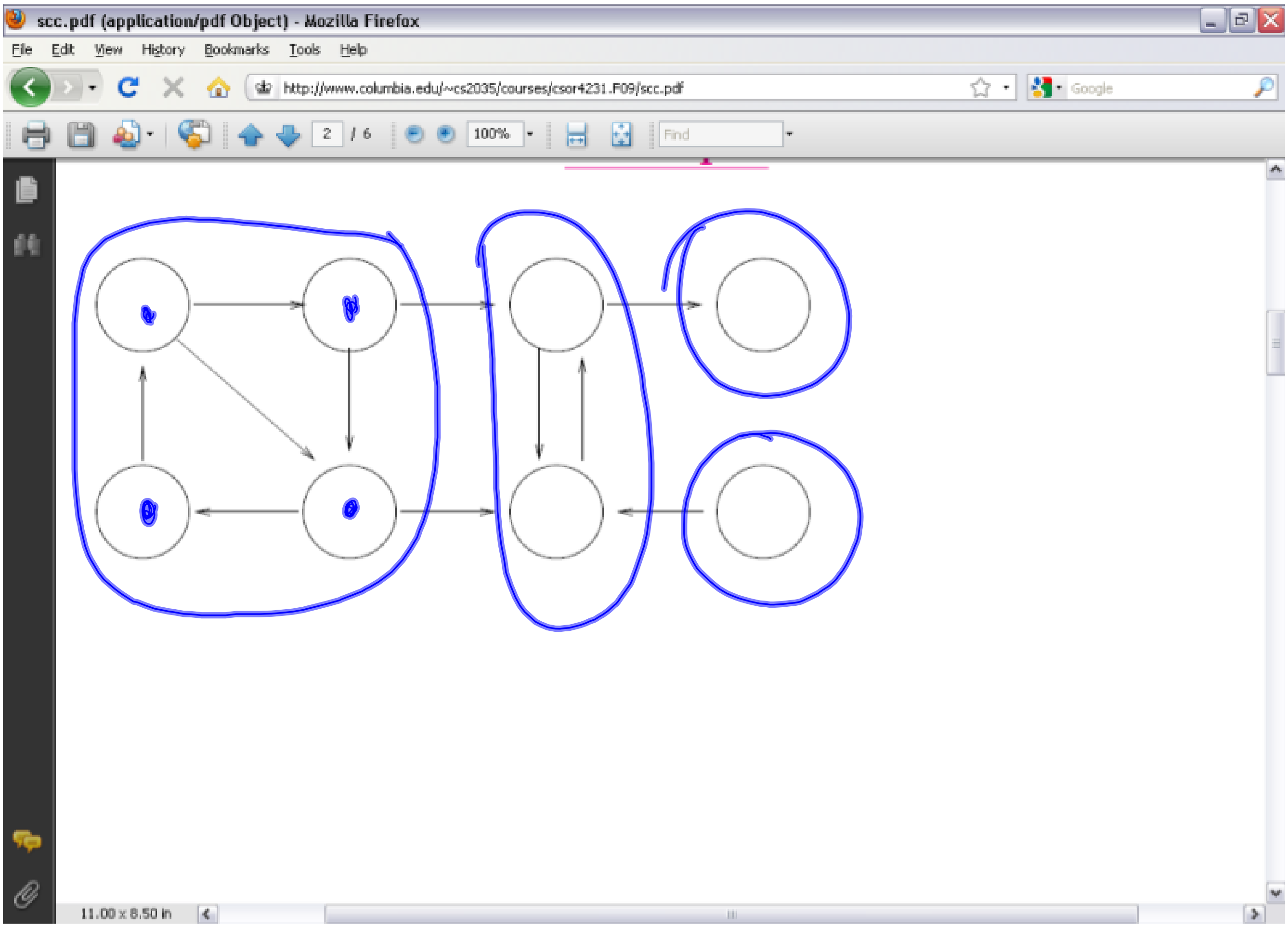
$d(x) < d(y) < f(y) < f(x)$

ii) $d(x) > d(y)$

Because G is a DAG, no path from y to $x \Rightarrow f(y) < d(x) < f(x)$ ☒

Connectivity

Given a undir graph, find connected components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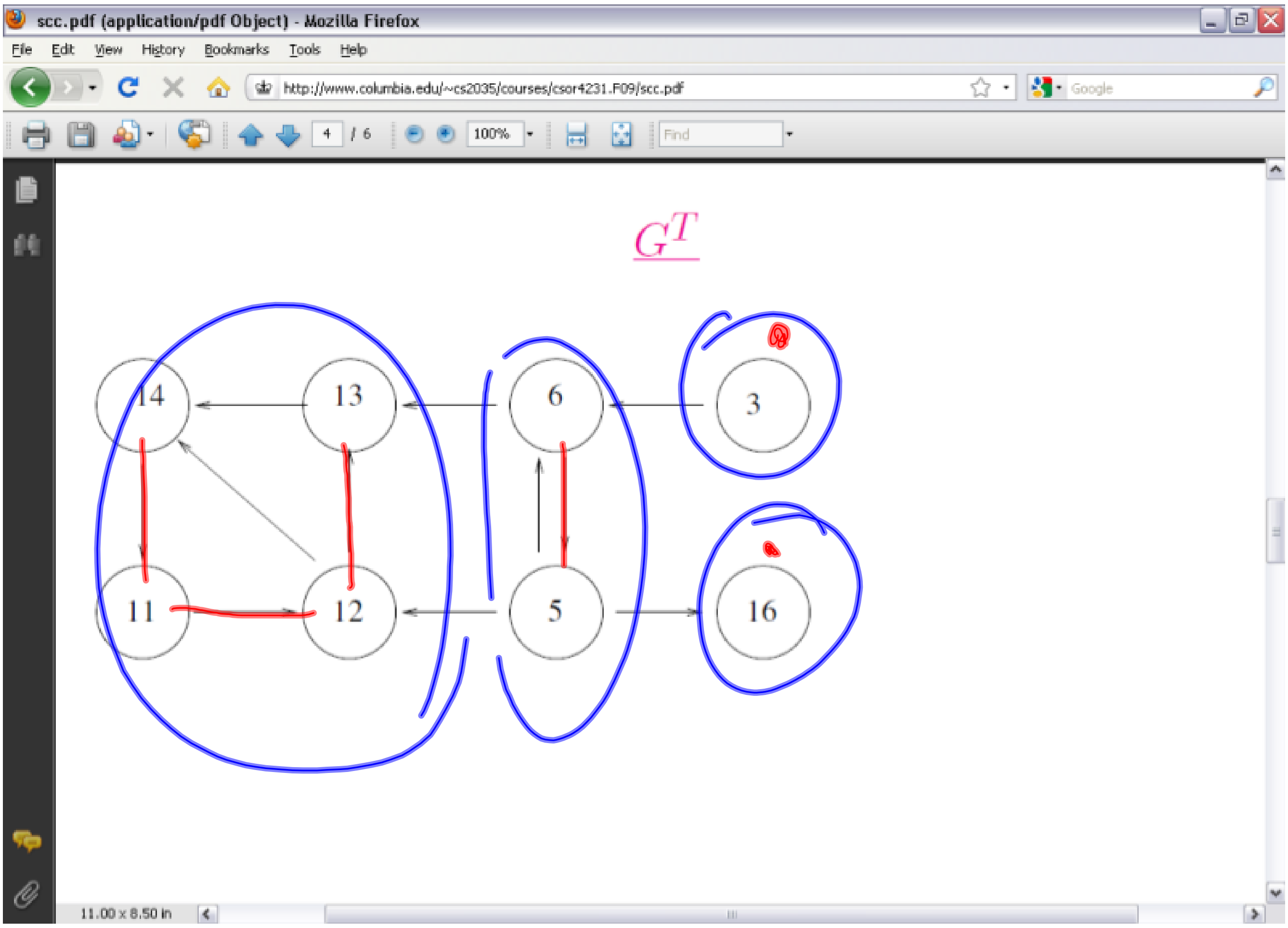
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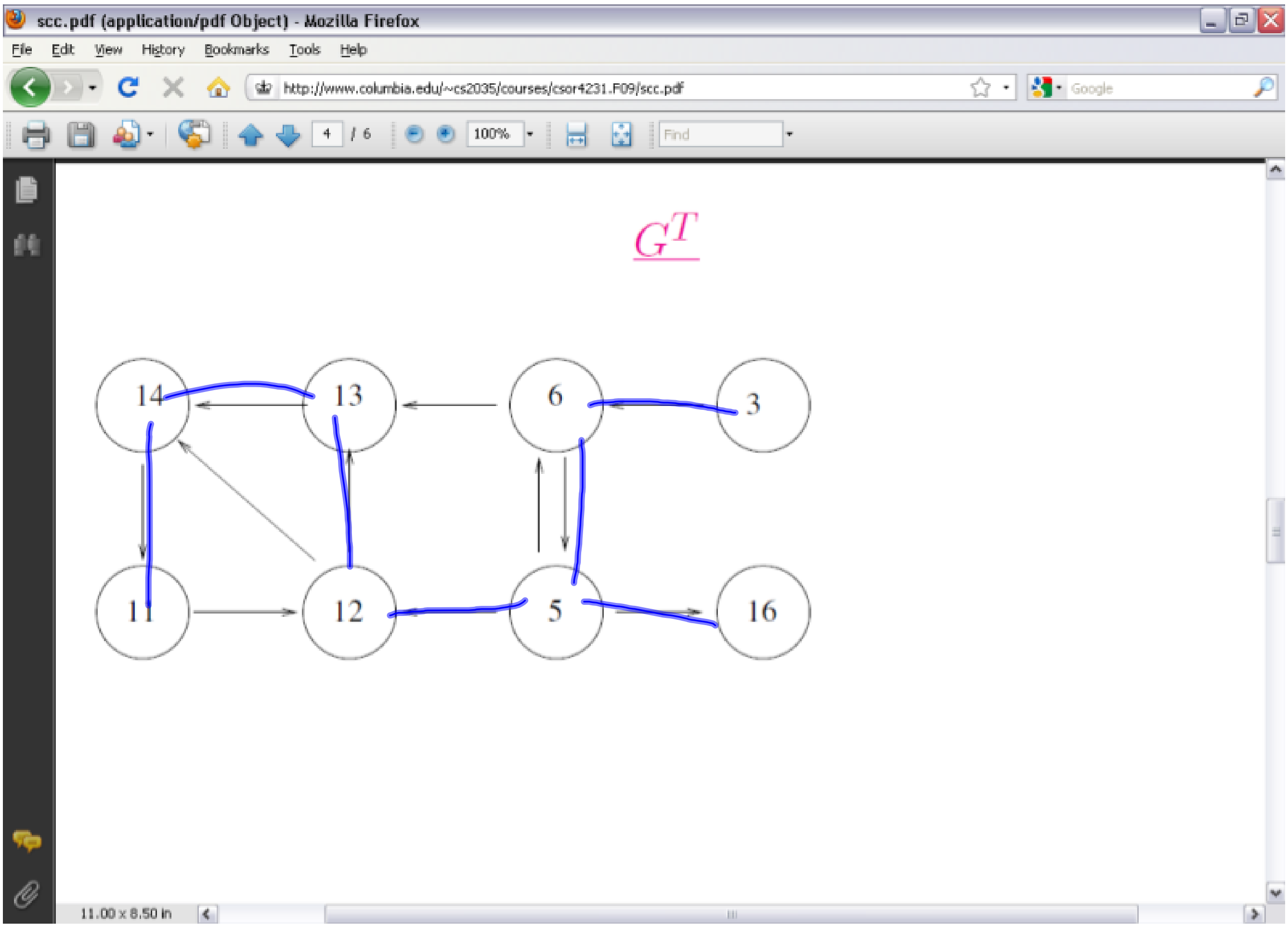

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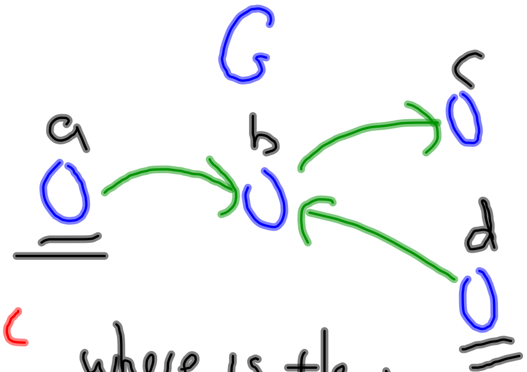
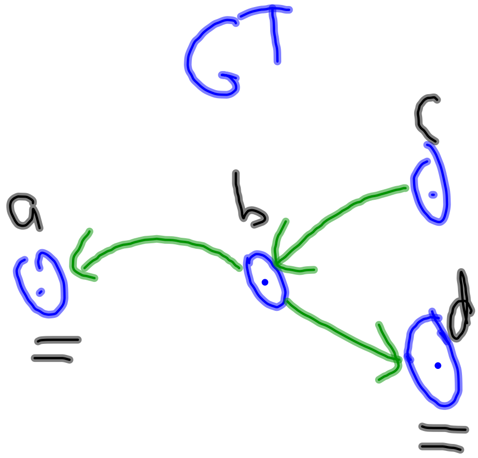
Acyclic Component Graph





adbc
dabc

where is the v w/ largest f(v)?

in a source → sink (only incoming edges)

at each step, we explore a (node) component of the component graph, & we do so in topological order of G, ∴ the component has no outgoing edges in G^T to an unexplored component.

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Strongly Connected Components

Definition A strongly connected component of a directed graph G is a maximal set of vertices $C \subseteq V$ such that for every pair of vertices u and v, there is a directed path from u to v and a directed path from v to u.

Strongly-Connected-Components(G)

1 call DFS(G) to compute finishing times $f[u]$ for each vertex u — O(E)
2 compute G^T — O(E)
3 call DFS(G^T), but in the main loop of DFS, consider the vertices in order of decreasing $f[u]$ (as computed in line 1) — O(E)
4 output the vertices of each tree in the depth-first forest formed in line separate strongly connected component — O(E)

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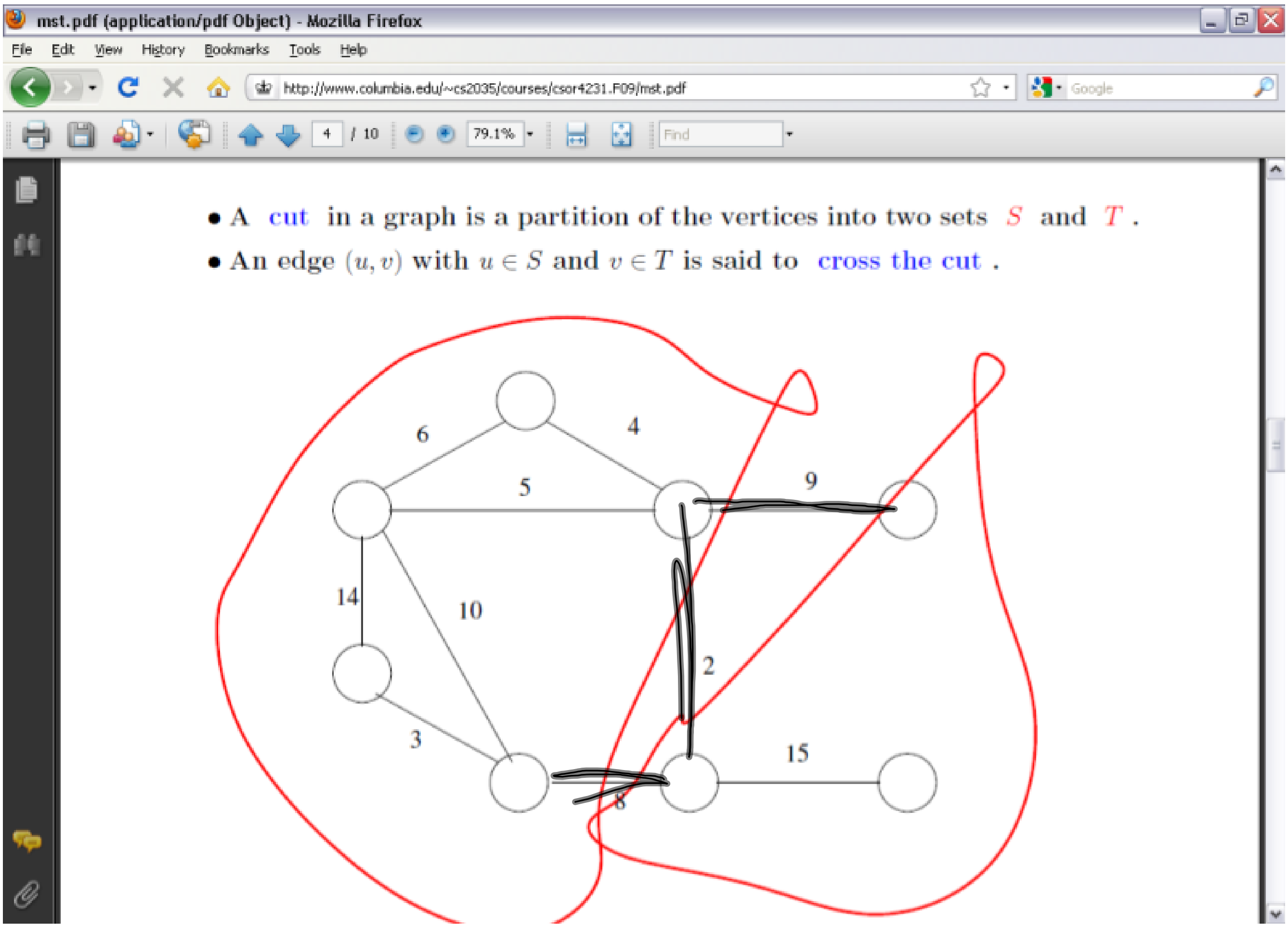
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• A cut in a graph is a partition of the vertices into two sets S and T.
• An edge (u, v) with $u \in S$ and $v \in T$ is said to cross the cut.
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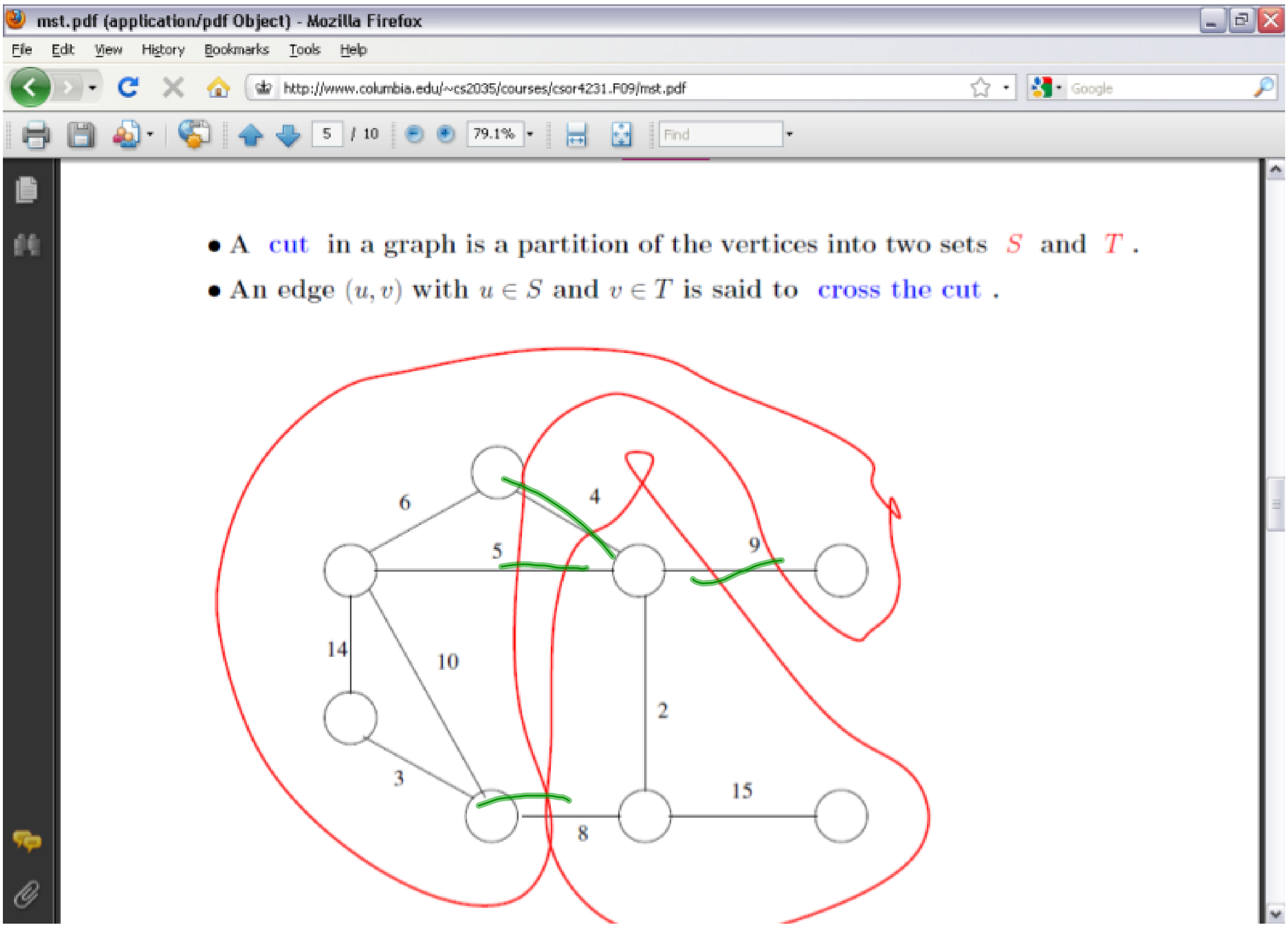
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• A cut in a graph is a partition of the vertices into two sets S and T.
• An edge (u, v) with u ∈ S and v ∈ T is said to cross the cut.
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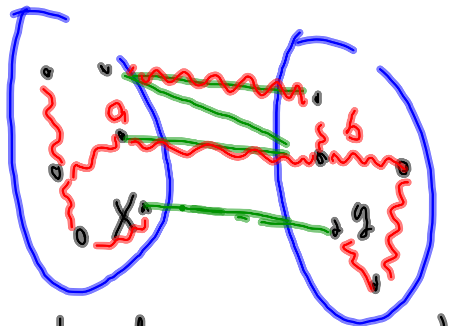
<u>Claim</u> For any cut (S,T), the minimum wt. edge crossing the cut is in the min. spanning tree.

<u>Pf</u> Assume not. There must be some cut for which the min wt. edge crossing the cut is not in the minimum spanning tree.



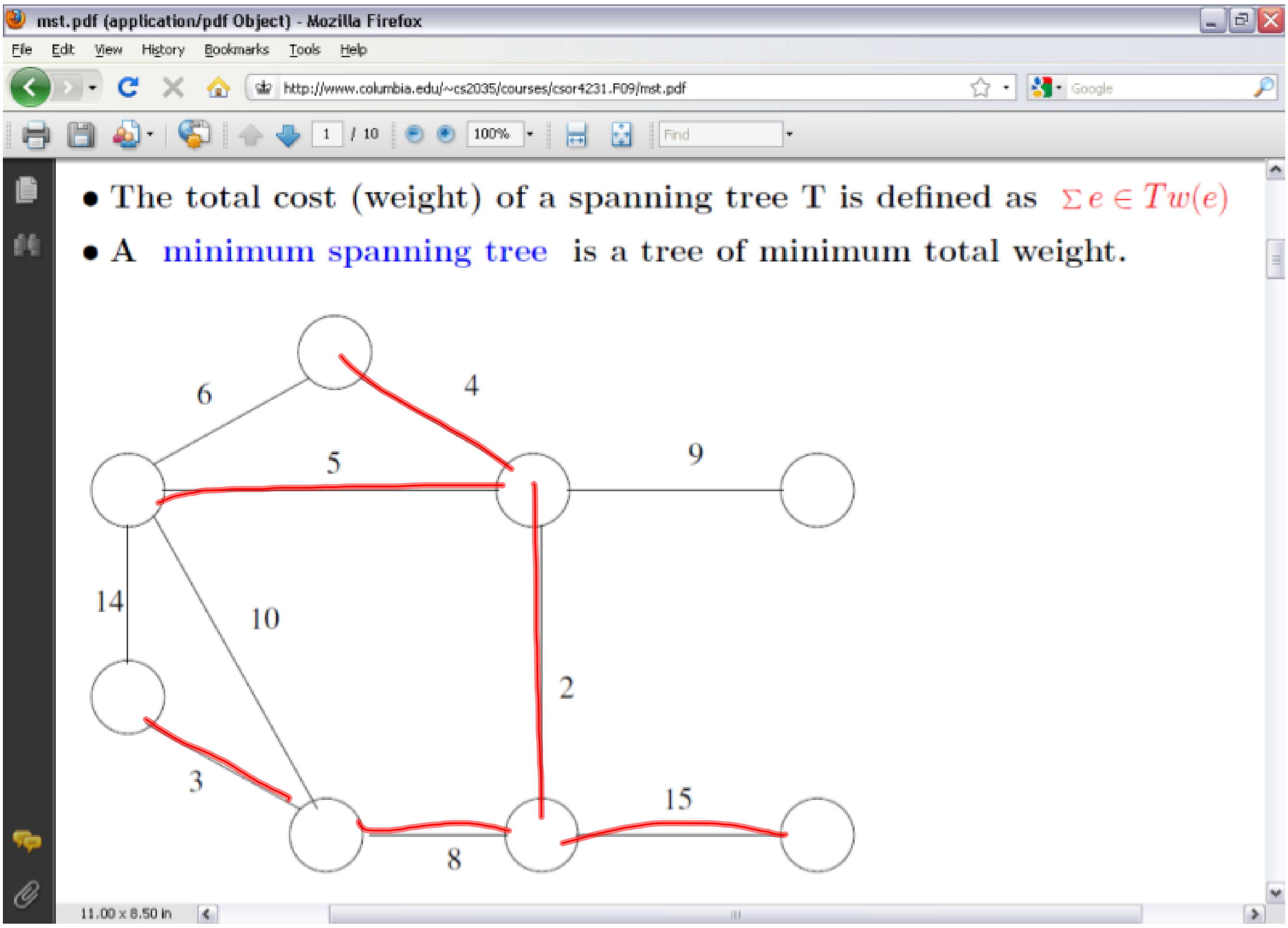
(x,y) is the min. wt edge crossing the cut $(\notin T)$.

Now let (a,b) be the edge in T on the path from x to y that crosses the cut. Consider $T' = T \cup \{(x,y)\} - (a,b)$.

$\therefore T'$ is a spanning tree. and

$$w(T') = w(T) + w(x,y) - w(a,b) < w(T).$$

which contradicts $w(T)$ being a <u>minimum</u> spanning tree. ⊠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Find
• The total cost (weight) of a spanning tree T is defined as $\Sigma e \in Tw(e)$
• A minimum spanning tree is a tree of minimum total weight.
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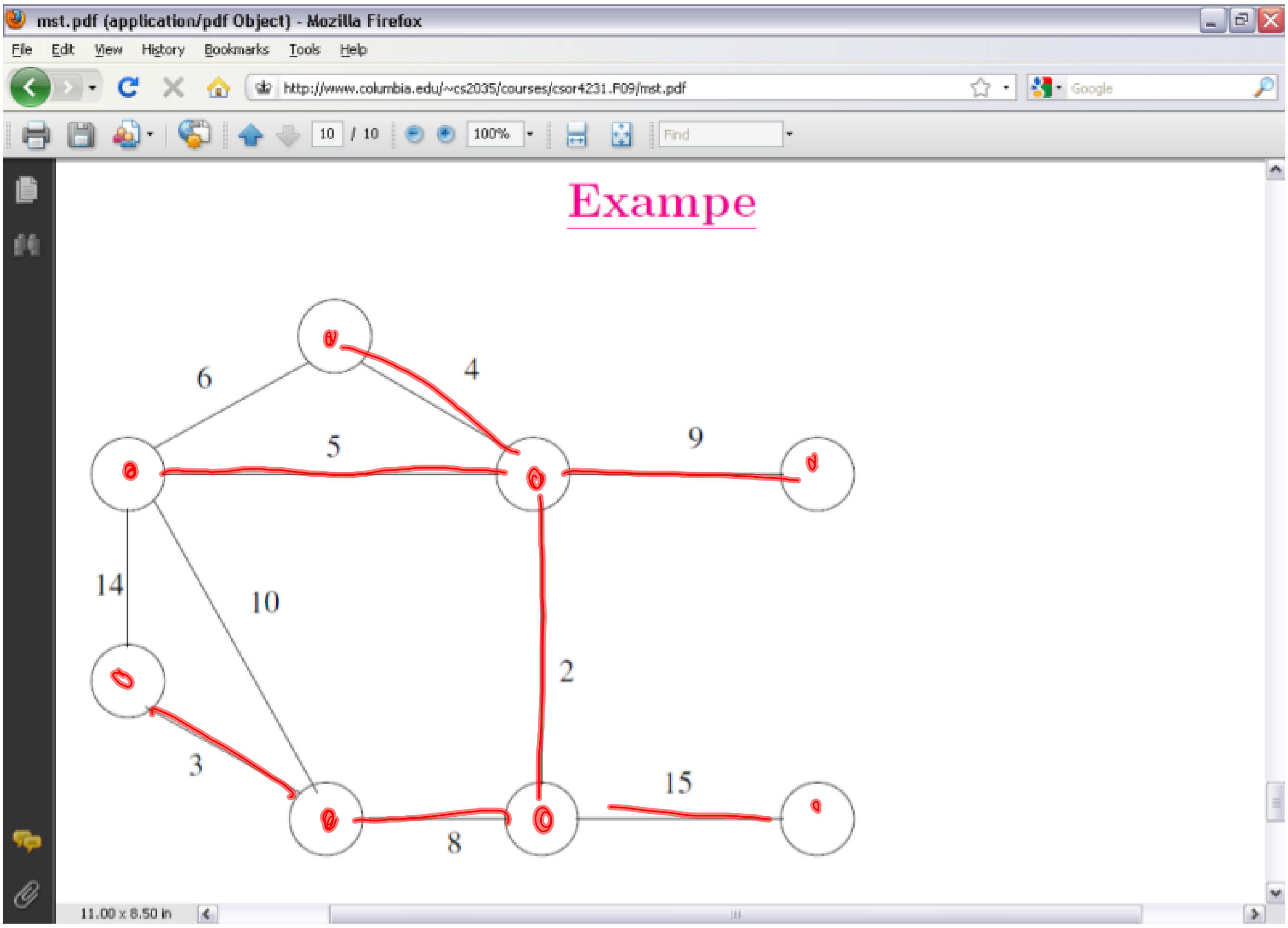
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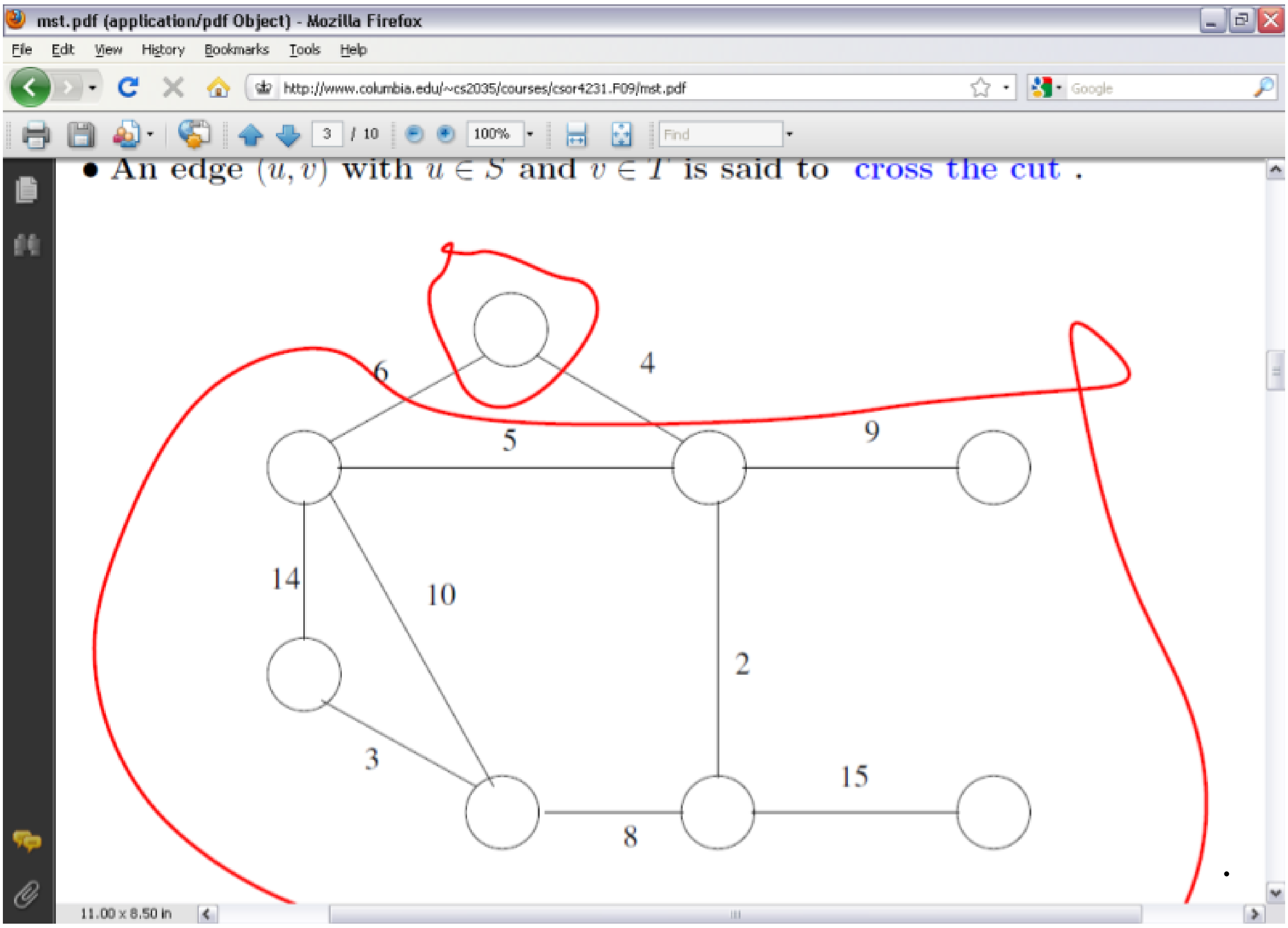
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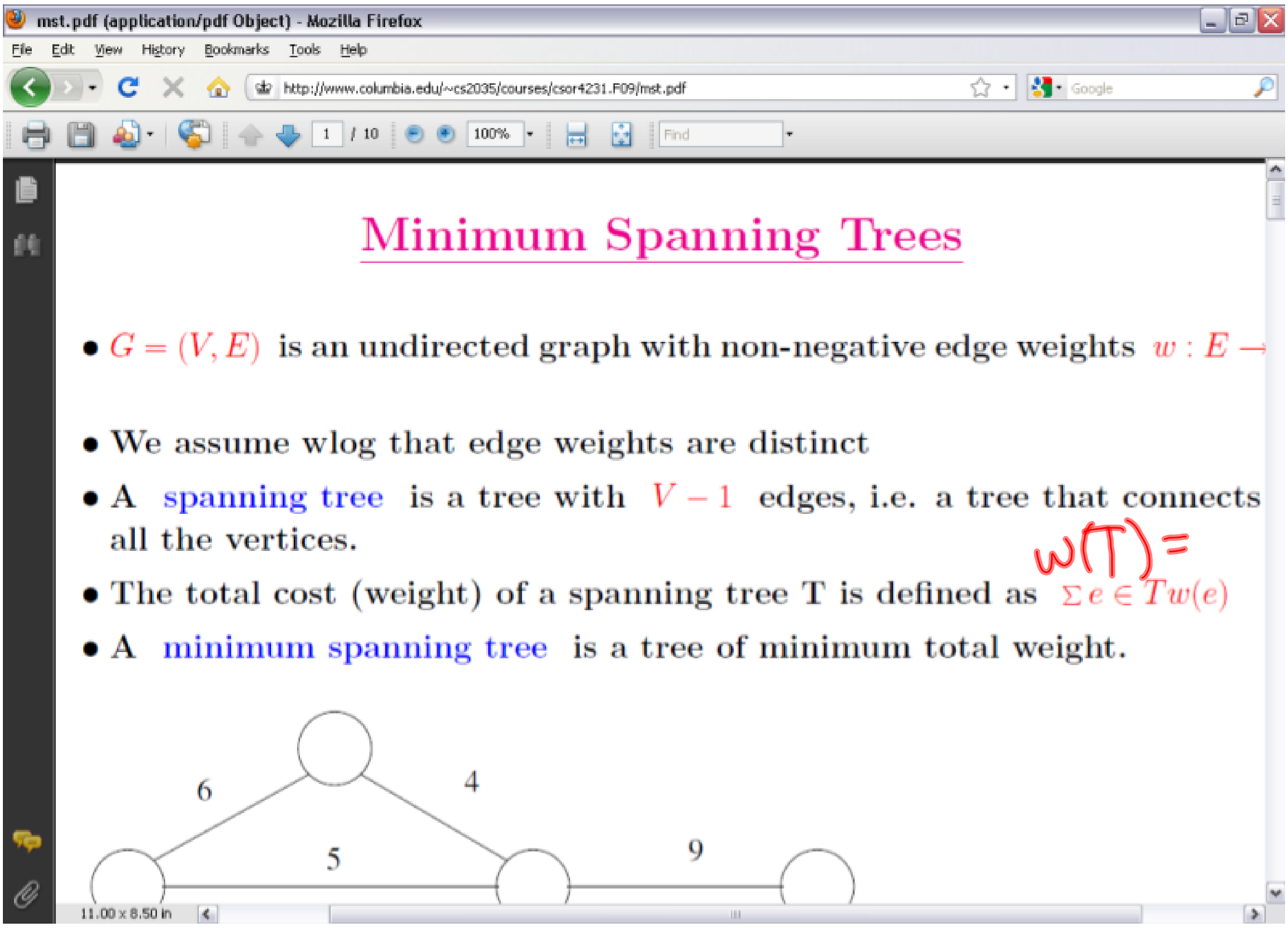
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Minimum Spanning Trees
• $G = (V, E)$ is an undirected graph with non-negative edge weights $w : E \rightarrow$
• We assume wlog that edge weights are distinct
• A spanning tree is a tree with $V - 1$ edges, i.e. a tree that connects all the vertices.
• The total cost (weight) of a spanning tree T is defined as $w(T) = \Sigma e \in Tw(e)$
• A minimum spanning tree is a tree of minimum total weight.
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