SAT

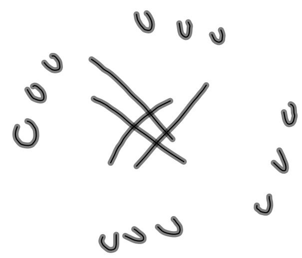
3-SAT

Clique

$3\text{-SAT} \leq \text{Clique}$

$(\quad) \wedge (\quad) \wedge (\quad) \wedge (\quad)$

$\xrightarrow{f}$



k

x is sat iff f(x) has a k-clique

Informall clique, where nodes are in groups of 3 w/ no intragroup edges is hard.

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Proof shows that a "special case" of clique is NP-complete $\Rightarrow$ that clique is NP-complete.

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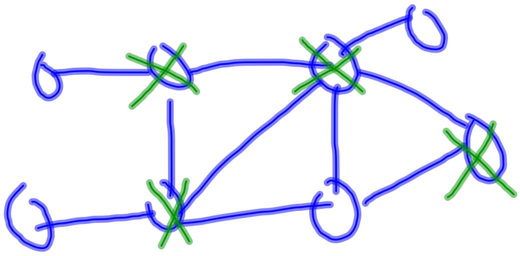
<u>Vertex Cover</u> Given a graph $G=(V,E)$ & an int k. A vertex cover $V' \subseteq V$ is a subset of the vertices s.t. $\forall (v,w) \in E$, $v \in V'$ or $w \in V'$ or both. Is there a vertex cover w/ $|V'| \leq k$.



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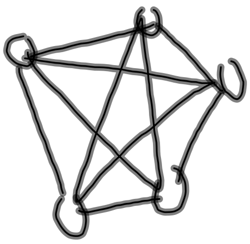
VC in NP-complete

<u>Pf</u>

1) $VC \in NP$

What is a VC for a clique



VC of a k-clique has k-1 vertices

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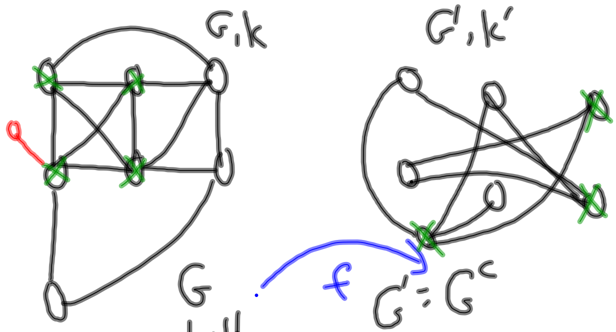

G Clique $k=4$ $\xrightarrow{f}$ $G' = G^c$, $k' = |V| - k$

Claim: G has a k-clique iff G' has a vertex cover of size k'.

Pf

$\Rightarrow$ let C be the vertices in the k-clique then C in G' has no edges in it.

$\therefore$ $V-C$ is a vertex cover of G'

$\Leftarrow$ let G' have a vertex cover D of size k'
Then $V-D$ is a subset of vertices w/ no edges in it.
$\therefore$ in G^c, $V-D$ is a clique of size $|V|-|D| = |V| - k'$
$= k$

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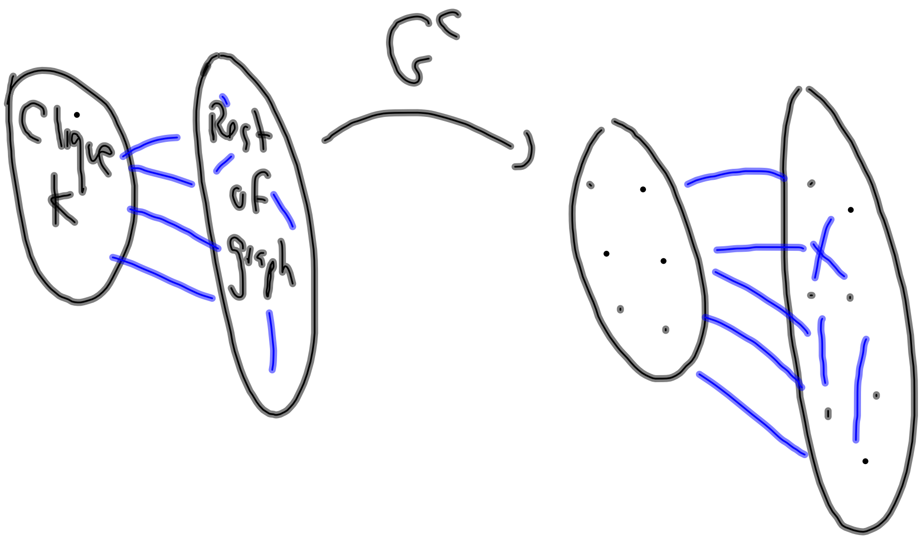

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Subset Sum

Given a set of integers $S = \{s_1, s_2 \dots s_n\}$ & a target int t.
Is there a subset $S' \subseteq S$ s.t.

$$\sum_{s_i \in S'} s_i = t.$$

$\{1, 4, 16, 64, 256, 1040, 1041, 1093, 1284, 1344\}$

$t = 3754$ $\quad SS \in NP$

$S' = \{1, 16, 64, 256, 1040, 1093, 1284\}$

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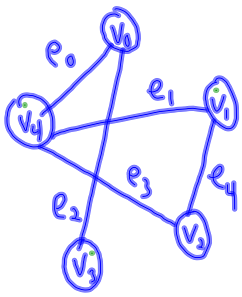
VC $\leq$ SS



$k = 3$

	e_4	e_3	e_2	e_1	e_0
v_0	0	0	1	0	1
v_1	1	0	0	1	0
v_2	1	1	0	0	0
v_3	0	0	1	0	0
v_4	0	1	0	1	1
	1	1	1	2	1

Main idea:
Think of rows as (binary) numbers, sum them, interpret the sum

VC is a set of rows s.t. each column has at least one 1.

Problems

1) Carries
2) What is t?
3) What about k?

$\Rightarrow$ Sum these row vectors, the sum should have all non-0 components

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Problems

1) Carries – use base 4
2) Target sum – introduce "dummy" entries
3) k – add a column to count

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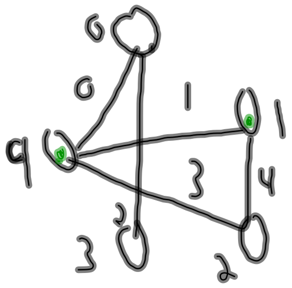

$k=3$

	vert		e_4	e_3	e_2	e_1	e_0	base 4
x_0	1		0	0	1	0	1	1041
→ x_1	1		1	0	0	1	0	1284
x_2	1		1	1	0	0	0	1344
→ x_3	1		0	0	1	0	0	1044
→ x_4	1		0	1	0	1	1	1093
1 per edge → y_0	0		0	0	0	0	1	1
y_1	0		0	0	0	1	0	4
→ y_2	0		0	0	1	0	0	16
→ y_3	0		0	1	0	0	0	64
→ y_4	0		1	0	0	0	0	256
			1	1	1	2		
$t=$	(3) k		2	2	2	2	2	3754

G has a vc of size k
⟺
SS has a subset summing to t.

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$\Rightarrow$ take the vertices of the vertex cover, of size k, they sum to

base 4 $\rightarrow$ (k) 1/2 1/2 1/2 1/2 1/2

addin y_i's corresponding to 1's, yielding

(k) 2 2 2 2 2

and this is t.

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$(\Leftarrow)$ SS instance is yes $\Rightarrow$ G has a VC of size k.

Pf

SS is yes, set of rows in the sum

- must include k x rows (k vertices)
- for each column, I must choose at lease one x that has a 1 in the col.

$\Rightarrow$ these vertices cover all the edges.

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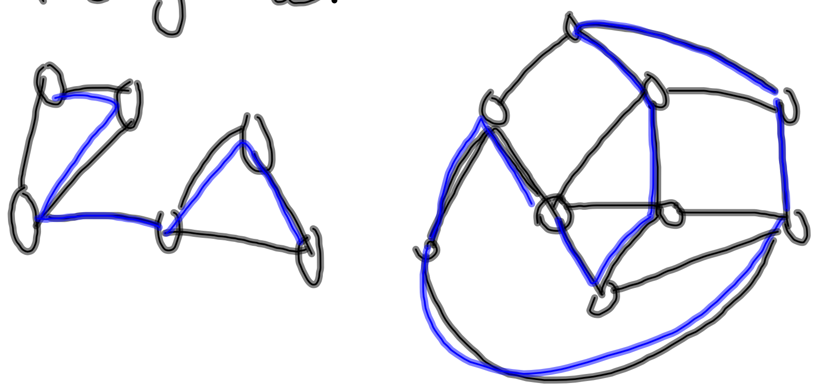
Hamiltonian Cycle

Given a graph $G=(V,E)$
is there a cycle visiting each vertex
exactly once.

no



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Traveling Salesman Problem

Given a graph $G=(V,E)$ w/ edge
weights w, int B. Is there
a Hamiltonina Cycle C s.t.

$$\sum_{e \in C} w_e \leq B.$$

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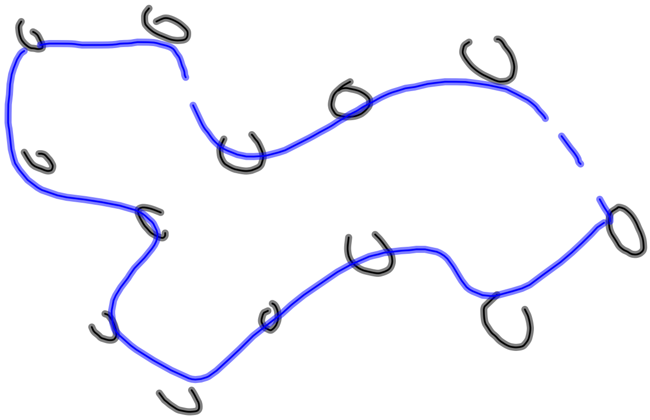

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