Solution to BreadCo

Variables

x_1 – **loaves of French Bread baked**
x_2 – **loaves of Sourdough Bread baked**
x_3 – **packets of yeast bought**
x_4 – **oz. of flour bought**

Objective:

$$\max 36x_1 + 30x_2 - 3x_3 - 4x_4$$

Constraints:

yeast used $\leq$ yeast on hand

$$x_1 + x_2 \leq x_3 + 5$$

flower used $\leq$ flower on hand

$$6x_1 + 5x_2 \leq x_4 + 10$$

Nonnegativity: **all variables**

LP

$$
\begin{array}{llllllll}
\textbf{maximize} & 36x_1 & + & 30x_2 & - & 3x_3 & - & 4x_4 & & \\
\textbf{subject to} & & & & & & & & & \\
& x_1 & + & x_2 & - & x_3 & & & \leq & 5 \\
& 6x_1 & + & 5x_2 & & & - & x_4 & \leq & 10 \\
& x_1 & , & x_2 & , & x_3 & , & x_4 & \geq & 0
\end{array}
$$

(1) (2) (3) (4)

Put into standard form:

$$
\begin{array}{lllllllllll}
z & = & & & 36x_1 & + & 30x_2 & - & 3x_3 & - & 4x_4 \\
s_1 & = & 5 & - & x_1 & - & x_2 & + & x_3 & & \\
s_2 & = & 10 & - & 6x_1 & - & 5x_2 & & & + & x_4
\end{array}
$$

(5) (6) (7)

Choose x_1 to enter
Choose s_2 to exit

Iteration 1

$$
\begin{aligned}
z &= \quad 36x_1 + 30x_2 - 3x_3 - 4x_4 && (8)\\
s_1 &= 5 - x_1 - x_2 + x_3 && (9)\\
s_2 &= 10 - 6x_1 - 5x_2 + x_4 && (10)
\end{aligned}
$$

Choose x_1 to enter
Choose s_2 to exit

$$
\begin{aligned}
z &= 60 - 6s_2 - 3x_3 + 2x_4 && (11)\\
x_1 &= \tfrac{5}{3} - \frac{s_2}{6} - \frac{5}{6}x_2 + \frac{x_4}{6} && (12)\\
s_2 &= \tfrac{10}{3} + \frac{s_2}{6} - \frac{x_2}{6} + x_3 - \frac{x_4}{6} && (13)
\end{aligned}
$$

Choose x_4 to enter
Choose s_1 to exit

Iteration 2

$$
\begin{aligned}
z &= 60 - 6s_2 - 3x_3 + 2x_4 && (14)\\
x_1 &= \tfrac{5}{3} - \frac{s_2}{6} - \frac{5}{6}x_2 + \frac{x_4}{6} && (15)\\
s_2 &= \tfrac{10}{3} + \frac{s_2}{6} - \frac{x_2}{6} + x_3 - \frac{x_4}{6} && (16)
\end{aligned}
$$

Choose x_4 to enter

Choose s_1 to exit

$$
\begin{aligned}
z &= 100 - 4s_2 - 2x_2 + 9x_3 - 12s_1 && (17)\\
x_1 &= 5 - x_2 + x_3 - s_1 && (18)\\
x_4 &= 20 + s_2 - x_2 + 6x_3 - 6s_1 && (19)
\end{aligned}
$$

Iteration 3

$$
\begin{aligned}
z &= 100 - 4s_2 - 2x_2 + 9x_3 - 12s_1 && (20)\\
x_1 &= 5 - x_2 + x_3 - s_1 && (21)\\
x_4 &= 20 + s_2 - x_2 + 6x_3 - 6s_1 && (22)
\end{aligned}
$$

Choose x_3 to enter.
Nothing limits its increase!!
Therefore, this LP is unbounded.

Note that it even gives us a particular unbounded solution. If we set $x_3 = L$, where L is a large number, $x_2, s_1, s_2 = 0$, and then use the equations to set $x_1 = L + 5$ and $x_4 = 6L + 20$,

Solution

we can go back to the original LP :

$$
\begin{array}{lrcrcrcrcl}
\textbf{maximize} & 36x_1 & + & 30x_2 & - & 3x_3 & - & 4x_4 & & \\
\textbf{subject to} & & & & & & & & & \\
& x_1 & + & x_2 & - & x_3 & & & \leq & 5 \\
& 6x_1 & + & 5x_2 & & & - & x_4 & \leq & 10 \\
& x_1 & , & x_2 & , & x_3 & , & x_4 & \geq & 0
\end{array}
$$

(23) (24) (25) (26)

and

$$(x_1, x_2, x_3, x_4) = (L + 5, 0, L, 6L + 20)$$

is a feasible solution with objective value

$$36(L + 5) - 3L - 4(6L + 2) = 9L - 172.$$

So, the problem is unbounded.