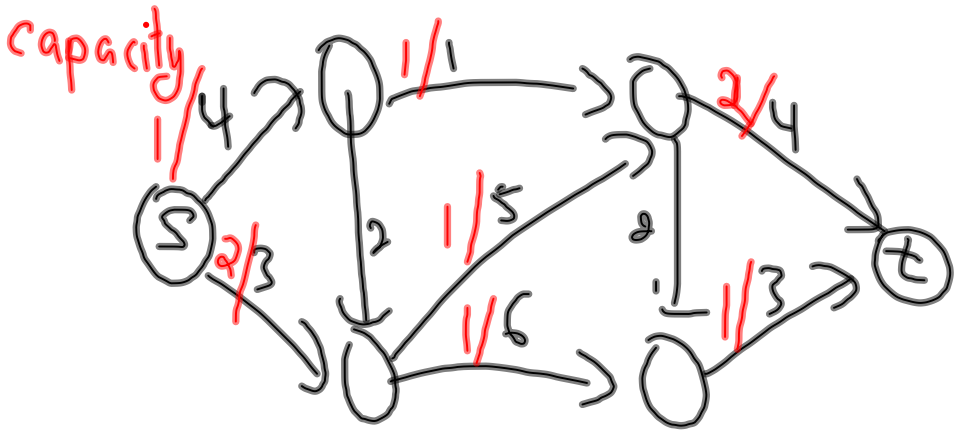
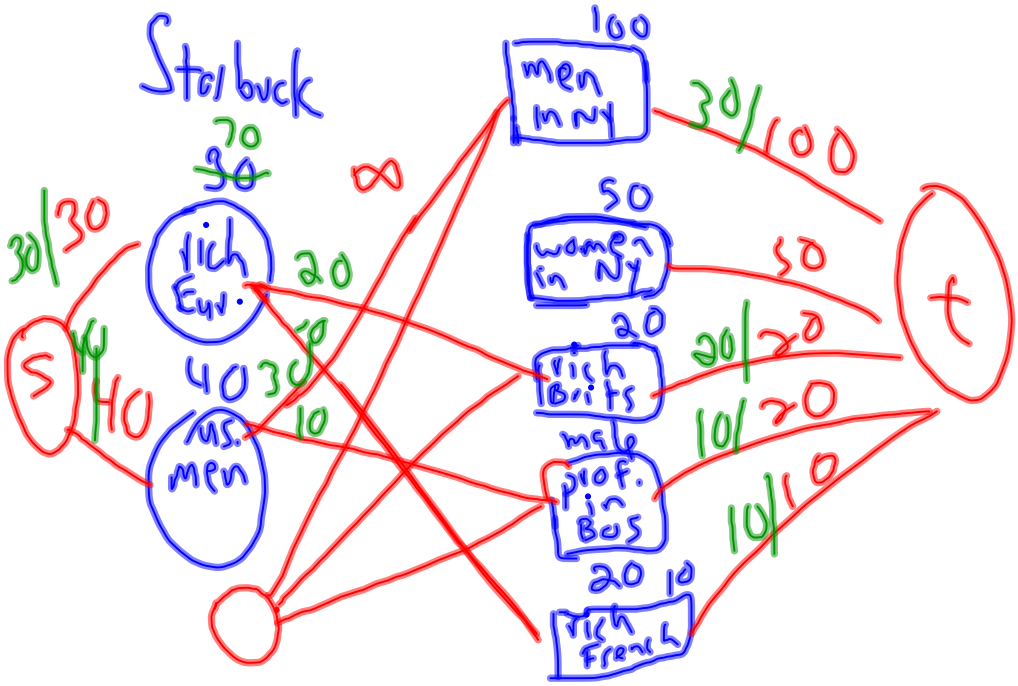
Maximum Flows



Nov 17-4:13 PM

NYTIMES Forecast



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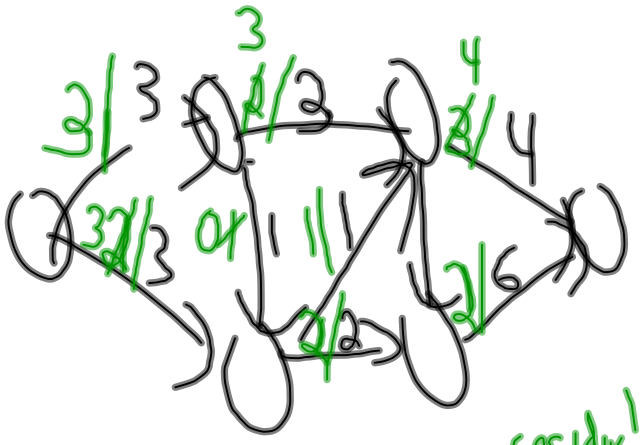
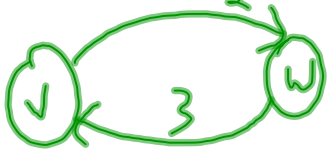


residual capacity

residual



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capacity of an s-t cut (S,T)

$$c(S,T) = \sum_{\substack{u \in S \\ v \in T}} c(u,v)$$

Flow across (S,T)

$$f(S,T) = \sum_{\substack{u \in S \\ v \in T}} f(u,v) - \sum_{\substack{u \in T \\ v \in S}} f(u,v)$$

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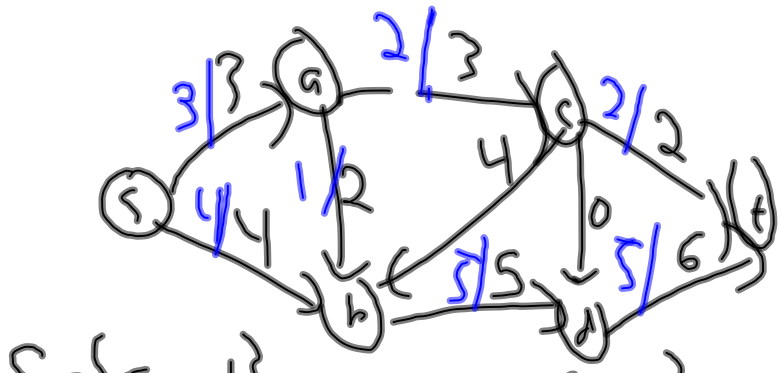

$S = s$

$c = 3+4 = 7$

$f = 3+4 = 7$

$S = \{s, a, b\}$

$c = 3+5 = 8$

$f = 2+5-0 = 7$

$S = \{s, b\}$

$c = 3+5 = 8$

$f = 3-1+5 = 7$

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for any cut

$$f(S,T) \le c(S,T)$$

0 1 2 3 4 5 6 | 7 8 10 ....

flows f | cuts (S,T) $c(S,T)$

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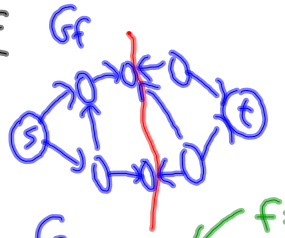
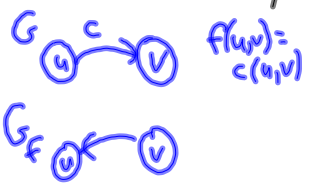
1 ⇒ 2 f is a max flow ⇒ G_f has no aug. paths

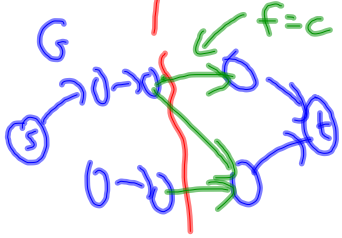
Pf ¬2 ⇒ ¬1 G_f has an aug. path ⇒ f is not a max flow

2 ⇒ 3 G_f has no aug. path ⇒ ∃ a cut c(S,T) w $|f| = c(S,T)$

Pf G_f







3 ⇒ 1) ∃ a cut $|f| = c(S,T)$ then f is a max flow.

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1 iteration of FF is O(E) time

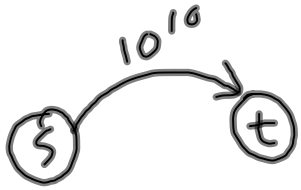
How many iterations?

assume caps are ints

each iteration sends ≥ 1 unit of flow

f^* = max flow

$|f^*|$ iterations



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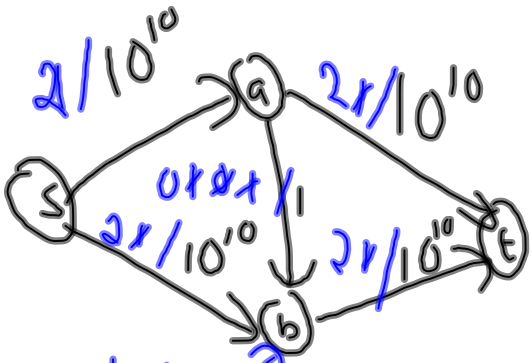

poly-time for flow

augment along a shortest path

$O(V^3E)$

s-a-b-t
s-b-a-t
s-a-b-t
s-b-a-t

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