	HEAP	FIB HEAP (amortized)
V INSERTS	$\lg V$	$\lg V$
V EXTRACT-MIN	$\lg V$	$\lg V$
E DECREASE-KEYS	$\lg V$	1
	$O(E \lg V)$	$O(E + V \lg V)$

faster for sparse graphs

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Disjoint Set Data Structure

Items : X

Maintain Sets $S_1 \ldots S_k$

Disjoint $S_i \cap S_j = \emptyset \quad \forall i \neq j$

MakeSet(x) - create a one element set

Find(x) - return the "name" of the set containing x

Union(x,y) - put together x's & y's sets

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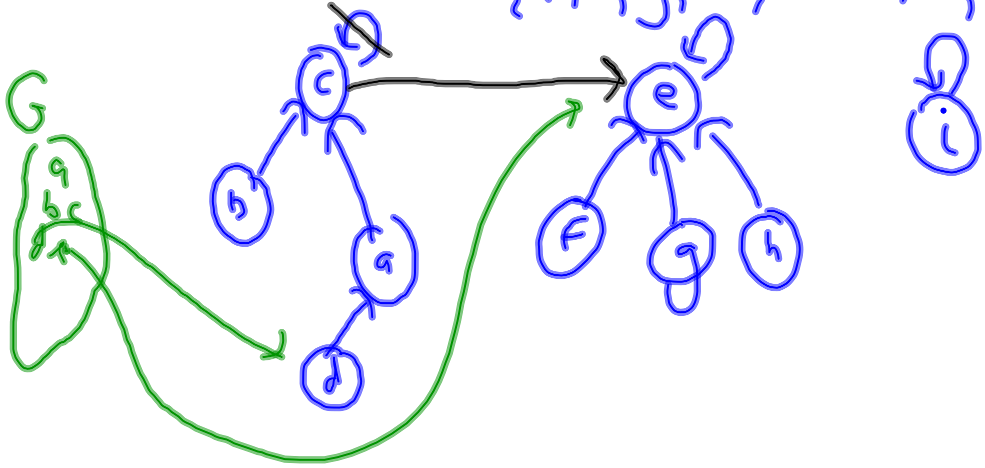
A set as a tree w/ nodes point to parents,
name of set = root

{a, b, c, d}   {e, f, g, h}   {i}



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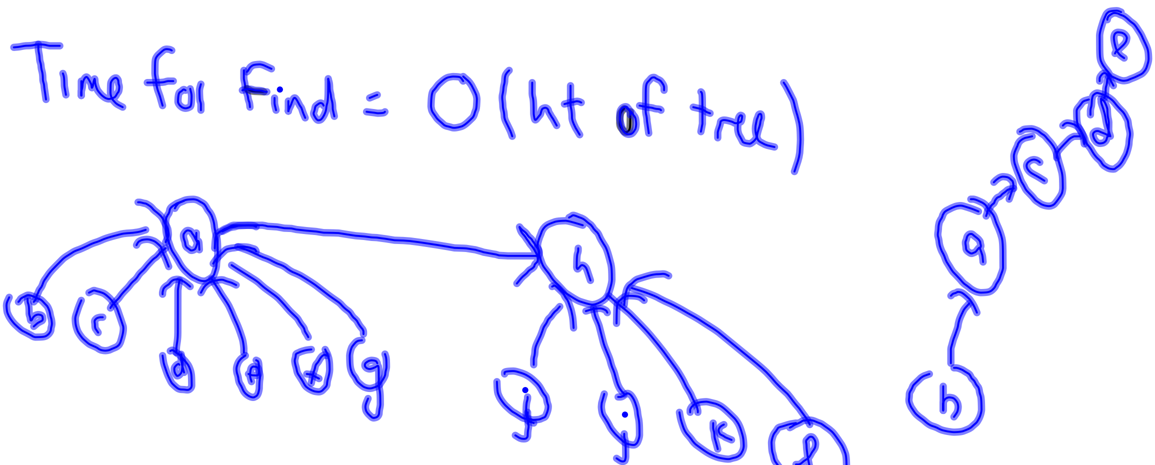
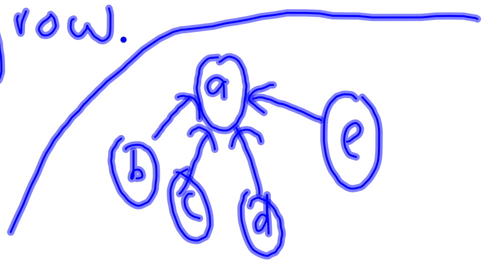
Time for Find = O(ht of tree)



Unions make trees grow.



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Union by rank

Make the tree w/ fewer nodes a child of the root of the tree w/ more nodes

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mst.pdf (application/pdf Object) - Mozilla Firefox

http://www.columbia.edu/~cs2035/courses/csor4231.F09/mst.pdf

Idea: Consider edges in increasing order.

```
MST-Kruskal(G, w)
A ← ∅
for each vertex v ∈ V[G]
      do Make-Set(v)                                          ✓
sort the edges of E into nondecreasing order by weight w
for each edge (u, v) ∈ E, taken in nondecreasing order by weig
      do if Find-Set(u) ≠ Find-Set(v)                         2E
            then A ← A ∪ {(u, v)}
                  Union(u, v)                                 ✓
return A
```

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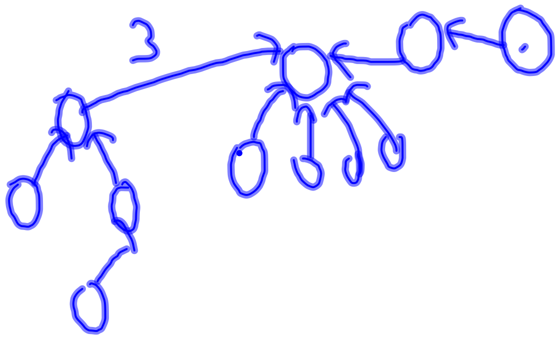
V MAKESETS
V UNIONS
E FINDS

How big does the ht of the tree get?



In order to increase the ht. of a tree, the # nodes has to at least double.

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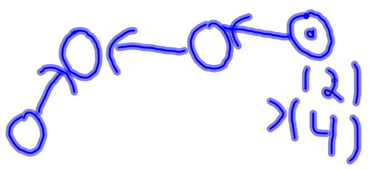
Look at node v, how many times can it be in the tree that is on the small side of a union

Between two instances of being on the small side of a union, your tree size must at least double $\therefore$ only on small side $< \lg V$ times



$\Rightarrow$ height $= O(\lg V)$

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Kruskal

Sort $O(E \lg V)$

$\longrightarrow O(E \lg V)$ rest of alg.

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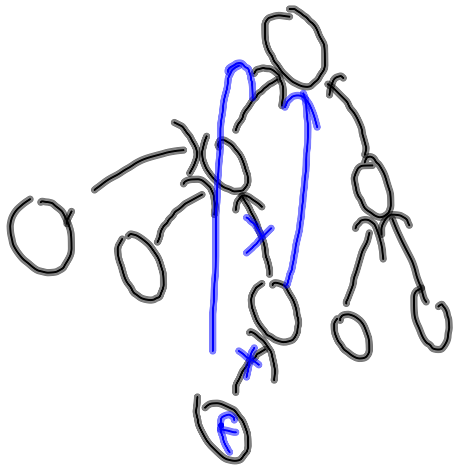

path compression

whenever you touch a node, make it a child of the root.

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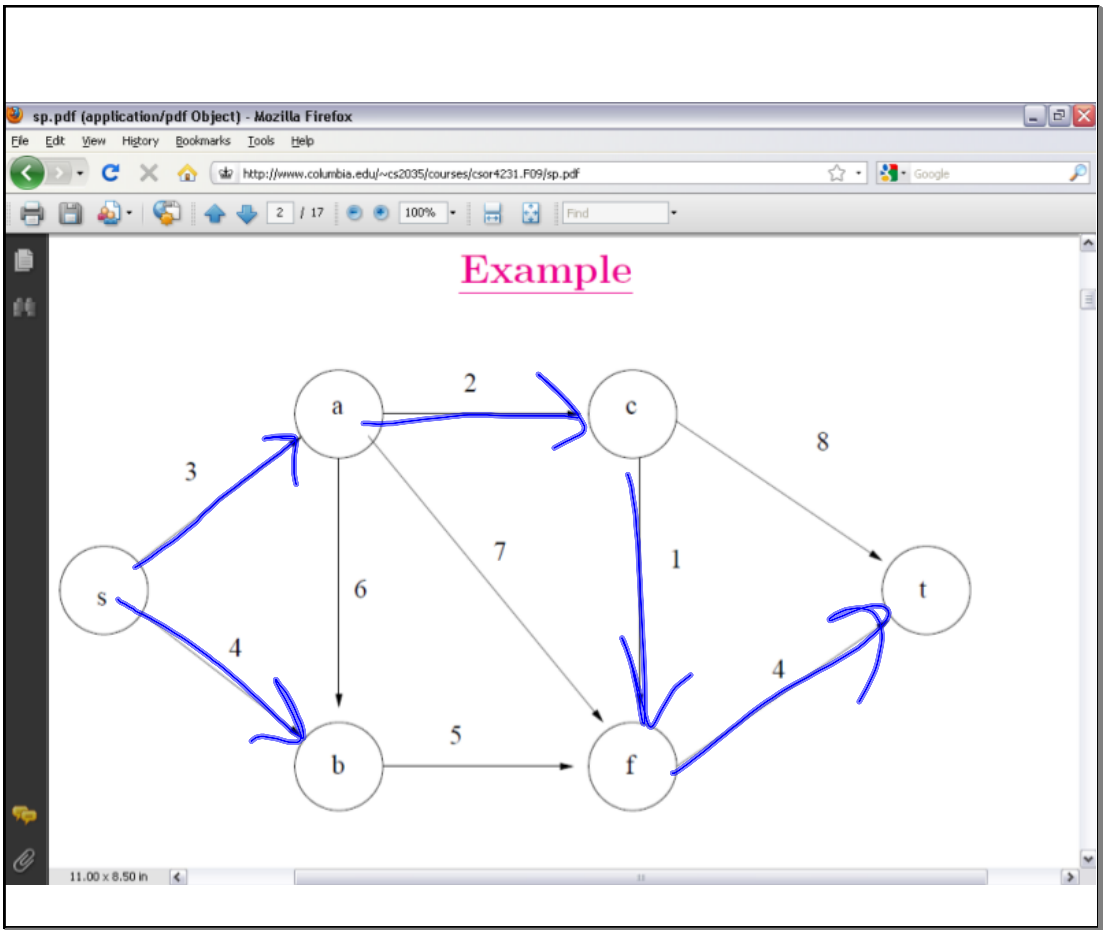

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Running time of one op. = $O(\lg^* V)$

(height tree)

x	$\lg^* x$
<1	0
1-2	1
2-4	2
4-16	3
$2^4 - 2^{16}$	4
$2^{16} - 2^{65536}$	5

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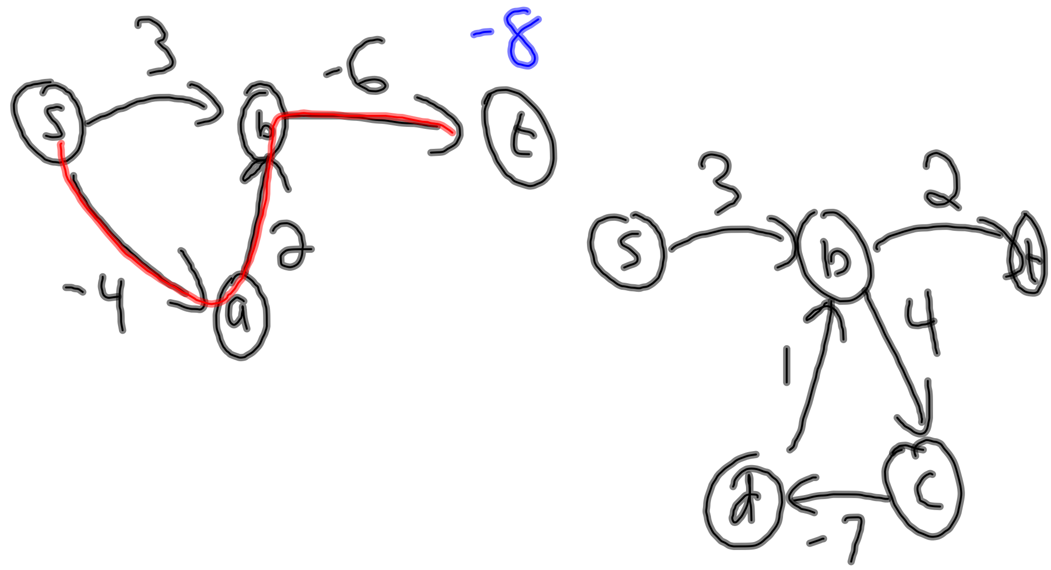

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90	13
80	17
70	20
60	17
50	16
40	6
<40	1

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