

apsp.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.columbia.edu/~cs2035/courses/csr4231.F09/apsp.pdf

2 / 8 79.1%

Solution

D

$$\begin{pmatrix} 0 & 3 & 15 & 8 \\ 7 & 0 & 12 & 5 \\ 1 & 4 & 0 & -1 \\ 2 & -4 & 8 & 0 \end{pmatrix}$$

Nov 12-4:06 PM

$d_{ij}^{(k)}$

uses k

$d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$

does not use k

$d_{ij}^{(k-1)}$

$> k$
($\neq i \neq j$)

Nov 12-4:16 PM

apsp.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.columbia.edu/~cs2035/courses/csr4231.F09/apsp.pdf

5 / 8 100%

$$D^0 = \begin{pmatrix} 0 & 3 & \text{blue} & \text{blue} \\ \text{blue} & 0 & 12 & 5 \\ 4 & \text{blue} & 0 & -1 \\ 2 & -4 & \text{blue} & 0 \end{pmatrix} \quad D^1 = \begin{pmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 12 & 5 \\ 4 & 7 & 0 & -1 \\ 2 & -4 & 0 & 0 \end{pmatrix} \quad D^2 = \begin{pmatrix} 0 & 3 & 15 & 8 \\ 0 & 0 & 12 & 5 \\ 4 & 7 & 0 & -1 \\ 2 & -4 & 8 & 0 \end{pmatrix}$$

(0 3 15 8) (0 3 15 8)

11.00 x 8.50 in

Nov 12-4:24 PM

apsp.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.columbia.edu/~cs2035/courses/csr4231.F09/apsp.pdf

5 / 8 79.1%

$O(V^3)$ space
if you only want D
matrix & not the paths,
can use $O(V^2)$ space.

$$D^0 = \begin{pmatrix} 0 & 3 & \text{blue} & \text{blue} \\ \text{blue} & 0 & 12 & 5 \\ 4 & \text{blue} & 0 & -1 \\ 2 & -4 & \text{blue} & 0 \end{pmatrix} \quad D^1 = \begin{pmatrix} 0 & 3 & 0 & 0 \\ \text{blue} & 0 & 12 & 5 \\ 4 & 7 & 0 & -1 \\ 2 & -4 & \text{blue} & 0 \end{pmatrix} \quad D^2 = \begin{pmatrix} 0 & 3 & 15 & 8 \\ \text{blue} & 0 & 12 & 5 \\ 4 & 7 & 0 & -1 \\ 2 & -4 & 8 & 0 \end{pmatrix}$$

$$D^3 = \begin{pmatrix} 0 & 3 & 15 & 8 \\ 16 & 0 & 12 & 5 \\ 4 & 7 & 0 & -1 \\ 2 & -4 & 8 & 0 \end{pmatrix} \quad D^4 = \begin{pmatrix} 0 & 3 & 15 & 8 \\ 7 & 0 & 12 & 5 \\ 1 & -5 & 0 & -1 \\ 2 & -4 & 8 & 0 \end{pmatrix}$$

Nov 12-4:30 PM

apsp.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.columbia.edu/~cs2035/courses/csr4231.F09/apsp.pdf

6 / 8 100%

RESET ALL DEFINITIONS OF D.

- Let w_{ij} be the length of edge ij
- Let $w_{ii} = 0$
- Let d_{ij}^m be the shortest path from i to j using m or fewer edges

use $\leq m$ edges $d_{ij}^1 = w_{ij}$

$$d_{ij}^m = \min\{d_{ij}^{m-1}, \min_{1 \leq k \leq n, k \neq j} d_{ik}^{m-1} + w_{kj}\}$$

use m edges

Combining these two, we get

$$d_{ij}^m = \min_{1 \leq k \leq n} \{d_{ik}^{m-1} + w_{kj}\}$$

This would give an $O(V^4)$ algorithm



11.00 x 8.50 in

Nov 12-4:36 PM

apsp.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.columbia.edu/~cs2035/courses/csr4231.F09/apsp.pdf

7 / 8 100%

Note the similarity of

$$d_{ij}^m = \min_{1 \leq k \leq n} \{d_{ik}^{m-1} + w_{kj}\}$$

with matrix multiplication:

$$c_{ij} = \sum_{1 \leq k \leq n} \{a_{ik} \cdot b_{kj}\}$$

Make the following substitutions (which have the right algebraic properties):

$$\begin{aligned} \text{sum} &\rightarrow \min \\ a_{ik} &\rightarrow d_{ik}^{m-1} \\ \cdot &\rightarrow + \\ b_{kj} &\rightarrow w_{kj} \\ c &\rightarrow d^m \end{aligned}$$

Using this matrix multiplication terminology, we have

11.00 x 8.50 in

Nov 12-4:41 PM

apsp.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.columbia.edu/~cs2035/courses/csr4231.F09/apsp.pdf

5 / 8 100%

Find

Handwritten notes:

$$(0 \ 3 \ 0 \ 0) \begin{pmatrix} 0 \\ 12 \\ 5 \\ 0 \end{pmatrix}$$

$$12 \ (D^0)^2 = \begin{pmatrix} & & 15 & \\ & & & \end{pmatrix}$$

$$\begin{pmatrix} \infty & 0 & 12 & 5 \end{pmatrix} \begin{pmatrix} \infty \\ 5 \\ 0 \\ -1 \end{pmatrix} = \min \begin{pmatrix} \infty & 5 \\ 11 & 5 \end{pmatrix}$$

$$D^0 = \begin{pmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 12 & 5 \\ 4 & 0 & 0 & -1 \\ 2 & -4 & 0 & 0 \end{pmatrix} \quad D^1 = \begin{pmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 12 & 5 \\ 4 & 7 & 0 & -1 \\ 2 & -4 & 0 & 0 \end{pmatrix} \quad D^2 = \begin{pmatrix} 0 & 3 & 15 & 8 \\ 0 & 0 & 12 & 5 \\ 4 & 7 & 0 & -1 \\ 2 & -4 & 8 & 0 \end{pmatrix}$$

11.00 x 8.50 in

Nov 12-4:45 PM

apsp.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://www.columbia.edu/~cs2035/courses/csr4231.F09/apsp.pdf

8 / 8 100%

Find

Handwritten notes:

"Mat. Mult." V times $O(V^4)$

$$D^m \quad D^2 \quad D^4 \quad D^8 \quad D^{16} \dots D^{128}$$

$$D^{150} = D^{128} \cdot D^{16} \cdot D^4 \cdot D^2$$

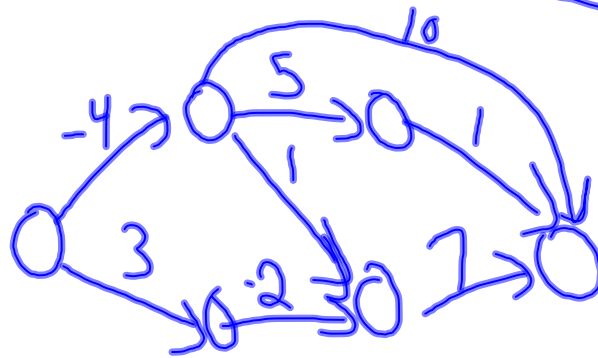
$D^m O(\lg m)$ matrix mult. $\Rightarrow$ $O(V^3 \lg V)$ time.

APSP

11.00 x 8.50 in

Nov 12-4:52 PM

Johnson's algorithm



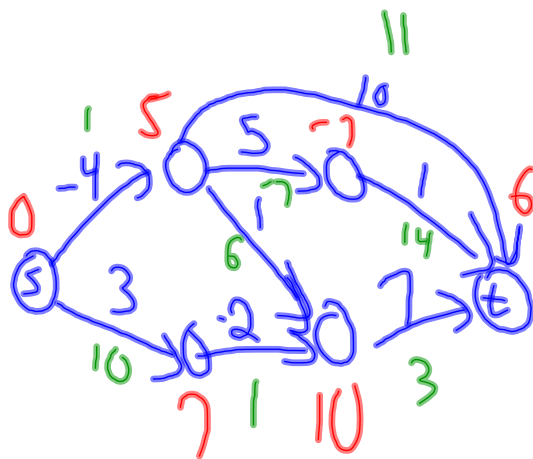
$$VE + \underline{VE + V^2 \lg V}$$

1. BF $O(VE)$

2. Use BF solution to change edge wts.

3. $V-1$ Dijkstra alg

Nov 12-4:53 PM



$w(u,v)$

$p(v)$

$$\bar{w}(u,v) = w(u,v) + p(v) - p(u)$$

$\bar{w}$ paths from s to t are longer than with w

$$\begin{aligned} \bar{w}(\text{path } p: v_1, v_2, v_3, v_4) &= \bar{w}(v_1, v_2) + \bar{w}(v_2, v_3) + \bar{w}(v_3, v_4) \\ &= w(v_1, v_2) - p(v_1) + p(v_2) \\ &\quad + w(v_2, v_3) - p(v_2) + p(v_3) \\ &\quad + w(v_3, v_4) - p(v_3) + p(v_4) \\ &= w(p) - p(v_1) + p(v_4) \end{aligned}$$

Nov 12-4:57 PM



$$d(v) \leq d(u) + w(u, v)$$

$$\Rightarrow \bar{w}(u,v) \geq 0.$$

set $p(v) = \frac{d(s,v)}{d(v)}$

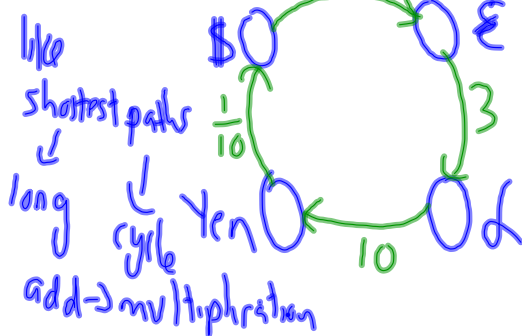
$$\bar{w}(u,v) = w(u,v) + d(u) - d(v)$$

$$\bar{\omega}(u,v) \geq 0 \quad \forall (u,v)$$

$w(u,v) = 0$ if (u,v) is on s.p. tree

Nov 12-5:06 PM

Arbitrage



cycles $C = e_1, e_2, e_3, e_4$
whos.t.

$$\begin{aligned} & r(i) \cdot r(e_1) \cdot r(e_2) \cdot r(e_3) \cdot r(e_4) \\ & \geq 1 \\ & \Rightarrow \text{cycles w/ large } r(i) \end{aligned}$$

$$r(e) \rightarrow -\lg(r(e)) \leftarrow$$

all pairs shortest paths in graph w/ edge wts

$\Rightarrow$ Neg. wt. cycle in graph

Nov 12-5:14 PM