In a graph, the distance between two vertices can be defined as follows:

\[ d(v) = d(u) + w(u,v) \quad \text{for any vertex } v \]  

Where \( d(v) \) is the distance of vertex \( v \) from some reference vertex \( u \), and \( w(u,v) \) is the weight of the edge connecting \( u \) and \( v \).

For any two vertices in a graph, the distance is given by:

\[ d(u, v) = \min_{p} \sum_{i=1}^{n} w(u, v_{i}) \]  

Where \( p \) is a path from \( u \) to \( v \).
1. $\mathbb{E}$

123 $\cdots \mathbb{E} 12 \cdots \mathbb{E} 12 \cdots \mathbb{E} \cdots \cdots \cdots \mathbb{E}$

\[ \text{5 times} \]

$\text{S.P. } 10^3$

12, 1, 100, 101, 0, 51, 17, 3
Example

Dijkstra(G, w, s)
1 INITIALIZE-SINGLE-SOURCE(G, s)
2 S ← ∅
3 Q ← V[G]
4 while Q ≠ ∅
5 do u ← EXTRACT-MIN(Q)
6 S ← S U {u}
7 for each vertex v ∈ Adj[u]
8 do RELAX(u, v, w)
Example

\[ \text{Dijkstra}(G, w, s) \]
\[
1. \text{INITIALIZE-SINGLE-SOURCE}(G, s) \\
2. S \leftarrow \emptyset \\
3. Q \leftarrow V[G] \\
4. \text{while } Q \neq \emptyset \\
   \text{do } u \leftarrow \text{EXTRACT-MIN}(Q) \\
   \text{S } \leftarrow S \cup \{u\} \\
   \text{for each vertex } v \in \text{Adj}[u] \\
      \text{do RELAX}(u, v, w) \\
5. \text{DECREASE-KEY}(E, \text{decrease-key}, O(E \lg V), \text{heap}) \\
6. \text{INSERTS}(V, \text{inserts}, O(V \lg V), \text{fib-heap}) \\
7. \text{EXTRACT-MIN}(V, \text{extract-min}, O(V \lg V), \text{fib-heap}) \
\]
Claim: When \( v \) is put in \( S \) (permanently labelled)
\[
d(v) = \delta(v).
\]

\[\text{Pf}\]
1. \( d(v) \geq \delta(v) \) because any alg that does a series of relax calls has this property
2. Assume \( \text{fpoc} \) that \( d(v) > \delta(v) \) and this is the first such vertex.

\[
\delta(x) = d(x) \\
\delta(y) = d(y) \quad \text{because} \quad (x,y) \text{ was relaxed when} \ x \text{ was added to} \ S.
\]

\[
d(y) = \delta(y) \leq \delta(v) < d(v) \quad \text{because} \quad y \in S \text{ and not} \ S_1 .
\]

\[
d(y) < d(v) \quad \text{and neither} \ y \text{ nor} \ v \text{ are in} \ S_1 . \text{ So}
\]

Diokstra's alg. will choose \( y \) \( \\neq v \).