4.10 – The Big M Method

Letting x1 = number of ounces of orange soda in a bottle of Oranj

x2 = number of ounces of orange juice in a bottle of Oranj

The LP is:

$$\min z = 2x_1 + 3x_2$$

$$\begin{aligned} \text{st} \quad & 0.5x_1 + 0.25x_2 \leq 4 && \text{(sugar constraint)} \\ & x_1 + 3x_2 \geq 20 && \text{(Vitamin C constraint)} \\ & x_1 + x_2 = 10 && \text{(10 oz in 1 bottle of Oranj)} \\ & x_1, x_2, > 0 \end{aligned}$$

The LP in standard form is shown on the next slide.

4.10 – The Big M Method

The LP in standard form has z and s_1 which could be used for BVs but row 2 would violate sign restrictions and row 3 no readily apparent basic variable.

Row 1: $z - 2x_1 - 3x_2 = 0$

Row 2: $0.5x_1 + 0.25x_2 + s_1 = 4$

Row 3: $x_1 + 3x_2 - e_2 = 20$

Row 4: $x_1 + x_2 = 10$

In order to use the simplex method, a bfs is needed. To remedy the predicament, **artificial variables** are created. The variables will be labeled according to the row in which they are used as seen below.

Row 1: $z - 2x_1 - 3x_2 = 0$

Row 2: $0.5x_1 + 0.25x_2 + s_1 = 4$

Row 3: $x_1 + 3x_2 - e_2 + a_2 = 20$

Row 4: $x_1 + x_2 + a_3 = 10$

4.10 – The Big M Method

In the optimal solution, all artificial variables must be set equal to zero. To accomplish this, in a min LP, a term Ma_i is added to the objective function for each artificial variable a_i. For a max LP, the term $-Ma_i$ is added to the objective function for each a_i. **M represents some very large number.** The modified Bevco LP in standard form then becomes:

Row 1:	z -	$2x_1$ -	$3x_2$		$-Ma_2$	$-Ma_3$		= 0
Row 2:		$0.5x_1$ +	$0.25x_2$ +	s_1				= 4
Row 3:		x_1 +	$3x_2$	$-e_2$ +	a_2			= 20
Row 4:		x_1 +	x_2	+		a_3		= 10

Modifying the objective function this way makes it extremely costly for an artificial variable to be positive. The optimal solution should force $a_2 = a_3 = 0$.

4.10 – The Big M Method

Description of the Big M Method

1. Modify the constraints so that the rhs of each constraint is nonnegative. Identify each constraint that is now an = or ≥ constraint.
2. Convert each inequality constraint to standard form (add a slack variable for ≤ constraints, add an excess variable for ≥ constraints).
3. For each ≥ or = constraint, add artificial variables. Add sign restriction $a_i \geq 0$.
4. Let M denote a very large positive number. Add (for each artificial variable) Ma_i to min problem objective functions or $-Ma_i$ to max problem objective functions.
5. Since each artificial variable will be in the starting basis, all artificial variables must be eliminated from row 0 before beginning the simplex. Remembering M represents a very large number, solve the transformed problem by the simplex.

4.10 – The Big M Method

If all artificial variables in the optimal solution equal zero, the solution is optimal. If any artificial variables are positive in the optimal solution, the problem is infeasible.

The Bevco example continued:

Initial Tableau

Row	z	x1	x2	s1	e2	a2	a3	rhs
0	1.00	-2.00	-3.00			-M	-M	0.00
1		0.50	0.25	1.00				4.00
2		1.00	3.00		-1.00	1.00		20.00
3		1.00	1.00				1.00	10.00

4.10 – The Big M Method

Pivot 1	z	x1	x2	s1	e2	a2	a3	rhs	ratio	ero
0	1.00	2m - 2	4M -3		-M			30M		Row 0 + M(Row 2) + M(Row 3)
1		0.50	0.25	1.00				4.00	16	
2		1.00	3		-1.00	1.00		20.00	6.67	
3		1.00	1.00				1.00	10.00	10	
ero 1	**z**	**x1**	**x2**	**s1**	**e2**	**a2**	**a3**	**rhs**		**ero**
0	1.00	2m - 2	4M -3		-M			30M		
1		0.50	0.25	1.00				4.00		
2		0.33	1		-0.33	0.33		6.67		Row 2 divided by 3
3		1.00	1.00				1.00	10.00		
ero 2	**z**	**x1**	**x2**	**s1**	**e2**	**a2**	**a3**	**rhs**		**ero**
0	1.00	(2M-3)/3			(M-3)/3	(3-4M)/3		(60+10M)/3		Row 0 - (4M-3)*(Row 2)
1		0.50	0.25	1.00				4.00		
2		0.33	1		-0.33	0.33		6.67		
3		1.00	1.00				1.00	10.00		
ero 3	**z**	**x1**	**x2**	**s1**	**e2**	**a2**	**a3**	**rhs**		**ero**
0	1.00	(2M-3)/3			(M-3)/3	(3-4M)/3		(60+10M)/3		
1		0.42		1.00	0.08	-0.08		2.33		Row 1 - 0.25*(Row 2)
2		0.33	1		-0.33	0.33		6.67		
3		1.00	1.00				1.00	10.00		
ero 4	**z**	**x1**	**x2**	**s1**	**e2**	**a2**	**a3**	**rhs**		**ero**
0	1.00	(2M-3)/3			(M-3)/3	(3-4M)/3		(60+10M)/3		
1		0.42		1.00	0.08	-0.08		2.33		
2		0.33	1		-0.33	0.33		6.67		
3		0.67			0.33	-0.33	1.00	3.33		Row 3 - Row 2

4.10 – The Big M Method

Pivot 2	z	x1	x2	s1	e2	a2	a3	rhs	ratio	
0	1.00	(2M-3)/3			(M-3)/3	(3-4M)/3		(60+10M)/3		
1		0.42		1.00	0.08	-0.08		2.33	5.60	
2		0.33	1		-0.33	0.33		6.67	20.00	
3		0.67			0.33	-0.33	1.00	3.33	5.00	

ero 1	z	x1	x2	s1	e2	a2	a3	rhs		ero
0	1.00	(2M-3)/3			(M-3)/3	(3-4M)/3		(60+10M)/3		
1		0.42		1.00	0.08	-0.08		2.33		
2		0.33	1		-0.33	0.33		6.67		
3		1.00			0.50	-0.50	1.50	5.00		(Row 3)*(3/2)

ero 2	z	x1	x2	s1	e2	a2	a3	rhs		ero
0	1.00				-0.50	(1-2M)/2	(3-2M)/2	25.00		Row 0 + (3-2M)*(Row 3)/3
1		0.42		1.00	0.08	-0.08		2.33		
2		0.33	1.00		-0.33	0.33		6.67		
3		1.00			0.50	-0.50	1.50	5.00		

ero 3	z	x1	x2	s1	e2	a2	a3	rhs		ero
0	1.00				-0.50	(1-2M)/2	(3-2M)/2	25.00		
1				1.00	-0.13	0.13	-0.63	0.25		Row 1 - (5/12)*Row 3)
2		0.33	1.00		-0.33	0.33		6.67		
3		1.00			0.50	-0.50	1.50	5.00		

ero 4	z	x1	x2	s1	e2	a2	a3	rhs		ero
0	1.00				-0.50	(1-2M)/2	(3-2M)/2	25.00		Optimal Solution
1				1.00	-0.13	0.13	-0.63	0.25		
2			1.00		-0.50	0.50	-0.50	5.00		Row 2 -(1/3)*Row 3
3		1.00			0.50	-0.50	1.50	5.00		