

Here is the LP that we discussed in class.

$$\begin{array}{llll}
 \max & 3x_1 & +x_2 & +2x_3 \\
 \text{s.t.} & & & \\
 & x_1 & +x_2 & +3x_3 \leq 30 \\
 & 2x_1 & +2x_2 & +5x_3 \leq 24 \\
 & 4x_1 & +x_2 & +2x_3 \leq 36 \\
 & x_1, x_2, x_3 & & \geq 0
 \end{array}$$

We convert this to standard form.

$$\begin{array}{rcccccl}
 z & = & 3x_1 & +x_2 & +2x_3 & & \\
 x_1 & & +x_2 & +3x_3 & +s_1 & & = 30 \\
 2x_1 & + & 2x_2 & +5x_3 & & +s_2 & = 24 \\
 4x_1 & + & x_2 & +2x_3 & & +s_3 & = 36
 \end{array}$$

Now we put the basic variables on the left, non-basic variables on the right:

$$\begin{array}{rcccccl}
 z & = & 3x_1 & +x_2 & +2x_3 & & \\
 s_1 & = 30 & -x_1 & -x_2 & -3x_3 & & \\
 s_2 & = 24 & -2x_1 & -2x_2 & -5x_3 & & \\
 s_3 & = 36 & -4x_1 & -x_2 & -2x_3 & &
 \end{array}$$

We choose x_1 to enter, since it has a positive coefficient. Its increase most limited by s_3 , so we resolve the last equation for x_1 and then plug into the other equations. This gives:

$$\begin{array}{rcccccl}
 z & = & 27 & +\frac{1}{4}x_2 & +\frac{1}{2}x_3 & -\frac{3}{4}s_3 \\
 x_1 & = & 9 & -\frac{1}{4}x_2 & -\frac{1}{2}x_3 & -\frac{1}{4}s_3 \\
 s_1 & = & 21 & -\frac{3}{4}x_2 & -\frac{5}{2}x_3 & +\frac{1}{4}s_3 \\
 s_2 & = & 6 & -\frac{3}{2}x_2 & -4x_3 & +\frac{1}{2}s_3
 \end{array}$$

Now we choose x_3 to enter, and it is most limited by s_2 . This yields:

$$\begin{array}{rcccccl}
 z & = & \frac{111}{4} & +\frac{1}{16}x_2 & -\frac{1}{8}s_2 & -\frac{11}{16}s_3 \\
 x_1 & = & \frac{33}{4} & -\frac{1}{16}x_2 & +\frac{1}{8}s_2 & -\frac{5}{16}s_3 \\
 x_3 & = & \frac{3}{2} & -\frac{3}{8}x_2 & -\frac{1}{4}s_2 & +\frac{1}{8}s_3 \\
 s_1 & = & \frac{69}{4} & +\frac{3}{16}x_2 & +\frac{5}{8}s_2 & -\frac{1}{16}s_3
 \end{array}$$

Finally, we choose x_2 to enter, and x_3 to leave, and obtain:

$$\begin{array}{rcll}
z & = & 28 & -\frac{1}{6}x_3 & -\frac{1}{6}s_2 & -\frac{2}{3}s_3 \\
x_1 & = & 8 & +\frac{1}{6}x_3 & +\frac{1}{6}s_2 & -\frac{1}{3}s_3 \\
x_2 & = & 4 & -\frac{8}{3}x_3 & -\frac{2}{3}s_2 & +\frac{1}{3}s_3 \\
s_1 & = & 18 & -\frac{1}{2}x_3 & +\frac{1}{2}s_2 &
\end{array}$$

All coefficients in the objective function are negative, so this is optimal.