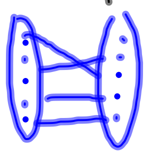
Review Session
Wed. 12/16
10AM
303 Mudd

Dec 8-4:07 PM

Dealing w/ NP-complete Problems

(go back to optimization problems)
NP-hard

SAT
3-SAT
2-SAT



- solve small instances
- maybe your input has special structure that makes it easy
- Heuristics (alg. that runs for a limited amt. of time, returns a sol'n that you hope is good) no guarantee on the quality of the sol'n.

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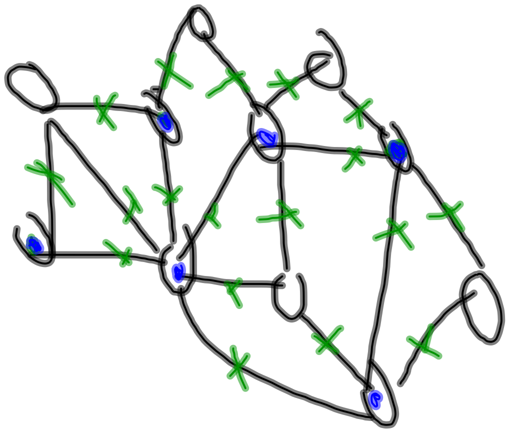

- Pick all nodes
- Pick max deg. vertex

- metaheuristic - simulated annealing
  tabu search
  genetic algs.
  GRASP
  greedy

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Approximation Algorithms (minimization problem)

Problem X, input I, alg. A

A is a ρ - approx. alg. if

1) A runs in poly time
2) $A(I) \leq \rho \, OPT(I)$,

where OPT(I) is the optimum sol'n on instance I.

$\rho \geq 1$, want ρ small

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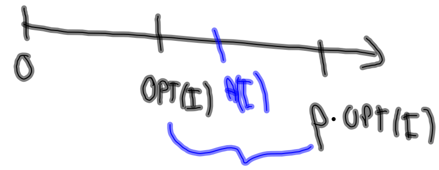
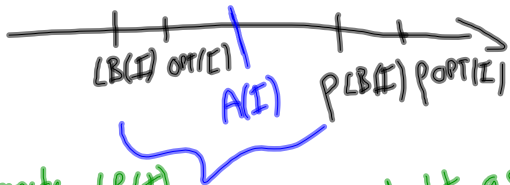


$\rho = 2$ is 100% error
$\rho = 1.01$ is 1% error

Lower bound $LB(I) \le OPT(I)$
& is not too far from OPT, and is easily computable.



1. Compute a $LB(I)$.
2. Design an alg. using the $LB(I)$, and obt. a sol'n w/ $A(I) \le \rho\, LB(I)$

$\Rightarrow$ conclude $\underline{A(I)} \le \rho LB(I) \le \underline{\rho OPT(I)}$

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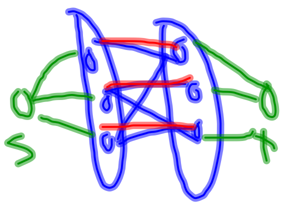
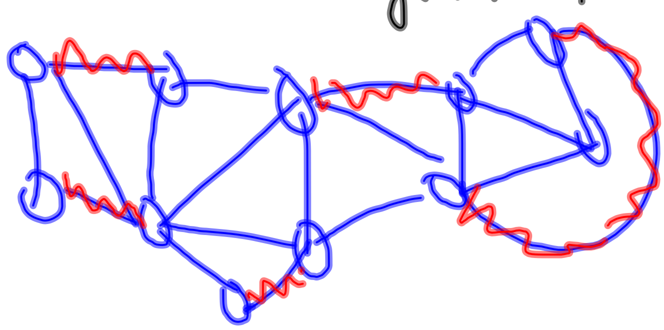
Matching

poly-time



A matching M in a graph is a subset of the edges $M \subseteq E$ s.t. each vertex $v \in V$ is incident to at most one edge in M

maximum matching



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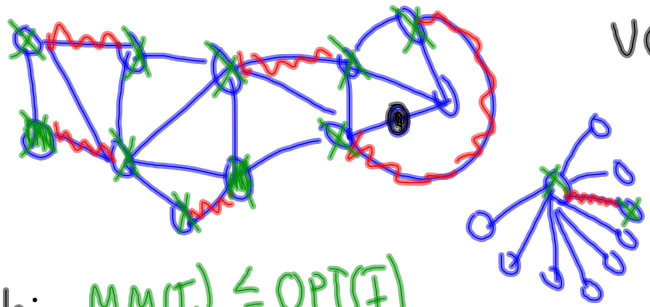
2-approx for VC

MM(I) - max. matching
OPT(I) - opt. vertex cover
VC(I) - vertex cover produced by my alg.



Claim $MM(I) \leq OPT(I)$

Pf Any vertex v that covers some edge e in MM cannot cover any other edge in MM. Therefore you need at least $MM(I)$ vertices.

Alg

1. Compute MM.
2. For each edge $(v,w) \in MM$, add BOTH v & w to VC.

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Claim VC is a vertex cover.

Pf If some edge (v,w) is not covered, then neither v nor w is incident to an edge in MM. which means I can add (v,w) to MM & get a larger matching which contradicts MM being a max matching

$$VC(I) = 2MM(I) \leq 2\,OPT(I)$$

$\therefore$ VC is a vertex cover & $VC(I) \leq 2\,OPT(I)$ ⊠

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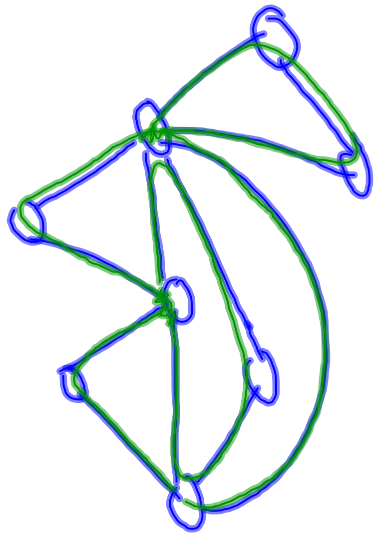
TSP

Euler Tour

Given an und. graph G w/ even degree, find a cycle (not simple) that visits each edge exactly once



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TSP

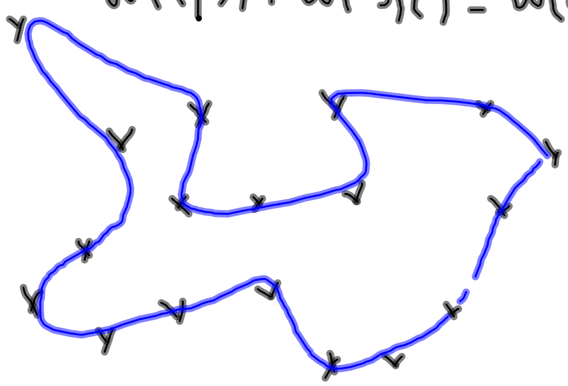
symmetric TSP w/ Δ-inequality

$w(a,b) = w(b,a)$

$w(a,b) + w(b,c) \geq w(a,c)$

w/o Δ-ineq. can't approx. TSP.

asymmetric $\log n / (\log\log n)$



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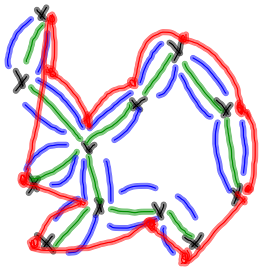
2-approx



Short
covers the vertices
~~cycle~~

MST is a lower bound

$MST(I) \le OPT(I)$ ↖ opt TSP

1. Compute MST
2. Double each edge in MST, to get ET
3. Trace the ET, but skip vertices that I have already visited to get $TSP(I)$

$MST(I) \le OPT(I)$

$ET(I) = 2\,MST(I)$

$TSP(I) \le ET(I)$

$\Rightarrow TSP(I) \le 2\,OPT(I)$

∴ I have a 2-approx.

$\frac{3}{2}$-approx is possible.

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If points are in plane, euclidean dist.
there is a $(1+\varepsilon)$-approx. for any $\varepsilon > 0$.

running time $\approx n^{O(\frac{1}{\varepsilon})}$

2-approx	n^1
$\frac{3}{2}$-approx	n^2
1.1-approx	n^{10}
1.01 ..	n^{100}

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