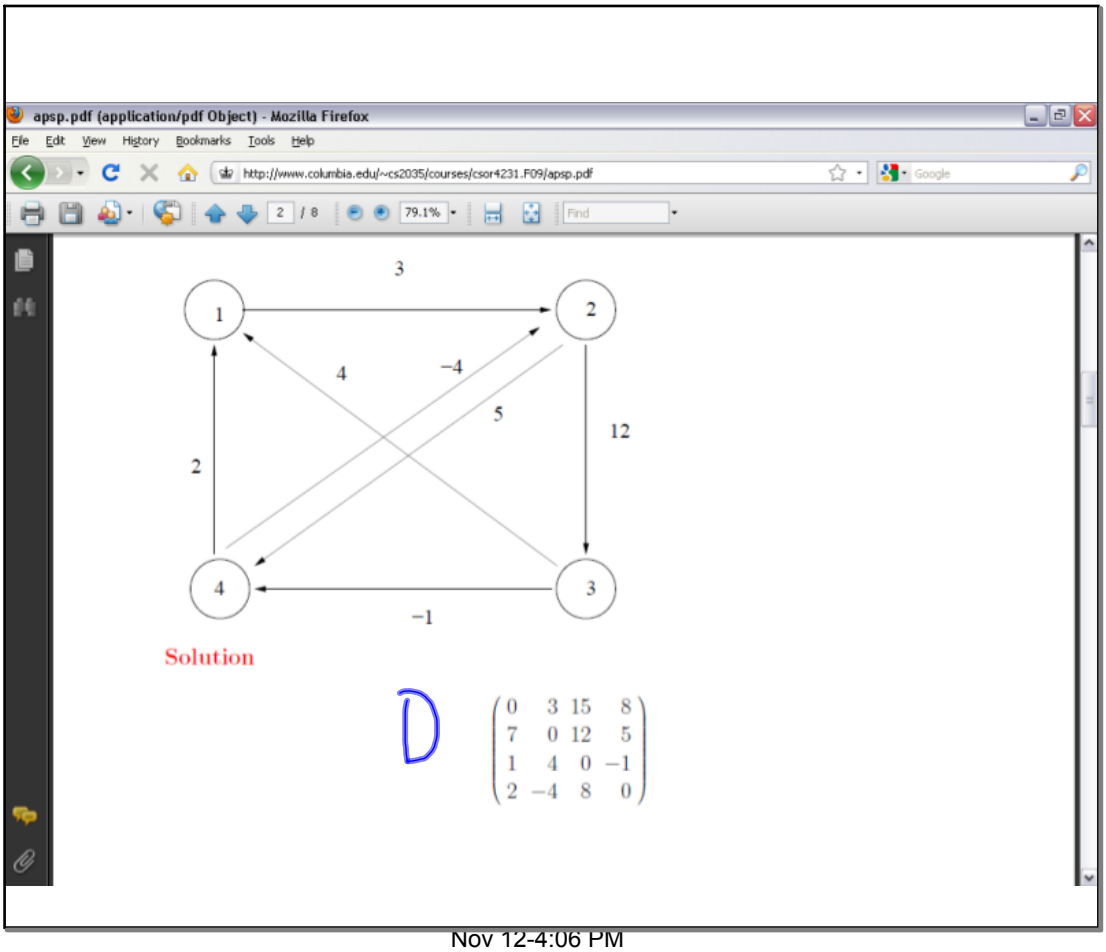

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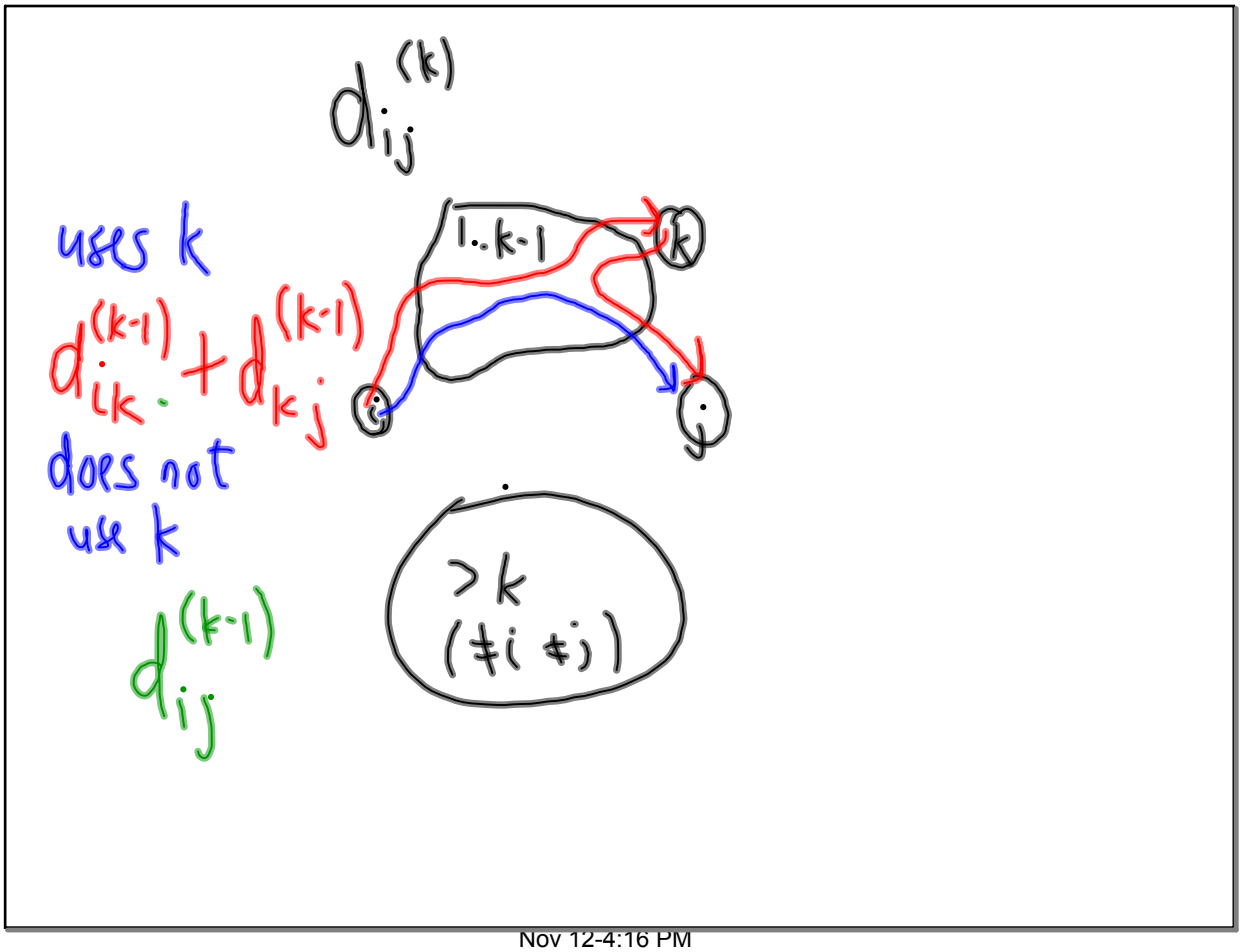

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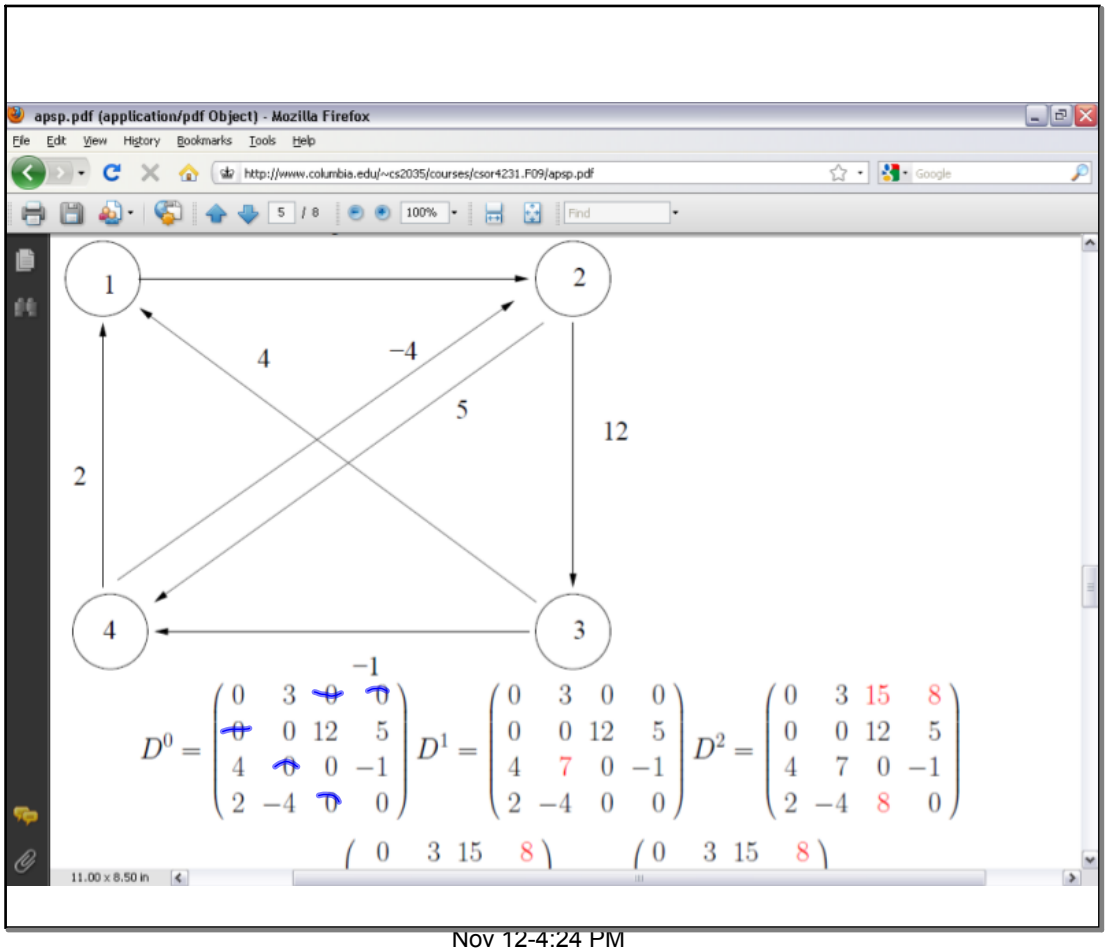

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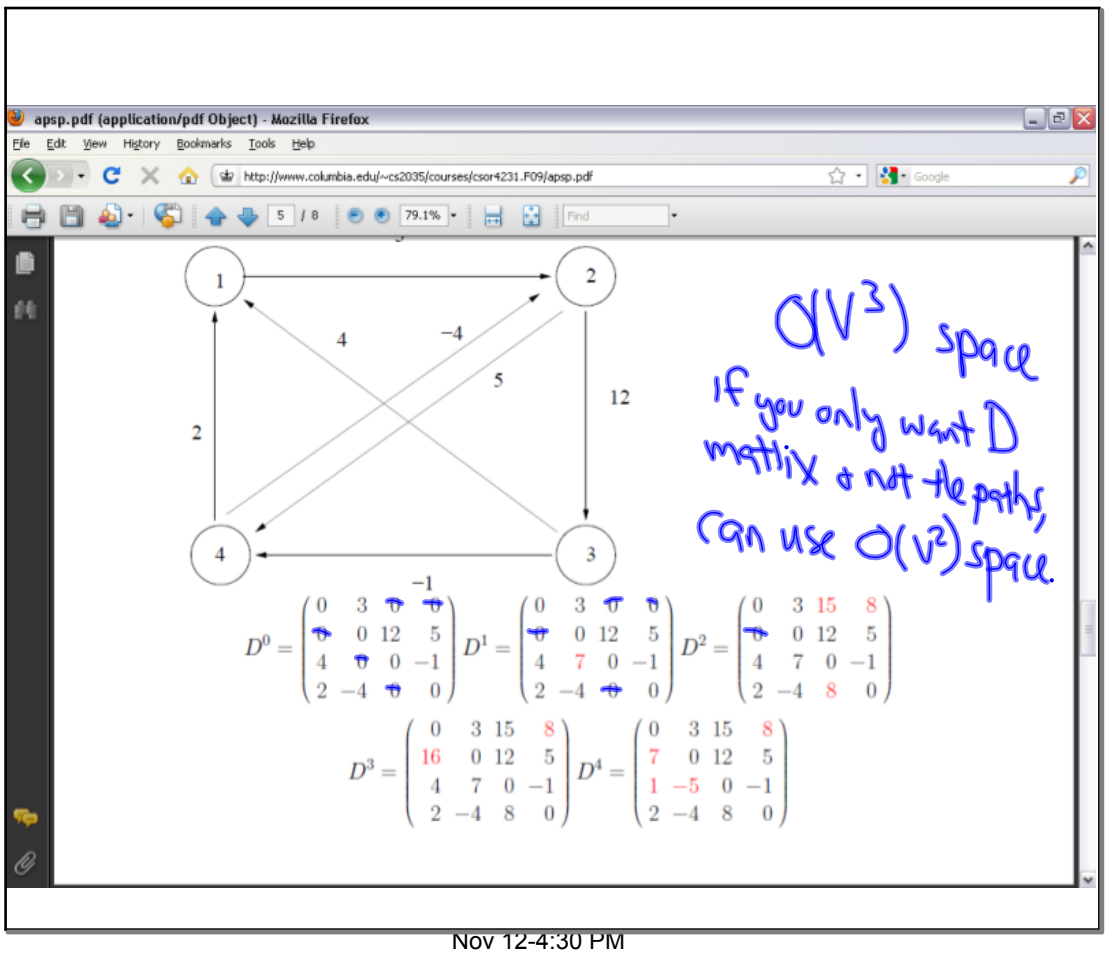

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RESET ALL DEFINITIONS OF D.

- Let w_{ij} be the length of edge ij
- Let $w_{ii} = 0$
- Let d_{ij}^m be the shortest path from i to j using m or fewer edges

use $\le m$ edges

$$d_{ij}^1 = w_{ij}$$

$$d_{ij}^m = \min\{d_{ij}^{m-1}, \min_{1 \le k \le n, k \ne j} d_{ik}^{m-1} + w_{kj}\}$$

use m edges

Combining these two, we get

$$d_{ij}^m = \min_{1 \le k \le n} \{d_{ik}^{m-1} + w_{kj}\}$$

This would give an $O(V^4)$ algorithm

11.00 x 8.50 in

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Note the similarity of

$$d_{ij}^m = \min_{1 \le k \le n} \{d_{ik}^{m-1} + w_{kj}\}$$

with matrix multiplication:

$$c_{ij} = \text{sum}_{1 \le k \le n}\{a_{ik} \cdot b_{kj}\}$$

Make the following substitutions (which have the right algebraic properties:

$$\begin{aligned} \text{sum} &\rightarrow \min \\ a_{ik} &\rightarrow d_{ik}^{m-1} \\ \cdot &\rightarrow + \\ b_{kj} &\rightarrow w_{kj} \\ c &\rightarrow d^m \end{aligned}$$

Using this matrix multiplication terminology, we have

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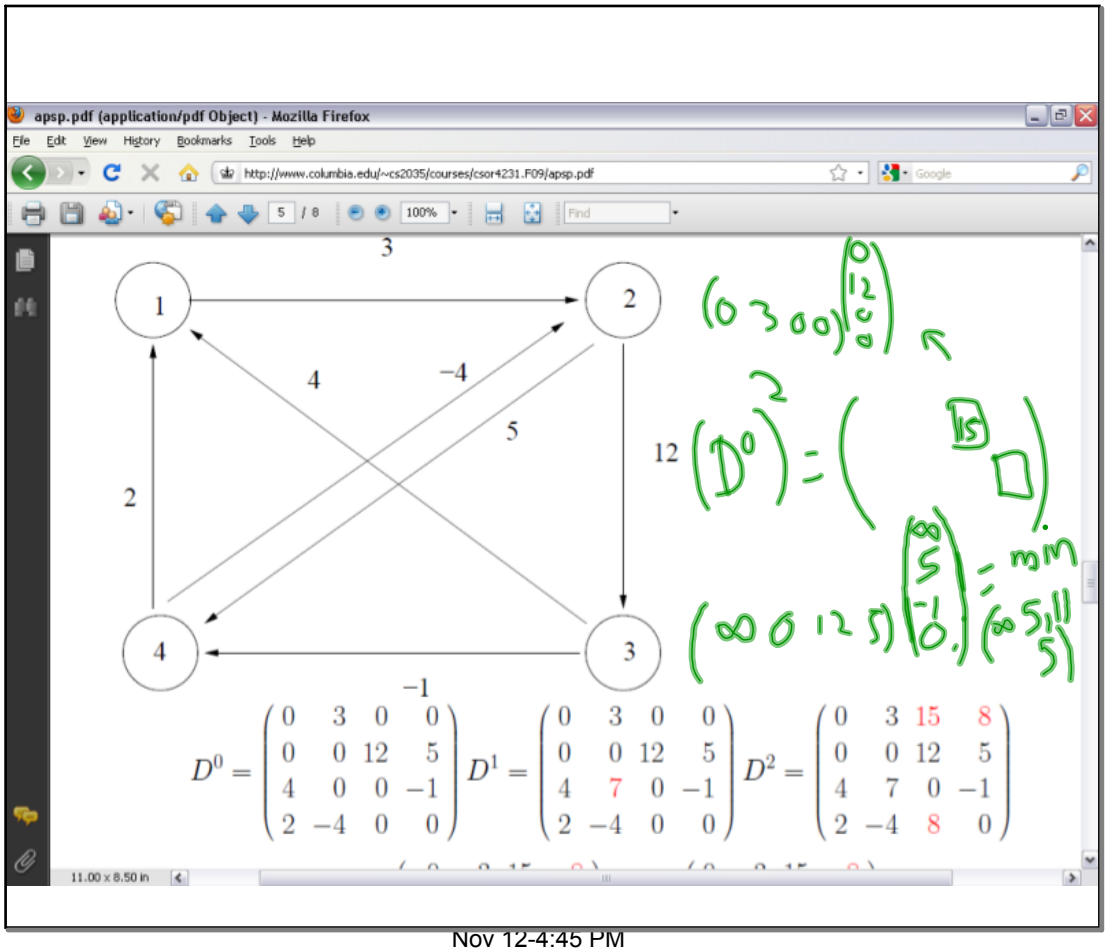

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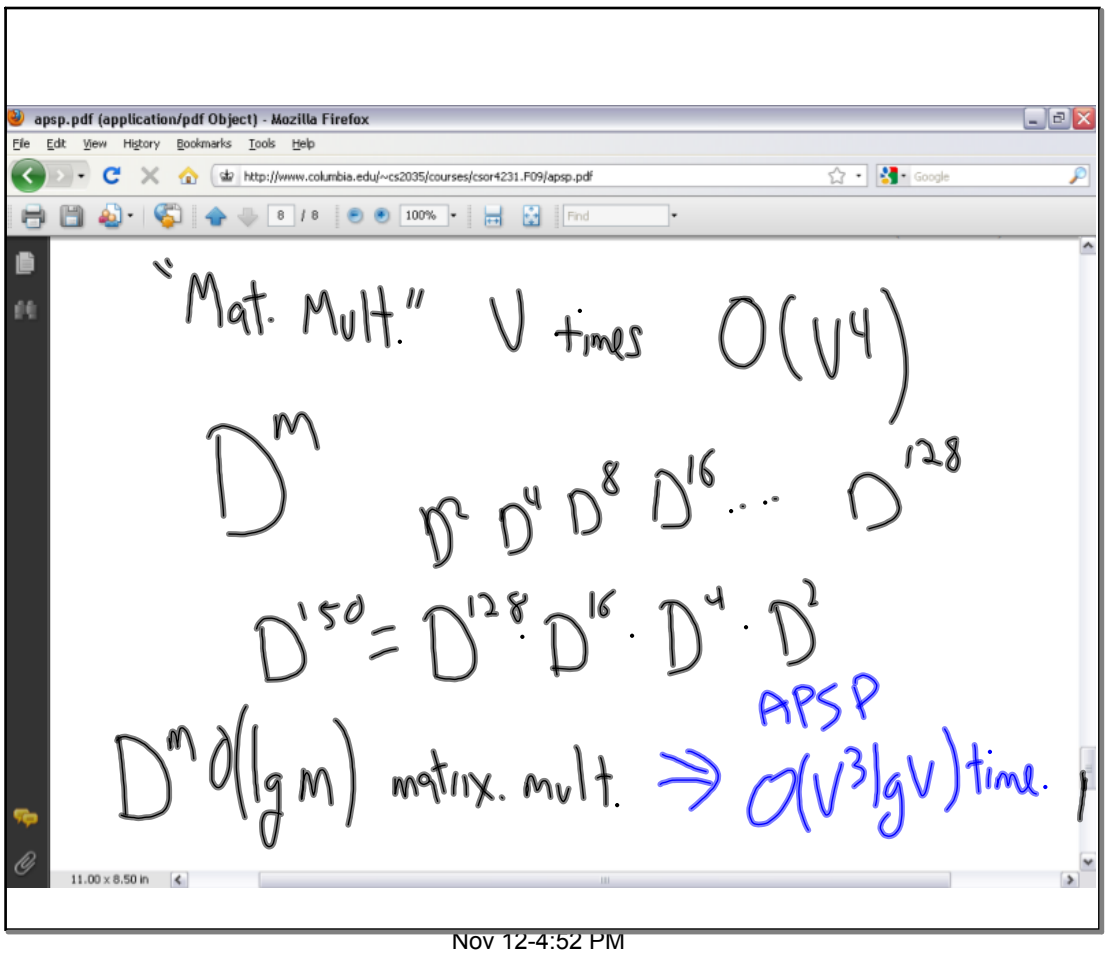

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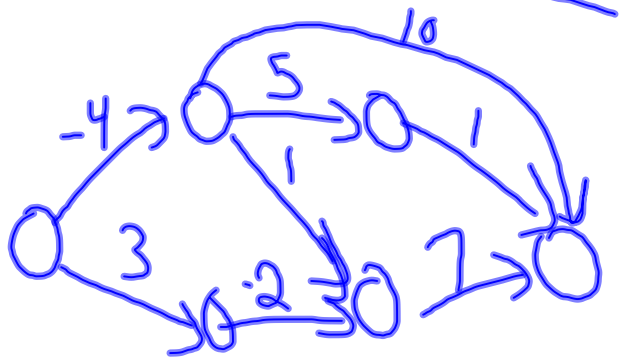
Johnson's algorithm



1. BF $O(VE)$
2. Use BF solution to change edge wts.
3. V-1 Dijkstra alg

$VE + VE + V^2 \lg V$

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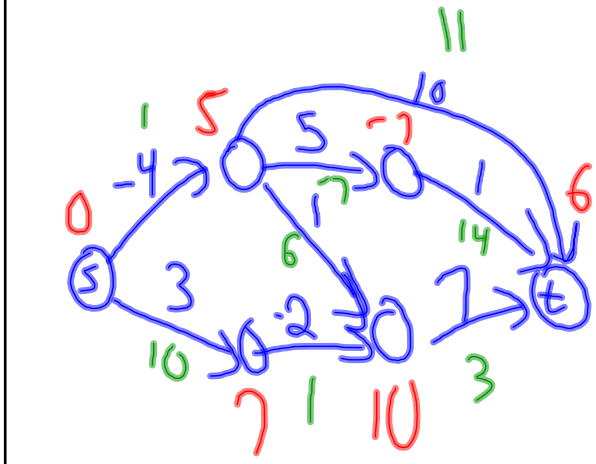

$w(u,v)$

$p(v)$

$\bar{w}(u,v) = w(u,v) + p(u) - p(v)$

$\bar{w}$ paths from s to t are 6 longer than w/ w

$\bar{w}(\text{path } p: v_1, v_2, v_3, v_4) = \bar{w}(v_1,v_2) + \bar{w}(v_2,v_3) + \bar{w}(v_3,v_4)$

$= w(v_1,v_2) - p(v_1) + p(v_2)$
$+ w(v_2,v_3) - p(v_2) + p(v_3)$
$+ w(v_3,v_4) - p(v_3) + p(v_4)$
$= w(p) - p(v_1) + p(v_4)$

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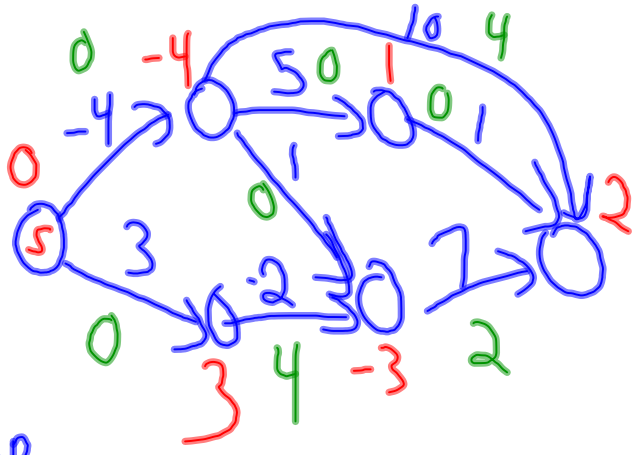

Set $p(v) = -d(s,v)$ ($d(v)$)

$\bar{w}(u,v) = w(u,v) + d(u) - d(v)$

$\bar{w}(u,v) \geq 0 \quad \forall (u,v)$

$w(u,v) = 0$ if (u,v) is in s.p. tree

s.p. opt.

$d(v) \leq d(u) + w(u,v)$

$\Rightarrow w(u,v) + d(u) - d(v) \geq 0$

$\Rightarrow \bar{w}(u,v) \geq 0.$

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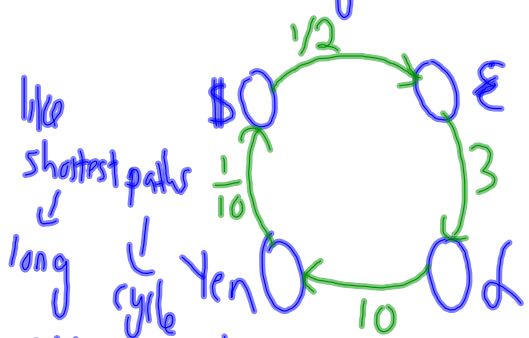
Shortest Path applications

Arbitrage



like shortest paths
↓
long cycle
add → multiplication

r

cycles $C = e_1 e_2 e_3 e_4$ whose s.t.

$r(C) = r(e_1) \cdot r(e_2) \cdot r(e_3) \cdot r(e_4) > 1$

$\Rightarrow$ cycles w/ large $r(C)$

$r(e) \rightarrow -\lg(r(e))$

all pairs shortest paths in graph w/ edge wts

$\Rightarrow$ neg. wt. cycle in graph

Nov 12-5:14 PM