

Definition of a Linear Program

Definition: A function $f(x_1, x_2, \dots, x_n)$ of $x_1, x_2, \dots, x_n$ is a **linear function** if and only if for some set of constants $c_1, c_2, \dots, c_n$,

$$f(x_1, x_2, \dots, x_n) = c_1x_1 + c_2x_2 + \dots + c_nx_n .$$

Examples:

- x_1
- $5x_1 + 6x_4 - 2x_2 + 1$
- 3

Non-examples:

- x_1^2
- $x_1 + 3x_2 - 4x_3^4$
- x_1x_2

Linear Inequalities

Definition: For any linear function $f(x_1, x_2, \dots, x_n)$ and any number b , the inequalities

$$f(x_1, x_2, \dots, x_n) \leq b$$

and

$$f(x_1, x_2, \dots, x_n) \geq b$$

are **linear inequalities**.

Examples:

- $x_1 + x_2 \leq 4$
- $5x_1 - 4 \geq 0$

Note: If an inequality can be rewritten as a linear inequality then it is one. Thus $x_1 + x_2 \leq 3x_3$ is a linear inequality because it can be rewritten as $x_1 + x_2 - 3x_3 \leq 0$. Even $x_1/x_2 \leq 4$ is a linear inequality because it can be rewritten as $x_1 - 4x_2 \leq 0$. Note that $x_1/x_2 + x_3 \leq 4$ is not a linear inequality, however.

Definition: For any linear function $f(x_1, x_2, \dots, x_n)$ and any number b , the equality

$$f(x_1, x_2, \dots, x_n) = b$$

is a **linear equality**.

LPs

Definition: A **linear programming problem (LP)** is an optimization problem for which:

1. We attempt to maximize (or minimize) a linear function of the decision variables. (**objective function**)
2. The values of the decision variables must satisfy a set of constraints, each of which must be a linear inequality or linear equality.
3. A sign restriction on each variable. For each variable x_i the sign restriction can either say
 - (a) $x_i \geq 0$,
 - (b) $x_i \leq 0$,
 - (c) x_i unrestricted (urs).

Definition: A **solution** to a linear program is a setting of the variables.

Definition: A **feasible solution** to a linear program is a solution that satisfies all constraints.

Definition: The **feasible region** in a linear program is the set of all possible feasible solutions.

Definition: An **optimal solution** to a linear program is the feasible solution with the largest objective function value (for a maximization problem).

Modeling Assumptions for Linear Programming

- **Proportionality.** If one item brings in a profit of x , then k items bring in a profit of kx . If one item use y units of resource R then k items use ky units of resource R .
- **Additivity.** The decisions made are independent, except as noted in the constraints. So, if we sell more trains, this does not decrease the demand for soldiers, in the Giapeto model.
- **Divisibility.** Decision variables can take on fractional values.
- **Certainty.** The values of various parameters are known with certainty.

Comments:

- Whether these assumptions hold is a feature of the model, not of linear programming itself.
- They often do not hold.
- They may be close to holding, or may hold in the region we are about:
e.g.
 - proportionally and additivity may hold in the feasible region
 - divisibility may not hold, but the conclusions of the model will be approximately sound anyway

- certainty may not hold, but we may have good estimates

Whenever we solve a model using linear programming, we should be aware of these assumptions, and ask ourselves whether they hold, and whether the solution makes sense.