Disjoint Sets

- **Set of items -** X **.**
- **Maintain disjoint sets** $S_1, \ldots, S_k$ **; i.e.** $S_i \cap S_j = \emptyset \;\; \forall i \neq j$
- **Operations:**
  - **MakeSet(** x **) - create a one-element set with** x
  - **Find-Set(** x **) - return the "name" of the set containing** x
  - **Union(** x **,** y **) - merge the set containing** x **and the set containing** y **into one set.**

Disjoint Set Code

Make-Set(x)

```
p[x] ← x
rank[x] ← 0
```

Union(x, y)

```
LINK(FIND-SET(x), FIND-SET(y))
```

Link(x, y)

```
if rank[x] > rank[y]
    then p[y] ← x
    else  p[x] ← y
            if rank[x] = rank[y]
               then rank[y] ← rank[y] + 1
```

Find-Set(x)

```
if x ≠ p[x]
    then p[x] ← FIND-SET(p[x])
return p[x]
```

Ackerman's Function

$$A_k(j) = \begin{cases} j+1 & \textbf{if } k = 0 \\ A_{k-1}^{(j+1)}(j) & \textbf{if } k \geq 1 \end{cases}$$

$$\alpha(n) = \min\{k : A_k(1) \geq n\}$$

$$\begin{aligned} A_0(j) &= j+1 \\ A_1(j) &= A_0^{(j+1)}(j) \\ &= 2j+1 \\ A_2(j) &= A_1^{(j+1)}(j) \\ &= 2(2(\cdots(2j+1)\cdots)+1)+1 \\ &= 2^{j+1}(j+1)-1 \end{aligned}$$

Ackerman

$$
\begin{aligned}
A_3(1) &= A_2^{(2)}(1) \\
&= A_2(A_2(1)) \\
&= A_2(7) \\
&= 2^8 \cdot 8 - 1 \\
&= 2^{11} - 1 \\
&= 2047
\end{aligned}
$$

$$
\begin{aligned}
A_4(1) &= A_3^{(2)}(1) \\
&= A_3(A_3(1)) \\
&= A_3(2047) \\
&= A_2^{(2048)}(2047) \\
&\gg A_2(2047) \\
&= 2^{2048} \cdot 2048 - 1 \\
&> 2^{2048} \\
&= (2^4)^{512} \\
&= 16^{512} \\
&\gg 10^{80} \ ,
\end{aligned}
$$