

# Quicksort

QUICKSORT( $A, p, r$ )

```
1  if  $p < r$ 
2      then  $q \leftarrow \text{PARTITION}(A, p, r)$ 
3          QUICKSORT( $A, p, q - 1$ )
4          QUICKSORT( $A, q + 1, r$ )
```

PARTITION( $A, p, r$ )

```
1   $y \leftarrow \text{RANDOM}(p, r)$ 
2  Exchange  $A[y]$  and  $A[r]$ 
3   $x \leftarrow A[r]$ 
4   $i \leftarrow p - 1$ 
5  for  $j \leftarrow p$  to  $r - 1$ 
6      do if  $A[j] \leq x$ 
7          then  $i \leftarrow i + 1$ 
8              exchange  $A[i] \leftrightarrow A[j]$ 
9  exchange  $A[i + 1] \leftrightarrow A[r]$ 
10 return  $i + 1$ 
```

# Partition Loop Invariant

PARTITION( $A, p, r$ )

```
1   $y \leftarrow \text{RANDOM}(p, r)$ 
2  Exchange  $A[y]$  and  $A[r]$ 
3   $x \leftarrow A[r]$ 
4   $i \leftarrow p - 1$ 
5  for  $j \leftarrow p$  to  $r - 1$ 
6      do if  $A[j] \leq x$ 
7          then  $i \leftarrow i + 1$ 
8              exchange  $A[i] \leftrightarrow A[j]$ 
9  exchange  $A[i + 1] \leftrightarrow A[r]$ 
10 return  $i + 1$ 
```

**Loop Invariant** At the beginning of each iteration of the for loop in Partition

1.  $A[p \dots i] \leq x$
2.  $A[i + 1 \dots j - 1] > x$
3.  $A[j \dots r - 1]$  is unexamined
4.  $A[r] = x$