Initial problem:

$$
\begin{aligned}
& \max z = 3x_1 + x_2 \\
\text{s.t.} \quad & x_1 + 4x_2 \leq 20 \\
& 2x_1 + x_2 \leq 12 \\
& x_1 + x_2 \leq 8 \\
& 2x_1 - x_2 \leq 8 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Initial problem:

$$\begin{aligned} \max z = \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & x_1 + 4x_2 \leq 20 \\ & 2x_1 + x_2 \leq 12 \\ & x_1 + x_2 \leq 8 \\ & 2x_1 - x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Convert to standard form:

$$\begin{aligned} \max z = \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & x_1 + 4x_2 + s_1 = 20 \\ & 2x_1 + x_2 + s_2 = 12 \\ & x_1 + x_2 + s_3 = 8 \\ & 2x_1 - x_2 + s_4 = 8 \\ & x_1, x_2, s_1, s_2, s_3, s_4 \geq 0 \end{aligned}$$

Initial dictionary:

$$\begin{aligned} z = \quad & 3x_1 + x_2 \\ s_1 = \quad & 20 - x_1 - 4x_2 \\ s_2 = \quad & 12 - 2x_1 - x_2 \\ s_3 = \quad & 8 - x_1 - x_2 \\ s_4 = \quad & 8 - 2x_1 + x_2 \end{aligned}$$

Idea of tableau:

$$\begin{aligned} & z - 3x_1 - x_2 = 0 \\ & x_1 + 4x_2 + s_1 = 20 \\ & 2x_1 + x_2 + s_2 = 12 \\ & x_1 + x_2 + s_3 = 8 \\ & 2x_1 - x_2 + s_4 = 8 \end{aligned}$$

Initial problem:

$$\begin{aligned} \max z = \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & x_1 + 4x_2 \leq 20 \\ & 2x_1 + x_2 \leq 12 \\ & x_1 + x_2 \leq 8 \\ & 2x_1 - x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Convert to standard form:

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Initial dictionary:

$$\begin{aligned} z = \quad & 3x_1 + x_2 \\ s_1 = \quad & 20 - x_1 - 4x_2 \\ s_2 = \quad & 12 + 2x_1 + x_2 \\ s_3 = \quad & 8 + x_1 + x_2 \\ s_4 = \quad & 8 + 2x_1 - x_2 \end{aligned}$$

Idea of tableau:

$$\begin{aligned} & z - 3x_1 - x_2 = 0 \\ & x_1 + 4x_2 + s_1 = 20 \\ & 2x_1 + x_2 + s_2 = 12 \\ & x_1 + x_2 + s_3 = 8 \\ & 2x_1 - x_2 + s_4 = 8 \end{aligned}$$

Initial *tableau*

z	x_1	x_2	s_1	s_2	s_3	s_4	RHS
1	-3	-1	0	0	0	0	0
0	1	4	1	0	0	0	20
0	2	1	0	1	0	0	12
0	1	1	0	0	1	0	8
0	2	-1	0	0	0	1	8

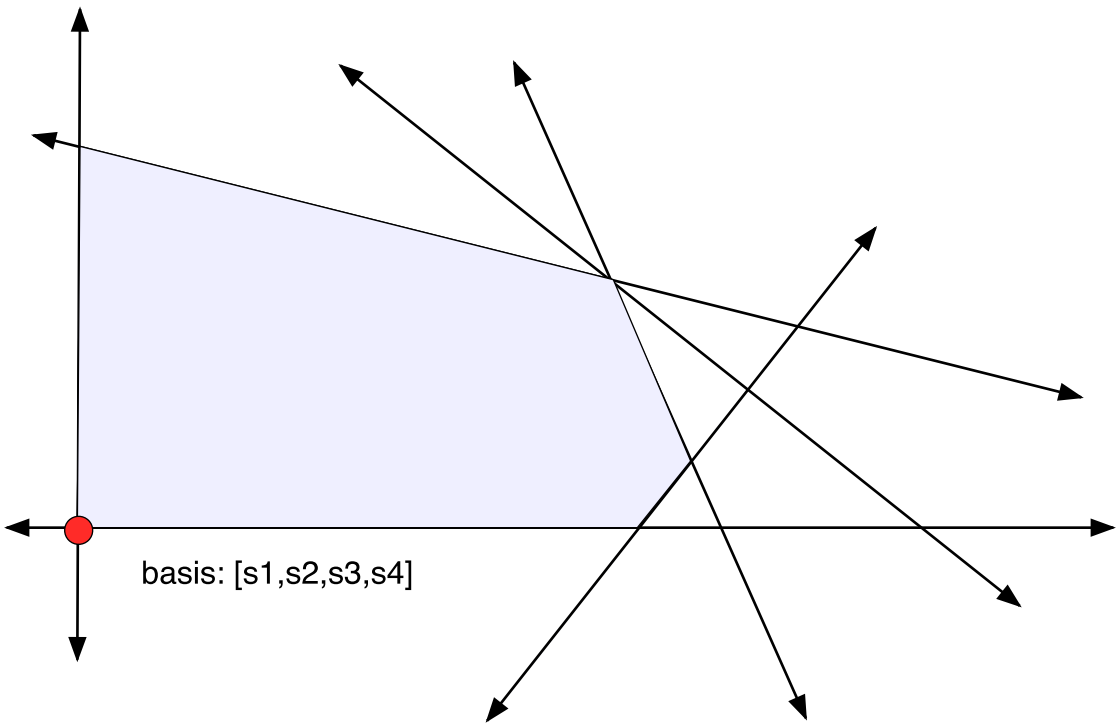

Figure 1: First pivot

Initial *tableau*

z	x_1	x_2	s_1	s_2	s_3	s_4	RHS
1	-3	-1	0	0	0	0	0
0	1	4	1	0	0	0	20
0	2	1	0	1	0	0	12
0	1	1	0	0	1	0	8
0	2	-1	0	0	0	1	8

Initial dictionary:

$$
\begin{aligned}
z &= 3x_1 + x_2 \\
s_1 &= 20 - x_1 - 4x_2 \\
s_2 &= 12 - 2x_1 - x_2 \\
s_3 &= 8 - x_1 - x_2 \\
s_4 &= 8 - 2x_1 + x_2
\end{aligned}
$$

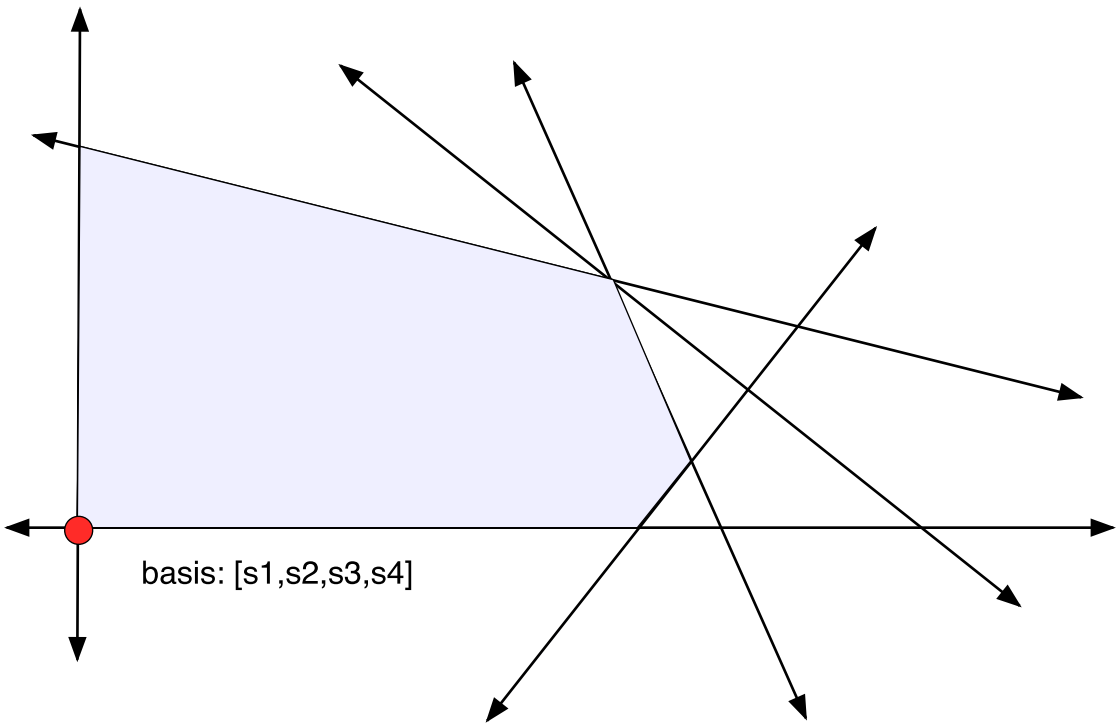

Figure 2: First pivot

Pivot in x_1

Pivoting on the *tableau*

z	x_1	$\mathbf{x_2}$	s_1	s_2	s_3	s_4	RHS	Min Ratio
1	-3	-1	0	0	0	0	0	
0	1	**4**	1	0	0	0	20	**5**
0	2	1	0	1	0	0	12	6
0	1	1	0	0	1	0	8	8
0	2	-1	0	0	0	1	8	n/a

Initial pivoting on the dictionary:

$$
\begin{aligned}
z &= 3x_1 + x_2 \\
x_2 &= 5 - \tfrac{1}{4}x_1 - \tfrac{1}{4}s_1 \\
s_2 &= 12 - 2x_1 - x_2 \\
s_3 &= 8 - x_1 - x_2 \\
s_4 &= 8 - 2x_1 + x_2
\end{aligned}
$$

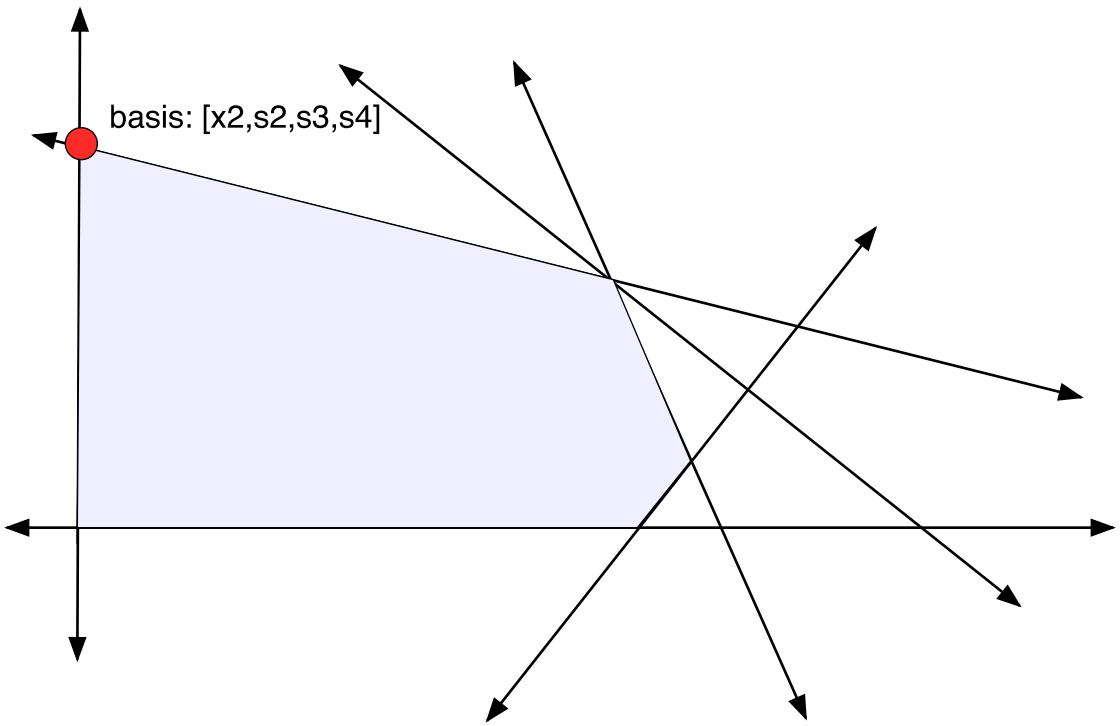

Figure 3: Second pivot

Second tableau

z	$\mathbf{x_1}$	x_2	s_1	s_2	s_3	s_4	RHS	Min Ratio
1	$-11/4$	0	$1/4$	0	0	0	5	
0	$1/4$	1	$1/4$	0	0	0	5	20
0	$7/4$	0	$-1/4$	1	0	0	7	**4**
0	$3/4$	0	$-1/4$	0	1	0	3	**4**
0	$9/4$	0	$1/4$	0	0	1	13	$52/9$

Second dictionary:

$$
\begin{aligned}
z &= 5 + 11/4x_1 - 1/4s_1 \\
x_2 &= 5 - 1/4x_1 - 1/4s_1 \\
s_2 &= 7 - 7/4x_1 + 1/4s_1 \\
s_3 &= 3 - 3/4x_1 + 1/4s_1 \\
s_4 &= 13 - 9/4x_1 - 1/4s_1
\end{aligned}
$$

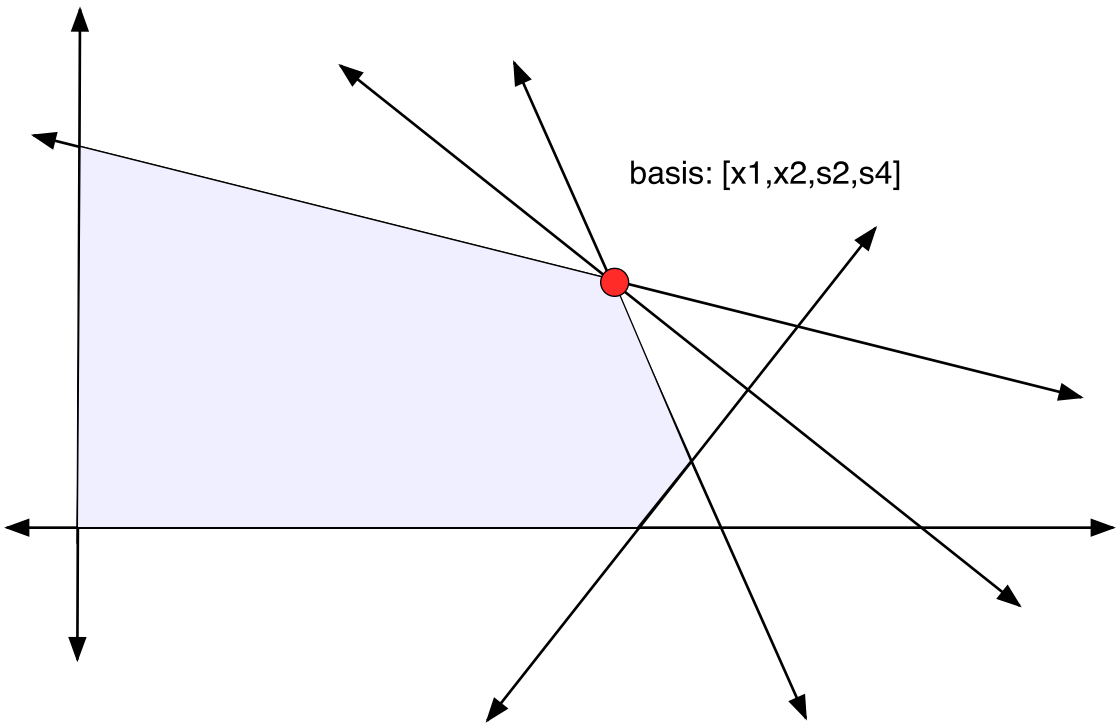

Figure 4: Third pivot

Third tableau

z	x_1	x_2	$\mathbf{s}_1$	s_2	s_3	s_4	RHS	Min Ratio
1	0	0	$-2/3$	0	$11/3$	0	16	
0	0	1	$1/3$	0	$-1/3$	0	4	12
0	0	0	$\mathbf{1}/3$	1	$-7/3$	0	0	**0**
0	1	0	$-1/3$	0	$4/3$	0	4	n/a
0	0	0	1	0	-3	1	4	4

Third dictionary:

$$
\begin{aligned}
z &= 16 + 2/3s_1 - 11/3s_3 \\
x_2 &= 4 - 1/3s_1 - 1/3s_3 \\
s_2 &= 0 - 1/3s_1 + 7/3s_3 \\
x_1 &= 4 + 1/3s_1 - 4/3s_3 \\
s_4 &= 4 - s_1 + 3s_3
\end{aligned}
$$

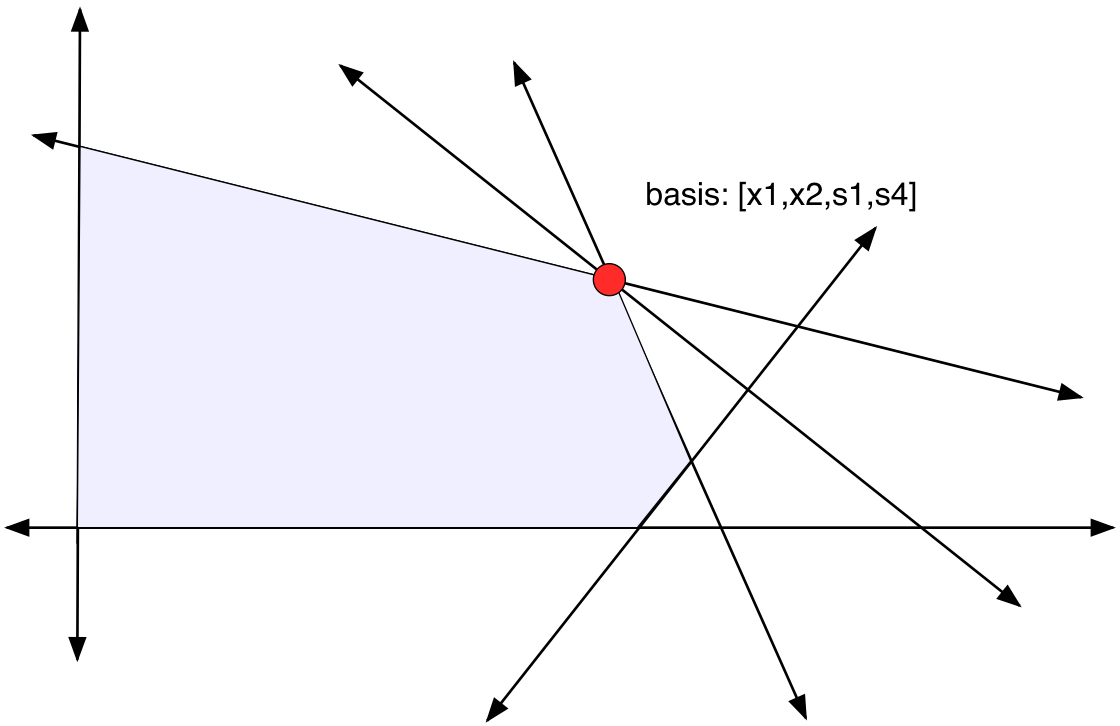

Figure 5: Fourth pivot

Fourth tableau

z	x_1	x_2	s_1	s_2	$\mathbf{s}_3$	s_4	RHS	Min Ratio
1	0	0	0	2	-1	0	16	
0	0	1	0	-1	2	0	4	2
0	0	0	1	3	-7	0	0	n/a
0	1	0	0	1	-1	0	4	n/a
0	0	0	0	-3	7/3	1	4	**12/7**

Fourth dictionary:

$$
\begin{aligned}
z &= \quad 16 - 2s_2 + s_3 \\
x_2 &= \quad 4 + s_2 - 2s_3 \\
s_2 &= \quad 0 - 3s_2 + 7s_3 \\
x_1 &= \quad 4 - 1s_2 + 1s_3 \\
s_4 &= \quad 4 + 3s_2 - 7/3s_3
\end{aligned}
$$

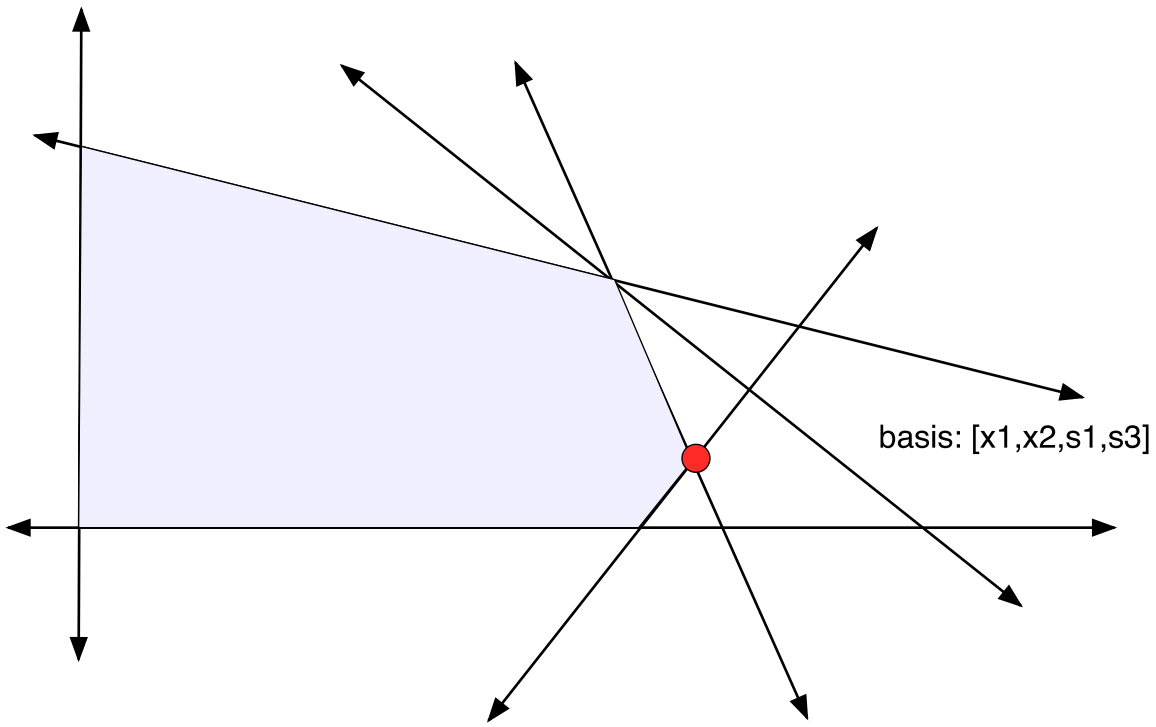

Figure 6: Fifth pivot

Final tableau

z	x_1	x_2	s_1	s_2	$\mathbf{s}_3$	s_4	RHS	Min Ratio
1	0	0	0	5/7	0	3/7	124/7	
0	0	1	0	11/7	0	−6/7	4/7	
0	0	0	1	−6	0	3	12	
0	1	0	0	−2/7	0	3/7	40/7	
0	0	0	0	−9/7	1	3/7	12/7	

Final dictionary:

$$
\begin{aligned}
z &= \quad 124/7 - 5/7s_2 - 3/7s_4 \\
x_2 &= \quad 4 - 11/7s_2 + 6/7s_4 \\
s_2 &= \quad 0 + 6s_2 - 3s_4 \\
x_1 &= \quad 4 + 2/7s_2 - 3/7s_4 \\
s_4 &= \quad 4 + 9/7s_2 - 3/7s_4
\end{aligned}
$$