x_1 = # soldiers manufactured
x_2 = # trains

$$\begin{aligned} \max\ & 3x_1 + 2x_2 \\ \text{s.t.}\ & \\ & 2x_1 + x_2 \le 100 \\ & x_1 + x_2 \le 80 \\ & x_1 \le 40 \\ & x_1, x_2 \ge 0 \end{aligned}$$

- feas. sol'n satisfies constraints
- opt. sol'n, feasible & maximizes obj. function

$x_1 = 10$ $x_2 = 10$ obj = 50 / $x_1 = 20$ $x_2 = 60$ obj = 180

Nov 19-4:12 PM

Fractional Knapsack

v_i pick some of each item, s.t.
w_i total wt. $\le W$, value maximized.
W

x_i = fraction of item i included.

linear program

$$\begin{aligned} \max\ & \sum_{i=1}^{n} v_i x_i \\ \text{s.t.}\ & \sum_{i=1}^{n} w_i x_i \le W \\ & 0 \le x_i \le 1 \quad i = 1 \ldots n \end{aligned}$$

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Solving a LP.

n vars. m constraints.

- Algs to solve LP in polynomial time
  (interior point, ellipsoid)

  3 4

- Simplex (not known polynomial time)
  Fast in practice ((CPLEX,)

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Integer Programming

LP w/ constraints on some vars that
x_i is an integer.

Integer Programming is NP-complete.

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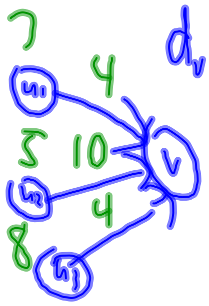

$$d_v \le d_{u_1} + w(u_1, v)$$
$$d_v \le d_{u_2} + w(u_2, v)$$
$$d_v \le d_{u_3} + w(u_3, v)$$

max d_v forces $d_v = \min \begin{cases} d_{u_1} + w(u_1,v) \\ d_{u_2} + w(u_2,v) \\ d_{u_3} + w(u_3,v) \end{cases}$

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Max flow &

$$f(a,b) + f(b,c) \le 2f(a,c)$$
$$f(e,f) \le 1$$
$$f(h,b) \ge 4$$

Still a LP

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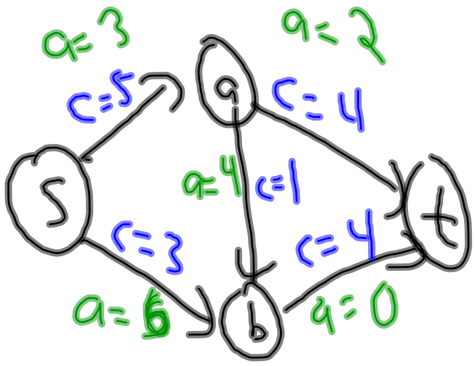
min cost flow



I want to route d units of flow from s to t as cheaply as possible.

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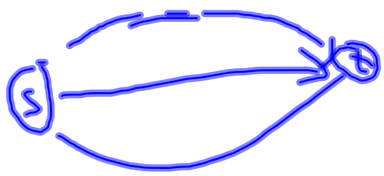
Flow



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max $3x_1+2x_2$

s.t.

$2x_1+1x_2 \le 100$

$1x_1+1x_2 \le 80$

$1x_1 \le 40$

$x_1, x_2 \ge 0$

$(1,1,0)$

180

min $100y_1+80y_2+40y_3$

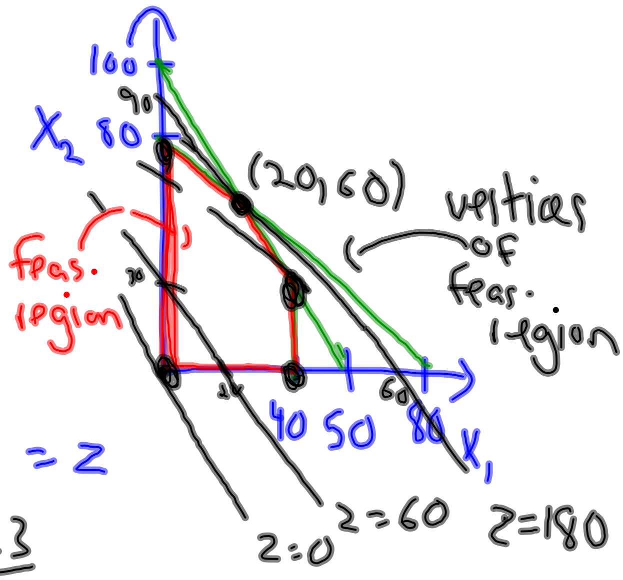
s.t.

$2y_1+y_2+y_3 \ge 3$

$y_1+y_2 \ge 2$

$3x_1+2x_2 = z$

slope $= \frac{-3}{2}$



If feas region is non-empty & bounded then $\exists$ an opt sol. which is a vertex of feas. region

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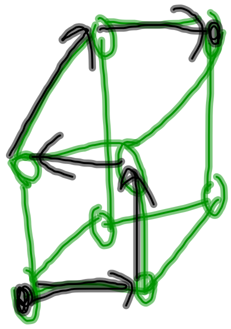
Simplex moves from vertex to adj. vertex in the feas. region, increasing (not decreasing) the obj. value at each step.



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