An example of the simplex algorithm

Original LP

$$\begin{aligned}
&\text{maximize} && 3x_1 + x_2 + 2x_3 && && (1)\\
&\text{subject to} \\
& && x_1 + x_2 + 3x_3 \leq 30 && && (2)\\
& && 2x_1 + 2x_2 + 5x_3 \leq 24 && && (3)\\
& && 4x_1 + x_2 + 2x_3 \leq 36 && && (4)\\
& && x_1, x_2, x_3 \geq 0 && . && (5)
\end{aligned}$$

Standard form.

$$\begin{aligned}
z &= 3x_1 + x_2 + 2x_3 && && (6)\\
x_4 &= 30 - x_1 - x_2 - 3x_3 && && (7)\\
x_5 &= 24 - 2x_1 - 2x_2 - 5x_3 && && (8)\\
x_6 &= 36 - 4x_1 - x_2 - 2x_3 && . && (9)
\end{aligned}$$

Pivot in x_1. Remove x_6 from the basis.

$$\begin{aligned}
z &= 27 + \frac{x_2}{4} + \frac{x_3}{2} - \frac{3x_6}{4} && && (10)\\
x_1 &= 9 - \frac{x_2}{4} - \frac{x_3}{2} - \frac{x_6}{4} && && (11)\\
x_4 &= 21 - \frac{3x_2}{4} - \frac{5x_3}{2} + \frac{x_6}{4} && && (12)\\
x_5 &= 6 - \frac{3x_2}{2} - 4x_3 + \frac{x_6}{2} && . && (13)
\end{aligned}$$

Pivor in x_3. Remove x_5.

$$\begin{aligned}
z &= \frac{111}{4} + \frac{x_2}{16} - \frac{x_5}{8} - \frac{11x_6}{16} && && (14)\\
x_1 &= \frac{33}{4} - \frac{x_2}{16} + \frac{x_5}{8} - \frac{5x_6}{16} && && (15)\\
x_3 &= \frac{3}{2} - \frac{3x_2}{8} - \frac{x_5}{4} + \frac{x_6}{8} && && (16)\\
x_4 &= \frac{69}{4} + \frac{3x_2}{16} + \frac{5x_5}{8} - \frac{x_6}{16} && . && (17)
\end{aligned}$$

Pivot in x_2. Remove x_3.

$$\begin{aligned}
z &= 28 - \frac{x_3}{6} - \frac{x_5}{6} - \frac{2x_6}{3} && && (18)\\
x_1 &= 8 + \frac{x_3}{6} + \frac{x_5}{6} - \frac{x_6}{3} && && (19)\\
x_2 &= 4 - \frac{8x_3}{3} - \frac{2x_5}{3} + \frac{x_6}{3} && && (20)\\
x_4 &= 18 - \frac{x_3}{2} + \frac{x_5}{2} && . && (21)
\end{aligned}$$