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- **Array $A[1 \ldots n]$ – holds input**
- **Array $C[1 \ldots k]$ – $C[j]$ holds number of elements of A less than or equal to j**

Example:

index	1	2	3	4	5	6	7	8	9
$A:$	2	9	1	8	6	5			

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A :	2	9	1	8	6	5			
C :	1	2	2	2	3	4	4	5	6

Questions

- **How do we compute C?**

Counting Sort

$Counting - Sort(A, B, k)$

for $i \leftarrow 0$ **to** k
    do $C[i] \leftarrow 0$
for $j \leftarrow 1$ **to** $length[A]$
    do $C[A[j]] \leftarrow C[A[j]] + 1$
$\triangleright$ $C[i]$ **now contains the number of elements equal to** i**.**
for $i \leftarrow 1$ **to** k
    do $C[i] \leftarrow C[i] + C[i-1]$
$\triangleright$ $C[i]$ **now contains the number of elements less than or equal to** i**.**
for $j \leftarrow length[A]$ **downto 1**
    do $B[C[A[j]]] \leftarrow A[j]$
        $C[A[j]] \leftarrow C[A[j]] - 1$

Analysis

- **Running Time** $O(n+k)$
- **No Comparisons**
- **Doesn't work on all data**
- **Good when** k **is small**
- **When** $k = O(n)$ **we have run-time** $O(n+k) = O(n)$
- **Examples?**

Stable Sorting

- **We want to sort** $x_1, x_2, ..., x_n$
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Question: **Is counting sort stable?**

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- **Running time is** $150 + 700 = 850$

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Improvement: Radix Sort

- **Sort second initial**
- **Then stable sort by first initial.**

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Improvement: Radix Sort

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Analysis

- **Sorting a single letter:** $150 + 27 < 200$
- **Total running time:** $2(150 + 27) < 400$

Radix Sort

$Radix - Sort(A, d)$

for $i \leftarrow 1$ **to** d
    do use a stable sort to sort array A **on digit** i

Example

379	**STABLE SORT**	912	**STABLE SORT**	802	**STABLE SORT**	258
912	$\Rightarrow$	802	$\Rightarrow$	803	$\Rightarrow$	259
258		823		804		269
269		803		912		279
823		804		823		379
259		258		258		802
803		379		259		803
279		269		269		804
804		359		379		823
802		279		279		912

Radix Sort Correctness

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Loop Invariant: **After the ith iteration of the loop, the elements are sorted by their last i digits.**

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Inductive Step:

- **Assume the invariant holds after** $i-1$ **iterations**
- **Need to prove that it holds after** i **iterations**

Radix Sort Analysis

- n elements

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- n **elements**
- **All elements have** d **digits**
  - **Initials:** $d = 2$
  - **SSN:** $d = 9$
  - **Dictionary Words:** $d = 30$

Radix Sort Analysis

- n elements
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  - Initials: $d = 2$
  - SSN: $d = 9$
  - Dictionary Words: $d = 30$
- Digits are in base b
  - Numbers: $b = 10$
  - Words: $b = 27$
  - UNI (letter/number): $b = 37$

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Counting Sort Running Time: $O(n+k) = O(n+b^d)$

Example

Setup : Sort everyone in columbia by UNI. Say $n = 40,000$

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Radix Sort:

- $d = 7$
- $b = 37$
- **Running Time:** $d(n + b) = 7(40,000 + 37) \sim 280,000$

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- **UNI = 7-digit number in base 37.**
- $k = b^d = 37^7 \sim 10^{11}$
- **Running Time:** $n + k = 40,000 + 37^7 \sim 10^{11}$

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Merge Sort

- **Running Time:** $nlog(n) = 40,000 \cdot \log(40,000) \sim 600,000$