MAX 3-SAT

Given a boolean formula ($\wedge$ of $\vee$), 3 literals per clause. Satisfy the max possible # of clauses.

$\frac{3}{4}$-approx $\quad (\overset{T}{x_1} \vee \overset{T}{x_2}) \wedge (\overset{T}{x_1} \vee \overset{F}{\bar{x}_2}) \wedge (\overset{F}{\bar{x}_1} \vee \overset{T}{x_2}) \wedge (\overset{F}{\bar{x}_1} \vee \overset{F}{\bar{x}_2})$

maximizing $\quad A(I) \geq \rho \, OPT(I) \qquad \rho \leq 1$

large ρ better

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Alg.

Randomly set each variable to T or F w/ $\Pr\{x_i := T\} = \frac{1}{2}$

— $\frac{7}{8}$-approximation alg.

$(\overset{F}{x_1} \vee \overset{T}{x_2} \vee \overset{F}{x_3}) \wedge (\overset{F}{x_1} \vee \overset{F}{\bar{x}_2} \vee \overset{F}{x_3}) \wedge (\overset{F}{x_1} \vee \overset{F}{\bar{x}_2} \vee \overset{F}{x_4}) \wedge (\overset{T}{x_2} \vee \overset{F}{x_3} \vee \overset{F}{x_4}) \wedge (\overset{T}{\bar{x}_3} \vee \overset{F}{x_4} \vee \overset{F}{x_1})$

$x_1 := F \quad x_2 := T \quad x_3 := F \quad x_4 := F$

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Analysis

Let Y = # clauses satisfied

Let m = # clauses

$$Y_i = I\{i^{th} \text{ clause being satisfied}\}$$

$$Y = \sum_{i=1}^{m} Y_i$$

$$E[Y] = E\left[\sum_{i=1}^{m} Y_i\right] = \sum_{i=1}^{m} E[Y_i]$$

$$= \sum_{i=1}^{m} \Pr\{i^{th} \text{ clause is true}\}$$

for an OR of 3 literals, $\Pr\{\text{clause is true}\} = \frac{7}{8}$

$$= 1 - \Pr\{\text{clause is false}\}$$

$$= 1 - \Pr\{\text{a literal is false}\}^3$$

$$= 1 - \left(\tfrac{1}{2}\right)^3 = \tfrac{7}{8}.$$

$$\sum_{i=1}^{m} \frac{7}{8} = \frac{7m}{8}$$

UPPER BOUND ON OPT = m

$\rho = \frac{7}{8}$ $\quad\|\quad \frac{7m}{8} \le \frac{7}{8} \cdot m$

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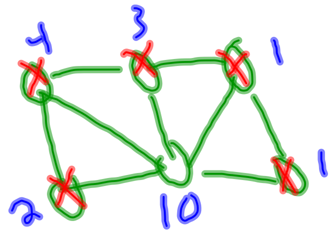
Weighted Vertex Cover

Graph $G=(V,E)$ $\quad w(v)$ on vertices.

Choose a vertex cover C s.t. $\sum_{v \in C} w(v)$ is minimized



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Use variables

$$x(v) = \begin{cases} 1 & \text{if } v \text{ in v.c.} \\ 0 & \text{o.w.} \end{cases}$$

$$\min \sum_{v \in V} w(v)\, x(v)$$

s.t.

$$x(u) + x(v) \geq 1 \quad \forall (u,v) \in E$$

$$\longrightarrow x(v) \in \{0,1\} \quad \forall v \in V$$

0-1 <u>Integer</u> Linear Programming NP-complete

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LP relaxation...

$$\min \sum w(v)\, x(v)$$

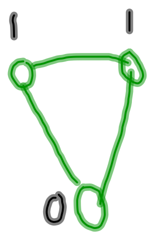
s.t.

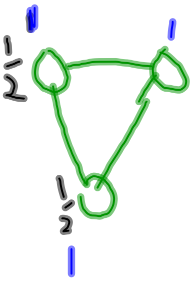
$$x(u) + x(v) \geq 1 \quad \forall (v,w) \in E$$

$$0 \leq x(v) \leq 1 \quad \forall v \in V$$

int.

≥ 2





$\frac{3}{2}$

<u>Approx Alg.</u>

1. Solve the LP
2. Round vals to ints (round 1/2 up)

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Let $\bar{x}(v)$ denote rounded solution.

- each edge is covered. ie. $\bar{x}(u)+\bar{x}(v) \geq 1$.

problem $\rightarrow$ 0 0

$\Rightarrow x(u), x(v) < \frac{1}{2}$

$\Rightarrow x(u)+x(v) < 1$

contradicts LP soln

soln $\sum w(v)\bar{x}(v)$

LB $\sum w(v)x(v)$

$\forall v \quad \bar{x}(v) \leq 2x(v)$

$x(v) < \frac{1}{2} \quad \bar{x}(v) = 0$

$x(v) \geq \frac{1}{2} \quad \bar{x}(v) = 1$

$\sum w(v)\bar{x}(v) \leq 2\sum w(v)x(v) \Rightarrow$ 2-approx.

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$1+\varepsilon$

$2, \frac{3}{2}$

$\lg n$

$n^{1-\varepsilon}$

can't be approx

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Modelers, Problem Solvers

Algorithms

Programming

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