Solution to Sunco

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What is the objective?

Maximize profit $= \textbf{income - cost}$

$= \textbf{income - (crude cost + production cost + advertising cost)}$

$= 70(x_{11} + x_{21} + x_{31}) + 60(x_{12} + x_{22} + x_{33}) + 50(x_{13} + x_{23} + x_{33})$

$- (45(x_{11} + x_{12} + x_{13}) + 35(x_{21} + x_{22} + x_{23}) + 25(x_{31} + x_{32} + x_{33}))$

$- 4 \Sigma_{i=1}^{3} \Sigma_{j=1}^{3} x_{ij}$

$- (a_1 + a_2 + a_3)$

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1. Gas produced meets demand
2. at most 5000 barrels of each crude
3. at most 14000 barrels a day
4. octane levels met
5. sulfur levels met

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1. Gas produced equals demand (no leftover allowed)

2. at most 5000 barrels of each crude

3. at most 14000 barrels a day

4. octane levels met

5. sulfur levels met

1)

$$\begin{aligned} x_{11} + x_{21} + x_{31} &= 3000 + a_1 \\ x_{12} + x_{22} + x_{33} &= 2000 + a_2 \\ x_{13} + x_{23} + x_{33} &= 1000 + a_3 \end{aligned}$$

2)

$$\begin{aligned} x_{11} + x_{12} + x_{13} &\leq 5000 \\ x_{21} + x_{22} + x_{23} &\leq 5000 \\ x_{31} + x_{32} + x_{33} &\leq 5000 \end{aligned}$$

3)

$$\sum_{i=1}^{3} \sum_{j=1}^{3} x_{ij} \leq 14000$$

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Average octane level = (total octane value) / number of barrels

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$$\frac{12x_{11} + 6x_{21} + 8x_{31}}{x_{11} + x_{21} + x_{31}} \geq 10$$

Is this linear? YES It is equivalent to

$$12x_{11} + 6x_{21} + 8x_{31} \geq 10(x_{11} + x_{21} + x_{31})$$

or

$$2x_{11} - 4x_{21} - 2x_{31} \geq 0$$

We can do the same for all the constraints, yielding the final linear program:

Final LP

maximize $21x_{11} + 11x_{12} + x_{13} + 31x_{31} + 21x_{22} + 11x_{23} + 41x_{31} + 31x_{32} + 21x_{33} - a_1 - a_2 - a_3$

s.t.

$$
\begin{aligned}
x_{11} + x_{21} + x_{31} - 10a_1 &= 3000 && \textbf{(demand for gas 1)} \\
x_{12} + x_{22} + x_{32} - 10a_2 &= 2000 && \textbf{(demand for gas 2)} \\
x_{13} + x_{23} + x_{33} - 10a_3 &= 1000 && \textbf{(demand for gas 3)} \\
x_{11} + x_{12} + x_{13} &\leq 5000 && \textbf{(limit on crude 1)} \\
x_{21} + x_{22} + x_{23} &\leq 5000 && \textbf{(limit on crude 2)} \\
x_{31} + x_{32} + x_{33} &\leq 5000 && \textbf{(limit on crude 3)} \\
x_{11} + x_{21} + x_{31} + x_{12} + x_{22} + x_{32} + x_{13} + x_{23} + x_{33} &\leq 14000 && \textbf{(limit on total production)} \\
2x_{11} - 4x_{21} - 2x_{31} &\geq 0 && \textbf{(l.b. on octane of gas 1)} \\
4x_{12} - 2x_{22} &\geq 0 && \textbf{(l.b. on octane of gas 2)} \\
6x_{13} + x_{33} &\geq 0 && \textbf{(l.b. on octane of gas 3)} \\
-.005x_{11} - .01x_{21} - .02x_{31} &\leq 0 && \textbf{(u.b. on sulphur in gas 1)} \\
.015x_{12} + .001x_{32} &\leq 0 && \textbf{(u.b. on sulphur in gas 2)} \\
-.005x_{13} - .01x_{23} + .02x_{33} &\leq 0 && \textbf{(u.b. on sulphur in gas 31)}
\end{aligned}
$$

What are the descisions :

- **The amount of crude i in gas j: x_{ij}.**
- **How much to advertise gas j: a_j.**

Solution

$$x = \begin{pmatrix} 2222.22 & 2111.11 & 666.67 \\ 444.44 & 4222.22 & 333.34 \\ 333.33 & 3166.67 & 0 \end{pmatrix}$$

$a = (0, 750, 0)$

objective $z = 287500$